

# ROBUST BOUNDS ON OPTIMAL TAX PROGRESSIVITY

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- Large literature on optimal tax design
  - theory: Ramsey, Mirrlees
  - applications: Diamond–Saez, Golosov–Troshkin–Tsyvinski
- Key predictions depend hard to measure objects
  - distribution of earning potentials (labor productivity)
  - distribution of preferences (labor supply elasticity)
- Optimal tax design acknowledging uncertainty about distribution of individual characteristics
  - combine robust control approach to model statistical uncertainty with Mirrlees
  - key sources of uncertainty: tail behavior and correlation of productivity and labor supply elasticity

## FRAMEWORK

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Household type indexed with  $s \sim G(s)$

Given a labor income tax function  $T(\cdot)$ , household of type  $s$  solves

$$\max_{c,n} u(c, n; s)$$

subject to

$$c = z(s)n(s) - T(z(s)n)$$

Indirect utility function:  $V(s; T)$

Uncertainty about  $s \sim G(s)$

- Represent a benchmark distribution  $G(s)$  as a mixture

$$G(s) = \sum_k p_k G_k(s)$$

- Parameterize cost of deviating from  $G$  to  $\tilde{G}$  using an entropy penalty with  $\{\theta_k\}$

$$\sum_k \theta_k \mathbb{E}_k m_k \ln(m_k)$$

where  $m_k = d\tilde{G}_k/dG_k$ .

- allows us to express misspecification concerns about specific parts of the distribution

A utilitarian government solves

$$\max_{T(\cdot)} \mathbb{E}V(T)$$

subject to

$$\mathbb{E}T(y) = g.$$

A **robust** utilitarian government solves

$$\min_{\tilde{G}_k} \max_{T(\cdot)} \tilde{\mathbb{E}}V(T) + \sum_k \theta_k \mathbb{E}_k m_k \ln(m_k)$$

subject to

$$\tilde{\mathbb{E}}T(y) = g.$$

Solution bounds welfare losses coming from specification errors in  $G$  in a flexible way.

## ANALYSIS

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Households only differ in productivity  $z$ .

The optimal tax problem can be cast as a mechanism design problem (Mirrlees)

$$\min_{\tilde{G}_R} \max_{c, y} \tilde{\mathbb{E}} u \left( c, \frac{y}{z} \right) + \sum_k \theta_k \mathbb{E}_k m_k \ln(m_k)$$

subject to incentive constraints

$$u \left( c(z), \frac{y(z)}{z} \right) \geq u \left( c(z'), \frac{y(z')}{z} \right) \quad \forall z, z'$$

and resource constraint

$$\tilde{\mathbb{E}}(y - c) = g.$$

Let  $\xi$  be the multiplier on the resource constraint. The allocation with full information satisfies

$$c(z; \{\theta_k\}) = C(z, \xi(\{\theta_k\}))$$

$$n(z; \{\theta_k\}) = N(z, \xi(\{\theta_k\}))$$

with

$$u_c(z)z = -u_n(z)$$

- concerns only appear via marginal value of public funds
- marginal labor income tax rates are zero

Marginal tax  $T'$  for type  $\hat{z}$  satisfies

$$\frac{T'}{1 - T'} [\hat{z}] = \left( \frac{1 + \varepsilon^u(\hat{z})}{\varepsilon^c(\hat{z})} \right) \left( \frac{\hat{z} d\tilde{G}(\hat{z})}{\tilde{G}(\hat{z})} \right)^{-1} \left[ \tilde{\mathbb{E}} \left( \frac{u_c(\hat{z})}{u_c} \middle| z \geq \hat{z} \right) - \tilde{\mathbb{E}} \left( \frac{u_c(\hat{z})}{u_c} \right) \right]$$

Minimizing distribution

$$\ln(m_k(\hat{z})) \propto -\frac{1}{\theta_k} [u(\hat{z}) + \xi(y(\hat{z}) - c(\hat{z}))]$$

- ex-post Bayesian given  $m$
- $m$  trades off utility and budgetary consequences

## QUANTITATIVE APPLICATION

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Preferences

$$u(c, n) = \frac{c^{1-\sigma}}{1-\sigma} - \psi \frac{n^{1+\gamma}}{1+\gamma}$$

- set  $\sigma = 1.5$ ,  $\gamma = 2.0$ , and  $\psi = 7.5$

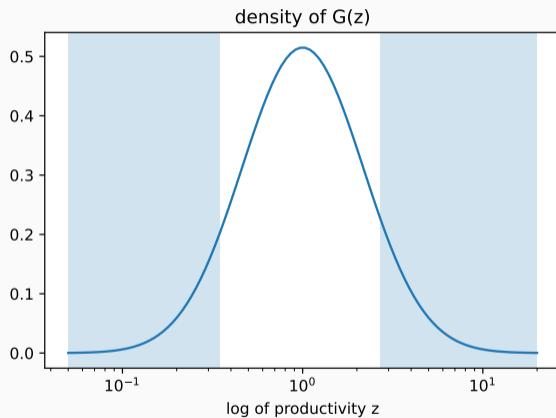
Shocks:  $\ln(z) \sim N(\mu, \nu^2)$

- set  $(\mu, \nu^2) = (0.0, 0.60)$

Benchmark model: mixture with 3 distributions

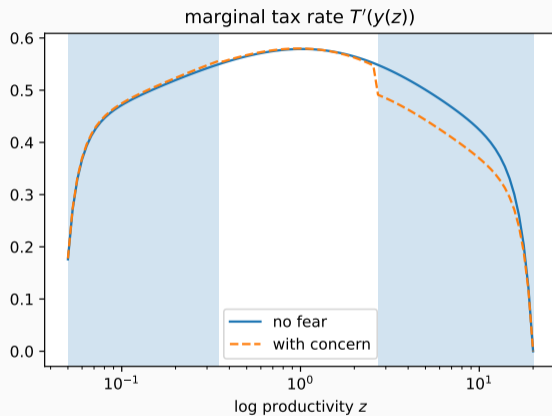
- partitions of  $G(z)$  representing 10% “left tail”, 10% “right tail”, and 10-90% “middle class”
- set  $\theta_{\text{left}} = \theta_{\text{right}} = \underline{\theta} < \bar{\theta} = \theta_{\text{middle}}$

# SHOCK DISTRIBUTION UNDER THE BENCHMARK MODEL



## RESULTS

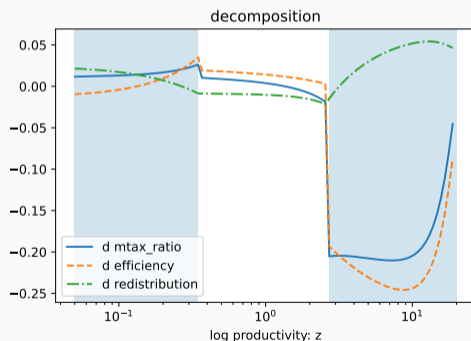
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Marginal tax rates largely unaffected in the left tail and substantially lower in the right tail.

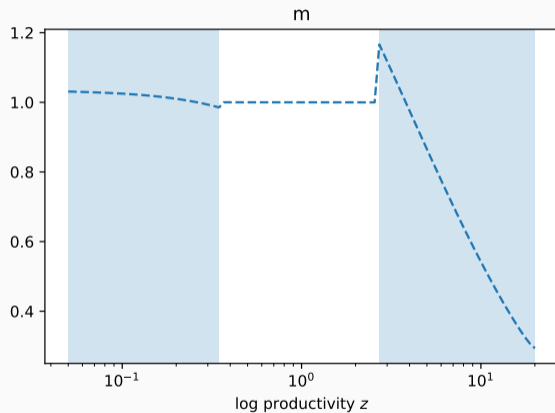


$$\ln \frac{T'}{1-T'} [\hat{z}] = \underbrace{\ln \left( \frac{1 + \varepsilon^u(\hat{z})}{\varepsilon^c(\hat{z})} \right) - \ln \left( \frac{\hat{z} d\tilde{G}(\hat{z})}{\tilde{G}(\hat{z})} \right)}_{\text{"efficiency"}} + \underbrace{\ln \left[ \tilde{\mathbb{E}} \left( \frac{u_c(\hat{z})}{u_c} \mid z \geq \hat{z} \right) - \tilde{\mathbb{E}} \left( \frac{u_c(\hat{z})}{u_c} \right) \right]}_{\text{"redistribution"}}$$



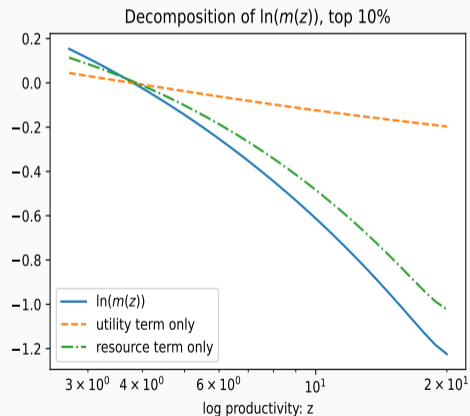
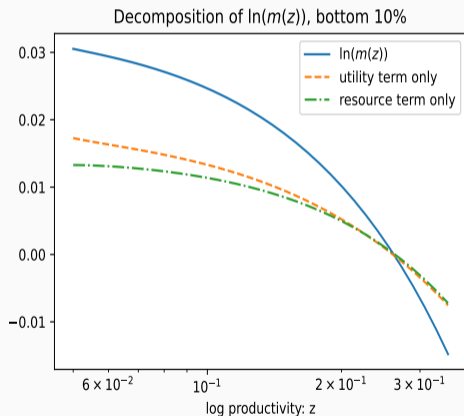
Virtually all the difference is due to “efficiency” terms in the robust Diamond–Saez formula.

# WHY ARE MARGINAL TAXES LOWER FOR RICH PEOPLE?



# WHY ASYMMETRY IN TAILS?

$$\ln(m_r(\hat{z})) \propto -\frac{1}{\theta_k} [u(\hat{z}) + \xi(y(\hat{z}) - c(\hat{z}))]$$



- Redistribution to low productivity agents
  - utilities and budgetary consequences “flat” on the left and “steep” on the right
- Fear of “too little” tax revenues  $>$  Fear of “too many” people who need transfers
  - lower progressivity

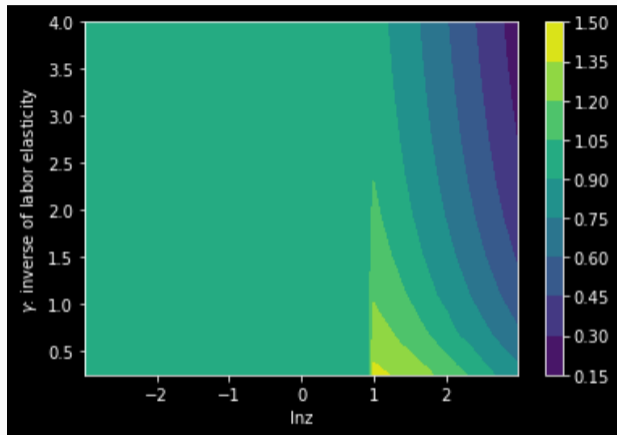
Households differ in productivity and labor supply elasticity

- Benchmark model: joint distribution  $(z, \gamma)$ 
  - productivity  $z$  as before
  - elasticity parameter  $\gamma \sim U[\underline{\gamma}, \bar{\gamma}]$  with  $\gamma \perp z$
  
- Approximate tax schedule with piecewise linear splines

Table 1: Optimal labor tax schedule

Labor income tax bracket $y \in$	[0, 0.1]	[0.1, 0.48]	[0.48, 0.86]	[0.86, 1.24]	[1.24, 1.62]	[1.62, 2.0]	[2.0, $\infty$ )
	Marginal tax rates						
No concern (%)	61.0	64.1	63.9	65.3	60.3	66.5	60.0
With concern (%)	59.6	62.9	63.6	61.5	60.1	57.7	53.6

## MINIMIZING DISTRIBUTION



Fear productive agents are scarce and more elastic.

## CONCLUSION

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- Acknowledging distributional uncertainty points to lower progressivity
- Insights robust to choices of benchmark distributions, restrictions on tax functions
- Working on dynamics