ROBUST BOUNDS ON OPTIMAL TAX PROGRESSIVITY

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INTRODUCTION

• Large literature on optimal tax design
  • theory: Ramsey, Mirrlees
  • applications: Diamond–Saez, Golosov–Troshkin–Tsyvinski

• Key predictions depend hard to measure objects
  • distribution of earning potentials (labor productivity)
  • distribution of preferences (labor supply elasticity)

• Optimal tax design acknowledging uncertainty about distribution of individual characteristics
  • combine robust control approach to model statistical uncertainty with Mirrlees
  • key sources of uncertainty: tail behavior and correlation of productivity and labor supply elasticity
FRAMEWORK
Household type indexed with $s \sim G(s)$

Given a labor income tax function $T(\cdot)$, household of type $s$ solves

$$\max_{c,n} u(c, n; s)$$

subject to

$$c = z(s)n(s) - T(z(s)n)$$

Indirect utility function: $V(s; T)$
Uncertainty about $s \sim G(s)$

- Represent a benchmark distribution $G(s)$ as a mixture

$$G(s) = \sum_k p_k G_k(s)$$

- Parameterize cost of deviating from $G$ to $\tilde{G}$ using an entropy penalty with $\{\theta_k\}$

$$\sum_k \theta_k \mathbb{E}_k m_k \ln(m_k)$$

where $m_k = \frac{d\tilde{G}_k}{dG_k}$.

- Allows us to express misspecification concerns about specific parts of the distribution
A utilitarian government solves

$$\max_{T(·)} \mathbb{E} V(T)$$

subject to

$$\mathbb{E} T(y) = g.$$
A robust utilitarian government solves

$$\min_{\tilde{G}_k} \max_{T(\cdot)} \mathbb{E}V(T) + \sum_k \theta_k \mathbb{E}_k m_k \ln(m_k)$$

subject to

$$\mathbb{E}T(y) = g.$$ 

Solution bounds welfare losses coming from specification errors in $G$ in a flexible way.
Analysis
Households only differ in productivity $z$.

The optimal tax problem can be cast as a mechanism design problem (Mirrlees)

$$\min_{c,y} \max_{G_R} \tilde{E} u \left( c, \frac{y}{z} \right) + \sum_k \theta_k \tilde{E} m_k \ln(m_k)$$

subject to incentive constraints

$$u \left( c(z), \frac{y(z)}{z} \right) \geq u \left( c(z'), \frac{y(z')}{z} \right) \quad \forall z, z'$$

and resource constraint

$$\tilde{E}(y - c) = g.$$
Let $\xi$ be the multiplier on the resource constraint. The allocation with full information satisfies

$$c(z; \{\theta_k\}) = C(z, \xi(\{\theta_k\}))$$

$$n(z; \{\theta_k\}) = N(z, \xi(\{\theta_k\}))$$

with

$$u_c(z)z = -u_n(z)$$

- concerns only appear via marginal value of public funds
- marginal labor income tax rates are zero
Marginal tax $T'$ for type $\hat{z}$ satisfies

$$\frac{T'}{1 - T'} [\hat{z}] = \left( 1 + \epsilon^u (\hat{z}) \right) \left( \frac{2d\tilde{G} (\hat{z})}{\tilde{G} (\hat{z})} \right)^{-1} \left[ \tilde{E} \left( \frac{u_c (\hat{z})}{u} \middle| z \geq \hat{z} \right) \right] - \tilde{E} \left( \frac{u_c (\hat{z})}{u} \right)$$

Minimizing distribution

$$\ln(m_k (\hat{z})) \propto -\frac{1}{\theta_k} [u (\hat{z}) + \xi (y (\hat{z}) - c (\hat{z}))]$$

- ex-post Bayesian given $m$
- $m$ trades off utility and budgetary consequences
QUANTITATIVE APPLICATION
Preferences

\[ u(c, n) = \frac{c^{1-\sigma}}{1-\sigma} - \frac{n^{1+\gamma}}{1+\gamma} \]

- set \( \sigma = 1.5 \), \( \gamma = 2.0 \), and \( \psi = 7.5 \)

Shocks: \( \ln(z) \sim N(\mu, \nu^2) \)

- set \( (\mu, \nu^2) = (0.0, 0.60) \)

Benchmark model: mixture with 3 distributions

- partitions of \( G(z) \) representing 10% “left tail”, 10% “right tail”, and 10-90% “middle class”

- set \( \theta_{\text{left}} = \theta_{\text{right}} = \underline{\theta} < \bar{\theta} = \theta_{\text{middle}} \)
SHOCK DISTRIBUTION UNDER THE BENCHMARK MODEL

![Graph of log of productivity z vs. density of G(z)](image)

- The x-axis represents the log of productivity z, ranging from $10^{-1}$ to $10^1$.
- The y-axis represents the density of G(z), ranging from 0 to 0.5.

The graph shows the distribution of shocks under the benchmark model, with the density of G(z) plotted against the log of productivity z.
Results
Marginal tax rates largely unaffected in the left tail and substantially lower in the right tail.
\[
\ln \frac{T'}{1-T'} [\hat{z}] = \ln \left( \frac{1 + \varepsilon^u(\hat{\tilde{z}})}{\varepsilon^c(\hat{\tilde{z}})} \right) - \ln \left( \frac{2d\tilde{G}(\hat{\tilde{z}})}{\tilde{G}(\hat{\tilde{z}})} \right) + \ln \left[ \tilde{E}\left( \frac{u_c(\hat{\tilde{z}})}{u_c} \right) \bigg| z \geq \hat{\tilde{z}} \right] - \tilde{E}\left( \frac{u_c(\hat{\tilde{z}})}{u_c} \right) \]
\]

"efficiency"

"redistribution"

Virtually all the difference is due to “efficiency” terms in the robust Diamond–Saez formula.
WHY ARE MARGINAL TAXES LOWER FOR RICH PEOPLE?
WHY ASYMMETRY IN TAILS?

\[
\ln(m_k(z)) \propto -\frac{1}{\theta_k} [u(z) + \xi(y(z) - c(z))]
\]

Decomposition of \( \ln(m(z)) \), bottom 10%

Decomposition of \( \ln(m(z)) \), top 10%

log productivity: \( z \)

(log productivity: \( z \))
Summing up

- Redistribution to low productivity agents
  → utilities and budgetary consequences “flat” on the left and “steep” on the right

- Fear of “too little” tax revenues > Fear of “too many” people who need transfers
  → lower progressivity
Households differ in productivity and labor supply elasticity

- Benchmark model: joint distribution \((z, \gamma)\)
  - productivity \(z\) as before
  - elasticity parameter \(\gamma \sim U[\gamma_l, \gamma_u]\) with \(\gamma \perp z\)

- Approximate tax schedule with piecewise linear splines
**Table 1: Optimal labor tax schedule**

<table>
<thead>
<tr>
<th>Labor income tax bracket $y \in$</th>
<th>[0, 0.1]</th>
<th>[0.1, 0.48]</th>
<th>[0.48, 0.86]</th>
<th>[0.86, 1.24]</th>
<th>[1.24, 2.0]</th>
<th>[2.0, ∞)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marginal tax rates</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No concern (%)</td>
<td>61.0</td>
<td>64.1</td>
<td>63.9</td>
<td>65.3</td>
<td>60.3</td>
<td>66.5</td>
</tr>
<tr>
<td>With concern (%)</td>
<td>59.6</td>
<td>62.9</td>
<td>63.6</td>
<td>61.5</td>
<td>60.1</td>
<td>57.7</td>
</tr>
</tbody>
</table>
Fear productive agents are scarce and more elastic.
Conclusion
• Acknowledging distributional uncertainty points to lower progressivity

• Insights robust to choices of benchmark distributions, restrictions on tax functions

• Working on dynamics