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BUMP Capstone Conference

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INTRODUCTION

- Hamilton (1790-1): US Federal government can lower borrowing costs by:
  - Sustaining reputation for timely debt service (*Report on Public Credit*),
  - Fostering a stable national currency (*Report on Establishment of a Mint*),
  - Chartering a monopoly Federal bank (*Report on the National Bank*).

- Wide range of institutional changes attempt to implement this agenda.

- Quantifying impact requires estimate of yield curve on US Federal debt.
  - Post-WW2: many estimates of the yield curve.
  - Pre-WW2: practically no estimates!

$\Rightarrow$ Need estimates to understand US borrowing costs across eras.

- We take on the challenges of estimating historical yield curves!
**This Paper**

- **Data:** new database of price, quantities, & cash-flows for all US Federal bonds
  - Available on github; see Hall et al. (2018).

- **Statistics:** propose a methodology to handle the limitations of historical data.
  - Challenge: long time series but sparse cross-section at many dates.
  - Response: statistical model with drifting parameters that interpolates gaps.

- **Output:** gold zero-coupon yield curve on US Federal bonds for 1791-1933.
  - + Greenback yield curve + gold-greenback exchange rate expectations (1862-1878).
  - + Real yield curve + inflation expectations. (1790-2020)
Literature

★ Analysis of historical interest rates

★ This paper: estimates full yield curve for all periods

★ Yield curve estimation

★ This paper: uses Hamiltonian MC with no U turns to compute posterior distribution of time-varying Nelson and Siegel (1987) parametrization

★ Long run inflation expectations
Mitchell (1903, 1908), Roll (1972), Sargent (1981), Gürkaynak et al. (2005), Cogley & Sargent (2005, 2015), Cogley (2005), Rudebusch & Swanson (2012), and Hazell (2020)

★ This paper: includes data covering episodes with debts denominated in different currencies
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US Monetary and Banking History

Yield Curves

Civil War and Greenbacks

Epilogue
US Monetary and Banking History

US Monetary and Banking Eras

★ 1791-1862: Bimetallism and state banks:
   ★ Federal government issued gold and silver coins; banks issued paper money.
   ★ First (1791-1811) and Second (1816-36) Banks of US “control” state banks.
   ★ “Free banking” state charters from 1837-63.

★ 1862-1913: Greenbacks, gold standard and the National Banking System.
   ★ 1862-78: Gov. issued inconvertible paper notes (“greenbacks”) as legal tender.
   ★ 1879: US treasury started converting greenbacks into gold dollars one-to-one.
   ★ 1863+: National Bank Acts introduced federally regulated banks:
     ★ National banks could issue bank notes backed by (long term) US Federal bonds.
     ★ National bank notes taxed at 1% p.a.; state bank notes at 10% p.a.

★ 1913: Federal Reserve System established as lender of last resort.

Deflation Until Civil War, Then Stable Prices Until 1900

Note: Gray intervals show recessions. Red intervals show major wars.

★ Fit inflation model with stochastic volatility + drifting mean + persistence
LONG RUN INFLATION ANCHOR UNTIL 1890s

- Long term inflation expectations
- First B.U.S.
- Second B.U.S.
- "free banking"
- National Banking Era
- FED
- Gold Coins, State Bank Notes
- Inconvertible Greenback
- Gold Standard
- FDR

Conditional Volatility (5 years ahead)
YIELD CURVES

**Our Dataset: Long But Shallow Panel**

(Note: Shaded areas show major American wars.)

Hall, Payne, Sargent, Szőke

Costs of Financing US Federal Debt: 1791-1933

September 21, 2022
**Nonlinear State Space Model**

\[ \tilde{p}^{(i)}_t = \langle q(\lambda_t, \tau), \bar{m}^{(i)}_t \rangle \quad \ldots \text{gold bond pricing} \]

★ where:

★ \( \tilde{p}^{(i)}_t \) = observed price of coupon bearing bond \( i \),

★ \( \bar{m}^{(i)}_t \) = payments of gold dollars promised by bond \( i \),

★ \( q(\lambda_t, \tau) \) = (parameterized) gold zero-coupon discount prices at all horizons,

★ **A.1.** Pricing kernel can be maturity specific but not bond specific, (Common “haircut risk” and “liquidity premium” across bonds not maturities.)

★ **A.2.** Parametrization follows Nelson & Siegel (1987), (Allows for monotonic, humped, and S-shaped curves.)
NONLINEAR STATE SPACE MODEL

\[ \tilde{p}^{(i)}_t = \left\langle q(\lambda_t, \tau), m^{(i)}_t \right\rangle \quad \ldots \text{gold bond pricing} \]

\[ \lambda_{t+1} = \bar{\lambda}_t + \varrho(\lambda_t - \bar{\lambda}_t) + \Sigma_t^{1/2} \varepsilon_{\lambda, t+1} \quad \ldots \text{yield curve parameters} \]

★ where:

★ \( \tilde{p}^{(i)}_t \) = observed price of coupon bearing bond \( i \),

★ \( m^{(i)}_t \) = payments of gold dollars promised by bond \( i \),

★ \( q(\lambda_t, \tau) \) = (parameterized) gold zero-coupon discount prices at all horizons,

★ A.1. Pricing kernel can be maturity specific but not bond specific, (Common “haircut risk” and “liquidity premium” across bonds not maturities.)


★ \( \Sigma_t \) governs pooling across time (\( \Sigma \to 0 \) ⇒ full pooling; \( \Sigma \to \infty \) ⇒ no pooling),
Yield Curves

**Nonlinear State Space Model**

\[
\tilde{p}_t(i) = \langle q(\lambda_t, \tau), \bar{m}_t(i) \rangle + \sigma_m \varepsilon_t(i) \quad \ldots \text{gold bond pricing}
\]

\[
\lambda_{t+1} = \bar{\lambda}_t + \varrho(\lambda_t - \bar{\lambda}_t) + \Sigma_t^{\frac{1}{2}} \varepsilon_{\lambda, t+1} \quad \ldots \text{yield curve parameters}
\]

* where:

* \( \tilde{p}_t(i) \) = observed price of coupon bearing bond \( i \),

* \( \bar{m}_t(i) \) = payments of gold dollars promised by bond \( i \),

* \( q(\lambda_t, \tau) \) = (parameterized) gold zero-coupon discount prices at all horizons,

  * **A.1.** Pricing kernel can be maturity specific but not bond specific,
    (Common “haircut risk” and “liquidity premium” across bonds not maturities.)

  * **A.2.** Parametrization follows Nelson & Siegel (1987),
    (Allows for monotonic, humped, and S-shaped curves.)

* \( \Sigma_t \) governs pooling across time \( (\Sigma \to 0 \Rightarrow \text{full pooling}; \Sigma \to \infty \Rightarrow \text{no pooling}) \),

* \( \varepsilon_t(i) \) is bond specific measurement error \( \text{(helps detect violations of A1)} \),

* Restrict sample to gold paying bonds with maturity greater than 1 year.
**Long End of the Yield Curve**

Note: Gray intervals show recessions. Red intervals show major wars. Black line is posterior mean with 5% – 95% iq-range.
Positive Spread Between US and UK Yields Until 1880s

Note: Gray intervals show recessions. Red intervals show major wars. Black line is posterior mean with 5% – 95% iq-range.
**“Liquidity” Premium on Short Term Bonds**

Note: Gray intervals show recessions. Red intervals show major wars. Pale blue dots depict the difference between model-implied and observed yield-to-maturities for bonds with *less than one year* to maturity.
Yield Curve Changes Sign During Civil War

Note: Gray intervals show recessions. Red intervals show major wars.
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Civil War and Greenbacks

Large greenback devaluation during the civil war.

1862-78: Gov. issued *inconvertible* paper notes ("greenbacks") as *legal tender*.
Large greenback devaluation during the Civil War

- Gold and greenback denominated bonds ⇒ can infer exchange rate expectations.
- Gold paying bonds converge to gold price near maturity ⇒ exchange rate anchor.
Non-linear State Space Model

\[ \tilde{p}_t^{(i)} = \left< q(\lambda_t, \tau), \overline{m}_t^{(i)} \right> + \sigma_m^{(i)} \varepsilon_t^{(i)} \quad \text{...gold bonds} \]

\[ p_t^{(i)} = \left< q(\lambda_t, \tau) \odot z(\theta_t), \overline{m}_t^{(i,d)} \right> + \sigma_m^{(i)} \varepsilon_t^{(i)} \quad \text{...greenback bonds} \]

★ where:

★ \( z(\theta_t) \) is the expected change in the gold-greenback exchange rate, \( P_t \).

★ A3. Interest rate parity holds. [Details] [Test]

★ A4. \( P_t \) follows state-space model with time varying parameters \( \theta_t \). [Details]

★ Other variables are as before.
“Heavy Nominal Anchor” During Civil War

Note: Gray intervals show recessions. Red intervals show major wars.
US Faced “High” Financing Costs Until 20th Century

Note: The light gray intervals depict recessions as dated by Davis (2006) for the 1796-1914 period and NBER recessions thereafter. The light red intervals depict wars.
Real Yield Low During Most of 20th Century

Note: The light gray intervals depict recessions as dated by Davis (2006) for the 1796-1914 period and NBER recessions thereafter.
CONCLUSION

- Estimated real and nominal yield curves for long but sparse sample.
- Sheds light on the long process of adopting a “Hamiltonian program”.
- Suggests strong connections and trade-offs between organizing monetary, financial, and fiscal institutions.
Thank you
**Correlation Between Output and Inflation Changes Signs**

![Graph showing correlation and inflation changes over time](image)

**Note:** The light gray intervals depict banking crises from Reinhart & Rogoff (2006). The light red intervals depict wars.
Spread Between Federal Bonds and AAA Corporate Bonds

Note: The light gray intervals depict recessions as dated by Davis (2006) for the 1796-1914 period and NBER recessions thereafter. The light red intervals depict wars.
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Debt/GDP and the Convenience Yield

![Graph showing the relationship between Debt/GDP and the 10-year corporate yield minus government spread from 1860 to 1925.]
## Our Historical Dataset

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\textbf{Inflation Model}

\begin{itemize}
    \item We impose that quarterly inflation $\pi_t$ obeys the state-space model:
    \begin{align*}
        \pi_{t+1} &= \alpha_t + x_{t}^{\pi} + \sigma_{\pi,t} \varepsilon_{\pi,t+1} \\
        x_{t+1}^{\pi} &= \rho_t x_{t}^{\pi} + \sigma_{x} \varepsilon_{\pi,t+1}
    \end{align*}
    \varepsilon_{\pi,t+1} \sim \mathcal{N}(0, 1), \ \forall t \geq 0
    \end{itemize}

where:

\begin{itemize}
    \item $x_{t}^{\pi}$ is a hidden state with a given initial $x_0$.
    \item Parameters $\alpha_t$ and $\sigma_{\pi,t}$ follow random walks:
    \begin{align*}
        \alpha_{t+1} &= \alpha_t + \sigma_{\alpha} \varepsilon_{\alpha,t+1} \\
        \log \sigma_{\pi,t+1} &= \log \sigma_{\pi,t} + \sigma_{\sigma_{\pi}} \varepsilon_{\sigma_{\pi},t+1}
    \end{align*}
    \varepsilon_{\alpha,t+1} \sim \mathcal{N}(0, 1) \quad \varepsilon_{\sigma_{\pi},t+1} \sim \mathcal{N}(0, 1)
    \item The persistence parameter $\rho_t$ follows a random walk with infrequent shocks:
    \begin{align*}
        \rho_{t+1} &= \begin{cases} 
            \rho_t + \sigma_{\rho} \varepsilon_{\rho,t+1} & \text{if } t = k\Delta \\
            \rho_t & \text{otherwise}
        \end{cases} \quad \varepsilon_{\rho,t+1} \sim \mathcal{N}(0, 1)
    \end{align*}
    \text{for } k \in \mathbb{N}
\end{itemize}
Inflation Model Priors

We use independent Gaussian priors for $\sigma_x$ and the initial parameters $\alpha_0$ and $\rho_0$:

$$
\sigma_x \sim \mathcal{N}(0, 0.5), \quad \alpha_0 \sim \mathcal{N}(0, 1), \quad \rho_0 \sim \mathcal{N}(0, 0.5)
$$

For the initial standard deviation $\sigma_{\pi, 0}$, we use prior $\sigma_{\pi, 0} \sim \log \mathcal{N}(0.015, 0.01)$.

For the standard deviations $\sigma_\alpha$, $\sigma_{\sigma_\pi}$, and $\sigma_\rho$, we use a common exponential prior with the rate parameter tuned so that a priori the probability that $\sigma_i > 0.3$ is lower than 5%. The corresponding prior mean is 0.1.
Conditional Moments of Inflation: 1800-2020

- Long term inflation expectations
- Short term inflation expectations (one year ahead)
- Conditional Volatility (5 years ahead)

Graph shows inflation expectations and volatility from 1800 to 2020, with significant spikes and trends over time.
**Inflation Model Summary Statistics**

- The top panel shows conditional inflation expectations:
  - Color grey refers to long term expectations (permanent component of inflation).
  - Color blue represents inflation expectations one year ahead.

- The bottom panel shows conditional volatility
  - The grey line depicts the posterior median estimate for the model implied 5 year ahead (annualized) conditional inflation volatility:
    \[ \sigma_{\pi, t}^{(j)} := \sqrt{\frac{1}{j} \left( E_t \left[ \exp \left( 2\pi_t^{(j)} \right) \right] - E_t \left[ \exp \left( \pi_t^{(j)} \right) \right]^2 \right)} \]

  - The purple line depicts the posterior median estimate for the 5-year-ahead smoothed conditional root mean square statistic:
    \[ crms_{\pi, t}^{(j)} := \sqrt{\frac{1}{j} E_t \left[ \exp \left( 2\pi_t^{(j)} \right) \right]} \]

- Gray intervals show recessions. Red intervals show major wars.
**PARAMETERIZED ZERO-CouPON YIELD CurVES**

★ Gold dollar \( j \)-maturity yield on US Federal debt is:

\[
y_t^{(j)} := -\log q_t^{(j)}/j.
\]

**A2. Parametric gold yield curve on US Federal debt.**

\[
y_t^{(j)} = \beta_{0,t} + (\beta_{1,t} + \beta_{2,t}) \left[ 1 - \exp \left( -\frac{j}{\tau} \right) \right] / \left( \frac{j}{\tau} \right) - \beta_{2,t} \exp \left( -\frac{j}{\tau} \right)
\]

★ This parametric specification follows Nelson & Siegel (1987):

★ \( \beta_{0,t} \) parametrizes asymptote (“level”)

★ \( \beta_{1,t} \) parametrizes short end of the yield curve (“slope”)

★ \( \beta_{2,t} \) parametrizes a potential hump at maturity \( \tau \) (“curvature”)
Nelson & Siegel Parametrization: Illustration

Components of the yield curve, \( \tau = 4 \)

Components of the yield curve, \( \tau = 1 \)

Components of the yield curve, \( \tau = 1 \)

Instantaneous forward curve and Yield curve

\( \beta_0 = 7.0, \beta_1 = -3.0, \beta_2 = 8.0, \tau = 4 \)

\( \beta_0 = 5.0, \beta_1 = 3.0, \beta_2 = 0.5, \tau = 2 \)

\( \beta_0 = 7.0, \beta_1 = -3.0, \beta_2 = 8.0, \tau = 1 \)

\[ y_t^{(j,g)} = \beta_{0,t} + (\beta_{1,t} + \beta_{2,t}) \left[ 1 - \exp \left( -\frac{j}{\tau} \right) \right] / \left( \frac{j}{\tau} \right) - \beta_{2,t} \exp \left( -\frac{j}{\tau} \right). \]

Note: Yield curve formula: \( y_t^{(j,g)} = \beta_{0,t} + (\beta_{1,t} + \beta_{2,t}) \left[ 1 - \exp \left( -\frac{j}{\tau} \right) \right] / \left( \frac{j}{\tau} \right) - \beta_{2,t} \exp \left( -\frac{j}{\tau} \right). \)

Contemporary Fed estimates follow Svensson (1994) and add a fourth term to allow for a double hump.
Estimate The Model Using a Bayesian Toolkit

- Monthly data for 140+ years ⇒ 7,500+ parameters to be estimated

- Study posterior distribution of model parameters conditional on the data.
  - Draw a random sample from posterior distribution, and
  - Use sample to compute means and quantiles of the posterior distribution.

- Technical details:
  - Code: Adapt the Julia DynamicHMC.jl package by Papp et al. (2021)
  - Priors: We use weakly informative priors to regularize the estimator
“Laboratory” Experiment: Setup

★ The Nelson-Siegel parametrization captures a wide range of yield curves

★ However, we have to try to recover the parameters with few price observations

★ As a “proof-of-concept” we generate an artificial sample of 4 bonds:
  ★ 6 % (semi-annual), 10 year maturity, $\sigma_m^{(i)} = 3$;
  ★ 3 % (semi-annual), 20 year maturity, $\sigma_m^{(i)} = 2$;
  ★ 5 % (semi-annual), 30 year maturity, $\sigma_m^{(i)} = 1$;
  ★ 2 % (semi-annual), 40 year maturity, $\sigma_m^{(i)} = 4$.

★ Then study whether we can recover the “true” yield curve
Note: Artificial sample with 4 bonds ($T = 20$ year): (i) 6 % 10 year maturity, $\sigma_m^{(i)} = 3$; (ii) 3 %, 20 year maturity, $\sigma_m^{(i)} = 2$; (iii) 5 %, 30 year maturity, $\sigma_m^{(i)} = 1$; (iv) 2 %, 40 year maturity, $\sigma_m^{(i)} = 4$. $\beta_0$ and $\Sigma$ values $\approx$ posterior median estimates.
Why Can we Recover The Short End of the Yield Curve?

★ We observe prices of bonds close to maturity (and we price them well)

★ We can learn about the co-movement between short and long end parameters

★ We observe coupon paying bonds with different coupon rates

★ To illustrate these points, we rerun the laboratory experiment without the 10-year bond . . .
“Laboratory” Experiment: Output

Note: Artificial sample with 4 bonds ($T = 20$ year): (i) 6 % 25 year maturity, $\sigma_{m}^{(i)} = 3$; (ii) 3 %, 33 year maturity, $\sigma_{m}^{(i)} = 2$; (iii) 5 %, 30 year maturity, $\sigma_{m}^{(i)} = 1$; (iv) 2 %, 40 year maturity, $\sigma_{m}^{(i)} = 4$. $\beta_0$ and $\Sigma$ values $\approx$ posterior median estimates.
**Bond Specific Pricing Errors**

Note: Each boxplot represents the interquartile range (IQR) and median (orange line) of the posterior distribution of $\sigma_{(i)}^{(m)}$—that is the standard deviation of the bond-specific pricing error—for each bond used in the estimation. Black crosses represent *mean absolute price errors* computed from the difference between observed and model-implied prices for each bond.
**Par Yield Curve Estimates vs. Yield-to-Maturities**

Note: The solid orange lines depict the median of our posterior for the gold dollar par yield yield curve at four specific dates (in gray boxes). The light orange bands around the posterior median depict the 95% interquantile ranges. Blue dots represent observed yield-to-maturities for bonds that are outstanding at the given period. Green stars depict model implied yield-to-maturities for the same bonds—computed from the posterior median price forecasts.
**Alternative Long-Term Yield Estimates**

**Note:** The solid black line depicts the mean of our posterior estimate for the 10-year, gold denominated, zero coupon yield. The dashed grey line depicts the mean of our posterior estimate for the 10-year, dollar denominated, zero coupon yield. The grey bands around the posterior mean depict the 95% interquantile range. The green line (bold and dotted) depicts the ‘US Government Bond Yield’ series from Homer (2004). The orange line (bold and dotted) depicts the New England Municipal Bond Yield reported by Homer (2004). The blue line depicts the Corporate Bond Yield reported by Homer (2004). The bold green-orange-blue line depicts the ‘composite’ bond series used by officer (2021). The light gray intervals depict recessions as dated by Davis (2006) for the 1796-1914 period and NBER recessions thereafter. The light red intervals depict wars (from left to right: the War of 1812, the Mexican-American War, the Civil War, the Spanish-American War, and World War I).
**Alternative Short-Term Yield Estimates**

Note: The solid black line depicts the mean of our posterior estimate for the 1-year, gold denominated, zero coupon yield. The dashed grey line depicts the mean of our posterior estimate for the 10-year, dollar denominated, zero coupon yield. The grey bands around the posterior mean depict the 95% interquantile range. The green dotted line depicts the US short term yield series (surplus funds, contemporary) used by Officer (2021) and Jorda (2019). The light gray intervals depict recessions as dated by Davis (2006) for the 1796-1914 period and NBER recessions thereafter. The light red intervals depict wars (from left to right: the War of 1812, the Mexican-American War, the Civil War, the Spanish-American War, and World War I).
Mean Absolute Price Error by Maturity Bins

Mean Error (yields)

Maturity bins (years)
0-1 1-5 5-10 10-15 15+

Mean Absolute Error (yields)

Maturity bins (years)
0-1 1-5 5-10 10-15 15+
“Liquidity” Premium on Short Term Bonds (More Detail)

First B.U.S.  Second B.U.S.  "Free banking"  National Banking Era  FED

Yield error

15-year moving average (maturity less than 1 year)
15-year moving average (maturity between 1 year and 5 year)
15-year moving average (maturity between 5 year and 10 year)
15-year moving average (maturity between 10 year and 15 year)
15-year moving average (maturity more than 15 years)

1790 1800 1810 1820 1830 1840 1850 1860 1870 1880 1890 1900 1910 1920 1930

Back

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Emergence of a Liquid Open Market for Short Debt

- During WWI US Treasury used Certificates of Indebtedness (interest-bearing securities with less than 1 year maturity) for cash management purposes

- Before 1920: Certificates did not have a secondary market, but the FED provided liquidity (for banks) via loan/discount window (secured by certificates)

- After 1920: the FED started to
  1. raise discount rates that forced the Treasury to let certificates be traded below par
  2. extend credit on repurchase agreements (“repos”) secured by certificates for non-member banks and non-bank dealers
  3. offer a service for the “wire transfer” of certificates

⇒ liquid secondary market for certificates quickly emerged

"Due to their high degree of security and wide market, certificates yield a lower rate than commercial paper of the same maturity..." (1925 Treasury Annual Report)

- In 1929 the Treasury introduced zero-coupon Treasury Bills
Yields Versus Tax Rates

Convertibility Reinstated

- 1-year zero-coupon yield (legal tender)
- 10-year zero-coupon yield (legal tender)
- Tax rate on notes outstanding
Profit on Note Issuance

Profit Rates of Note Issuance using Cagan's Formula

10y Gov. Yield
Unweighted Average of Eligible Bonds
Weighted Average (Champ 1990)
4s of 1925
Yield Curve Changes Sign During Civil War

- Spread between 10-year and 2-year (gold yields)
- Moving average (15-year, centered) of the spread

- Correlation
  - Rolling correlation (15-year, centered) between inflation and GDP growth (left)
  - Spread between 10-year and 2-year cond inflation volatility (right)
Term Spread: 1800-2020

Note: Gray intervals show recessions. Red intervals show major wars. Pale blue dots depict the difference between model-implied and observed yield-to-maturities for bonds with less than one year to maturity.
Term Spread and Inflation Volatility: 1800-2020

- Spread between 10-year and 2-year (gold yields)
- Spread between 10-year and 2-year (dollar yields)
- Moving average (15-year, centered) of the spread

- Rolling correlation (15-year, centered) between inflation and GDP growth (left)
- Spread between 10-year and 2-year conditional inflation volatility (right)
Forecasts of GDP Growth From Term Spreads

\[ g_{t+k} = \alpha_k^{(j)} + \beta_k^{(j)} \left( y_t^{(10)} - y_t^{(j)} \right) + \varepsilon_{t+k,k} \]

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The table reports the coefficient \( \beta_k^{(j)} \) and \( R^2 \) for the regression \( g_{t+k} = \alpha_k^{(j)} + \beta_k^{(j)} \left( y_t^{(10)} - y_t^{(j)} \right) + \varepsilon_{t+k,k} \) where \( g_{t+k} \) is the annual percentage growth of real GDP over the next \( k \) years and \( y_t^{(j)} \) denotes the annualized \( j \)-year zero coupon yield. We annualize the yields by taking the arithmetic average for each year. Newey and West heteroskedasticity- and autocorrelation-consistent standard errors with lag order one in parentheses.
## Forecasts of Real GDP Growth From First Differenced Term Spreads

\[
g_{t+k} = \alpha^{(j)}_k + \beta^{(j)}_k \left( y^{(10)}_t - y^{(j)}_t \right) - \left( y^{(10)}_{t-1} - y^{(j)}_{t-1} \right) + \varepsilon_{t+k,k}
\]

<table>
<thead>
<tr>
<th></th>
<th>1797-1860</th>
<th></th>
<th>1866-1933</th>
<th></th>
<th>1950-2000</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10y - 1y</td>
<td>10y - 5y</td>
<td>10y - 1y</td>
<td>10y - 5y</td>
<td>10y - 1y</td>
<td>10y - 5y</td>
</tr>
<tr>
<td><strong>Horizon</strong></td>
<td><strong>R2</strong></td>
<td><strong>R2</strong></td>
<td><strong>R2</strong></td>
<td><strong>R2</strong></td>
<td><strong>R2</strong></td>
<td><strong>R2</strong></td>
</tr>
<tr>
<td><strong>k-years</strong></td>
<td>(\beta^1_k)</td>
<td>(\beta^5_k)</td>
<td>(\beta^1_k)</td>
<td>(\beta^5_k)</td>
<td>(\beta^1_k)</td>
<td>(\beta^5_k)</td>
</tr>
<tr>
<td>1-year</td>
<td>-0.20</td>
<td>0.015</td>
<td>-0.42</td>
<td>0.011</td>
<td>-0.01</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>(0.13)</td>
<td>(0.31)</td>
<td>(0.61)</td>
<td>(1.53)</td>
<td>(0.32)</td>
<td>(1.22)</td>
</tr>
<tr>
<td>3-year</td>
<td>0.57</td>
<td>0.022</td>
<td>0.54</td>
<td>0.003</td>
<td>0.63</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td>(0.30)</td>
<td>(1.03)</td>
<td>(1.55)</td>
<td>(3.64)</td>
<td>(0.66)</td>
<td>(2.28)</td>
</tr>
</tbody>
</table>

The table reports the coefficient \(\beta^{(j)}_k\) and \(R^2\) for the regression \(g_{t+k} = \alpha^{(j)}_k + \beta^{(j)}_k \left( y^{(10)}_t - y^{(j)}_t \right) - \left( y^{(10)}_{t-1} - y^{(j)}_{t-1} \right) + \varepsilon_{t+k,k}\) where \(g_{t+k}\) is the annual percentage growth of real GDP over the next \(k\) years and \(y^{(j)}_t\) denotes the annualized \(j\)-year zero coupon yield. We annualize the yields by taking the arithmetic average for each year. Newey and West heteroskedasticity- and autocorrelation-consistent standard errors with lag order one in parentheses. \(* * * 1\%\), \(* * 5\%\), and \(* 10\%\) significance.
Fama Bliss (1987) Regression

\[ h(x, x - 1 : t + 1) - r(1 : t) = \alpha + \beta[f(x, x - 1 : t) - r(1 : t)] + u(t + 1) \]

<table>
<thead>
<tr>
<th>Maturity ( n )</th>
<th>1797-1860</th>
<th>1866-1933</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n )</td>
<td>( \beta )</td>
<td>( R^2 )</td>
</tr>
<tr>
<td>5</td>
<td>0.405 (0.10)</td>
<td>0.026</td>
</tr>
<tr>
<td>10</td>
<td>0.518 (0.11)</td>
<td>0.033</td>
</tr>
</tbody>
</table>

The table reports the coefficient \( \beta \) and \( R^2 \) for the regression \( h(x, x - 1 : t + 1) - r(1 : t) = \alpha + \beta[f(x, x - 1 : t) - r(1 : t)] + u(t + 1) \) from Fama & Bliss (1987) where \( h(x, x - 1 : t + 1) \) is the return on a \( x \)-year discount bond bought at year \( t \) and sold at \( t + 1 \), \( r(1 : t) \) is the 1-year yield at time \( t \) and \( f(x, x - 1 : t) \) is the time \( t \) forward rate for an \( x \)-year discount bond. Standard errors in parentheses.
**Predictive Power?**

★ Term spread is predictive of wars and banking crises over 1791-1934
  ★ P(Financial Crisis|Term Spread = −1%) = 8.4%
  ★ P(War|Term Spread = −1%) = 12.2%

★ Somewhat predictive of recessions and inflation

\[ z_{t+k} = \alpha_k + \beta_k (y_t^{(5)} - y_t^{(1)}) + \epsilon_{t+k,k} \]

<table>
<thead>
<tr>
<th></th>
<th>1791-1860</th>
<th>1866-1933</th>
<th>1946-2018</th>
</tr>
</thead>
<tbody>
<tr>
<td>( z_{t+2} = GDP/N ) Growth (( k = 2 ) years)</td>
<td>0.51 (0.41)</td>
<td>0.42 (1.40)</td>
<td>1.72 (0.53)</td>
</tr>
<tr>
<td>( z_{t+1} = ) Inflation (( k = 1 ) year)</td>
<td>0.98 (0.48)</td>
<td>-0.62 (1.31)</td>
<td>-1.92 (0.66)</td>
</tr>
</tbody>
</table>

Notes: Standard errors in parentheses

★ Puzzle? Some yield curve predictive power before active monetary policy
**Relating Greenback and Gold Yield Curves**

**A3. Conditional Independence (on time \( t \) info)**

The gold-greenback exchange rate \( P_t \) is conditionally independent of:

(i) SDF, \( S_{t+j}^{(t)} \)
(ii) gold price of goods, \( e_{t+j}^{(g)} \)
(iii) haircut risk, \( \xi_{t+j} \)

★ Greenback discount price is given by an “interest rate parity” formula:

\[
q_t^{(j,d)} = q_t^{(j,g)} \frac{\mathbb{E}_t [P_{t+j}]}{P_t} \quad \Rightarrow \quad y_t^{(j,d)} - y_t^{(j,g)} \approx -\frac{1}{j} \log \left( \frac{\mathbb{E}_t [P_{t+j}]}{P_t} \right)
\]

★★ We then impose a statistical model on the joint price dynamics \([P_t, e_t^{(g)}]'\) (with parameters denoted by \( \theta_t \))
**Statistical Price Model**

**A4. Statistical Model of Prices**

Define $v_t := [P_t, \epsilon^{(g)}_t]'$. Then:

$$
v_{t+1} = \mu_t + x_t + F\epsilon_{v,t+1}
$$

$$
x_{t+1} = A_t x_t + K\epsilon_{v,t+1}
$$

$$
\epsilon_{v,t+1} \sim \mathcal{N}(0, I_2), \ \forall t \geq 0
$$

where $x_t$ is a 2-vector, and the time varying parameters $\mu_t$ and $A_t$ follow:

$$
\mu_t = \mu_{t-1} + \Sigma_{\mu} \epsilon_{\mu,t}
$$

$$
\text{vec}(A_t) = \text{vec}(A_{t-1}) + \Sigma_A \epsilon_{A,t}
$$

- Parameter vector:

$$
\theta_t := [\mu', \ \text{vec}(A_t)', \ \text{vec}(F)', \ \text{vec}(K)']'.
$$

- Interpret $\theta_t$ as agents’ time $t$ belief about law of motion for prices.

(As in Cogley-Sargent (2009) and Kreps (1998))
Covariance is Small Relative to Expected Price Movement

- Estimate fixed parameter version of the price model \( (\Sigma_\mu = \Sigma_A = 0) \)
- Take model implied conditional moments at two dates
- Observation 1: Covariance is small relative to expected price
- Observation 2: Expected prices move a lot
Civil War: Anticipation of Fast Return to Gold Standard

Graph showing economic data with key dates:
- Jul 1862
- Feb 1863
- Aug 1864
- Apr 1865
Post Civil War: Slower Anticipated Return
**Greenbacks Devalued During The Civil War**

Note: Red shaded areas show major American wars. Gold / Greenback exchange rate is the number of units of gold required to purchase one greenback.
Note: Red shaded areas show major American wars.
US Debt to GDP Ratio From 1790-2021

Note: Calculations from Hall and Sargent (2020).
Revenue and Expenditures

![Graph showing revenue and expenditures over time, with peaks corresponding to major historical events such as the Revolutionary War, War of 1812, Civil War, World War I, and World War II. The graph illustrates the percent of GDP for expenditures and receipts from 1775 to 2025.]
**Connection to Theory**

- Well known complications in reconciling time varying Nelson & Siegel parametrization with no-arbitrage models.

- We leave the SDF process very general for this section because we are not going to directly specify and estimate it

- *SDF process(es)*. Suppose for each $t \geq 0$, there exists a non-negative stochastic discount factor stochastic process $S^{(t)}$ that can price all government bonds. (Anticipated utility approach, Cogley-Sargent (2009), Kreps (1998))

- Then the price of a gold denominated zero coupon bond is:
  
  (if there is no convenience yield)

  $$
  p^{(i,j)}_t = q^{(j)}_t \overline{m}^{(i)}_{t+j} \text{ where } q^{(j)}_t := \mathbb{E}_t \left[ \left( \frac{S^{(t)}_{t+j}}{S^{(t)}_t} \right) \left( \frac{e^{(g)}_{t+j}}{e^{(g)}_t} \right) \xi_{t+j} \right].
  $$

  where $e^{(g)}_t$ is the goods to gold exchange rate.
Let \( X_t \) denote underlying states. Suppose that the risk free short rate satisfies:

\[
r_t = \rho_0 t + \rho_1 t X_t
\]

Let \( h_t \) and \( L_t \) denote hazard rate and loss under default. The default adjusted short rate is (Duffie & Singleton (1999)):

\[
R_t = r_t + h_t L_t
\]

There exist \( B_t(j), C_t(j), \) and \( A_t(j) \) such that:

\[
y_t(j) = -(B_t(j)/j)X_t - (C_t(j)/j)h_t L_t - (A(j)/j)
\]

Connection to Nelson-Seigel if \( B_t, C_t \) such that (Christensen et al. (2011)):

\[
y_t(j) = (X_1^1 + h_t L_t) + \left( \frac{1 - e^{-\lambda(j)}}{\lambda(j)} \right) X_t^2 + \left( \frac{1 - e^{-\lambda(j)}}{\lambda(j)} - e^{\lambda(j)} \right) X_t^3 - \frac{A(j)}{j}
\]
Options For Reconciling with $A(j)/j$

★ Absorb $A(j)$ into a collection of SDF or haircut risk processes

★ Argue that $A(j)$ disappears in an “eigenvalue approximation” to the arbitrage-free affine asset pricing model (Krippner (2015))

★ Estimate $A(j)$ term (Christensen et al. (2011))

★ In practice, doesn’t seem to make much difference whether $A(j)$ is included
Anticipated Utilities

For all $t \geq 0$, $\mathbb{E}_t$ is evaluated with respect to frozen parameters of $S^{(t)}$, parameters of price processes and parameters of other risk processes.

- Pervasive in machine learning and AI: Sebastian Jaimungal’s group.
- “Bounded rationality”?
- “Bounded irrationality”? (Chen, Hansen, Hansen (PNAS 2021))
Parameter Calculations

★ Gold denominated yield curve (YC):
   ★ 143 years × 12 months × 4 Nelson & Siegel parameters per month
   ★ + 255 bond specific measurement error volatilities
   ★ + 10 from covariance matrix of parameter shocks
   ★ = 7129 parameters

★ Greenback denominated yield curve (YC):
   ★ 9 bond specific measurement error volatilities
   ★ 408 + 13 (fixed) parameters for price state space model
   ★ = 430 parameters
Currency Risk Premia

★ The difference between a dollar $n$ yield and the risk free real yield is:

$$y_t^{(j,n)} - \hat{y}_t^{(j)} \approx -\frac{1}{j} \log \left( \frac{E_t \left[ e_{t+j}^{(n)} \right]}{e_t^{(n)}} \right) + \frac{-1}{j} \log E_t \left[ \xi_{t+j} \right]$$

- Expected dollar $n$ inflation

$$+ \left( \frac{1}{j} \text{Cov}_t \left( \frac{\xi_{t+j}}{E_t \left[ \xi_{t+j} \right]}, \frac{e_{t+j}^{(n)}/e_t^{(n)}}{E_t \left[ e_{t+j}^{(n)}/e_t^{(n)} \right]} \right) \right) + \left( \frac{1}{j} \text{Cov}_t \left( \frac{S_{t+j}/S_t}{E_t \left[ S_{t+j}/S_t \right]}, \frac{\xi_{t+j}}{E_t \left[ \xi_{t+j} \right]} \right) \right)$$

- Risk from haircut & inflation comovement

$$+ \left( \frac{1}{j} \text{Cov}_t \left( \frac{S_{t+j}/S_t}{E_t \left[ S_{t+j}/S_t \right]}, \frac{e_{t+j}^{(n)}/e_t^{(n)}}{E_t \left[ e_{t+j}^{(n)}/e_t^{(n)} \right]} \right) \right)$$

- Risk premium on dollar $n$ inflation

★ Assumption A4.* implies that the risk premia related to inflation are zero
**Hamiltonian Monte Carlo**

- Draw random sample from posterior distribution
  - ... Markov Chain with ergodic distribution = posterior

- Standard Random Walk Metropolis-Hastings
  - Random walk proposals with a rejection rule (small steps $\rightarrow$ slow exploration)
  - High dimensional models tend to have complicated *typical set*

- Hamiltonian Monte Carlo
  - Use logposterior’s gradient to generate proposals through Hamilton’s diff equation (energy conserving dynamics = move along level curves)
  - Proposes *distant but probable* points in the typical set (fast exploration)
  - Automatic tuning of the algorithm: No U-Turn Sampler (NUTS) (Hoffman and Gelman (2014), Betancourt (2017), Stan Team (2020))
**Random Walk Metropolis-Hastings**

**Source:** Alex Rogozhnikov (Brilliantly Wrong blog) Link
Source: Alex Rogozhnikov (Brilliantly Wrong blog) Link
Q & A: Why do we Not Use Stan?

- **Stan** has well-developed software that provides an efficient implementation of the HMC-NUTS sampler.

- However, applying *Stan* to our model is non-trivial for many reasons:
  - The number of observed assets change over time,
  - There are periods without price observations,
  - The relevant set of bond-specific pricing errors changes over time, and
  - We want to estimate exchange rate expectations only for a specific sub-period, ...

- Instead we code the log posterior in Julia and feed it into *DynamicHMC.jl* (a robust Julia implementation of the HMC-NUTS by Papp et al. (2021))

- This allows us to easily enter the Jacobian of the log-posterior manually rather than relying on automatic differentiation
Weakly Informative Priors

- Initial yield curve parameters: use (independent) Gaussian priors:
  \[ \lambda_{0,0} \sim \log \mathcal{N}\left(10 - \beta, 6\right), \quad \lambda_{1,0} \sim \log \mathcal{N}\left(10 - \beta, 6\right), \quad \lambda_{2,0} \sim \log \mathcal{N}\left(10 - \beta, 15\right), \quad \tau \sim \mathcal{N}\left(0, 6\right). \]

- Covariance matrix \( \Sigma \): we use the decomposition:
  \[
  \Sigma = \begin{pmatrix}
  \sigma_1 & 0 & 0 \\
  0 & \sigma_2 & 0 \\
  0 & 0 & \sigma_3
  \end{pmatrix}
  \begin{pmatrix}
  1 & \omega_1 & \omega_2 \\
  \omega_1 & 1 & \omega_4 \\
  \omega_2 & \omega_4 & 1
  \end{pmatrix}
  \begin{pmatrix}
  \sigma_1 & 0 & 0 \\
  0 & \sigma_2 & 0 \\
  0 & 0 & \sigma_3
  \end{pmatrix}
  = : \Xi = : \Omega
  \]

- \( \Xi \): use *common* exponential prior with the rate parameter tuned s.t. *a priori* \( \mathbb{P}(\sigma_i > 0.3) < 5\% \)

- \( \Omega \): use the LKJ prior with concentration parameter \( \eta = 5.0 \uparrow \)

- Standard deviation of measurement errors, \( \sigma^{(i)}_m \): use common exponential priors s.t. *a priori* \( \mathbb{P}(\sigma^{(i)}_m > 20) < 5\% \)

1. The LKJ distribution is defined by \( p(\Omega|\eta) \propto \det(\Omega)^{\eta-1} \). For \( \eta = 1 \), this is a uniform distribution over correlation matrices. For \( \eta > 1 \), the density increasingly concentrates mass around the unit matrix, i.e., favoring less correlation.
**Implied Prior Distributions**

Left panel: The solid grey line depicts the mean, dotted lines depict the 20% and 80% percentiles of the prior distribution. Right panel: The solid grey line depicts the median, dotted lines depict the 20% and 80% percentiles of the prior distribution. Shaded areas represent interquantile ranges so that dark areas are indicative of concentrated prior probability.
The solid black line depicts the mean of our posterior estimate for the 1-year, gold denominated, zero coupon yield. The grey bands around the posterior mean depict the 95% interquantile range. The green dotted line depicts the US short term yield series used by Officer et. al. (2021) and Jorda et. al (2019). The light gray intervals depict recessions, and the light red intervals depict wars.