OPTIMAL EXCHANGE RATE POLICY

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Abstract

We develop a general policy analysis framework that features nominal rigidities and financial frictions with endogenous PPP and UIP deviations. The goal of the optimal policy is to balance output gap stabilization and international risk sharing using a mix of monetary policy and FX interventions. The nominal exchange rate plays a dual role. First, it allows for the real exchange rate adjustments when prices are sticky, which are necessary to close the output gap. Monetary policy can eliminate the output gap, but this generally requires a volatile nominal exchange rate. Volatility in the nominal exchange rate, in turn, limits the extent of international risk sharing in the financial markets with limits to arbitrage. Optimal monetary policy closes the output gap, while optimal FX interventions eliminate UIP deviations. When the natural real exchange rate is stable, both goals can be achieved by a fixed exchange rate policy — an open-economy divine coincidence. Generally, this is not the case, and the optimal policy requires a managed peg by means of a combination of monetary policy and FX interventions, without requiring the use of capital controls. We explore various constrained optimal policies, when either monetary policy or FX interventions are restricted, and characterize the possibility of central bank’s income gains and losses from FX interventions.

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1 Introduction

What is the optimal exchange rate policy? Should exchange rates be optimally pegged, managed or allowed to freely float? What defines a freely floating exchange rate? Do open economies face a trilemma constraint in choosing between inflation and exchange rate stabilization? These classic questions in international macroeconomics are generally difficult to address, as the exchange rate is neither a policy instrument, nor a direct objective of the policy, but rather an endogenous general equilibrium variable with direct equilibrium links in both product and financial markets. At the same time, equilibrium exchange rate behavior features a variety of puzzles from the point of view of conventional business cycle models, which thus casts doubt on their exchange rate policy implications.

We address these questions by developing a general policy analysis framework with nominal rigidities and financial frictions that are both central for equilibrium exchange rate determination and result in an empirically realistic model of exchange rates. We extend the framework in Itskhoki and Mukhin (2021b), where we study positive implications of a switch between floating and fixed exchange rate regimes, to allow for explicit policy analysis using both conventional monetary policy and foreign exchange interventions (FXI) in the financial market. We show that this framework is easily amenable to normative analysis and characterize the optimal exchange rate policies implied by the model.

We focus on a problem of a small open economy with tradable and non-tradable goods with a segmented international financial market resulting in endogenous uncovered interest rate parity (UIP) deviations. Productivity shocks determine the value of the frictionless real exchange rate, or departures from purchasing power parity (PPP). Nominal rigidities constitute another — frictional — source of PPP violations. The presence of both endogenous PPP and UIP violations is essential for the optimal exchange rate policy analysis, as exchange rates are key determinants of both deviations. Historically, the main frameworks for the exchange rate policy analysis features PPP deviations due to sticky prices, but assumed that UIP holds, resulting in a Trilemma constraint on feasible exchange rate policies (Fleming 1962, Mundell 1963, Dornbusch 1976, Obstfeld and Rogoff 1995, Galí and Monacelli 2005).

The nominal exchange rate plays a dual role — in the goods and asset markets. First, it allows for expenditure switching and the real exchange rate adjustment when prices (or wages) are sticky, and in the absence of such nominal exchange rate movements, the economy features an output gap resulting in welfare losses. Monetary policy can eliminate the output gap, but this generally requires a volatile nominal exchange rate that accommodates fundamental macroeconomic shocks. Volatility in the nominal exchange rate, in turn, limits the extent of international risk sharing in the financial market, as international financial flows must be intermediated by risk-averse market makers who hold the nominal exchange rate risk. This also leads to welfare losses. Financial market interventions can shift this risk away from arbitrageurs, stabilizing the resulting equilibrium UIP deviations and improving the extent of international risk sharing. Thus, the goal of the optimal policy is to balance output gap

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1 Trilemma states that in the absence of capital controls, the policy must choose between an independent monetary policy or a nominal exchange rate targeting. In other words, without capital controls, inward looking monetary policy uniquely determines the exchange rate outcomes, and vice versa exchange rate targeting fully ties hands of monetary policy from the perspective of inflation and output and output gap. UIP deviations can relax the Trilemma constraint on the nominal exchange rate imposed by inward-looking monetary policy.
Figure 1: Exchange rate policy tradeoffs

Note: The figure plots frontiers of output gap and exchange rate volatility, namely menus of \((\sigma_x, \sigma_e)\) that can be chosen by monetary policy, in three types of models: (a) classic trilemma models, (b) models with endogenous UIP deviations driven by exchange rate risk, and (c) models with exogenous UIP (or CIP) shocks. FB corresponds to the first best (or a “Friedman float”) with \(\sigma_x = 0\) and \(\sigma_e = \sigma_q\), the volatility of the first-best real exchange rate. The line segmented connecting FB and Peg corresponds to classic Trilemma models in which UIP (and CIP) holds. Free Float in models with financial (UIP) shocks features \(\sigma_e\) that combines macro-fundamental (blue) and financial (red and yellow) exchange rate volatility. In these models, first best is feasible only when monetary policy is combined with FXI that offset financial volatility. Dotted-dashed curves are indifference curves for the welfare loss function, and Managed Float (or crawling peg) is the optimal monetary policy rule in the absence of FXI, in a model with endogenous UIP deviations. See text for Divine (coincidence) and Mussa Puzzle points.

stabilization and international risk sharing.

We begin our analysis by characterizing the optimal allocation, which ensures efficient level of production and optimal risk sharing in the tradable sector. We then show how an unconstrained joint use of monetary policy and FX interventions allows to implement the optimal allocation, with monetary policy eliminating the output gap and FX interventions eliminating the intermediation wedge and the resulting UIP deviation in the international financial market. Exchange rate stabilization is not a direct goal of a welfare maximizing policy. The resulting equilibrium generally features volatile nominal exchange rate and inflation targeting, with financial interventions targeting UIP deviations as their policy goal. Such policy mix allows the exchange rate to accommodate fundamental macroeconomic shocks and optimal expenditure switching, while it neutralizes the effects of non-fundamental currency demand shocks in the financial market on exchange rate volatility. This is the sense in which economies with a segmented financial market do not feature a trilemma constraint, as market segmentation offers the financial regulator an additional instrument to stabilize market volatility, even when monetary policy focuses exclusively on domestic inflation and output gap stabilization.

Figure 1 provides an illustration to the policy tradeoff and the optimal policy choice, comparing
our framework with endogenous UIP deviations to two alternative classes of models, namely classic Trilemma models without UIP deviations and alternative models with exogenous financial shocks resulting in UIP (or CIP) deviations.\textsuperscript{2} Specifically, the figure plots the policy tradeoffs in the space of output gap and nominal exchange rate volatility. The first best corresponds to a fully eliminated output gap and a nominal exchange rate that, under sticky prices, must accommodate the volatility of the first-best real exchange rate that ensures efficient expenditure switching. In trilemma models without UIP deviations, a freely floating exchange rate under inward-looking inflation-stabilizing monetary policy and no FXI achieves just that, as suggested by Friedman (1953). More generally, when UIP deviations are featured in equilibrium, a laissez-faire float results in excessive exchange rate volatility, which reflects both macro-fundamental and non-fundamental financial volatility, consistent with exchange rate disconnect. In such models, both free floats and full pegs are, generally, suboptimal, and the optimal policy requires an additional use of FXI to offset financial volatility.

Implementing the optimal allocation in the goods and asset market, in general, requires an unconstrained use of both monetary and FX instruments. There exists, however, an important special case when addressing both frictions could be done with a nominal exchange rate peg by means of monetary policy alone. We refer to this case as “divine coincidence” in an open economy, by analogy with a closed-economy divine coincidence. Indeed, if the natural real exchange rate that ensures efficient risk sharing is stable, then there is no tradeoff from the points of view of the goods and asset markets. Specifically, a fixed nominal exchange rate is consistent with efficient expenditure switching under sticky prices in the goods market, as well as eliminates risk in the international financial market allowing for frictionless intermediation. Direct nominal exchange rate targeting is favored over inflation stabilization in this case as it guarantees a unique optimal equilibrium. While our analysis is consistent with the optimal currency areas logic, it identifies not only circumstances when the costs of a fixed exchange rate are low in the goods market, but also the risk-sharing benefits associated with a fixed exchange rate. In Figure 1, the case of the Divine coincidence corresponds to the situation when \(\sigma_q = 0\), and the entire blue area collapses to the origin, making the Peg and the first best (FB) coincide.\textsuperscript{3}

Next, we explore circumstances where either monetary policy is constrained (e.g., due to the zero lower bound) or the financial interventions are constrained (e.g., due to non-negative requirement on central bank foreign reserves or value-at-risk constraints for the central bank portfolio). In this case, there are two independent policy goals — the output gap and the risk sharing wedge — and only one unconstrained policy instrument, thus making it generally impossible to replicate the optimal allocation. Fixing the exchange rate using the monetary policy tool is generally feasible, but is also generally

\textsuperscript{2}The latter class includes models with exogenous UIP shocks (e.g. Devereux and Engel 2002, Kollmann 2005, Farhi and Werning 2012), convenience yield (e.g. Jiang, Krishnamurthy, and Lustig 2021), and financial frictions in the form of balance sheet constraints (e.g. Gabaix and Maggiori 2015, Basu, Boz, Gopinath, Roch, and Unsal 2020). In Itskhoki and Mukhin (2021a), we show that all such models can be equally successful in explaining the general exchange rate disconnect, yet unlike the model with endogenous UIP deviations due to limits to arbitrage these models cannot readily explain the Mussa facts (see Itskhoki and Mukhin 2021b), essential for the optimal exchange rate policy analysis. Mussa Puzzle point in Figure 1 illustrates the challenge for models with exogenous financial shocks, where a monetary peg counterfactually absorbs all the floating exchange rate volatility into inflation and output gap.

\textsuperscript{3}This is the case both in trilemma models and in a model with endogenous UIP deviations, but not in models with exogenous financial shocks that do not disappear under a fixed nominal exchange rate.
suboptimal outside the case of divine coincidence. Similarly, targeting the output gap alone is also
suboptimal, and monetary policy trades-off output gap and exchange rate stabilization (managed and
crawling pegs) in the absence of FX interventions. Managed peg and dirty floats with monetary policy
may emerge as the second best policy, even when divine coincidence is not satisfied, yet there are tight
constraint on the balance sheet of the central bank making effective FX interventions infeasible. Using
financial interventions to stabilize output gap is generally infeasible.

Lastly, we explore the monopoly power of the government in the international financial market
and the ability of the central bank to earn rents without compromising the expenditure switching and
risk sharing goals of optimal exchange rate policy. When the financial sector is offshore, the policy-
maker can compete with financial intermediaries for rents (international transfers) that emerge from
exogenous shifts in currency demand. In the presence of an additional capital control instrument, it is
possible to extract maximum rents without compromising the other objectives of the policy. In partic-
ular, FX interventions are used only part way in this case, without offsetting excess currency demand
and eliminating the entire intermediation rent, while capital controls are used in addition to eliminate
the effect of rents on the optimal risk sharing. This, however, requires a flexible state-contingent use
of capital controls, which may be infeasible. Without capital controls, the policymaker can still use FX
interventions to implement the frictionless allocation at no expected financial costs. This, however, is
generally suboptimal as it fails to take advantage of trading off frictionless risk sharing for financial
market rents from undersupplying currency reserves.

We finish our analysis with a number of extensions. In particular, we extend our small open econ-
omy model to global equilibrium with a continuum of small open economies, one of which issues a
dominant funding currency that is used for international borrowing and lending against other national
currencies. Unconstrained use of non-cooperative monetary policy and FX interventions eliminates all
international risk sharing wedges and ensures efficient level of output in every country. When policies
are constrained, however, international spillovers can no longer be internalized by non-cooperative
policies. We characterize such spillovers that emerge in both dominant and non-dominant countries,
and show how a cooperative policy of international FX interventions can address these spillovers. We
also discuss costs and benefits associated with a global currency union and a gold standard.
Related literature  We build on a vast literature studying the role of exchange rates in both goods and financial markets, as well as the optimal macroeconomic and financial policies in an open economy. Our model is most closely related to two strands of literature. On the one hand, our emphasis on the role of demand for and supply of currency in financial markets and the modelling of financial intermediaries follows the tradition of Kouri (1983), Driskill and McCafferty (1987), Dornbusch (1988) (Chapter 7) and a more recent work of Jeanne and Rose (2002), Blanchard, Giavazzi, and Sa (2005), Camanho, Hau, and Rey (2022), Gourinchas, Ray, and Vayanos (2019), Greenwood, Hanson, Stein, and Sunderam (2020). In contrast to these papers, we embed the financial sector into a realistic general equilibrium model, which is a prerequisite for any policy analysis. On the other hand, we share the assumption of segmented asset markets with Alvarez, Atkeson, and Kehoe (2009), Gabaix and Maggiori (2015). However, guided by the evidence form Itskhoki and Mukhin (2021b), we assume that limits to arbitrage come from the risk aversion of intermediaries rather than the borrowing constraints or convenience yields, which considerably changes the optimal policy and distinguishes our analysis from otherwise closely related work by (see also Basu, Boz, Gopinath, Roch, and Unsal 2020, henceforth IPF for Integrated Policy Framework) and the other normative papers listed below.


Kareken and Wallace (1981)
Ilzetzki, Reinhart, and Rogoff (2018) + “fear of floating”
2 Modeling Framework

This section introduces the baseline theoretical framework and derives the optimal policy problem. Building on Itskhoki and Mukhin (2021b), we choose the ingredients of the model with an eye to the main empirical properties of exchange rates and intentionally make several strong assumptions to keep the policy problem as simple as possible. We derive a novel linear-quadratic approximation to the planner’s problem, which allows us to characterize optimal policies in Section 3. Sections 4 and 5 generalize the setup in several dimensions and consider a number of extensions.

2.1 Setup

We consider a small open economy with tradable and non-tradable goods. There are two frictions — sticky prices and a segmented financial market — that distort the equilibrium allocation, justify government interventions, and give rise to a policy tradeoff. The policymaker can choose the path of nominal interest rates and carry out FX interventions in the currency market.

**Real sector** The households have log-linear preferences over consumption of tradables $C_T$, non-tradables $C_N$, and hours worked $L$:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \gamma \log C_T + (1 - \gamma)(\log C_N - L) \right],$$

where $\gamma$ is the expenditure share on the tradable good capturing the openness of the economy. Households receive labor income $W_L$, firm profits $\Pi$, and transfers $T$, and can borrow or lend using a one-period risk-free home-currency bond $B$:

$$P_T C_T + P_N C_N + \frac{B}{R} = B_t - 1 + W_L + \Pi + T,$$

where $R$ is the gross nominal interest rate.

The endowment of tradable goods $Y_T$ is exogenous and stochastic generating demand for international risk sharing. The prices of tradables are flexible and satisfy the law of one price:\footnote{Exogenous terms of trade due to a homogenous tradable good eliminate the beggar-thy-neighbour policy motive that typically complicates the normative analysis (see Corsetti and Pesenti 2001) and make the international risk sharing dependent on the structure of the asset markets despite logarithmic preferences (cf. Cole and Obstfeld 1991).}

$$P_T = \mathcal{E}_t P_T^*,$$

where $P_T^*$ is the international price of the tradable good and $\mathcal{E}_t$ is the nominal exchange rate in units of home currency for one unit of foreign currency (i.e., an increase in $\mathcal{E}_t$ corresponds to a home depreciation). We assume a stable price level in the foreign country, $P_T^* = 1$, and therefore the home-currency tradable price tracks the nominal exchange rate, $P_T = \mathcal{E}_t$. 

4Exogenous terms of trade due to a homogenous tradable good eliminate the beggar-thy-neighbour policy motive that typically complicates the normative analysis (see Corsetti and Pesenti 2001) and make the international risk sharing dependent on the structure of the asset markets despite logarithmic preferences (cf. Cole and Obstfeld 1991).
Output of non-tradables is endogenous and depends on the labor input and productivity shock:

\[ Y_{Nt} = A_t L_t. \]

We assume that prices are permanently sticky at an exogenous level, \( P_{Nt} = 1 \), and output is demand determined, \( C_{Nt} = Y_{Nt}. \) Total profits in the economy are given by \( \Pi_t = P_{Tt} Y_{Tt} + P_{Nt} Y_{Nt} - W_t L_t. \)

The equilibrium in the goods sector is characterized by two optimality conditions. Given that households split their consumption between tradables and non-tradables according to \( \gamma P_{Nt} C_{Nt} = \frac{(1 - \gamma)}{P_{Tt}} C_{Tt} \), and goods prices are \( P_{Tt} = \mathcal{E}_t P^e_{Tt} = \mathcal{E}_t \) and \( P_{Nt} = 1 \), the equilibrium expenditure switching condition is given by:

\[ \frac{\gamma}{1 - \gamma} \frac{C_{Nt}}{C_{Tt}} = \frac{\mathcal{E}_t P^e_{Tt}}{P_{Nt}} = \mathcal{E}_t. \] (2)

The relative demand for goods depends on their relative price, \( \mathcal{E}_t P^e_{Tt} / P_{Nt} \), which under fully sticky prices of non-tradables is equal to the nominal exchange rate \( \mathcal{E}_t \). The optimal consumption-savings decision of households is described by a standard Euler equation:

\[ \beta R_t \mathcal{E}_t \frac{C_{Nt}}{C_{Nt+1}} = 1, \] (3)

and depends on the nominal interest rate \( R_t \) set by the policymaker. Finally, the optimality condition for labor supply, \( C_{Nt} = W_t / P_{Nt} = W_t \), determines the equilibrium nominal wage.

**Financial sector** While the equilibrium in the goods market is conventional to the open-economy sticky-price models, our analysis deviates from this literature by introducing segmentation in global asset markets. In particular, we assume that home households have access exclusively to local-currency bonds, and hence all international capital flows have to be intermediated by specialized financial traders.\(^6\)

Household demand for the home-currency bond \( B_t \) reflects fundamental macroeconomic forces and shapes the equilibrium path of net exports and net foreign assets. Additionally, there are three types of agents that can trade home and foreign currency bonds in the international financial market — the government, noise traders and intermediaries (arbitrageurs) — all residing in the home economy. For these agents who have access to foreign-currency (dollar) saving and borrowing, the dollar bond is in a perfectly elastic international supply at an exogenous interest rate \( R^*_t \). Section 4 considers extensions that allow for foreign intermediaries and noise traders resulting in cross-border financial income transfers, as well as an endogenize \( R^*_t \) in a multi-country global economy.

Each period, arbitrageurs choose a zero capital portfolio \( (D_t, D^*_t) \) such that \( D_t / R_t = -\mathcal{E}_t D^*_t / R^*_t \), where \( 1 / R_t \) and \( 1 / R^*_t \) are prices of the two bonds. The dollar net income of arbitrageurs from such investments...
a carry trade is given by $\pi_{t+1} = D_t - D_t/E_t = \tilde{R}_{t+1} - R_t E_t$, where $\tilde{R}_{t+1} = R_t + \xi_{t+1}$ is a one-period return on one dollar holding of a carry trade portfolio. This income is transferred lump-sum to households.

Arbitrageurs choose their portfolio to maximize min-variance preferences, $\mathbb{E}_t \left\{ \Theta_{t+1} \pi_{t+1} \right\} - \frac{\omega}{2} \text{var}_t (\pi_{t+1})$, where $\Theta_{t+1} = \beta \frac{C_{t+1}}{C_{t+1} - 1}$ is the stochastic discount factor of home households and $\omega$ is the risk aversion parameter of arbitrageurs. The second term in the objective function is the additional risk penalty due to intermediary frictions that create limits to arbitrage. The optimal portfolio choice satisfies:

$$\frac{D_t}{\tilde{R}_t} = \frac{\mathbb{E}_t \left\{ \Theta_{t+1} \tilde{R}_{t+1} \right\}}{\omega \text{var}_t (\tilde{R}_{t+1})},$$

(4)

where $\text{var}_t (\tilde{R}_{t+1}) = R_t^2 \cdot \text{var}_t (E_t/E_{t+1})$ is a measure of the nominal exchange rate volatility. As $\omega \to 0$, the risk-absorption capacity of arbitrageurs increases unboundedly, and the uncovered interest rate parity (UIP) holds in equilibrium, $\mathbb{E}_t \left\{ \Theta_{t+1} \tilde{R}_{t+1} \right\} = 0$.\(^7\)

Noise traders also hold a zero capital portfolio $(N_t, N_t^*)$, such that $N_t/R_t = -E_t N_t^*/R_t^*$, and $N_t^*$ is an exogenous liquidity demand shock for foreign currency that is uncorrelated with macroeconomic fundamentals. A positive $N_t^*$ means that noise traders short home-currency bonds to buy foreign-currency bonds, and vice versa. Noise traders’ net income and losses are transferred to the households. Although difficult to measure in the data, these shocks are necessary to match the disconnect properties of the exchange rate. Importantly, our normative results do not require that $N_t^*$ is pure noise, and go through when one assumes that currency demand is driven by household preference shock for foreign-currency bonds (see Section 5.3).

Finally, the government holds a portfolio $(F_t, F_t^*)$ of home- and foreign-currency bonds with the net value of the portfolio given by $F_t/R_t = E_t F_t^*/R_t^*$. Changes in $F_t$ and $F_t^*$ correspond to open market operations of the government. The net government income and losses are also transferred to the households. Therefore, net transfers of income to the households from financial transactions of the government, noise traders and arbitrageurs are equal to:

$$T_t = \left( F_{t-1} - F_t/R_t \right) + E_t \left( F_{t-1}^* - F_t^*/R_t^* \right) + E_t \tilde{R}_t \cdot \frac{N_{t-1}^* + D_t^*}{R_{t-1}^*}.$$

The financial market clearing requires that the home-currency bond positions of all four types of agents balance out, $B_t + N_t + D_t + F_t = 0$. We define $B_t^*$ to be the net foreign asset (NFA) position of the home country, expressed in foreign currency, such that:

$$\frac{B_t^*}{R_t^*} = \frac{1}{E_t} \frac{B_t + F_t}{R_t} + \frac{F_t^*}{R_t^*}.$$

Thus, home NFA is the value of the combined position of home households and the government, as the remaining agents in the financial market hold zero value portfolios, albeit exposed to currency risk.

Using this definition and the zero value portfolios of noise traders and arbitrageurs, we rewrite the

\(^7\)More precisely, in this limit, the household SDF $\Theta_{t+1}$ prices the exchange rate risk, and the expected return on the carry trade is given by $E_t \tilde{R}_{t+1} = R_{t+1} \cdot \text{cov}_t (\Theta_{t+1}, E_t/E_{t+1})$, a property of the optimal international risk sharing.
financial market clearing condition as:

$$B_t^* = F_t^* + N_t^* + D_t^*.$$  \hfill (5)

In words, the NFA position of the country equals the combined foreign-currency bond position in the financial market. That is, currency market equilibrium requires that currency supply $B_t^*$ from accumulated NFA equals aggregate currency demand, $F_t^* + N_t^* + D_t^*$.

**Equilibrium** Two international conditions — the country budget constraint and international risk sharing — complete the description of the equilibrium system. To derive the country budget constraint, we substitute the expressions for profits $\Pi_t$ and financial transfers $T_t$ into the household budget constraint. This yields:

$$\frac{B_t^*}{R_t^*} - B_{t-1}^* = Y_{Tt} - C_{Tt},$$ \hfill (6)

where the right-hand side is net exports expressed in dollars (or in terms of tradables, since $P_t^* = 1$).

Intuitively, trade surpluses lead to the accumulation of net foreign assets — a macro-fundamental source of currency supply to the home market.

To derive the international risk-sharing condition, we combine household optimality (2) and (3) with the equilibrium conditions in the financial market (4) and (5). This results in:

$$\beta R_t^* \mathbb{E}_t \frac{C_{Tt}}{C_{Tt+1}} = 1 + \omega \sigma_t^2 \frac{B_t^* - N_t^* - F_t^*}{R_t^*}, \quad \text{where} \quad \sigma_t^2 = R_t^* \cdot \text{var}_t \left( \frac{\mathbb{E}_t}{\mathbb{E}_{t+1}} \right).$$ \hfill (7)

If households had direct access to dollar bonds, then a conventional Euler equation $\beta R_t^* \mathbb{E}_t \frac{C_{Tt}}{C_{Tt+1}} = 1$ would hold. Instead, household positions need to be intermediated by the financial sector which charges a risk premium — a risk-sharing wedge. This risk premium depends both on the size of the currency exposure of arbitrageurs, $D_t^* = B_t^* - F_t^* - N_t^*$, and the price of risk $\omega \sigma_t^2$ per dollar of the exposure.

Currency outflows — due to both fundamental ($B_t^* < 0$) and non-fundamental ($N_t^* > 0$) reasons — require intermediation ($D_t^* < 0$) and expose arbitrageurs to currency depreciation risk, resulting in an equilibrium risk premium and a risk-sharing wedge. Greater exchange rate volatility $\sigma_t^2$ increases the price of risk and the resulting risk-sharing wedge for given gross currency positions. A policymaker can intervene either by reducing expected exchange rate volatility or by absorbing the currency risk into the government balance sheet with FX interventions ($F_t^* \downarrow$), as we study in the next section.

Finally, we define the equilibrium in this economy. Given the stochastic path of exogenous shocks $\{A_t, Y_{Tt}, R_t^*, N_t^*\}$, sticky non-tradable prices $P_{Nt} \equiv 1$, and the path of policies $\{R_t, F_t, F_t^*\}$, an equi-
librium vector \( \{C_{Nt}, C_{Tt}, B_t^*, D_t^*, \epsilon_t\} \) and the implied \( \{\sigma_t^2\} \) solve the dynamic system (2)–(7) with the initial condition \( B^*_{t-1} \) and the transversality condition \( \lim_{T \to \infty} B^*_T / \prod_{t=0}^T R_t^* = 0 \).\(^{11}\) Note that Ricardian equivalence does not hold vis-à-vis foreign currency position \( F_t^* \), as households cannot directly hold foreign currency bonds. As a result, both the country’s NFA position \( B_t^* \) and the arbitrageurs’ currency exposure \( D_t^* = B_t^* - N_t^* - F_t^* \) are endogenous state variables of the equilibrium system. In contrast, the model features Ricardian equivalence for home-currency bonds — a change in \( F_t \) merely crowds out private \( B_t \), and hence it is not a state variable for the equilibrium allocation.

### 2.2 Policy problem

In our baseline analysis, we focus on the Ramsey problem of choosing a sequence of government policies that maximize welfare under commitment. Given the equilibrium definition above, the government chooses a feasible path of monetary policy and FX interventions, \( \{R_t, F_t^*\} \), that maximizes household welfare (1).\(^{12}\) We set up the exact non-linear policy problem in Appendix A1, which allows for characterization of the first-best allocation and the policies that decentralize it. To make progress for the main cases of interest, where the first-best allocation is not feasible given the available policy instruments, we work with a linear-quadratic approximation to the policy problem around the first-best allocation.

In this section, we derive the approximate policy problem. In doing so, we address two major challenges associated with the transition to a linear-quadratic environment. The first challenge relates to the quadratic approximation of the welfare function in an open economy, and in particular where the best possible risk sharing is not full insurance, as the international financial market is incomplete and features risk free bonds only. The second challenge arises due to the risk-sharing friction driven by time-varying risk premium in the currency market that disappears in conventional linear approximations. Our approach ensures that the risk-sharing friction remains in the linear-quadratic environment, preserving the key policy tradeoff between output gap stabilization and international risk sharing.

**First-best allocation** The first-best allocation is the path of tradable and non-tradable consumption and labor, which we denote tildes \( \{\tilde{C}_{Tt}, \tilde{C}_{Nt}, \tilde{L}_t\} \), that maximizes the household welfare in (1) subject to the country budget constraint (6) and the non-tradable production possibility frontier \( C_{Nt} = Y_t = A_t L_t \), taking as given the path of shocks \( \{Y_{Tt}, A_t, R_t^*\} \) and the initial net foreign assets \( B^*_{t-1} \), as well as the NPGC for \( B^*_{\infty} \). This problem abstracts from both the sticky price friction in the goods market and the intermediation friction in the financial market. Furthermore, the local planner takes as given the structure of the international financial market which provides a perfectly elastic supply of dollar risk-free

\(^{11}\)Note that (4) is redundant given (7) and (5). Hence, we have four independent equilibrium conditions, (2)–(3) and (6)–(7), to solve for four endogenous variables \( \{C_{Nt}, C_{Tt}, B_t^*, \epsilon_t\} \) and a side equation (5) to solve for \( D_t^* \). The other endogenous variables \( \{W_t, L_t, Y_{Nt}, B_t\} \) are recovered from static equilibrium conditions outlined above. Specifically, from the goods market clearing and labor supply \( Y_{Nt} = C_{Nt} \) and \( W_t = C_{Nt} \); from production function \( L_t = Y_{Nt} / A_t \); and \( B_t \) can be backed out from \( \{B_t^*, F_t^*, F_t\} \) given the definition of NFA \( B_t^* \).

\(^{12}\)The government does not take the welfare of arbitrageurs and noise traders into account, as these agents pass on all their financial incomes and losses to the households. Yet, their behavior — namely, exogenous currency demand of noise traders and endogenous inelastic currency supply by arbitrageurs — affects the equilibrium allocation in the financial market via a wedge in the risk-sharing condition (7). This is akin to the behavior of a monopolist in the goods market that creates a markup wedge and passes on all profits back to the households.
bonds at an exogenous interest rate $R^*_t$.

Given the log-linear utility (1), the first-best allocation features a constant labor supply $\tilde{L}_t = 1$ yielding $\tilde{C}_{Nt} = A_t$, and a path of $\tilde{C}_{Tt}$ that solves a frictionless Euler equation $\beta R^*_t \mathbb{E}_t \{ C_{Tt}/C_{Tt+1} \} = 1$ together with the country budget constraint (6). Therefore, $\tilde{C}_{Tt}$ is a function of shocks $\{ Y_{Tt}, R^*_t \}$ and the initial net foreign assets $B^-_\infty$. The first-best path of NFA satisfies the country budget constraint (6), that is $\tilde{B}^*_t = R^*_t ( \tilde{B}^-_t + Y_{Tt} - \tilde{C}_{Tt} )$.

With fully sticky non-tradable prices, the decentralization of the first-best allocation involves a path of nominal wages $\tilde{W}_t = A_t$ to ensure the first-best labor supply, and a path of the nominal exchange rate $\tilde{E}_t = \tilde{Q}_t = \gamma_1 - \gamma_1 C_{N_t} \tilde{C}_{T_t} (8)$ to ensure the first-best relative price and expenditure allocation between tradables and non-tradables in (2). Equation (8) defines $\tilde{Q}_t$ to which we refer as the first-best, or natural, real exchange rate. It is the value of international relative prices that ensures the optimal expenditure allocation between tradables and non-tradables in our economy.\(^{13}\)

**Second-order approximation to the welfare function** We evaluate welfare losses due to a departure of the equilibrium allocation from the first best. To do so, we derive a second-order approximation to the objective function in (1) around a non-stochastic steady state and evaluate the welfare loss relative to the first-best allocation $\{ \tilde{C}_{Tt}, \tilde{C}_{Nt}, \tilde{L}_t \}$ characterized above.

To this end, we introduce two wedges central to our analysis — the output gap $x_t$ and the risk-sharing wedge $z_t$ — defined by:

$$x_t \equiv \log C_{Nt} - \log \tilde{C}_{Nt} \quad \text{and} \quad z_t \equiv \log C_{Tt} - \log \tilde{C}_{Tt}.$$  

(9)

The output gap $x_t$ emerges as a result of sticky non-tradable prices, and it measures the gap in non-tradable consumption relative to $\tilde{C}_{Nt} = A_t$. This also corresponds to the departure of labor supply $L_t = C_{Nt}/A_t$ from $\tilde{L}_t = 1$. The risk-sharing wedge $z_t$ is the result of a violation of the first-best risk sharing. Specifically, an intermediation wedge in (7) causes a risk-sharing wedge. Note that all feasible paths of $C_{Tt}$, and hence $z_t$, must still satisfy the country budget constraint (6).

To make the analysis tractable, the innovation of our approach is to focus only on budget-feasible allocations $\{ C_{Tt}, C_{Nt}, L_t \}$ that satisfy the production possibilities frontier for non-tradables, $C_{Nt} = A_t L_t$, and the country budget constraint for tradables, that is (6) together with the NPGC for $B^-_\infty$ and given $B^-_\infty$. For every such allocation that results in wedges $x_t$ and $z_t$ defined in (9), the welfare loss relative to the

\(^{13}\)Formally, the real exchange rate is $E_t P^*_t/P_t = E_t^{1-\gamma}$ (with $P^*_t = P^T_t = 1$ and $P_t = P^T_t P^N_t = E_t^{1-\gamma}$), while $E_t P^T_t/P^N_t = E_t$ is the relative price of non-tradables; the two are proportional to each other in logs. More generally, in every economy, one can define a relevant concept for the first-best real exchange rate that, given goods market clearing condition, ensures an efficient expenditure allocation between home and foreign goods.
first best is given by (see Appendix A2):

\[
\text{Loss} = \frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \gamma z_t^2 + (1 - \gamma) x_t^2 \right].
\] (10)

Note that the approximation to the welfare function relative to the first best has no first order terms by construction. Intuitively, the weight on the output gap equals \(1 - \gamma\) and the weight on the risk-sharing wedge equals \(\gamma\) — the expenditure weights on non-tradables and tradables, respectively.

The path of the risk-sharing wedge \(z_t\) must be consistent with the budget constraint (6), which in deviations from the first best is given by:

\[
\beta b^*_t - b^*_{t-1} = -z_t,
\] (11)

where \(b^*_t \equiv (B^*_t - \tilde{B}^*_t)/\tilde{Y}_T\) is the deviation of NFA from its first-best path scaled by the steady-state level of tradable output \(\tilde{Y}_T\), and \(\beta R^* = 1\) in steady state. The initial condition is \(b^*_{-1} = 0\) (as \(\tilde{B}^*_0 = B^*_0\)), the NPGC is \(\lim_{t \to \infty} \beta^t b^*_t = 0\), and thus \(z_t = b^*_t = 0\) for all \(t \geq 0\) is a feasible allocation corresponding to the first-best risk sharing. Note that \(z_t\) acts simultaneously as the risk-sharing wedge and as the deviation of net exports from their first-best path, \(z_t = -(nx_t - n\bar{x}_t)\) where \(n\bar{x}_t \equiv (Y_{tT} - \tilde{C}_T)/\tilde{Y}_T\). Cumulated deviations of net exports \(z_t\) result in deviations of NFA \(b^*_t\), as summarized by (11).

**First-order approximation to the equilibrium system** Minimizing the welfare loss (10) subject to the budget constraint (11) alone poses no policy tradeoff as \(x_t = z_t = 0\) for all \(t \geq 0\) is a budget-feasible allocation. In addition to the budget constraint (11), the first-order approximation to the exact equilibrium system (2)–(7) involves two additional conditions — one that characterizes equilibrium in the goods market and the other that characterizes equilibrium in the financial market.

In the goods market, the expenditure allocation condition (2) can be written in log deviations as:

\[
et_t = \tilde{q}_t + x_t - z_t,
\] (12)

where \(e_t = \log \mathcal{E}_t\) and \(\tilde{q}_t = \log \tilde{Q}_t\) is the first-best real exchange rate defined in (8), and the two wedges \(x_t\) and \(z_t\) as defined in (9). Given sticky prices, the nominal exchange rate must accommodate movements in the first-best real exchange \(\tilde{q}_t\), otherwise one or both wedges open up. Indeed, if the relative price of non-tradables is off its first-best level, either tradable or non-tradable consumption (or both) must deviate from their first-best levels as well. Equation (12) captures the locus of possible equilibrium allocations in the goods market shaped by expenditure switching between tradables and non-tradables.\(^{15}\)

The remaining condition characterizes equilibrium in the financial (currency) market. The risk-sharing friction emphasized in (7) corresponds to the risk premium charged by arbitrageurs for inter-
mediating currency trades and holding the associated exchange rate risk. In conventional linear approximations, risk premia go to zero with second moments such as $\sigma_t^2$. We consider an alternative point of approximation in which risk premia remain first-order objects and, hence, affect first-order dynamics of the equilibrium system. To this end, we let the risk aversion parameter $\omega$ to increase as $\sigma_t^2$ decreases, keeping the price of risk $\omega \sigma_t^2$ non-zero in the limit. We provide formal details in Appendix A2, where we show that our first-order approximation to (7) results in:

$$\mathbb{E}_t \Delta z_{t+1} = \bar{\omega} \tilde{\sigma}_t^2 (n_t^* + f_t^* - b_t^*) \quad \text{with} \quad \tilde{\sigma}_t^2 = \text{var}_t(e_{t+1}), \quad (13)$$

where $\bar{\omega} \equiv \omega \bar{Y}_T/\beta$, $f_t^* \equiv F_t^*/\bar{Y}_T$ are FXI scaled by tradable output, and $n_t^* \equiv (N_t^* - \tilde{B}_t^*)/\bar{Y}_T$ is a combined exogenous currency demand shock. Like a conventional first-order approximation, our approximation scales linearly with the size of exogenous shocks that drive $n_t^*$ in (13) and $\tilde{q}_t$ in (12). However, due to an unconventional point of approximation in which the risk-bearing capacity of the financial sector $1/\omega \to 0$, the equilibrium system is not linear in shocks (and hence state variables), and in particular features time-varying volatility that affects first-order equilibrium dynamics.

Condition (13), together with the budget constraint (11), characterizes the equilibrium path of tradable consumption relative to its first-best level, $z_t \equiv \log(C_{T_t}/\bar{C}_{T_t})$, from the point of view of households. Alternatively, it also determines the path of UIP deviations from the point of view of the financial market (recall conditions (4) and (5)). Indeed, we have that the UIP deviation equals:

$$i_t - i_t^* - \mathbb{E}_t \Delta c_{t+1} = \mathbb{E}_t \Delta z_{t+1}, \quad (14)$$

where $i_t - i_t^* = \log(R_t/R_t^*)$. Therefore, the risk-sharing wedge in (13), $\bar{\omega} \tilde{\sigma}_t^2 (n_t^* + f_t^* - b_t^*)$, is also the frictional UIP wedge. It is a first-order object that comoves with the intermediated demand for currency, $n_t^* + f_t^* - b_t^*$, and with the unit price of the exchange rate risk, $\bar{\omega} \tilde{\sigma}_t^2$. Thus, variation in the conditional exchange rate volatility $\tilde{\sigma}_t^2$ is both an equilibrium outcome and has a direct first-order feedback into equilibrium dynamics. We argue that this provides a superior point of approximation for models that focus on the joint dynamics of macroeconomic variables and risk premia.

In Appendix A2, we prove two additional results. First, the dynamic system (11)–(13) provides an accurate first-order approximation to the exact equilibrium dynamics. That is, taking the path of exogenous shocks $\{\tilde{q}_t, n_t^*\}$ and policies $\{x_t, f_t^*\}$, this system characterizes the path of endogenous equilibrium outcomes $\{z_t, b_t^*, e_t, \tilde{\sigma}_t^2\}$. Note that we take output gap $x_t$ as the policy variables since it is directly controlled by the monetary policy instrument $i_t$, and we discuss the implementation below.

Second, we prove that minimizing the second-order welfare loss in (10) with respect to $\{x_t, f_t^*, z_t, b_t^*, e_t\}$

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16Note that $n_t^*$ features both noise trader demand for foreign currency ($N_t^* > 0$) net of supply of foreign currency accumulated from the first-best path of net exports (that is, NFA $\tilde{B}_t^*$ > 0); of course, these variables can take both positive and negative values with corresponding interpretations.

17From the frictionless international Euler equation, $i_t^* = \log(R_t^*/R^*) = \mathbb{E}_t \Delta \tilde{c}_{T_t+1}$. The home Euler equation (3) together with (2), in turn, implies $i_t = \log(R_t/R) = \mathbb{E}_t \{\Delta c_{T_t+1} + \Delta e_{t+1}\}$. Subtracting one from the other, and using the fact that $z_t = c_{T_t} - \tilde{c}_{T_t}$ according to the definition in (9), yields the UIP expression in the text. Note also that the equilibrium path of the local interest rate can be recovered from (3) as $i_t = \tilde{r}_t + \mathbb{E}_t \Delta x_{t+1}$, where $\tilde{r}_t = \mathbb{E}_t \Delta a_{t+1} = \mathbb{E}_t \Delta \tilde{c}_{N_{t+1}}$ is the natural real interest rate and $x_t = c_{N_t} - a_t$ is the output gap.
and subject to the linearized equilibrium system (11)–(13) results in a first-order accurate description of the optimal policies in the exact non-linear problem. While the equilibrium system is non-linear in the path of shocks and state variables due to the presence of $\bar{\sigma}^2_t$ in (13), the policy problem scales proportionally with the general magnitude of shocks, and in this narrow sense one may refer to this policy problem as linear-quadratic.\footnote{Formally, we let $\nu$ scale all exogenous shocks in the exact non-linear economy $\{A_t, Y^*_T, R^*_t, N^*_t\}$ with $1/\omega$ scaled by $\nu^2$ to keep the unit price of risk $\omega\sigma^2_t$ stable. Then, the linearized system (11)–(13) characterizes the first-order component of the non-linear system, which scales proportionally with $\nu$, while the welfare loss in (10) scales proportionally with $\nu^2$. The first-order optimal policy and the equilibrium price of risk $\omega\sigma^2_t$ do not change with $\nu$, and are generally time-varying.}

## 3 Optimal Policies

Given the results in Section 2.2, we restate here the baseline Ramsey policy problem (10)–(13) of a small-open economy policymaker in deviations from the first-best allocation:

$$\min_{\{x_t, f^*_t, z_t, e_t, b^*_t, \sigma^2_t\}} \frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \gamma z^2_t + (1 - \gamma) x^2_t \right]$$  \hspace{1cm} (15)

subject to $\beta b^*_t = b^*_{t-1} - z_t,$

$$e_t = \tilde{q}_t + x_t - z_t$$

$$\mathbb{E}_t \Delta z_{t+1} = \tilde{\omega}\sigma^2_t (n^*_t + f^*_t - b^*_t)$$ with $\sigma^2_t = \text{var}_t(e_{t+1}),$

given the initial condition $b^*_{-1} = 0$ and transversality condition $\lim_{t \to \infty} \beta^t b_t = 0$. The policymaker directly controls the path of output gap and FX interventions, $\{x_t, f^*_t\}$. Additional constraints may restrict the path of policies, e.g. under a binding zero-lower bound, or with non-negativity or value-at-risk constraints on foreign reserves $f^*_t$, as we discuss below.

All exogenous shocks affecting equilibrium dynamics are summarized by two variables — the natural (first-best) real exchange rate $\tilde{q}_t$ defined in (12) and the exogenous net currency demand shock $n^*_t$ defined in (13). In particular, $\tilde{q}_t$ is a sufficient statistic for all macroeconomic shocks $\{A_t, Y^*_t, R^*_t\}$ that shape the first-best path of tradable and non-tradable consumption. In turn, $n^*_t$ summarizes currency demand shocks of noise traders $N^*_t$ and households $B^*_t$, with the latter shaped by the path of the first-best $\tilde{N}X_t \equiv Y^*_T - \tilde{C}_T$. Departures from the first-best path of tradable consumption result in risk-sharing wedges $z_t \equiv \log(C_{Tt}/\tilde{C}_T)$, which via (11) lead to deviations of NFA $b^*_t$ that also feed back as additional endogenous currency supply (or demand, if negative) in (13).

The goal of the policy (15) is to minimize deviations from the first-best allocation — namely, eliminate to the extent possible the output gap $x_t$ and the risk-sharing wedge $z_t$, with the relative weight on the latter given by the openness of the economy $\gamma$. Policies shape the equilibrium path of the exchange rate $e_t$, and thus its conditional volatility $\sigma^2_t$, which in turn feeds back into the dynamics of the equilibrium system via (13).

We note that a particular level or volatility of the exchange rate is not a policy goal in itself. Nonetheless, the exchange rate $e_t$ emerges as the key equilibrium variable linking the financial and the goods markets. This puts the exchange rate at the center of the policy tradeoff. When prices are sticky,
movements in the nominal exchange rate are necessary to accommodate the adjustment of relative prices in the goods market (12). Yet, volatility of the exchange rate is also a source of the risk-sharing wedge in the financial market with imperfect intermediation (13).

**Relaxed Trilemma** An important feature of the model is that the planner can sidestep the standard trade-off between an independent monetary policy and a managed exchange rate. Even in the absence of capital controls, the government can choose the path of the output gap $x_t$ with an inward-looking interest rate policy (e.g., ensure $x_t = 0$), and simultaneously manipulate the path of the exchange rate via sterilized interventions in the currency market (by means of $f_t^*$ in (13)). This result does not contradict the trilemma: FX interventions have real effects because market segmentation limits capital mobility and does not allow households to undo the open market operations of the central bank. As a result, the policymaker can move exchange rate risk between balance sheets of arbitrageurs and households, and thus change the equilibrium outcome in the currency market (cf. Wallace 1981, Silva 2016, Kekre and Lenel 2022). Similarly to how nominal rigidities allow monetary policy to affect real outcomes, intermediation frictions give rise to an additional policy instrument $f_t^*$ in (13). Note also that FX reserves are not essential for interventions as the same outcomes can be achieved using FX derivatives to absorb risk from the balance sheet of market participants. This result is consistent with a wide use of instruments such as currency swaps in central bank interventions (Patel and Cavallino 2019) and contrasts with the mechanism based on CIP deviations (e.g., Gabaix and Maggiori 2015, Fanelli and Straub 2021, IPF).

By this logic, the central bank can peg the nominal exchange rate in two different ways — using either monetary policy or FX interventions, affecting $e_t$ in (12) by means of $x_t$ and $z_t$, respectively. The policy problem (15) identifies the costs and benefits associated with each of these implementations. On the one hand, monetary policy has the advantage that there are no restrictions on the implementable paths of the exchange rate. However, this comes at a cost of opening the output gap $x_t$. Unless prices are fully flexible, a monetary peg drives a wedge between the real exchange rate and its natural level $\tilde{q}_t$, resulting in suboptimal expenditure switching in the goods market (cf. “divine coincidence” below).

On the other hand, FX interventions can be used to manipulate the path of the exchange rate without any output gap side effects. However, there are important limits on the possible paths of the exchange rate that can be implemented with FXI. First, for a given monetary policy, FX interventions affect the nominal exchange rate by changing the real exchange rate, net exports and net foreign asset dynamics, via $z_t$ in (11)–(13). Therefore, while FXI can temporarily alter the dynamics of the exchange rate, it is, for example, impossible to use them to generate a permanent appreciation, as it would result in a permanent trade deficit. Second, FXI become entirely ineffective when monetary policy fully (and

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19Formally, if monetary policy stabilizes the output gap, $x_t = 0$, then from (12) the nominal exchange rate must equal $e_t = \tilde{q}_t - z_t$. This, in general, results in $\sigma_t^2 = \text{var}_t(e_t+x_t) > 0$, and hence a non-zero risk-sharing wedge $z_t$ from (13). Conversely, optimal risk sharing $z_t \equiv 0$ can only be achieved with $\sigma_t^2 = 0$ in the absence of FX interventions ($f_t^* = 0$), which in turn requires $e_t = \tilde{q}_t + x_t = 0$, and thus in general a non-zero output gap, $x_t = -\tilde{q}_t$.

20It is the presence of a non-zero price of currency-holding risk, $\omega \sigma_t^2 > 0$ in (13), that relaxes the trilemma constraint on FXI and allows FXI to affect the equilibrium currency risk premium and thus the exchange rate.

21What happens when a policymaker attempts to fix the exchange rate at a level above what is consistent with a long-run steady state, $\bar{e} > \tilde{q}$ in (12)? Statically, it can either result in a negative output gap, $x < 0$, or a positive risk-sharing
credibly) stabilizes the nominal exchange rate, \( e_t = \bar{e} \) and hence \( \sigma_t^2 = 0 \) in (13), bringing back the classic trilemma constraint. This is the case because the currency supply by arbitrageurs becomes perfectly elastic in the absence of exchange rate risk, and arbitrageurs fully neutralize the effects of any open market operations on the exchange rate.

**Optimal monetary policy** We introduce here a general characterization of the optimal monetary policy for any given path of FX interventions, which nests as special cases the specific results that we consider in turn in Sections 3.1–3.3. We prove in Appendix A3 that the solution to the policy problem (15) involves the following optimality condition:

**Theorem 1** For any given path of FX interventions \( \{f^*_t\} \), the Ramsey optimal monetary policy sets the path of the output gap to satisfy \( \mathbb{E}_t x_{t+1} = 0 \) and:

\[
\beta(1 - \gamma)x_{t+1} = -2\gamma\tilde{\omega}\mu_t(n^*_t + f^*_t - b^*_t)(e_{t+1} - \mathbb{E}_te_{t+1}),
\]

where \( \mu_t \) is the Lagrange multiplier on the risk sharing constraint (13).

The optimality condition (16) connects the optimal path of monetary policy, summarized by the path of the output gap \( \{x_{t+1}\} \), with three properties of the exchange rate and the currency market:

(i) exchange rate surprises, \( \{e_{t+1} - \mathbb{E}_te_{t+1}\} \);

(ii) capital outflows, or currency demand \( \{n^*_t + f^*_t - b^*_t\} \);

(iii) departures from the first-best risk sharing and UIP, \( \mathbb{E}_t \Delta z_{t+1} \neq 0 \), as captured by the sequence of respective Lagrange multipliers \( \{\mu_t\} \) on the risk sharing constraint (13).

It is the interaction of these three features that determines the optimal monetary policy response, emphasizing already the role of non-linearity in the optimal exchange rate analysis captured by our approximation approach.

In general, according to (16), the optimal monetary policy in an open economy (\( \gamma > 0 \)) responds to exchange rate surprises and, hence, deviates from the exclusive inward-looking goal of stabilizing inflation and output gap (optimal in the closed economy when \( \gamma \to 0 \)). Specifically, output gap in each period is stabilized on average, \( \mathbb{E}_t x_{t+1} = 0 \), but generally not state by state. In what follows, we first focus on two cases where output gap is fully stabilized, \( x_{t+1} \equiv 0 \), either because capital outflows are fully accommodated with FXI, or because there are no exchange rate shocks in equilibrium. Then we consider the general case that can be described as the optimal crawling peg (or a dirty float), whereby the optimal monetary policy compromises full output gap stabilization to smooth out exchange rate surprises.

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\( z > 0 \) (i.e., excess tradable consumption). However, the latter is inconsistent with the intertemporal budget constraint (11), and specifically the NPGC on \( b^*_t \) which explodes to negative infinity. Therefore, unless the monetary authority permits a negative output gap, an equilibrium with \( \bar{e} > \hat{q} \) and \( z > 0 \) can be sustained only temporarily, until a run on the government FX reserves (cf. Krugman 1979). Since targeting \( z > 0 \) is not part of an optimal policy profile, we do not explore this case further.

Note that a (gross) currency demand shock \( n^*_t > 0 \), unaccommodated with FXI, results in a (net) capital outflow, \( z_t < 0 \), at least when \( \tilde{\omega}\sigma_t^2 > 0 \). Therefore, we occasionally refer to \( n^*_t \) as capital outflow shocks.
3.1 Unconstrained optimal policy

When both policy instruments — monetary policy that controls the path of $x_t$ and FXI $f^*_t$ — are available and unconstrained, the first-best allocation is feasible and, thus, is implemented by the optimal policy. Indeed, this corresponds to the special case of Theorem 1 where FXI ensure $n^*_t + f^*_t - b^*_t = 0$ in (16), which both results in $z_t = b^*_t = 0$ from (11)–(13), and renders optimal an inward-looking monetary policy that targets $x_t = 0$.\footnote{Proposition 1, as well as Proposition 2 below, holds without approximation in the exact policy problem, as we show in Appendix A1. This is because the exact first-best allocation is feasible when the two policy instruments are available and unconstrained; similarly, it is feasible with a single monetary policy tool under the “divine coincidence” introduced below.}

**Proposition 1** If both policy instruments are available and unconstrained, the optimal policy fully stabilizes both wedges, the output gap $x_t = 0$ and the risk sharing wedge $z_t = 0$, by targeting output gap with monetary policy and demand for currency with FX interventions ($f_t = -n^*_t$). This solution is unique, time consistent, and its implementation requires no commitment.

Though the fact that two policy instruments are sufficient to implement the first-best allocation in the presence of two frictions is perhaps not surprising, the proposition shows that there is a one-to-one mapping between instruments and optimal targets (cf. Mundell 1962).\footnote{Another notable feature of this result is that capital controls are not needed for implementation, as FX interventions are sufficient to achieve the first best allocation when combined with the optimal monetary policy (see Section 4.1).} In particular, monetary policy closes the output gap $x_t = 0$ and stabilizes producer prices, while optimal FX interventions eliminate frictional UIP deviations, $E_t \Delta z_{t+1} = i_t - i^*_t - E_t \Delta e_{t+1} = 0$, and thus close the risk-sharing wedge $z_t = 0$. Crucially, neither policy instrument targets the exchange rate directly, nor fully stabilizes it. Instead, the optimal policy ensures $x_t = z_t = 0$, which in turn implies that the nominal exchange rate tracks the natural real exchange rate, $e_t = \tilde{q}_t$, and hence generally $\sigma_t^2 = \text{var}_t(\Delta \tilde{q}_{t+1}) > 0$.

The proposition also provides a complementary characterization of the optimal policy in terms of responses to different types of shocks. Using the language of CGG, FX interventions offset currency demand shocks $f^*_t = b^*_t - n^*_t = -n^*_t$, as $b^*_t \equiv 0$ along the first best, while allowing the exchange rate to accommodate fundamental macroeconomic shocks $\{A_t, Y_t, R^*_t\}$ that drive the natural real exchange rate $\tilde{q}_t$. To the extent financial intermediation is frictional and results in risk-sharing wedges, FX interventions should step in to eliminate the associated UIP deviations. In practice, this means providing FX liquidity to the market to offset currency demand shocks, alleviating the need for costly intermediation by absorbing the exchange rate risk exposure from the arbitrageurs’ and into the government balance sheet. The fact that interventions offset liquidity shocks state by state and are independent of expectations about future outcomes explains why the optimal policy is time consistent and does not require commitment on the part of the government.

An important feature of this setup is that it allows us to distinguish between UIP and CIP deviations, and to show that the optimal policy should target the former. This contrasts with the conclusions of the previous literature where the limits to arbitrage arise due to financially-constrained arbitrageurs and CIP wedges are the only source of UIP deviations (Fanelli and Straub 2021, IPF). This difference is important from the practical perspective given that, in the data, UIP deviations are an order of magnitude larger than CIP deviations.
More generally, FX interventions should be used to eliminate all rents in the currency market due to intermediation frictions, including the monopoly power of intermediaries. Because the policymaker is the agent on behalf of the households who cannot directly participate in FX trading, the portion of UIP deviations due to risk that is priced by the household SDF (e.g., default risk) should not be eliminated with FXI. Consistent with Friedman (1953), the policymaker should take positions in the currency market as long as they are deemed profitable from the point of view of the households. As a result, the central bank should be making money, at least on average, from its FXI activity.

**Implementation** The optimal policy can be implemented using a conventional Taylor interest rate rule targeting the output gap and a similar policy rule for FXI targeting ex ante UIP deviations. For example, adopting a rule \( f_t^* = -\alpha E_t \Delta z_{t+1} \) results in \( E_t \Delta z_{t+1} = \frac{\omega \sigma_t^2}{1 + \alpha \sigma_t^2} (n_t^* - b_t^*) \) from (13), which converges to \( E_t \Delta z_{t+1} = 0 \) as \( \alpha \to \infty \), and \( f_t^* = b_t^* - n_t^* \) in this limit. In words, FX interventions should lean against the wind intensively enough until the UIP wedge is entirely eliminated.

Despite its simplicity, the optimal FX policy might be hard to implement in practice. The challenge is that neither the UIP wedge \( E_t \Delta z_{t+1} = i_t - i_t^* - E_t \Delta e_{t+1} \) nor liquidity shocks \( n_t^* \), nor the natural level of the real exchange rate \( \hat{q}_t \) are directly measurable in the data. One solution to this problem is to condition the policy rule on easily measurable variables such as the ex post UIP realization, \( i_{t-1} - i_{t-1}^* - \Delta e_t \), or on a noisy measure of the ex ante UIP wedge, \( E_t \Delta z_{t+1} + u_t \), where \( u_t \) is a measurement/expectational error. In both cases, the first-best implementation is infeasible, and there is an internal optimum for the intensity of the policy response to avoid overreaction to noise in the measurement of the target. Another possibility is to condition the policy rule on the level of the exchange rates, \( f_t^* = -\alpha e_t \), which corresponds to a second-best implementation if currency demand shocks \( n_t^* \) are the dominant source of the exchange rate volatility.25 If instead most of the exchange rate volatility is due to macro shocks affecting \( \hat{q}_t \), then the optimal FXI do not respond to the exchange rate and set \( \alpha = 0 \).

It is worth noting that the challenge of unobserved targets and shocks is, of course, not unique to FXI. It is a common feature of the optimal monetary policy in a closed economy, where the policymaker needs to make a judgement call about the natural rate of interest, potential output and NAIRU, to offset shocks to aggregate demand and accommodate productivity shocks (see CGG). Even though not directly observable in the data, these concepts are useful in guiding the decisions of policymakers.

3.2 Divine coincidence

We consider now a special case of Theorem 1, whereby it is possible to achieve both policy objectives — in the goods and in the financial market — with a single monetary instrument, without recurring to capital flow or exchange rate management using FXI. By analogy with the closed-economy New Keynesian literature, we refer to this case as *divine coincidence*, and we further show in Section 5.1 how it generalizes the closed economy case.

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25There is an upper bound on the value of \( \alpha \) beyond which the policy becomes inconsistent with the budget constraint (11) resulting in an explosive path of net foreign assets. Intuitively, a very large \( \alpha \) implements a nearly constant exchange rate, which generally is inconsistent with the intertemporal budget constraint. However, if the peg is set at a sufficiently depreciated level that it guarantees persistent trade surpluses and hence, accumulation of NFA and FX reserves, such policy does not violate NPGC and is feasible in the long run, albeit generally suboptimal due to \( z_t < 0 \) on average.
The open-economy divine coincidence obtains when the first best (natural) real exchange rate is stable at some level, $\tilde{q}_t = \bar{q}$. In this case, allowing for an arbitrary path of FXI $\{f^*_t\}$, a monetary policy rule that targets the same level of the nominal exchange rate, $e_t = \bar{q}$, both ensures zero output gap and eliminates the risk-sharing wedge, $x_t = z_t = 0$, delivering the first best outcome. Indeed, in this case, $\sigma^2_t = \text{var}_t(\Delta e_{t+1}) = 0$, and thus $z_t = 0$ is the unique solution of (11) and (13) independently of the path of $(n^*_t, f^*_t)$. Given $z_t = 0$ and the fact that $\tilde{q}_t = \bar{q}$, expenditure allocation in the goods market (12) eliminates the output gap, $x_t = 0$, as the unique equilibrium outcome. We summarize this result in:

**Proposition 2** If the natural real exchange rate is stable, $\tilde{q}_t = \bar{q}$, then monetary policy that fully stabilizes the nominal exchange rate, $e_t = \bar{q}$, ensures the first best allocation with $x_t = z_t = 0$, for any path of FX interventions, including $f^*_t = 0$. An exchange rate peg is superior to inflation or output gap targeting, as it rules out multiplicity of exchange rate equilibria.

In general, our model emphasizes the tension between the need for exchange rate adjustment in the goods market with sticky prices and the risk-sharing consequences of a volatile exchange rate unaccommodated by FXI. This dual role of the exchange rate, generally, makes a single policy instrument insufficient to attain efficiency in both goods and financial markets at once, as suggested by Theorem 1.

Divine coincidence is the situation when this policy tradeoff disappears, as the natural real exchange rate — a goods-market proxy for the desirable nominal exchange rate adjustment — is stable. In turn, a pegged nominal exchange rate encourages intermediaries to supply currency more elastically and, in the limit, offset any demand shocks in the currency market. Thus, a fixed nominal exchange rate comes at no cost from the perspective of the goods market and delivers the first-best risk sharing from the perspective of the financial market. In fact, the fixed exchange rate policy is implied by Theorem 1, as $x_{t+1} = 0$ is consistent with $e_{t+1} = E_t e_{t+1} = \bar{q}$ in this case, making sure the optimality condition (16) holds under a monetary peg.

Divine coincidence provides a rationale for pegging the exchange rate. Moreover, in this case, a nominal exchange rate peg by means of monetary policy is not only efficient, but also effective, as it immediately eliminates the possibility of multiple equilibria. Consider the alternative policy of output gap (inflation) targeting that ensures $x_t = 0$ independently of the path of $z_t$. Under divine coincidence, such policy is consistent with an equilibrium with $e_t = \bar{q}$ and $z_t = \sigma^2_t = 0$. However, this is not a unique equilibrium, as there exists another equilibrium with arbitrageurs uncertain about the future exchange rate, $\sigma^2_t > 0$, which makes them charge a risk premium in response to currency demand shocks $n^*_t$, resulting in a self-fulfilling volatile exchange rate equilibrium. The positive volatility equilibrium is suboptimal as it features $E_0 z_t^2 > 0$ in contrast to the first best with $z_t = 0$. Thus, under divine coincidence, an exchange rate peg dominates inflation targeting, even though the result of the exchange rate peg is also a zero inflation and a zero output gap (cf. Marcet and Nicolini 2003, Atkeson, Chari, and Kehoe 2010, Bianchi and Coulibaly 2023).

How special is the open-economy divine coincidence? On the one hand, this result extends immediately to various generalizations of the goods market with expenditure switching between varieties.

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26If $n^*_t$ follows an AR(1), then $e_t = \bar{q} - z_t$ follows an ARMA(2,1) with innovations proportional to the innovation of $\bar{\omega} \sigma^2 n^*_t$, where $\sigma^2 = \text{var}_t(e_{t+1}) > 0$ is a fixed point, in addition to the other fixed point with $\sigma^2 = 0$. 

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of home and foreign tradable goods. In each such model, one can define a concept of the natural real exchange rate that delivers efficient expenditure switching. A stable natural real exchange rate implies that a fixed nominal exchange rate does not come into conflict with the objectives of inflation and output gap stabilization in the goods market. At the same time, a stable natural real exchange rate is, of course, a knife-edge case which we do not expect to systematically hold in practice, yet it provides a useful benchmark and a stark illustration to the model’s mechanism.\footnote{In our baseline model of the goods market, an example with a stable natural real exchange rate is \( i_t^* = 0 \) (i.e., \( R_t^* = 1/\beta \)), and \( y_{Tt} = a_t \) and both follow the same random walk. In this case, \( \tilde{q}_t = a_t - \tilde{c}_{Tt} = 0 \), as \( \tilde{c}_{Tt} = y_{Tt} \). Note that Proposition 2 generalizes to any deterministic path of the natural real exchange rate without unexpected surprises, that is any \( \tilde{q}_t = E_{t-1} \tilde{q}_t \), so that targeting \( e_t = E_{t-1} \tilde{q}_t \) maintains \( \sigma_t^2 = \text{var}_t (\Delta e_{t+1}) = 0 \).}

On the other hand, the divine coincidence result is quite specific to the particular structure of the financial market that we assume in our framework. In particular, in this framework an ex post stable exchange rate, \( e_{t+1} \equiv 0 \), implies ex ante certainty, namely \( \sigma_t^2 = \text{var}_t (e_{t+1}) = 0 \), and this in turn guarantees that UIP holds and risk sharing is undistorted. This nests two assumptions. First, it requires that a commitment to a peg is ex ante credible. Second, it relies on the structure of the model in which a fully stabilized exchange rate eliminates UIP deviations via the endogenous response of arbitrageurs who are willing to supply currency with infinite elasticity in the absence of exchange rate risk. If either the peg is not credible, or UIP deviations may coexist with \( \sigma_t^2 = 0 \), then divine coincidence result breaks down. For example, this is the case when risk-sharing frictions are driven by balance sheet constraints rather than risk, and UIP and CIP deviations are closely linked. To the extent a credible peg eliminates a large portion of UIP deviations, while leaving CIP deviations intact, — as the data seem to suggest (see Itskhoki and Mukhin 2021b) — this result offers a useful quantitative benchmark.

**Optimal currency areas** The divine coincidence result also provides an important benchmark for common currency areas, which are optimal when the natural real exchange rate between member countries is stable. In particular, this is the case when member countries share correlated fundamental shocks (see Section 4.2), confirming the logic of Mundell (1961). What is new to our result is that it not only identifies the cases when the goods-market costs of a fixed exchange rate are low, but it also emphasizes the benefits of a fixed exchange rate from the perspective of the financial market. These benefits include reduced financial volatility and improved risk sharing between member countries. The benefits are larger the more the member countries trade with each other, as captured by the openness weight \( \gamma \) in the welfare loss function (10).\footnote{This insight is consistent with the experience of the Euro Zone, where the cost of borrowing was harmonized across countries and the cross-country financial flows increased significantly since the introduction of the euro in 1999 (see the discussion of the reduced borrowing wedge in Blanchard and Giavazzi 2002). Of course, an alternative interpretation is that these capital flows were excessive and driven by inefficient risk pricing of borrowing in Southern Europe, a case that may also arise in our model environment augmented with a possibility of default on net foreign liabilities (see also Fornaro 2021).} Furthermore, we expect a fixed exchange rate — or a formation of the currency union — to dominate the alternative of an unmanaged (free) float, when the volatility of the bilateral nominal exchange rate under the float is dominated by non-fundamental currency demand shocks \( n_t^* \) relative to fundamental macro-trade shocks \( \tilde{q}_t \), provided that \( \gamma \) is sufficiently large (i.e., the exchange rate volatility is consequential for aggregate allocations).
3.3 Crawling peg

Proposition 1 suggests that it is generally optimal to combine conventional monetary policy with FX interventions. However, in practice, it is not uncommon for countries to abstain from using FXI. This may be due to incomplete information about shocks and optimal targets in the currency market, as we discuss below in Section 3.4. It also may reflect additional constraints on the central bank’s balance sheet making it costly to intervene when FX reserves are too low or too high (Krugman 1979, Amador, Bianchi, Bocola, and Perri 2016). In both cases, the central bank is prone to negative valuation effects, which can undermine its credibility and lead to a loss of independence. Thus, we now study general implications of Theorem 1 for the optimal monetary and exchange rate policy away from the first best, when FX interventions follow an arbitrary given path \{f_t^*\}, including \(f_t^* = 0\) as one possibility.

**Discretionary monetary interventions** Before turning to the discussion of Ramsey-optimal policy, we first consider briefly the effects of discretionary ex post monetary interventions to stabilize the exchange rate and capital flows. We find that, without commitment, the optimal policy is always inward looking and focuses exclusively on the output gap. This is the case, even though, according to Theorem 1, an inward-looking monetary policy is generally suboptimal, provided there are departures from the first-best risk sharing. Ex post discretionary monetary interventions are allocative in the goods market and, in particular, affect the exchange rate. However, they cannot improve the allocation in the financial market — that is, they neither prevent capital outflows, nor improve international risk sharing or eliminate UIP wedges.

**Proposition 3** Without commitment, the optimal discretionary monetary policy stabilizes the output gap, \(x_t = 0\). Discretionary ex post interventions that depart from \(x_t = 0\), affect the exchange rate \(e_t\), but do not change capital flows, UIP deviations or the risk-sharing wedge \(z_t\).

Proposition 3 shows that, in general, the optimal discretionary monetary policy should not respond to the exchange rate, capital flows, or risk sharing wedges. To see the intuition, consider an ex post monetary tightening and an associated reduction in output \(x_t\) done in response to a capital outflow shock — namely, an increase in currency demand \(n_t^*\) in (13) resulting in \(z_t < 0\) and an exchange rate depreciation. Monetary tightening with \(x_t < 0\) leads to an appreciation of the exchange rate \(e_t\), offsetting the effect of \(z_t < 0\), and an expenditure switching away from home non-tradables in the goods market, according to the equilibrium condition (12).

This might be misleading for a policy success to fend off the capital outflow shock. However, this outcome results only in costs in the goods market \((x_t < 0)\) and no benefits in the financial market (as \(z_t < 0\) still). Indeed, the equilibrium in the financial market, and in particular the path of the risk-sharing wedge \(z_t\), is characterized by (11) and (13), which remain unaffected by a discretionary monetary tightening. Neither the size of the capital outflow, \(n_t^* + f_t^* - b_t^*\), nor the unit price of risk \(\bar{\omega}\sigma_t^2\) in (13) respond to ex post monetary tightening. Discretionary policy affects the path of \(e_t\) and

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29Two types of constraints are a lower bound of FX reserves, \(f_t^* \geq 0\), or a value-at-risk constraint on the balance sheet, \(\sigma_t^2 \cdot (f_t)^2 \leq M\), where \(\sigma_t^2 \cdot (f_t)^2\) is the variance of returns on the gross position of the central bank.
Proposition 4

Consider a case in which FXI \( x \), rate, \( e \) the optimal monetary policy has a structure of a managed \( f_\text{loat} \), or a crawling peg. Specifically, it a pure peg (\( e_0 > 0 \) open economy, \( \gamma > 0 \)), and study the general implications of the Ramsey-optimal monetary policy in an tional on the state of the economy. In other words, we step outside of the special cases considered but the monetary authority can commit to a policy rule to respond to exchange rate surprises condi-

| Commitment to a crawling peg | We now return to Theorem 1, and consider the case when FXI do not ensure the first-best risk sharing (\( n_t^* + f_t^* - b_t^* \neq 0 \)) and divine coincidence does not apply (\( \bar{q}_t \neq \bar{q} \)), but the monetary authority can commit to a policy rule to respond to exchange rate surprises condi-

| **Proposition 4** | Consider a case in which FXI \( \{ f_t^* \} \) are unconstrained in every period but \( t \geq 0 \). Then \( x_t = 0 \) in every period except:

\[
x_{t+1} = -2\gamma \frac{\omega^2 \sigma^2}{1 - \gamma} \left( n_t^* + f_t^* - b_t^* \right)^2 (e_{t+1} - \mathbb{E}_t e_{t+1}),
\]

that is (16) applies with \( \mu_t = \beta \frac{\omega^2 \sigma^2}{1 + \gamma} \left( n_t^* + f_t^* - b_t^* \right) \).

To see the intuition, consider a state of the world with non-zero intermediated capital flows and a binding risk-sharing condition (13), so that \( \mu_t (n_t^* + b_t^* - f_t^*) \neq 0 \). As discussed above, adjusting monetary policy in period \( t \) does not affect contemporaneous capital flows. Instead, the policymaker can only indirectly mitigate the risk-sharing wedge by encouraging arbitrageurs to take larger positions and lowering the required risk premium. Monetary policy achieves this by leaning against surprise exchange rate innovations at \( t + 1 \) and lowering the perceived conditional variances of the exchange rate, \( \sigma^2 = \text{var}_t (\Delta e_{t+1}) \). This makes financial intermediation less risky and relaxes the risk-sharing constraint (13). In particular, this implies that an unexpected depreciation, \( e_{t+1} > \mathbb{E}_t e_{t+1} \), requires a monetary tightening that results in an output gap, \( x_{t+1} < 0 \). Importantly, this commitment does not depend on the source of volatility in the exchange rate at \( t + 1 \) — namely, whether exchange rate surprises are driven by financial noise shocks \( n_{t+1}^* \) or fundamental macro shocks \( \bar{q}_{t+1} \). Thus, optimal

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30The result that monetary policy has no effect on capital flows whatsoever relies on the assumption that preferences are separable in tradables and non-tradables and no foreign intermediates are used in production. Despite being a special case, this provides a benchmark that illustrates the limited capacity of conventional monetary policy in capital flow management. Note that this result also generalizes to the case where noise shocks \( n_t^* \) are partially elastic to the expected UIP deviation, as we show that it remains unchanged in equilibrium.

31Rewriting \( e_{t+1} = \bar{q}_{t+1} - z_{t+1} + x_{t+1} \) in unexpected changes and using (16) results in:

\[
e_{t+1} - \mathbb{E}_t e_{t+1} = \frac{(\bar{q}_{t+1} - \mathbb{E}_t \bar{q}_{t+1}) - (\Delta z_{t+1} - \mathbb{E}_t \Delta z_{t+1})}{1 + \frac{\omega^2 \sigma^2}{(1-\gamma)}} \left( n_t^* + f_t^* - b_t^* \right),
\]

22
monetary policy is no longer inward-looking and limits the free float of the exchange rate.

Proposition 4 has several important implications. First, the optimal monetary policy always stabilizes the expected output gap, \( E_t x_{t+1} = 0 \), irrespective of the path of the exchange rate \( e_t \) and the risk sharing wedge \( z_t \). Symmetrically, any expected changes in the exchange rate, \( E_t \Delta e_{t+1} \), do not require accommodation with a monetary policy response. In other words, it is only exchange rate surprises, \( e_{t+1} - E_t e_{t+1} \), that require a policy response. Therefore, the optimal policy rule has the structure of a crawling peg — it fully allows for expected exchange rate adjustment and responds only to unexpected exchange rate movements. The implication is that any medium-run exchange rate adjustment can be accommodated with expected exchange rate changes, a managed float, without resulting in welfare costs in goods or financial markets.\(^{32}\)

Second, optimal monetary interventions in the currency market are state-contingent. Exploiting non-linearity allowed by our approximation, Proposition 4 shows that the intensity of the optimal policy lean increases with the size of the frictional UIP wedge (13) and the size of the capital (out)flow shock. Specifically, rewrite the general optimality condition (16) in the form of a "lean against the wind" policy rule:

\[
x_{t+1} = -\delta_t \cdot (e_{t+1} - E_t e_{t+1}).
\]

A constant-intensity policy rule, \( \delta_t \equiv \delta \), is feasible, and results in a constant conditional exchange rate volatility, \( \sigma_t^2 = \sigma^2 \).\(^{33}\) However, it is suboptimal and dominated, in the context of Proposition 4, by a state-contingent policy rule with (see Appendix A3):

\[
\delta_t = \frac{2\gamma}{1 - \gamma} \cdot \frac{\bar{\omega}^2 \sigma_t^2}{1 + \beta + \bar{\omega}^2 \sigma_t^2} (n^*_t + f^*_t - b^*_t)^2,
\]

which is both increasing in the unit price of exchange rate risk, \( \bar{\omega}^2 \sigma_t^2 \), and increasing and convex in the size of unaccommodated capital outflow shocks, \( |n^*_t + f^*_t - b^*_t| \). Recall that the size of the frictional UIP deviation is given by \( \bar{\omega}^2 \sigma_t^2 (n^*_t + f^*_t - b^*_t) \).

It follows that the crawling peg is more relevant for countries with a larger tradable sector \( \gamma \) and, thus, higher welfare costs of capital flow shocks. Furthermore, periods with larger expected exchange rate volatility, \( \sigma_t^2 \), and larger excess demand or supply of currency that requires intermediation, \( |n^*_t + f^*_t - b^*_t| \), call for a commitment to a stronger future response of monetary policy, \( x_{t+1} \), to unexpected exchange rate movements, \( e_{t+1} - E_t e_{t+1} \). This suggests a state-contingent policy approach to financial market volatility, which can be ignored when it causes no spikes in risk premia (intermediation wedges), but should be smoothed out with monetary policy when such volatility distorts risk sharing and direct financial market interventions (FXI) are limited.

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\(^{32}\) Some oil exporting countries, such as Saudi Arabia, follow this type of exchange rate policy, in parallel accumulating an FX sovereign wealth fund for the future when global demand for oil declines. If the present value of all future oil revenues were known, this would indeed be the first best policy.

\(^{33}\) In this class of policies, one can optimize over \( (\delta, \sigma^2) \) to show that \( \delta \) is increasing in openness \( \gamma \) and in the ratio of volatilities of noise \( n^*_t \) to fundamental \( q_t \) shocks.
**Numerical solution** We next go beyond the one-time deviation from the first-best FXI and generalize the insight of Proposition 4 to an environment when monetary policy faces a more complicated intertemporal trade-off. We show that under certain conditions, a closed-form solution can be obtained with optimal policy characterized by two equations: a forward-looking condition that pins down the equilibrium volatility of the exchange rate and a backward-looking policy that respects previous promises. The standard methods of iterating policy functions can then be used to obtain the numerical solution.

To this end, we focus on a case when monetary policy responds symmetrically to capital inflows and outflows, i.e. the policy depends on the size of shocks $n_t^*$, but not on their sign. In addition, assume that the government does not use FX interventions $f_t^* = 0$, the currency demand shocks are dominated by noise traders $b_t^*/n_t^* \approx 0$, and shocks $n_t^*$ are i.i.d. across periods and symmetrically distributed around zero. The latter condition simplifies the analysis: iterating forward the risk-sharing condition (13) and imposing the country’s budget constraint (11), we get the following stochastic process for the risk-sharing wedge

$$z_t = (1 - \beta)b_{t-1}^* - \bar{\omega}\sigma_t^2 n_t^*.$$  

According to Theorem 1, the optimal policy rule can be expressed as

$$x_{t+1} = -\delta_t[\tilde{q}_{t+1} - z_{t+1} - E_t(\tilde{q}_{t+1} - z_{t+1})],$$

where $\delta_t$ reflects the commitment of monetary authorities to stabilize the exchange rate in period $t+1$ and depends only on information available in period $t$. Combining these two equations, it is possible to solve for the conditional equilibrium exchange rate volatility $\sigma^2_t$ as well as the unconditional moments $Ez_t^2$ and $Ex_t^2$. The planner’s problem is then to minimize the resulting loss function subject to the constraint on the equilibrium exchange rate volatility (see Appendix A3 for details).

**Lemma 1** Assume no FXI $f_t^* = 0$, small fundamental demand for currency $b_t^*/n_t^* \approx 0$, and i.i.d. shocks $n_t^*$ symmetrically distributed around zero. Then the optimal policy $\{\sigma^2_t, \delta_t\}$ is characterized by the equilibrium exchange rate volatility condition

$$\sigma^2_t = (1 - \delta_t)^2\left[\sigma^2_q + E_t(\bar{\omega}\sigma_{t+1}^2 n_{t+1}^*)^2\right],$$  

(17)

where $\sigma^2_q \equiv \text{var}_t(\tilde{q}_{t+1})$, and the optimal policy rule $\delta_t \in [0, 1)$ such that:

$$\frac{\delta_t}{1 - \delta_t} = \frac{1}{\beta}\left(\frac{\gamma}{1 - \gamma} + \delta_{t-1}(2 - \delta_{t-1})\right)(\bar{\omega}\sigma_t n_t^*)^2.$$  

(18)

To understand the result, consider first constraint (17). Intuitively, there are two sources of the exchange rate volatility $e_{t+1} -$ fundamental shocks $\tilde{q}_{t+1}$ and noise trader shocks $n_{t+1}^*$ with the pass-through that depends on the future currency risk $\bar{\omega}\sigma_{t+1}^2$. The latter channel implies that the equilibrium volatility in period $t$ depends on the future volatility of the exchange rate and gives rise to a recursive forward-looking determination of $\sigma^2_t$. In contrast, the size of contemporaneous shocks $n_t^*$ has no direct
Figure 2: Optimal policy $\delta_t$ and equilibrium volatility $\sigma_t$ as functions of...

(a) shocks $n_t^*$

(b) past policy $\delta_{t-1}$

Note: the figure shows the optimal monetary interventions parametrized by $\delta_t$ from the policy rule $x_{t+1} = -\delta_t [\tilde{q}_{t+1} - z_{t+1} - E_t(\tilde{q}_{t+1} - z_{t+1})]$ (blue lines) and the equilibrium exchange rate volatility $\sigma_t$ (red lines) as functions of two state variables — the capital flow shocks $n_t^2$ and past interventions $\delta_{t-1}$.

Effect on $\sigma_t^2$. These factors are then scaled by the aggressiveness of the monetary policy with $\delta_t = 0$ corresponding to a floating regime and $\delta_t = 1$ representing a fully fixed exchange rate. Importantly, the future policy determines $\sigma_{t+1}^2$ and therefore, implicitly affects the equilibrium volatility in period $t$. From a planner’s perspective, this means that commitment to future interventions can be used to fight present shocks and opens room for exchange rate forward guidance.

The optimal policy rule (18) formalizes this intuition. On the one hand, the aggressiveness of interventions in the currency market $\delta_t$ increases in the absolute size of the capital flows $n_t^*$ and the riskiness of carry trade for arbitrageurs $\bar{\omega}\sigma_t^2$. The equation also shows that for any finite values of $\sigma_t^2$, the optimal policy implements a partial peg $\delta_t \in [0, 1)$. Thus, the main insight of Proposition 4 carries over. On the other hand, the policy also depends on the previous history of shocks as summarized by the lagged intervention $\delta_{t-1}$. Furthermore, $\delta_t$ is increasing in $\delta_{t-1}$ reflecting the forward guidance motive: if faced with a capital flow shock, the planner dampens the volatility of the exchange rate by promising to intervene not only in the next period but in all further periods. The same dampening of $\sigma_t^2$ can then be achieved at lower costs of distorting the output gap.\(^{34}\)

The non-linear system of equations (17) and (18) can then be solved numerically. The policy function $\delta_t$ and $\sigma_t$ can be expressed in terms of two state variables — the lagged interventions $\delta_{t-1}$ and exogenous capital flows $n_t^*$ — and then iterated until convergence. Figure 2 provides a numerical illustration. Large capital flows $n_t^*$ require more aggressive interventions $\delta_t$ and therefore, result in lower equilibrium volatility of the exchange rate $\sigma_t^2$. Similarly, a history of large shocks and interventions $\delta_{t-1}$ in the past imply more active policy $\delta_t$ and more stable exchange rates $\sigma_t^2$.

\(^{34}\)This mechanism resembles the optimal response to markup shocks in a closed economy when monetary policy allows for higher inflation expectations to smooth out the negative effects of shocks on present output (see CGG).
3.4 Optimal FX interventions

Consider next the opposite situation when monetary policy is constrained and the planner can only choose FX interventions. No divine coincidence with one instrument closing the two gaps emerges in this case. This is because FX interventions have only indirect effect on output gap choosing consumption of tradables, while the path of $x_t$ is determined by the response of monetary policy to movements in $z_t$ (and/or $e_t$). Under the zero lower bound (ZLB) — a particularly relevant constraint on monetary policy for many developed countries — the path of $x_t$ is determined by the Euler equation $E_t \{ \Delta x_{t+1} + \Delta \log C_{N_{t+1}} \} = 0$ and is independent from $z_t$. As a result, FX interventions cannot close the output gap and optimally focus on closing the risk-sharing wedge $z_t = 0$. This contrasts with the case of a currency union where a fixed nominal exchange rate $e_t = \tilde{q}_t + x_t - z_t = 0$ induces an endogenous response of output gap $x_t$ to consumption of tradables $z_t$. Yet, as discussed above, FX policy becomes irrelevant in this limit when carry trade is risk free and government interventions crowd out positions of arbitrageurs without any affect on risk premia or allocations.\(^{35}\)

Another important difference from monetary policy is that for any given path of $x_t$, FX interventions are time consistent, so that the optimal discretionary policy still closes UIP deviations and implements $z_t = 0$. This does not mean, however, that commitment on behalf of the planner provides no additional benefits. The gains from commitment arise when FX interventions are subject to occasionally binding constraints. As discussed above, the latter may arise when governments are unable to take negative reserve positions in foreign currency at the world interest rate, $f_t^* \geq 0$, or when taking large FX positions can lead to losses which would require a bailout by households compromising central bank independence.

If either of these constraints binds, the FX instrument cannot be used to fully offset liquidity shock $n_t^*$ opening a risk-sharing wedge. In this case, the commitment technology allows the planner to improve the allocation by offering forward guidance about the future path of $f_t^*$. Similarly to conventional monetary guidance, the planner can exploit the fact that $z_t$ is a forward-looking variable and depends on its own future expectation, $E_t z_{t+1}$ as a result of consumption smoothing for imported goods. For example, consider a capital outflow shock $n_t^* > 0$ that depresses the present consumption of tradable goods $z_t$. Even if reserves $f_t^*$ are subject to a non-negative constraint, a planner can promise to tolerate future capital inflows $n_{t+1}^* < 0$, which stimulate $z_{t+1}$ and consequently $z_t$ (cf. Werning 2011).

Furthermore, differently from conventional forward guidance, future FX interventions can be used to smooth out the variation of $z_{t+1}$ around the same mean $E_t z_{t+1}$, thus reducing conditional volatility of the exchange rate $\sigma^2$ and encouraging arbitrageurs to offset the present distortional shock $n_t^*$.\(^{36}\)

In this sense, commitment to future interventions that lean against the wind — especially in periods of large shocks — mitigates the present deviations from the optimal risk sharing and reduces the size

\(^{35}\)This illustrates an important limitation of FX interventions relative to capital controls in eliminating aggregate demand externality (Farhi and Werning 2016).

\(^{36}\)Formally, we can rewrite the risk sharing condition (13) as:

$$z_t = E_t z_{t+1} + \omega \sigma_t^2 (t b_t - n_t^* - f_t^*), \quad \text{where} \quad \sigma_t^2 = E_t \left\{ (\tilde{q}_{t+1} - E_t \tilde{q}_{t+1}) - (z_{t+1} - E_t z_{t+1}) \right\}^2,$$

and we assumed for simplicity that $x_{t+1} = 0$ is ensured by monetary policy. When $f_t^*$ is constrained, forward guidance using $f_{t+1}^*$ can impact both $E_t z_{t+1}$ and $\sigma_t^2$ via $(z_{t+1} - E_t z_{t+1})$ on the right hand side, thus affecting $z_t$. 

26
of required interventions today. This mechanism rationalizes the signalling channel of FXI, which is often viewed by central bankers as the key part of monetary transmission in the currency market (see Patel and Cavallino 2019).

4 Extensions

4.1 International Transfers and Capital Controls

The analysis above shows that two instruments — nominal interest rate and FX interventions — are sufficient to implement the constrained optimal allocation. In this section, we complement conventional monetary policy and quantitative interventions in the FX market with capital controls. The goal is twofold. First, we discuss whether macroprudential policy can substitute for other instruments when the latter are constrained. Second, we show that under certain conditions, all three types of instruments are necessary to support the optimal allocation.

To this end, assume that the planner sets state-contingent agent-specific taxes on holding assets with net income transferred lump-sum to households. In particular, let $\tau^h_t$ denote a tax on household positions in local bonds and let $\tau^a_t$ and $\tau^{a*}_t$ denote respectively taxes on home and foreign bonds held by financial agents — arbitrageurs and noise traders. To keep the problem interesting, we restrict the set of available instruments by excluding taxes on foreign households, which would allow the planner to directly extract surplus from foreigners’ positions that are unlikely to be in its jurisdiction. For the same reasons, we do not allow for discriminatory taxes on noise traders.

To see how the equilibrium system changes, notice that capital controls only affect intertemporal decisions, and static conditions remain unchanged, including the expenditure switching condition for the nominal exchange rate (2). The household intertemporal consumption smoothing depends on net returns on home bonds:

$$\frac{\beta R_t}{1 + \tau^h_t} \mathbb{E}_t \frac{C_{Nt}}{C_{Nt+1}} = 1,$$

and for any given path of $\tau^h_t$, monetary policy $R_t$ can still implement any demand for non-tradable goods and effectively controls the output gap $x_t$.

Solving portfolio problem of arbitrageurs and combining it with the market clearing for assets and the household Euler equation, we get a modified international risk-sharing condition:

$$\beta R^*_t \mathbb{E}_t \frac{C_{Tt}}{C_{Tt+1}} = \frac{(1 + \tau^h_t)(1 + \tau^{a*}_t)}{1 + \tau^a_t} + \omega \sigma_t^2 \frac{B^*_t - N^*_t - F^*_t}{R^*_t}.$$  (19)

This equation clarifies two important properties of capital controls in the economy. From theoretical perspective, taxes and quantity interventions in FX markets are largely isomorphic and can be used interchangeably to implement the optimal risk sharing. In particular, any of the three taxes is sufficient to offset distortionary effects of liquidity shocks $N^*_t$. At the same time, the use of capital controls might be complicated in practice. First, the optimal risk sharing requires state-contingent instruments and cannot be implemented with slow-moving taxes. Second, the planner might be unable to distinguish
different types of agents and impose agent-specific capital controls. As the expression above makes clear, setting a uniform tax on home bonds for all agents $\tau^h_t = \tau^o_t$ is isomorphic to a change in local interest rate and cannot be used to eliminate the UIP deviations. Finally, imposing a tax on holdings of foreign assets is complicated by the fact that some of these agents are foreigners and are outside of home jurisdiction. In short, the optimal risk sharing requires complicated capital controls that have to be state-contingent as well as agent- and asset-specific and can be challenging to implement in practice (Rebucci and Ma 2020).

With these caveats in mind, we proceed under the assumption that both arbitrageurs and noise traders are foreigners and the only form of capital controls used by the planner is a tax on their positions in home bonds $\tau^o_t$. Using the optimal portfolio choice of arbitrageurs, the country’s budget constraint (6) can be expressed as

$$\frac{B_t^s}{R_t} = B^s_{t-1} + (Y_{Tt} - C_{Tt}) - \left[ \frac{E_{t-1} \Theta_t \tilde{R}^o_t}{\omega \sigma^2_{t-1}} + \frac{N^a_{t-1}}{R^a_{t-1}} \right] \tilde{R}^o_t,$$

where $\tilde{R}^o_{t+1} = R^o_t - \frac{R}{1 + R} E_{t+1} f^e_{t+1}$ are net returns on carry trade after paying taxes. The term in bracket is demand of foreign arbitrageurs and noise traders, which is multiplied by the ex-post return $\tilde{R}^o_t$ to obtain a net transfer to the rest of the world. For example, a depreciation of the exchange rate generates a positive valuation effect when foreign traders have long positions in home bonds. While it is easy to generate such transfers from inelastic noise traders, the arbitrageurs invest in assets with a higher expected returns and make positive profits in expectation, which puts a limit on how much rents the planner can extract from foreigners.

Following the same approach as in Section 2.2, we next take the second-order approximation around the optimal allocation with the maximum extraction of foreign rents. The extended policy problem can be written as:\footnote{The log-linearized version of (19), when $\tau^h_t = \tau^o_t = 0$, is given by $E_t \Delta z_{t+1} = \psi_t + \tau^o_t$, where as before $z_t = \log(C_{T_t}/\tilde{C}_{T_t})$, and we denote $\psi_t = -\tilde{\omega} b^*_t (n^*_t - f^*_t)$ as it is the term that emerges in the loss function (20).}

$$\min_{\{x_t, z_t, b^*_t, f^*_t, \psi_t, \tau^o_t, \sigma^2_t\}} \frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ (1 - \gamma)x^2_t + \gamma z^2_t + 2\beta \gamma \left( \frac{1}{\omega \sigma^2_t} \psi_t - n^*_t \right) \psi_t \right]$$

s.t. $E_t \Delta z_{t+1} = \psi_t + \tau^o_t$,

$$\psi_t = -\tilde{\omega} \sigma^2_t (ib^*_t - n^*_t - f^*_t),$$

$$b^*_t = b^*_{t-1} - z_t,$$

$$\sigma^2_t = \text{var}_t (\tilde{q}_{t+1} + x_{t+1} - z_{t+1}),$$

where $E_t \Delta x_{t+1} = i_t - i^*_t - \Delta E_t e_{t+1} = \psi_t + \tau^o_t$ is the UIP deviation evaluated from the point of view of households (i.e., the risk sharing wedge), while $\psi_t = i_t - i^*_t - \Delta E_t e_{t+1} - \tau^o_t$ is the after-tax expected carry trade return for arbitrageurs. Two main differences from the benchmark problem (15) stand out. First, in addition to output gap and the risk-sharing wedge, the objective function also includes transfers from foreign traders. Interestingly, to the second-order approximation, these rents
depend only on expected returns, while any variation in ex-post valuation effects is of a higher order.\footnote{This implies that given the structure of international asset markets and the order of approximation, the planner does not aim to use state-contingent valuation effects to “complete the markets” (cf. Fanelli 2017).} As a result, the expression for transfers is largely isomorphic to the one in a deterministic case with CIP deviations replaced by UIP deviations (cf. Fanelli and Straub 2021). Second, capital controls $\tau_t$ provide an additional degree of freedom to the planner breaking a tight link between consumption of tradables $z_t$ and returns on carry trade $\tau_t$.

Given the three motives in the loss function, three instruments are required in general case to achieve the optimal allocation. As before, the interest rate policy controls the aggregate demand and closes the output gap $x_t = 0$. In contrast to the baseline model, however, FX interventions are reserved to extract rents rather than to close the risk-sharing wedge. Transfers are a quadratic function of $\psi_t$ and attain the maximum value when $\psi_t = \frac{ωσ_t^2}{2} n_t^*$, which in turn, requires that FX interventions partially satisfy demand of noise traders $f_t^* = \frac{1}{2} n_t^*$. Finally, the efficient risk sharing $z_t = 0$ (and consequently $b_t^* = 0$) still requires closing the UIP wedge (from the household perspective), but it is now implemented using capital controls, $\tau_t = -\psi_t$.\footnote{Consider, for example, a response to a liquidity demand for foreign currency (dollar), $n_t^* > 0$, that is noise traders borrow in home currency to invest in dollars. The planner responds with a capital control tax on borrowing in home currency, $\tau_t < 0$, which in turn is pocketed by intermediaries who extend (part of the) home currency lending, $\psi_t = -\tau_t > 0$, while UIP still holds from the perspective of households, $i_t - \bar{i}_t - E_t \Delta x_{t+1} = 0$. To maximize rents, the government optimally satisfies half of the currency demand of the noise traders, $f_t^* = - n_t^*/2$, which results in $\psi_t = \frac{ωσ_t^2}{2} n_t^*/2$ and rents $\frac{ωσ_t^2 (n_t^*)^2}{4}$.} Neither policy instrument explicitly targets the exchange rate and its dynamics is still determined by $\bar{q}_t$.

**Proposition 5** Assume that arbitrageurs and noise traders are foreign agents. Then implementing the constrained optimal allocation requires closing the output gap with monetary policy $x_t = 0$, partially offsetting demand of noise traders with FX interventions $f_t^* = - n_t^*/2$, and closing the UIP deviations and the risk-sharing wedge $z_t = 0$ with capital controls, $\tau_t = -\frac{ωσ_t^2}{2} n_t^*/2$.

While fairly simple, this optimal policy has a few interesting properties. First, it follows that whenever $n_t^* \neq 0$, a country can generate a positive transfer from the rest of the world by exploiting the monopoly power in the home currency market.\footnote{Most of the existing literature focuses on the case of $n_t^* = 0$ and foreign arbitrageurs when any FX interventions result in negative rents (Jeanne 2012, Amador, Bianchi, Bocola, and Perri 2019, Fanelli and Straub 2021).} The optimal FX interventions always lean against the wind, but offset only a part of the liquidity demand of noise traders leaving the rest to be absorbed by arbitrageurs to ensure positive equilibrium rents. Echoing the recent experience of Switzerland, this result implies that a positive demand for home currency should be addressed by issuing reserves and accumulating assets in foreign currency.

Second, there remains a one-to-one mapping between policy instruments and optimal targets. In particular, FX interventions still address noise trader demand, however, now it is no longer optimal to fully offset it, as it would then eliminate all rents (that is, $f_t^* = - n_t^*$ results in $\psi_t = 0$). Optimal FX interventions partially offset noise trader demand, leaving an opportunity for positive rents, $\psi_t \neq 0$. However, if left unaddressed by capital controls, this would result in a risk sharing wedge. Therefore, capital controls are optimally used to close the risk sharing wedge, $\tau_t^a = -\psi_t$, without eliminating equilibrium rents. As a result, UIP (optimal risk sharing) holds from the perspective of households
\((i_t - i_t^* - E_t \Delta e_{t+1} = 0)\), while arbitrages receive equilibrium rents in proportion with \(\psi_t = -\tau_t^a\). 41

Note that capital controls cannot substitute for FX interventions, as they do not change demand for domestic currency and, therefore, cannot be used to collect rents from foreign traders: Similarly to interest rate shocks \(R_t\), any changes in capital controls \(\tau_t^a\) are absorbed by expected depreciation \(E_t \Delta e_{t+1}\) and do not affect net carry trade returns \(\psi_t\), which are pinned by the balance of supply and demand in the currency market and determine, in turn, the size of the international transfer.

Lastly, if for the reasons discussed above, capital controls are not available to the planner \(\tau_t^a = 0\), closing the risks sharing wedge (UIP deviation) with FX interventions is still feasible and associated with no additional costs as the net transfer is equal to zero when \(\psi_t = 0\). At the same time, the optimal policy in this case balances between closing the risk-sharing wedge and collecting international rents and. That is, unlike in Proposition 1, the optimal policy does not fully offset the noise trader demand, leaving a non-zero equilibrium UIP deviation.

Does the divine coincidence still hold in a model with transfers? Consider again the case of a stable natural real exchange rate \(\tilde{q}_t = 0\) and no FX interventions \(f_t^* = 0\) or capital controls \(\tau_t^a = 0\). It follows from the equilibrium system that a nominal peg ensures the optimal risk sharing \(z_t = 0\) and closes the output gap \(x_t = 0\). At the same time, international income loss can be evaluated using the equation for expected returns \(\psi_t\):

\[
2\gamma \omega \sigma_t^2 (ib_t^* - f_t^*) (ib_t^* - n_t^* - f_t^*)
\]

and are equal to zero given \(\sigma_t^2 = 0\). Thus, for any value of \(n_t^*\), closing the two gaps with a nominal peg comes at no extra cost in terms of international income loss, yet requires leaving out potential rents in the currency market. Such rents may offer a reason to abandon a nominal peg even when the conditions for divine coincidence are satisfied.

\[\text{International Cooperation}\]

So far, we focused on the optimal policy in a small open economy that takes as given global economic conditions. This section studies the international dimensions of monetary policy, the spillovers across countries, and a classical question about international cooperation.

To this end, consider a world comprised of a continuum of small open economies index by \(i \in [0, 1]\), each one isomorphic to a country in the baseline model, and country \(i = 0\) (the US) issues the global funding currency (the dollar), and we denote this country with \(*\). There is a global market for the tradable good and a non-tradable sector in each economy. The law of one price still holds for tradables, and now we write it in logs as \(p_{Tit} = p_{T*}^* + e_{it}\) for all \(i \in (0, 1]\) with \(e_{it}\) denoting the country \(i\) nominal exchange rate against the dollar. We allow for \(p_{T*}^* \neq 0\) and \(\pi_{T*}^* = \Delta p_{T*}^*\) to denote the US tradable inflation. The expenditure switching condition (2) in this case can be written as

\[
e_{it} = \tilde{q}_{it} - p_{T*}^* + x_{it} - z_{it}, \tag{21}
\]

\[\text{Recall that arbitrageur’s carry trade return is } i_t - i_t^* - E_t \Delta e_{t+1} - \tau_t^a = \psi_t, \text{ while households cannot take carry trade positions, yet nonetheless their equilibrium risk sharing is still governed by } E_t \Delta z_{t+1} = i_t - i_t^* - E_t \Delta e_{t+1} = \psi_t + \tau_t^a.\]
with the wedges \(x_{it}\) and \(z_{it}\) and the natural real exchange rate \(\tilde{q}_{it} = \tilde{c}_{Nit} - \tilde{c}_{Tit}\) still defined as before.

We make two assumptions about the structure of asset markets. First, only nominal dollar bonds are available for international risk sharing, which as we will see, generates an asymmetry between the US and other economies. Second, for each currency there is a separate market, in which agents can trade it against dollars. This segmentation of currency markets is in line with the fact that the dollar accounts for 88% of the global FX market turnover, but it is not crucial for our results which remain largely unchanged if one assumes that arbitrageurs can invest simultaneously in a portfolio of currencies. For simplicity, we assume local financial markets to exclude the redistributive motive in the national policies (see discussed in Section 4.1). Appendix A4.1 provides detailed derivations.

The equilibrium conditions for a given economy are the same as in the baseline model. Instead, the main difference is that the international interest rate, \(i^*_t \equiv \log R^*_t - \log \tilde{R}^*_t\), is endogenous and is shaped by the dollar inflation and the global market clearing condition for tradables:

\[
\int_0^1 c_{Tit} \, di = \int_0^1 y_{Tit} \, di \equiv y_{Tt},
\]

where \(y_{Tt}\) is the global tradable output endowment. We denote \(\tilde{r}^*_t = \log \tilde{R}^*_t\) the world interest rate that obtains in a global constrained optimum allocation with zero tradable inflation \((\pi^*_T = 0)\), that is:

\[
\tilde{r}^*_t = \mathbb{E}_t \Delta y_{Tit+1} = \mathbb{E}_t \Delta \tilde{c}_{Tit+1} \quad \text{for all } i \in [0, 1],
\]

where \(\{\tilde{c}_{Tit}\}\), now corresponds to the allocation with constrained optimum risk sharing among \(i \in (0, 1]\) rather than to a small open economy constrained optimum which takes \(R^*_t\) as given.

With this, we log-linearize the international risk-sharing condition (the analog to (7) which now also features \(P^*_T/P^*_{Tit+1}\) inside the expectation) to obtain:

\[
\mathbb{E}_t \Delta z_{it+1} = i^*_t - \mathbb{E}_t \pi^*_{Tit+1} + \psi_{it}, \quad \text{where} \quad \psi_{it} \equiv -\tilde{\omega}_t \sigma^2_{it}(ib^*_it - n^*_it - f^*_it),
\]

and as before \(\sigma^2_{it} = \text{var}(\Delta e_{it})\) and \(\beta b^*_it - b^*_{it-1} = -z_{it}\). We now use a short-hand \(\psi_{it}\) for currency \(i\) UIP wedge equal to the product of the unit price of risk of currency \(i\), \(\tilde{\omega}_t \sigma^2_{it}\), and excess demand for the dollar relative to currency \(i\) that needs to be absorbed by the intermediaries, \(n^*_it + f^*_it - ib^*_it\).

Tradable market clearing with global tradable endowment \(y_{Tt}\) ensures that \(\int_0^1 z_{it} \, di = 0\), since market clearing must hold for both frictional \(\{c_{Tit}\}_i\) and optimal \(\{\tilde{c}_{Tit}\}_i\) allocations (and \(z_{it} \equiv c_{Tit} - \tilde{c}_{Tit}\)). Therefore, we can solve for the equilibrium interest rate deviation by integrating (22):

\[
i^*_t - \mathbb{E}_t \tilde{\pi}^*_{Tit+1} = -\tilde{\psi}_t, \quad \text{where} \quad \tilde{\psi}_t \equiv \int_0^1 \psi_{it} \, di = -\int_0^1 \tilde{\omega}_t \sigma^2_{it}(ib^*_it - n^*_it - f^*_it) \, di.
\]

Therefore, global excess demand for the dollar, \(\tilde{\psi}_t > 0\), creates a force that depresses the global dollar real interest rate, \(i^*_t - \mathbb{E}_t \pi^*_{Tit+1}\). Finally, substituting (23) back into (22) yields \(\mathbb{E}_t \Delta z_{it+1} = (\psi_{it} - \tilde{\psi}_t)\),

\footnote{A correlated excess demand for dollar relative to other currencies that results in \(\tilde{\psi}_t > 0\) creates correlated UIP premia on non-dollar currencies, which in turn result in \(z_{it} = c_{Tit} - \tilde{c}_{Tit} < 0\) on average across \(i \in (0, 1]\). This creates an excess}
which generalizes condition (13) in a small open economy with an endogenous $i^*_t$.

National policymakers take $i^*_t$ as given and their problems (15) remain unchanged. However, a

First thing to note about this problem is that the optimal non-cooperative policies from Proposition 1 translate into a globally optimal outcome: that is, the Nash equilibrium played by the national policymakers results in zero output gap and optimal risk sharing between all economies. Elimination of UIP deviations with privately optimal FX interventions country-by-country, $\psi_{it} = 0$ for all $i \in (0, 1)$, also eliminates the pressure on the global real interest rate beyond its efficient level $\bar{\pi}_t^* = \mathbb{E}_t \Delta y_{T+1}$. Indeed, there are no externalities when countries choose consumption of tradables subject to intertemporal budget constraint. Although international asset markets are incomplete, the fact that there is only one tradable good implies that there is no pecuniary externality as in Geanakoplos and Polemarchakis (1986) and Greenwald and Stiglitz (1986). Similarly, there is no aggregate demand externality in the risk sharing as long as the monetary policy closes the output gap (Farhi and Werning 2016).

On the other hand, when FX policies of a subset of countries are constrained and shocks have a correlated component resulting in $\bar{\psi}_t \neq 0$, this creates negative international spillovers that are not internalized by national policymakers that take $\psi_{it}^* = 0$ as given. The unconstrained national policymakers use FX interventions according to Proposition 1 to target $\psi_{it} = 0$, while the optimal cooperative policy prescription for such countries is to target $\bar{\psi}_{it} = \bar{\psi}_t$ to ensure $\mathbb{E}_t \Delta z_{it+1} = 0$ in (22). This cooperative policy eliminates the risk sharing wedge between the group of unconstrained and constrained countries $i \in (0, 1]$. Intuitively, a correlated global demand shock for dollars, $\bar{n}_t^* = \int_0^1 n_{it}^* di > 0$, if not offset with FX interventions in a subset of countries results in $\bar{\psi}_t > 0$ and depresses the world dollar interest rate $i_t^* < 0$ in (23). Without taking the endogeneity of $i_t^*$ into account, this creates a wedge in the path of tradable consumption that is depressed in the constrained economies (due to $\psi_{it}$ shocks) and expands in unconstrained economies (due to lower $i_t^*$). A cooperative policy aims to close this gap by under-reacting to the $\psi_{it}$ shock in unconstrained economies to curb capital inflows, emphasizing the complementarity in the use of FX interventions across countries.

Fixing the exchange rate to the dollar $\sigma_{it}^2 = \text{var}_i(\Delta e_{it+1}) = 0$, if done by all countries $i \in (0, 1)$,
eliminate risk sharing wedges across countries by ensuring $\psi_{it} = \tilde{\psi}_t = 0$ for all $i \in (0, 1]$. The reason why the peg to the dollar has such an effect is not a particular form of currency market segmentation, but rather the assumption that the dollar is the international funding currency, i.e. the dollar bond is the internationally traded asset. This explains the central role of the bilateral exchange rates against the dollar and that pegging other bilateral or weighted exchange rates is suboptimal and can potentially exacerbate risk sharing wedges by increasing $\sigma_{it}^2$. Thus, taking as given the dominance of the dollar in international borrowing and lending (Maggiori, Neiman, and Schreger 2020), the model explains why most countries in the world — including the ones with weak trade linkages to the US — use the dollar as an anchor currency in their monetary and FX policies (Ilzetzki, Reinhart, and Rogoff 2019).

The divine coincidence of Proposition 2, however, may fail for two reasons. First, a peg to the dollar by an individual country ensures $\psi_{it} = 0$, but not $\psi_{it} = \tilde{\psi}_t$, and thus fails to eliminate capital flow externalities among $i \in (0, 1]$ when some countries are constrained and $\tilde{\psi}_t \neq 0$, as discussed above. In addition to this, there are highly asymmetric spillover effects of US monetary policy via tradable inflation $\pi^*_T$ that affects $\Delta e_{it}$ according to (21). This is inconsequential under the optimal policy that ensures $x_{it} = z_{it} = 0$ country-by-country, as $\Delta e_{it}$ simply accommodates shocks to $\pi^*_T$. In contrast, the peg to the dollar, or even a partial peg in (16), imports the US monetary policy stance and results in the output gap $x_{it}$ trailing US inflation $\pi^*_T$. For example, consider a tightening of US monetary policy that leads to an appreciation of the dollar ($\Delta e_{it} > 0$) and lowers the price of tradables ($\pi^*_T < 0$). By leaning against the wind and also raising interest rates, other countries partially stabilizes their exchange rates against the dollar at the expense of a negative output gap $x_{it} < 0$. Thus, despite a zero mass of the US economy, all countries import its monetary stance giving rise to the global monetary cycle (Rey 2013b, Egorov and Mukhin 2021).

**Proposition 6** Cooperation is not required when all countries follow unconstrained privately optimal monetary and FX policies. Under constrained policies, global dollar demand shocks and US monetary policy shocks result in international spillovers.

Lastly, we briefly comment on the optimal US policy and an alternative of a global gold standard. With constrained policies, an inward looking US policy fails to internalize the spillovers associated with global dollar demand shocks and the global monetary cycle induced by its monetary policy. While the latter requires some compromise between output gaps in the US and in the rest of the world, the former can be accommodated with a supply of dollar liquidity, e.g. in the form of currency swap lines between the Federal Reserve and monetary authorities in the rest of the world to eliminate wedges in international risk sharing. As an alternative, consider a global financial system dominated by gold. Equilibrium risk sharing conditions remain the same as before, except that the relevant source of risk $\sigma_{it}^2$ is now the volatility of exchange rates against gold. In this case, the gold price of tradables $P^*_T$, is pinned down by the market clearing condition for gold, which does not depend directly on any monetary policy, yet depends on the supply of gold reserves around the world. This leads to less asymmetric spillovers than under a US-centric financial markets, however at the cost of potentially greater volatility in $P^*_T$.

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45 Similarly to Hassan, Mertens, and Zhang (2021), the goal of the peg in our model is to eliminate the UIP deviation, but the anchor status of the dollar is due to the structure of financial markets, not the size of the US economy.
5 Robustness

The baseline model makes several stark assumptions to get a sharp characterization of the optimal policy. This section relaxes some of them — in particular, allowing for sluggish price adjustment, expenditure switching in tradables, and alternative structures of UIP deviations — to evaluate robustness of our main results. Detailed derivations are relegated to Appendix A4.

5.1 Staggered prices

The assumption of fully rigid prices in our baseline analysis provides emphasis to our main focus on the trade-off between output gap and international risk sharing, yet is admittedly very stark, and in particular removes domestic inflation as a policy consideration. We now generalize our results to an environment with staggered price adjustment. In particular, we assume that there is a continuum of varieties of non-tradable goods with an elasticity of substitution equal $\varepsilon > 1$ that are produced by monopolistic competitors. Firms are subject to a Calvo (1983) friction and update prices with probability $1 - \lambda$. We allow for markup shocks $\nu_t$ and assume that a constant production subsidy is used to eliminate the steady state markup wedge. The resulting planner’s problem is largely isomorphic to the baseline (15), but features both inflation $\pi_{Nt}$ and output gap $x_t$ in the objective function:

$$\min_{x_t, \pi_{Nt}, z_t, b_t^*, f_t^*, \sigma_t^2} \frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \gamma z_t^2 + (1 - \gamma)(x_t^2 + \alpha \pi_{Nt}^2) \right],$$

subject to

$$\mathbb{E}_t \Delta z_{t+1} = -\bar{\omega} \sigma_t^2 (ib_t^* - n_t^* - f_t^*),$$

$$\beta b_t^* = b_{t-1}^* - z_t,$$

$$\sigma_t^2 = \text{var}_t (\tilde{q}_{t+1} - x_{t+1} + \pi_{Nt+1}),$$

$$\pi_{Nt} = \kappa x_t + \beta \mathbb{E}_t \pi_{Nt+1} + \nu_t,$$

where $\alpha \equiv \varepsilon/\lambda$ is the relative weight on welfare losses from inflation and the set of constraints now additionally features a standard NKPC with $\kappa \equiv \frac{(1-\lambda)(1-\beta\lambda)}{\lambda}.$

The first thing to notice is that the results about the first-best policies remain largely unchanged. When two policy instruments are available, the FX interventions $f_t^* = ib_t^* - n_t^*$ eliminate the risk-sharing wedge and the interest rates implements the optimal path of inflation $\pi_{Nt}$ and output gap $x_t$, as in the closed economy, generalizing Proposition 1. Similarly, by adopting an exchange rate peg, monetary policy on its own can implement the optimal allocation with $z_t = x_t = \pi_{Nt} = 0$, if firms’ markups are constant, $\nu_t = 0$, and the natural real exchange rate is stable, $\tilde{q}_t = 0$. Hence, open economy divine coincidence requires that closed economy divine coincidence is satisfied, that is there is no conflict between output gap and inflation stabilization, and additionally that a fixed exchange rate does not interfere with efficient expenditure switching. This generalizes Proposition 2.

This isomorphism to the baseline model extends further and applies also to the second-best policies. To see this, notice that the only interaction between the two sectors comes from the nominal exchange rate via expenditure switching (2), which results in the $x_{t+1} + \pi_{Nt+1}$ term in the definition of $\sigma_t^2$ in the constraint set. This implies that the planner’s problem can be broken into two sequential steps: first,
solve for the optimal path \( \{ x_t, \pi_{Nt} \} \) given shocks to aggregate demand \( m_t \equiv x_t + \pi_{Nt} \), and second, solve for the optimal trade-off between risk sharing \( z_t \) and domestic conditions summarized by \( m_t \). The latter problem is the same as in the baseline model, except that the output losses \( x_t^2 \) are replaced with the overall welfare losses due to output gap and inflation from suboptimal monetary response to markup innovations. This implies that results about the second-best policies, including the optimal partial peg (16), extend to the setup with adjusting prices.\(^{46}\)

5.2 Terms of trade

Another important limitation of the baseline model are constant terms of trade and no expenditure switching in exports. Following the previous normative open-economy literature (Gali and Monacelli 2005, Devereux and Engel 2003, Benigno and Benigno 2003), this extension replaces tradables and non-tradables with a home good consumed locally \( C_{Ht} \) and exported abroad \( C_{Ht}^* \) and with an imported foreign good \( C_{Ft} \). We keep the assumption of log-linear preferences with \( C_t = C_{Ht}^{1-\gamma} C_{Ft}^\gamma \), linear technology, and CES demand for exports:

\[
A_t L_t = C_{Ht} + C_{Ht}^*, \quad C_{Ht}^* = \gamma P_{Ht}^* C_t^\gamma,
\]

where \( P_{Ht}^* \) is the export price in foreign currency, \( \varepsilon > 1 \) is the elasticity of foreign demand, and \( C_t^\gamma \) is the global demand shifter. For simplicity, all prices are fully sticky in the currency of invoicing. We assume that domestic prices are set in local currency and consider two alternatives for export prices: producer currency (PCP) with rich terms-of-trade dynamics and dollar pricing, which provides a better description of the current international price system (Gopinath, Boz, Casas, Diez, Gourinchas, and Plagborg-Møller 2020).

**Producer currency pricing**  When export prices are sticky in the currency of exporter, the monetary policy can generate expenditure switching in the market of destination and simultaneously close the output gap in domestic and export sectors. As a result, the loss function can be written in terms of the total output gap \( x_t \) and the deviations of imports from the optimal level \( z_t \):

\[
\min_{\{ x_t, z_t, b_t^*, f_t^* \}} \frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \kappa z_t^2 + x_t^2 \right]
\]

s.t. \( \mathbb{E}_t \Delta z_{t+1} = -\bar{\omega} \sigma_t^2 (b_t^* - n_t^* - f_t^*) \),

\[
\beta b_t^* = b_{t-1}^* + \frac{\varepsilon - 1}{\varepsilon} x_t - z_t,
\]

\[
\sigma_t^2 = \text{var}_t (\tilde{q}_{t+1} + x_{t+1} - (1 - \bar{\gamma}) z_{t+1}),
\]

where \( \kappa \equiv \frac{\varepsilon^2 \gamma}{\varepsilon - \gamma} \) and \( \bar{\gamma} \equiv \frac{2(\varepsilon - 1)}{\varepsilon - \gamma} \) is the steady-state share of exports in total output. The only substantial difference from the baseline problem (15) is that the monetary policy affects exports via expenditure

\[\text{46Interestingly, in contrast to the prescriptions of the standard New-Keynesian model (Gali 2008), the optimal Ramsey policy does not target the long-run price level, and shocks in both sectors have permanent effects on price levels.}\]
switching channel and therefore, $x_t$ appears in the country’s budget constraint with a multiplier that depends on the elasticity of substitution $\varepsilon$. This additional channel does not change the main results about the first-best policies. When two instruments are available, the planner can implement efficient allocation by closing the output gap $x_t = 0$ with interest rate policy and eliminating the risk-sharing wedge with the FX interventions $f^*_t = \tau b^*_t - n^*_t$. Moreover, the divine coincidence still holds when efficient real exchange rate is constant: by stabilizing the nominal exchange rate, monetary policy alone can close both wedges $x_t = z_t = 0$. The condition that $\tilde{q}_t = 0$ is satisfied when local productivity shocks move one-to-one with global demand shocks $a_t = c^*_t$ and both shocks follow a random walk.

Moving to the second-best policies, because of the effect of monetary policy on country’s exports, a nominal peg $\sigma^2_t = 0$ is no longer sufficient to implement $z_t = 0$. However, for any given path of $x_t$, it is still optimal to close the UIP deviations — either using the FX interventions or by stabilizing the nominal exchange rate. In particular, a partial peg remains optimal when FX interventions are not available. While the monetary policy can also stimulate exports to increase country’s imports, the effect is relatively weak because of consumption smoothing and is not very useful to offset financial shocks.

**Dominant currency pricing**  When export prices are sticky in foreign currency, the law of one price does not hold creating an additional gap in the planner’s problem:

$$
\begin{align*}
\min_{\{x_t, z_t, b^*_t, f^*_t, \sigma^2_t\}} \quad & \frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \gamma z^2_t + (1 - \gamma) x^2_t + \gamma (\varepsilon - 1) \tilde{q}^2_t \right] \\
\text{s.t.} \quad & \mathbb{E}_t \Delta z_{t+1} = -\bar{\omega} \sigma^2_t (ib^*_t - n^*_t - f^*_t), \\
& \beta b^*_t = b^*_{t-1} - (\varepsilon - 1) \tilde{q}_t - z_t, \\
& \sigma^2_t = \text{var}_t (\tilde{q}_{t+1} + x_{t+1} - z_{t+1}),
\end{align*}
$$

where $x_t$ is the output gap in domestic sector and $z_t$ is the deviation of consumption of foreign goods from the optimal level. Because the export prices do not respond to shocks, the deviations from the optimal exports, $(1 - \varepsilon)\tilde{q}_t$, fluctuate together with the optimal level of the real exchange rate. As a result, the exports are exogenous to monetary policy and neither interest rates nor FX interventions can close the output gap in the export sector (see Egorov and Mukhin 2021). Moreover, the suboptimal exports imply that it is impossible to achieve the efficient level of imports $z_t = 0$. Yet, when two policy instruments are available, the optimal targets are the same as in the baseline model: the monetary policy closes domestic output gap $x_t = 0$ and the FX interventions offset financial shocks $f^*_t = \tau b^*_t - n^*_t$.

Interestingly, the divine coincidence from the baseline model is still valid under DCP: if the real exchange rate is stable, the monetary policy alone can implement the first-best allocation. Indeed, if $\tilde{q}_t = 0$, then there is no need for export prices to adjust and exports are efficient. Pegging a nominal exchange rate encourages arbitrageurs and eliminates the risk-sharing wedge, while simultaneously closing the output gap. Away from this knife-edge case, a partial peg balances $x_t$ and $z_t$. 

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5.3 Financial shocks

While the analysis above focuses on noise trader shocks as the main source of volatility in financial markets, the previous literature suggests that other shocks may play an important role as well. To study robustness of the optimal policy, we augment the model with three additional financial shocks. The first one is the expectation error of arbitrageurs $\xi_t$ in the spirit of Gourinchas and Tornell (2004), which implies that the subjective beliefs are given by $\tilde{E}_t \Delta e_{t+1} = E_t \Delta e_{t+1} - \xi_t$. Second, following Brunnermeier, Nagel, and Pedersen (2009) and Gabaix and Maggiori (2015), we allow for risk-appetite shocks to $\omega_t$. Finally, assume that there is a time-varying probability of default modelled as a shock $\delta_t$ to the returns on the home currency bond. Because this latter shock applies for both households and arbitrageurs, it is absorbed by the equilibrium interest rate and does not directly affect UIP deviations. Instead, it creates an additional source of carry trade risk. Combining these pieces together, the new risk-sharing condition is:

$$E_t \Delta z_{t+1} = \xi_t - \bar{\omega}_t \sigma^2_t (\iota^* b_t - n^*_t - f^*_t), \quad \sigma^2_t = \text{var}_t (e_{t+1} + \delta_{t+1}).$$

The rest of the equilibrium system and the objective function remain the same as in the baseline model.

It follows that the first-best policy remains largely unchanged in the presence of additional shocks. In particular, it remains optimal to target UIP deviations with FX interventions and offset both demand shocks of noise traders and expectational errors of arbitrageurs, $f^*_t = \iota b^*_t - n^*_t - \xi_t / \bar{\omega}_t \sigma^2_t$, aiming to implement $E_t \Delta z_{t+1} = 0$. In contrast, the divine coincidence result holds with respect to the risk-appetite shocks $\omega_t$, but does not apply more generally as stabilizing the nominal exchange rate is no longer sufficient to eliminate the UIP wedge in the presence of $\xi_t$ and $\delta_t$ shocks. Nonetheless, an exchange rate peg still eliminates a part of the UIP wedge associated with the noise trader shocks.

6 Conclusion

This paper studies optimal exchange rate policy in an open economy with frictional goods and asset markets. In contrast to the previous normative literature, we use a framework that is consistent with the major exchange rate puzzles, including the change in macroeconomic dynamics after a switch from a peg to a float associated with the end of the Bretton-Woods system. The model is tractable and allows for an intuitive linear-quadratic approximation of the planner’s problem, yet rich enough to accommodate interesting policy trade-offs and multiple policy instruments.

We show that the constrained optimum can be implemented with monetary policy closing the output gap under sticky prices in goods markets and FX interventions targeting UIP deviations due to intermediary frictions in asset markets. In addition, when foreign agents participate in financial intermediation, the government can collect monopoly rents in home currency markets and the optimal mix of policy tools includes capital controls. The open-economy divine coincidence holds when the natural real exchange rate is constant and allows closing the two wedges with one monetary instrument by pegging the nominal exchange rate. More generally, when FX interventions are subject to additional constraints, the planner can use a crawling peg and/or FX forward guidance to mitigate fi-
nancial distortions. International cooperation is not required when countries follow the unconstrained privately optimal policies, but helps mitigate international spillovers from global liquidity shocks and US monetary shocks under constrained policies.
A Appendix

A1 Exact non-linear policy problem

As described in Sections 2.1–2.2, the Ramsey problem maximizes the household welfare in (1) over policies \( \{R_t, F_t^*\} \) and the equilibrium allocation \( \{C_{Nt}, C_{Tt}, B_t^*, \mathcal{E}_t, \sigma_t^2\} \), subject to the equilibrium system (2)–(3) and (6)–(7) (including the definition of \( \sigma_t^2 \)), given the stochastic path of exogenous variables \( \{A_t, Y_{Tt}, R_t^*, N_t^*\} \) and subject to initial and transversality conditions on \( B_t^* \):

\[
\max_{\{R_t, F_t^*, C_{Nt}, C_{Tt}, B_t^*, \mathcal{E}_t, \sigma_t^2\} \geq 0} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \gamma \log C_{Tt} + (1 - \gamma) \left( \log C_{Nt} - \frac{C_{Nt}}{A_t} \right) \right] \tag{A1}
\]

subject to

\[
\frac{B_t^*}{R_t} - B_{t-1} = Y_{Tt} - C_{Tt},
\]

\[
\beta R_t^* \mathbb{E}_t \frac{C_{Tt}}{C_{Tt+1}} = 1 + \omega \sigma_t^2 \frac{B_t^* - N_t^* - F_t^*}{R_t^*},
\]

\[
\beta R_t \mathbb{E}_t \frac{C_{Nt}}{C_{N_t+1}} = 1,
\]

\[
\mathcal{E}_t = \frac{\gamma}{1 - \gamma} \frac{C_{Nt}}{C_{Tt}}.
\]

\[
\sigma_t^2 = R_t^2 \cdot \text{var}_t \left( \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} \right),
\]

where we used the non-tradable production function and market clearing \( C_{Nt} = Y_t = A_t L_t \) to substitute for \( L_t \) in the welfare function.\(^{47}\)

**First best** The first-best allocation maximizes (A1) with respect to \( \{C_{Nt}, C_{Tt}, B_t^*+1\} \) and subject to the budget constraint only, removing the remaining four constraints. The optimality conditions for this problem imply \( \tilde{C}_{Nt} = A_t \) and \( \{\tilde{C}_{Tt}, \tilde{B}_{t+1}^*\} \) such that:

\[
\beta R_t^* \mathbb{E}_t \frac{C_{Tt}}{C_{Tt+1}} = 1
\]

and the budget constraint holds, \( B_t^*/R_t - B_{t-1} = Y_{Tt} - C_{Tt} \).

When the two policy instruments — monetary policy and FXI, \( \{R_t, F_t^*\} \) — are available and unconstrained, the first best allocation is feasible. This is because the two constraints on the policy problem (A1) — namely, the two Euler equations (with \( R_t \) and \( R_t^* \), respectively), with the last two constraints being static side equations defining \( \mathcal{E}_t \) and \( \sigma_t^2 \) — each feature an independent policy instrument which can ensure that the respective constraint is relaxed.

\(^{47}\)Note that one can alternatively rewrite the problem in terms of wages \( W_t \), as in an equilibrium with sticky prices \( P_{Nt} = 1 \) the labor supply condition implies \( W_t = P_{Nt} C_{Nt} = C_{Nt} \), and monetary policy by controlling aggregate nominal expenditure \( P_t C_t \), controls also the path of nominal wages, \( W_t = P_{Nt} C_{Nt} = (1 - \gamma) P_t C_t \).
Specifically, decentralizing the first-best allocation requires a path of \( \{R_t, F_t^c\} \) such that:

\[
\beta R_t \mathbb{E}_t \frac{A_t}{A_{t+1}} = 1, \\
\omega \sigma_t^2 \frac{B_t^c - N_t^c - F_t^c}{R_t^c} = 0
\]

and the implies path of the nominal exchange rate given by \( \hat{E}_t = \frac{\gamma}{1-\gamma} \frac{A_t}{C_{T_t}} \). The two displayed equations characterize the necessary path of policy outcomes \( \tilde{R}_t \) and \( \tilde{F}_t^c \), leaving aside the conventional issue of uniqueness of the decentralized equilibrium (see Atkeson, Chari, and Kehoe 2010). Therefore, the first-best monetary policy eliminates the output gap, that is, ensures optimality conditions of uniqueness of the decentralized equilibrium (see Proposition 1), while the first-best financial market policy ensures a zero risk-sharing wedge. This happens when either \( \omega \sigma_t^2 = 0 \), or when \( F_t^c = B_t^c - N_t^c \); the latter corresponds to the case of Proposition 1, while the former to divine coincidence of Proposition 2 (or trilemma models with \( \omega = 0 \)).

The first-best path of NFA according to the budget constraint is \( \tilde{B}_t^c = R_t^c (\tilde{B}_{t-1}^c + Y_{T_t} - \tilde{C}_{T_t}) \), and hence the optimal FXI is \( \tilde{F}_t^c = \tilde{B}_t^c - \tilde{N}_t^c \) when \( \sigma_t^2 = \tilde{\sigma}_t^2 = \tilde{R}_t^2 \cdot \text{var}(\tilde{E}_t/\tilde{E}_{t+1}) \neq 0 \).

**Optimality conditions** We make the following substitution of variables:

\[
\Gamma_t \equiv \frac{1}{C_{N_t}}, \quad \beta R_t \mathbb{E}_t \frac{\Gamma_{t+1}}{\Gamma_t} = 1, \quad \mathcal{E}_t = \frac{\gamma}{1-\gamma} \frac{1}{\Gamma_t C_{T_t}}.
\]

(A2)

where the last two conditions are implied by the constraints in (A1). We can thus recover the path of \( \{R_t, C_{N_t}, \mathcal{E}_t\} \) from the path of \( \{\Gamma_t, C_{T_t}\} \). As a result, the original policy problem (A1) is equivalent to:

\[
\max_{\{\Gamma_t, C_{T_t}, B_t^c, \sigma_t^2\}_{t \geq 0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \gamma \log C_{T_t} - (1-\gamma) \left( \log \Gamma_t + \frac{1}{A_t \Gamma_t} \right) \right]
\]

(A1′)

subject to \( \frac{B_t^c}{R_t^c} - B_{t-1}^c = Y_{T_t} - C_{T_t} \),

\[
\beta R_t^c \mathbb{E}_t \frac{C_{T_t}}{C_{T_{t+1}}} = 1 + \omega \sigma_t^2 \frac{B_t^c - N_t^c - F_t^c}{R_t^c},
\]

\[
\sigma_t^2 \beta^2 C_{T_t}^2 (\mathbb{E}_t \Gamma_{t+1})^2 = \mathbb{E}_t (\Gamma_{t+1} C_{T_{t+1}})^2 - (\mathbb{E}_t \Gamma_{t+1} C_{T_{t+1}})^2,
\]

where we used (A2) to solve out \( \{R_t, \mathcal{E}_t\} \) from the definition of \( \sigma_t^2 \):

\[
\sigma_t^2 = \frac{\mathbb{E}_t \left[ \frac{\Gamma_{t+1}}{\Gamma_t} \right]^2 \cdot \left( \Gamma_t C_{T_t} \right)^2 - \mathbb{E}_t \left( \Gamma_{t+1} C_{T_{t+1}} \right)^2 - (\mathbb{E}_t \Gamma_{t+1} C_{T_{t+1}})^2}{\text{var}(\Gamma_{t+1} C_{T_{t+1}})}
\]

resulting in the final constraint in (A1′). Note that we characterize the planner’s optimality conditions for an arbitrary path of \( \{F_t^c\} \), and then discuss the optimal path of \( \{F_t^c\} \).

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48 Divine coincidence, of course, requires that \( \hat{E}_t = \frac{\gamma}{1-\gamma} \frac{A_t}{C_{T_t}} = \text{const} \); otherwise, at least one of the two wedges cannot be eliminated — either \( \sigma_t^2 \neq 0 \) and hence \( C_{T_t} \neq \tilde{C}_{T_t} \) (for an arbitrary path of \( F_t^c \neq \tilde{F}_t^c \)), or \( C_{N_t} \neq \tilde{C}_{N_t} = A_t \) under the peg (with \( \sigma_t^2 = 0 \) that ensures \( C_{T_t} = \tilde{C}_{T_t} \) for any path of \( F_t^c \)).
We write the Lagrangian for \((A1')\) as follows:

\[
L_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \gamma \log C_{Tt} - (1 - \gamma) \left( \log \Gamma_t + \frac{1}{A_t \Gamma_t} \right) + \lambda_t \left( B_{t-1}^* + Y_{Tt} - C_{Tt} - \frac{B_t^*}{R_t^*} \right) + M_t \left( 1 + \omega \sigma_t^2 \frac{B_t^* - N_t^* - F_t^*}{R_t^*} - \beta R_t^* E_t C_{Tt+1} \right) + D_t \left( \beta^2 \sigma_t^2 C_{Tt}^2 (E_t \Gamma_{t+1})^2 - E_t (\Gamma_{t+1} C_{Tt+1})^2 + (E_t \Gamma_{t+1} C_{Tt+1})^2 \right) \right],
\]

where \(\{\lambda_t, M_t, D_t\}\) is the sequence of Lagrange multipliers on the respective constraints. The first order conditions with respect to \(\{\Gamma_t, C_{Tt}, B_t^*, \sigma_t^2\}\) are as follows:\(^{49}\)

\[
0 = -(1 - \gamma) \frac{1}{\Gamma_t} + (1 - \gamma) \frac{1}{A_t \Gamma_t^2} + 2 \beta^{-1} D_{t-1} C_{Tt} \left( \beta^2 \sigma_{t-1}^2 C_{Tt-1} \frac{C_{Tt-1}}{C_{Tt}^2} \right) + \lambda_t C_{Tt} - \frac{1}{C_{Tt+1}} + M_{t-1} R_{t-1} C_{Tt-1} - 2 \beta D_{t-1} \Gamma_{t-1} (\Gamma_{t-1} C_{Tt-1})^2 - 2 \beta^{-1} D_{t-1} \Gamma_{t-1} (\Gamma_{t-1} C_{Tt-1}),
\]

\[
0 = \frac{\gamma}{C_{Tt}} - \lambda_t - M_t R_t^* E_t \frac{1}{C_{Tt+1}} + M_{t-1} R_{t-1} C_{Tt-1} - 2 \beta D_{t-1} \Gamma_{t-1} (\Gamma_{t-1} C_{Tt-1})^2 - 2 \beta^{-1} D_{t-1} \Gamma_{t-1} (\Gamma_{t-1} C_{Tt-1}),
\]

\[
0 = -\lambda_t + \beta E_t \lambda_{t+1} + M_t \omega \frac{\sigma_t^2}{R_t^*},
\]

\[
0 = M_t \omega B_t^* - N_t^* - F_t^* + D_t \beta^2 C_{Tt}^2 (E_t \Gamma_{t+1})^2.
\]

where we define \(D_{-1} = M_{-1} = 0\), and we use the fact that operator \(E_t \{\cdot\}\) sums across future realizations of uncertainty using conditional probabilities \(\pi(h^{t+1})/\pi(h^t)\) for any history of exogenous states \(h^{t+1} \equiv \{A_s, Y_{T^s}, R_s\}_{s=0}^{t+1}\).

We simplify the conditions as follows. The last two conditions allow to relate the Lagrange multipliers \(\lambda_t\) and \(D_t\) with \(M_t\):

\[
\lambda_t - \beta R_t^* E_t \lambda_{t+1} = M_t \omega \sigma_t^2, \tag{A3}
\]

\[
D'_t = M_t \omega (E_t R_t)^2 \frac{N_t^* + F_t^* - B_t^*}{R_t^*}, \tag{A4}
\]

where we used definitions (A2) in the second line and substituted \(D_t' \equiv (\frac{\gamma}{1 - \gamma})^2 D_t\). Next we simplify the first optimality condition by substituting out \(\sigma_{t-1}^2\) using its definition (the third constraint of the problem):

\[
\beta (1 - \gamma) (X_t - 1) = 2 D'_{t-1} \left[ \frac{1}{E_t^2} - \frac{E_t^{-1} E_t^{-1}}{E_t} - \frac{\Gamma_t}{E_t^{-1} \Gamma_t} \left( E_t^{-1} E_t^{-1} - (E_t^{-1} E_t^{-1})^2 \right) \right], \tag{A5}
\]

where we defined \(X_t \equiv C_{Nt}/A_t = 1/(A_t \Gamma_t)\) so that \(X_t - 1\) corresponds to the output gap. Note that (A5) implies \(E_{t-1} X_t = 1\), as the conditional expectations of the right-hand side is zero. The final optimality condition:

\[^{49}\text{Note that with an optimal unconstrained choice of } F_t^* \text{ at } t, \text{ we additionally have that } M_t = 0, \text{ and therefore } D_t = 0, \text{ } C_{Nt+1} = 1/\Gamma_{t+1} = A_{t+1}, \text{ } \lambda_t = \beta R_t^* E_t \lambda_{t+1} \text{ and } \beta R_t^* E_t [C_{Tt}/C_{Tt+1}] = 1, \text{ consistent with conditions for the best allocation.}\]
\[
\gamma \left(1 - \frac{A_t}{\gamma / C_{Tt}}\right) - (1 - \gamma)(X_t - 1) = M_t \beta R^*_t E_t \frac{C_{Tt}}{C_{Tt+1}} - M_{t-1} R^*_{t-1} \frac{C_{Tt-1}}{C_{Tt}} - \frac{2D'_t \sigma^2_t}{(\varepsilon_t R_t)^2} + \frac{2D'_{t-1} \sigma^2_{t-1}}{E_{t-1} R_{t-1}} \Gamma_t - \frac{\Gamma_t}{T_{t-1}} \right) \tag{A6}
\]

where we used \(D^*_t = B^*_t - N^*_t - F^*_t\).

Conditions (A3)–(A6) together with definitions in (A2) and constraints in (A1') characterize the optimal monetary policy for a given path of FXI \(\{F^*_t\}\) and the associated allocation.

## A2 Approximations

### A2.1 Second-order approximation to the objective function

Consider any allocation \(\{C_{N_t}, C_{Tt}, L_t, B^*_t\}\) that satisfies production possibilities frontier for non-tradables, \(C_{N_t} = A_t L_t\), and the country budget constraint (6):

\[
\frac{B^*_t}{R^*_t} = B^*_{t-1} + Y_{Tt} - C_{Tt},
\]

for a given \(B^*_{t-1}\) and a transversality condition on \(B^*_\infty\), and corresponding to a stochastic path of shocks \(\{A_t, Y_{Tt}, R^*_t\}\). We refer to all such allocation as resource- and budget-feasible. The first best allocation corresponding to the same path of shocks is denoted with \(\{\bar{C}_{Nt}, \bar{C}_{Tt}, \bar{L}_t, \bar{B}^*_t\}\), it is also resource- and budget-feasible, and satisfies the following optimality conditions (see Appendix A1):

\[
\bar{C}_{Nt} = A_t, \quad \bar{L}_t = 1, \quad \beta R^*_t E_t \frac{\bar{C}_{Tt}}{\bar{C}_{Tt+1}} = 1.
\]

A non-stochastic zero-NFA steady state corresponding to \((\bar{A}, \bar{Y}_T, \bar{R}^*)\) such that \(\bar{R}^* = 1/\beta\), is given by \((\bar{C}_N, \bar{C}_T, \bar{L}, \bar{B}^*)\):

\[
\bar{C}_N = \bar{A}, \quad \bar{L} = 1, \quad \bar{C}_T = \bar{Y}_T, \quad \bar{B}^* = 0,
\]

which implies \(\bar{N}\bar{X} = \bar{Y}_T - \bar{C}_T = 0\) and the steady state budget constraint is satisfied. Finally, the welfare function is given by (1).

**Lemma A1** The second order Taylor expansion around a zero-NFA steady state \((\bar{C}_N, \bar{C}_T, \bar{L})\) of the welfare loss for any budget- and resource-feasible allocation \(\{C_{Nt}, C_{Tt}, L_t\}\) relative to the first-best allocation \(\{\bar{C}_{Nt}, \bar{C}_{Tt}, \bar{L}_t\}\) is given by:

\[
\frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \gamma z^2_t + (1 - \gamma) z^2_t \right],
\]

where \(z_t = \log(C_{Tt}/\bar{C}_{Tt})\) and \(z_t = \log(C_{Nt}/\bar{C}_{Nt})\). Therefore, it is sufficient to know the first-order dynamics of the two wedges \(\{x_t, z_t\}\) to evaluate the second-order welfare loss.

**Proof:** We take a second order Taylor expansions of (1) for any resource- and budget-feasible allocation \(\{C_{Nt}, C_{Tt}, L_t, B_t; A_t, Y_{Tt}, R^*_t\}\) around a zero-NFA steady state \((\bar{C}_N, \bar{C}_T, \bar{L}, \bar{B}^*) = (\bar{A}, \bar{Y}_T, 1, 0)\),
using log deviations:

\[ c_{Nt} = \log(C_{Nt}/C_N), \quad c_{Tt} = \log(C_{Tt}/C_T), \quad a_t = \log(A_t/A), \quad y_{Tt} = \log(Y_{Tt}/Y_T), \quad r_t^* = \log(R_t^*/R^*), \]

and for NFA we use a proportional deviation relative to \( Y_T \):

\[ \tilde{b}_t^* = B_t^*/\bar{Y}_T. \]

Note that the first best deviation and the wedge for non-tradables are \( \tilde{c}_{Nt} = \log(\tilde{C}_{Nt}/\bar{A}) = \log(A_t/\bar{A}) = a_t \) and \( x_t = \log(C_{Nt}/\tilde{C}_{Nt}) = c_{Nt} - \tilde{c}_{Nt} = c_{Nt} - a_t \). We also use the fact that for any resource-feasible allocation \( L_t = C_{Nt}/A_t \), and hence we solve out \( L_t \) from the welfare function.

We, therefore, can rewrite the welfare function (1) in terms of deviations as:

\[ W_0 = E_0 \sum_{t=0}^{\infty} \beta_t \left[ \gamma \log \bar{Y}_T + (1-\gamma)(\log \bar{A} - 1) + \gamma c_{Tt} + (1-\gamma) \left[ \log c_{Nt} - (e^{c_{Nt}-a_t} - 1) \right] \right], \]

as well as the flow budget constraint (6) as:

\[ \tilde{b}_{t-1}^* + e^{y_{Tt}} - e^{c_{Tt}} - \beta e^{-r_t^*} \tilde{b}_t^* = 0, \]

using the fact that \( \bar{C}_T = \bar{Y}_T \) and \( \bar{R}^* = 1/\beta \). We characterize the welfare loss in two steps:

1. The second-order Taylor expansion for the non-tradable terms in \( W_0 \) is:

\[ E_0 \left[ c_{Nt} - (e^{c_{Nt} - a_t} - 1) \right] = E_0 \left[ c_{Nt} - \left( \underbrace{c_{Nt} - a_t}_{=a_t} \right) - \frac{1}{2} \left( \underbrace{c_{Nt} - a_t}_{=x_t} \right)^2 \right] + h.o.t. \]

and in the first best allocation \( x_t = 0 \) as \( \tilde{c}_{Nt} = a_t \).

2. The second-order Taylor expansion to the flow budget constraint is:

\[ 0 = \tilde{b}_{t-1}^* + y_{Tt} + \frac{1}{2} y_{Tt}^2 - c_{Tt} - \frac{1}{2} c_{Tt}^2 - \beta \tilde{b}_t^* + \beta r_t^* \tilde{b}_t^* + h.o.t., \]

which we use to express:

\[ E_0 \sum_{t=0}^{\infty} \beta^t c_{Tt} = \tilde{b}_{-1}^* + E_0 \sum_{t=0}^{\infty} \beta^t \left[ y_{Tt} + \frac{1}{2} y_{Tt}^2 - \frac{1}{2} c_{Tt}^2 + \beta r_t^* \tilde{b}_t^* \right] + h.o.t., \]

using the transversality condition for NFA deviations, \( \lim_{j \to \infty} \beta^j \tilde{b}_{t+j} = 0 \). Evaluating relative
to the first-best allocation, we have:

\[
\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \gamma c_{T_t} - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \gamma c_{\bar{T}_t} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \gamma \left[ \frac{1}{2} c_{T_t}^2 - \frac{1}{2} \bar{c}_{T_t}^2 - \beta r_t^* b_t^* \right] + \text{h.o.t.}
\]

\[
= \frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \gamma z_t^2 + \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \gamma \left[ (c_{T_t} - \bar{c}_{T_t}) \bar{c}_{T_t} - \beta r_t^* b_t^* \right] + \text{h.o.t.},
\]

where we expanded \( c_{T_t} = z_t + \bar{c}_{T_t} \) and denoted with \( b_t^* = \bar{b}_t^* - \hat{b}_t^* = (B_t^* - \bar{B}_t^*)/\bar{Y}_T \) the proportional deviation of the NFA position from the first best NFA. Finally, we show that:

\[
0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \gamma [(c_{T_t} - \bar{c}_{T_t}) \bar{c}_{T_t} - \beta r_t^* b_t^*] + \text{h.o.t.}
\]

\[
= \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [(b_t^* - \beta b_t^*) \bar{c}_{T_t} - \beta r_t^* b_t^*] + \text{h.o.t.}
\]

\[
= b_{t-1}^* \cdot \bar{c}_{T_0} + \beta \sum_{t=0}^{\infty} \beta^t \mathbb{E}_0 \left[ b_t^* (\Delta \bar{c}_{T+1} - r_t^*) \right] + \text{h.o.t.}
\]

The second line uses the expansion of the flow budget constraint for \( c_{T_t} \) and \( \bar{c}_{T_t} \), which implies:

\[
(c_{T_t} - \bar{c}_{T_t}) \bar{c}_{T_t} = (b_{t-1}^* + \frac{1}{2} c_{T_t}^2 - \frac{1}{2} \bar{c}_{T_t}^2 - \beta b_t^* + \beta r_t^* b_t^* + \text{h.o.t.}) \bar{c}_{T_t} = (b_{t-1}^* - \beta b_t^*) \bar{c}_{T_t} + \text{h.o.t.}
\]

The third lines uses the fact that \( b_{t-1}^* = \bar{b}_{t-1}^* - \hat{b}_{t-1}^* = 0 \) by the initial condition, and the optimality condition (Euler equation) for the first-best consumption growth, which we rewrite in log deviations as \( e^{r_t^*} \mathbb{E}_t e^{-\Delta \bar{c}_{T+1}} = 1 \), and take the following second-order Taylor expansion:

\[
\mathbb{E}_t \Delta \bar{c}_{T+1} - r_t^* = \frac{1}{2} (r_t^*)^2 + \frac{1}{2} \mathbb{E}_t (\Delta \bar{c}_{T+1})^2 - r_t^* \mathbb{E}_t \Delta \bar{c}_{T+1} + \text{h.o.t.}
\]

and therefore using the law of iterated expectations:

\[
\mathbb{E}_0 \left[ b_t^* (\Delta \bar{c}_{T+1} - r_t^*) \right] = \mathbb{E}_0 \left[ b_t^* (\mathbb{E}_t \Delta \bar{c}_{T+1} - r_t^*) \right] = 0 + \text{h.o.t.}
\]

Combining these results, we evaluate the welfare loss relative to the first-best allocation to be given by:

\[
\bar{W}_0 - W_0 = \frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \gamma z_t^2 + (1 - \gamma) x_t^2.
\]

A2.2 First-order approximation to the equilibrium system

Non-linear equilibrium system (2)–(3) and (6)–(7), and non-linear wedges:

\[
X_t = C_{Nt}/\bar{C}_{Nt} = C_{Nt}/A_t \quad \text{and} \quad Z_t = C_{Tt}/\bar{C}_{Tt}.
\]

Steady state given by:

\[
\bar{B}^* = \bar{F}^* = \bar{N}^* = 0, \quad \bar{R} = \bar{R}^* = 1/\beta, \quad \bar{C}_T = \bar{Y}_T, \quad \bar{C}_N = \bar{A}.
\]
and the associated exchange rates:
\[ \bar{\varepsilon} = \bar{Q} = \frac{\gamma \bar{C}_N}{1 - \gamma \bar{C}_T}, \]
as well as no steady state wedges, \( \bar{X} = \bar{Z} = 1 \).

**Deviations** Define for any (endogenous or exogenous) variable \( Y_t \) with a non-zero steady state value its log steady-state deviation \( y_t \) as:
\[ Y_t = \bar{Y} e^{\nu_y t} \quad \text{for} \quad \nu = 1, \]
and for net foreign assets \( B_t^* \) with a zero steady state value its deviation proportional to tradable steady state output \( b_t^* \) as:
\[ B_t^* = \bar{Y}_T \nu b_t^* \quad \text{for} \quad \nu = 1. \]

For \( \nu = 0 \), we get the steady state values of variables. We take the first order Taylor expansion of the equilibrium system in \( \nu \) around steady state \( \nu = 0 \) and evaluated at \( \nu = 1 \). The approximate system is linear (scales) in \( \nu \), but is not necessarily linear in variables (deviations \( y_t \)), as we see below.

**First best** allocation \( \{ \bar{C}_{Nt}, \bar{C}_{Tt}, \bar{B}_{t}^*, \bar{Q}_t \} \) solves \( \bar{C}_{Nt} = A_t \) and:
\[ \bar{Q}_t = \frac{\gamma \bar{C}_N}{1 - \gamma \bar{C}_T}, \]
\[ \frac{\bar{B}_{t}^* - \bar{B}_{t-1}^*}{R_t} = Y_{Tt} - \bar{C}_{Tt}, \]
\[ \beta R_t^* \bar{E}_t \frac{\bar{C}_{Tt}}{\bar{C}_{Tt+1}} = 1. \]

The first order Taylor expansion in \( \nu \) to this system is given by \( \tilde{c}_{Nt} = a_t \) and:
\[ \tilde{q}_t = a_t - \bar{c}_{Tt}, \]
\[ \beta \tilde{b}_{t}^* - \bar{b}_{t-1}^* = y_{Tt} - \bar{c}_{Tt}, \]
\[ \bar{E}_t \Delta \tilde{c}_{Tt+1} = r_t^*, \]
where \( \{ a_t, y_{Tt}, r_t^* \} \) are stochastic shocks (in proportional deviations) determining the dynamics of the first-best allocation.

**Proof:** Substitute the definitions of variables in terms of \( \nu \)-deviations into the non-linear system describing the first-best allocation
\[ \tilde{Q}_e^{\nu_{\tilde{q}_t}} = \frac{\gamma \bar{C}_N}{1 - \gamma \bar{C}_T} e^{\nu_\left(\tilde{c}_{Nt} - \bar{c}_{Tt}\right)}, \]
\[ \beta e^{-\nu_{r_t^*}} \bar{Y}_T \nu \tilde{b}_{t}^* - \bar{Y}_T \nu \bar{b}_{t-1}^* = \bar{Y}_T e^{\nu y_{Tt}} - \bar{C}_T e^{\nu \bar{c}_{Tt}}, \]
\[ e^{\nu_{r_t^*}} \bar{E}_t e^{-\nu \Delta \tilde{c}_{Tt+1}} = 1, \]
where we used the fact that $\bar{R}^* = 1/\beta$. Using the steady state value of $\bar{Q}$, and the fact that $\bar{c}_{Nt} = a_t$ (as $\bar{C}_{Nt} = A_t$), the first equation is immediately log-linear, $\bar{q}_t = a_t - \bar{c}_{Tt}$. Dividing the second equation by $\bar{Y}_T$, using the fact that $\bar{C}_{T} = \bar{Y}_T$, and taking the Taylor expansion, we have:

$$(1 - \nu r_t^* + O(\nu^2))\nu/\bar{b}_t^\prime - \nu \bar{b}_t^\prime = \nu \bar{y}_{Tt} - \nu \bar{c}_{Tt} + O(\nu^2),$$

where $O(\nu^2)$ denotes terms of order $\nu^2$ or higher (around $\nu = 0$). Dividing by $\nu$ and eliminating remaining $O(\nu)$ terms results in the first-order approximate equation. The final equation is expanded as follows:

$$1 = \mathbb{E}_t\{(1 + \nu r_t^* + O(\nu^2))(1 - \nu \Delta \bar{c}_{Tt+1} + O(\nu^2))\} = \mathbb{E}_t\{1 + \nu r_t^* - \nu \Delta \bar{c}_{Tt+1} + O(\nu^2)\}. $$

Subtracting 1 on both sides, dividing through by $\nu$, and eliminating the remaining $O(\nu)$ terms results in the first-order approximate equation. ■

**Equilibrium system**  (2) and (6)–(7) is reproduced here as:

$$\mathcal{E}_t = \frac{\gamma}{1 - \gamma} \frac{C_{Nt}}{C_{Tt}},$$

$$\frac{B_t^*}{R_t^*} - B_{t-1}^* = Y_{Tt} - C_{Tt},$$

$$\beta R_t^* \mathbb{E}_t \frac{C_{Tt}}{C_{Tt+1}} = 1 + \omega \sigma_t^2 \frac{B_t^* - N_t^* - F_t^*}{R_t^*}, \quad \sigma_t^2 = R_t^2 \cdot \text{var}_t\left(\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right).$$

Rewrite this system in deviations from the first-best system:

$$\mathcal{E}_t = \bar{Q}_t \frac{X_t}{Z_t},$$

$$\frac{B_t^* - \bar{B}_t^*}{R_t^*} - (B_{t-1}^* - \bar{B}_{t-1}^*) = -(C_{Tt} - \bar{C}_{Tt}),$$

$$\beta R_t^* \mathbb{E}_t \frac{Z_t \bar{C}_{Tt}}{Z_{t+1} \bar{C}_{Tt+1}} = \beta R_t^* \mathbb{E}_t \frac{\bar{C}_{Tt}}{\bar{C}_{Tt+1}} = \omega \sigma_t^2 \frac{B_t^* - N_t^* - F_t^*}{R_t^*}, \quad \sigma_t^2 = R_t^2 \cdot \text{var}_t\left(\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right).$$

Define the following additional proportional deviation terms:

$$B_t^* - \bar{B}_t^* = Y_T \nu b_t^\prime, \quad N_t^* - \bar{N}_t^* = Y_T \nu n_t^\prime, \quad F_t^* = Y_T \nu f_t^\prime$$

and

$$\omega = \omega_0/\nu^2,$$

for some $\omega_0 \geq 0$, so that $\omega$ is the value of risk-aversion at $\nu = 1$, and the value of risk-aversion increases as $\nu$ decreases towards 0 (the value of risk-aversion in steady state is irrelevant given the exact absence of risk). Given this definitions, the first-order Taylor approximation in $\nu$ to the non-linear equilibrium
system around \( \nu = 0 \) is given by:

\[
e_t = \tilde{q}_t + x_t - z_t,
\]

\[
\beta b_t^* - b_{t-1}^* = -z_t,
\]

\[
\mathbb{E}_t \Delta z_{t+1} = \tilde{\omega} \bar{\sigma}_t^2 (n_t^* + f_t^* - b_t^*), \quad \bar{\sigma}_t^2 = \text{var}_t(\Delta e_{t+1}).
\]

where \( \tilde{\omega} = \omega_0 \tilde{Y}_T / \beta. \)

**Proof:** Following similar steps as above, we substitute the definitions of variables in terms of deviations. The first line immediately results in \( e_t = \tilde{q}_t + x_t - z_t \), as the non-linear equation is, in fact, log linear in variables. The second equation yields \( \beta b_t^* - b_{t-1}^* = -z_t \) following similar steps as above, and additionally noting that

\[
\frac{C_{Tt} - \tilde{C}_{Tt}}{\bar{Y}_T} = e^{\nu c_{Tt}} - e^{\nu \tilde{c}_{Tt}} = \nu z_t + O(\nu^2),
\]

where \( z_t = c_{Tt} - \tilde{c}_{Tt} \) by the definition of variables. Finally, the last equilibrium condition is expressed as follows:

\[
e^{\nu r_t} \mathbb{E}_t \left[ e^{-\nu (\Delta z_{t+1} + \Delta \tilde{c}_{Tt+1})} - e^{-\nu \Delta \tilde{c}_{Tt+1}} \right] = (\omega_0 \tilde{Y}_T / \beta) \frac{1}{\nu^2} e^{\nu (2r_t - r_t^*)} \text{var}_t(e^{-\nu \Delta e_{t+1}}) \nu (b_t^* - n_t^* - f_t^*),
\]

after substituting in the expression for \( \bar{\sigma}_t^2 \) and using \( \tilde{R} = \tilde{R}^* = 1 / \beta \) to simplify. Dividing both sides by \( e^{\nu r_t} \), substituting in \( \tilde{\omega} = \omega_0 \tilde{Y}_T / \beta \), and taking a first-order Taylor expansion in \( \nu \) yields:

\[
\mathbb{E}_t \left[ -\nu \Delta z_{t+1} + O(\nu^2) \right] = \tilde{\omega}(1 + 2\nu(r_t - r_t^*) + O(\nu^2)) \frac{1}{\nu^2} \text{var}_t(1 - \nu \Delta e_{t+1} + O(\nu^2)) \nu (b_t^* - n_t^* - f_t^*),
\]

Dividing both sides by \( \nu \) and simplifying:

\[
-\mathbb{E}_t \Delta z_{t+1} + O(\nu) = \tilde{\omega}(1 + O(\nu))(\text{var}_t(\Delta e_{t+1}) + O(\nu^2))(b_t^* - n_t^* - f_t^*)
\]

Eliminating the remaining \( O(\nu) \) terms yields the first-order approximate equation in \( \nu \), which is however not linear in variables (deviations). \( \blacksquare \)

We also note that the side equation (3) that determines the path of \( R_t \) is approximated in the same way as the other equations of the equilibrium system:

\[
r_t = \mathbb{E}_t \Delta c_{Nt+1} = \mathbb{E}_t \Delta x_{t+1} + \mathbb{E}_t \Delta a_{t+1}.
\]

Combining the equation for \( r_t \) with \( \mathbb{E}_t \Delta \tilde{c}_{Tt+1} = r_t^* \) results in the UIP deviation expression (14):

\[
r_t - r_t^* - \mathbb{E}_t \Delta e_{t+1} = \mathbb{E}_t \Delta x_{t+1} + \mathbb{E}_t \Delta a_{t+1} - \mathbb{E}_t \Delta \tilde{c}_{Tt+1} - \mathbb{E}_t \Delta e_{t+1}
\]

\[
= \mathbb{E}_t \{ \Delta x_{t+1} + \Delta \tilde{q}_{t+1} - \Delta e_{t+1} \} = \mathbb{E}_t \Delta z_{t+1},
\]

where we used the facts that \( e_t = \tilde{q}_t + x_t - z_t \) and \( \tilde{q}_t = a_t - \tilde{c}_{Tt} \). Note: since in our baseline model
there is no inflation, \( P_{Tt}^* = P_{Nt} = 1 \), we have that \( i_t = r_t \) and \( i_t^* = r_t^* \), i.e. nominal and real interest rates coincide.

**Lemma A2** The solution to the approximate equilibrium system characterizes an \( O(\nu) \) accurate dynamics of the non-linear equilibrium system. Furthermore, \( \omega \sigma_t^2 - \dot{\omega}' \tilde{\sigma}_t^2 = O(\nu) \), where \( \dot{\omega}' = \beta^3 \dot{\omega}/\bar{Y}_T \).

**Proof:** We first formalize the claim. Consider an exact equilibrium path \( \{C_{Nt}, C_{Tt}, \xi_t, B_t^*, \sigma_t^2\} \) that corresponds to policies \( \{X_t, F_t^*\} \) and shocks \( \{A_t, Y_{Tt}, R_t^*, N_t^*\} \), which also determine the first-best allocation \( \{\hat{C}_{Nt}, \hat{C}_{Tt}, \hat{B}_t^*, \hat{Q}_t\} \). Note that \( X_t = C_{Nt}/\hat{C}_{Nt} \) and \( Z_t = C_{Tt}/\hat{C}_{Tt} \). Define the exact deviations from the steady state \( \{\hat{x}_t, \hat{f}_t^*, \hat{z}_t, \hat{b}_t^*, \hat{n}_t^*, \hat{q}_t\} \) as:

\[
X_t = e^{\nu \hat{z}_t}, \quad F_t^* = \bar{Y}_T \nu \hat{f}_t^*, \quad Z_t = e^{\nu \hat{z}_t}, \quad \xi_t = \bar{E} e^{\nu \hat{z}_t}, \quad B_t^* - \bar{B}_t^* = \bar{Y}_T \nu \hat{b}_t^*, \quad N_t^* - \bar{B}_t^* = \bar{Y}_T \nu \hat{n}_t^* \quad \text{and} \quad \hat{Q}_t = \bar{Q} e^{\nu \hat{q}_t} \quad \text{for} \quad \nu = 1.
\]

We define similarly \( \{\bar{c}_{Tt}, \hat{r}_t, \hat{r}_t^*, \hat{y}_{Tt}, \hat{\alpha}_t\} \).

Consider now an approximate equilibrium path \( \{z_t, e_t, b_t^*, \bar{\sigma}_t^2\} \) that emerges as a result of policies \( \{\hat{x}_t, \hat{f}_t^*\} \) in response to shocks \( \{\hat{n}_t^*, \hat{q}_t\} \). Then:

\[
\nu \hat{e}_t = \nu (\hat{q}_t + \hat{x}_t - \hat{z}_t),
\]

\[
\beta e^{\nu \hat{z}_t} \nu \hat{b}_t^* - \nu \hat{b}_{t-1}^* = - (e^{\nu (\hat{x}_t+\hat{b}_t^*)} - e^{\nu \hat{b}_t^*}),
\]

\[
E_t e^{-\nu (\Delta \hat{z}_{t+1} + \Delta \hat{b}_{Tt+1})} - E_t e^{-\nu \Delta \hat{b}_{Tt+1}} = \frac{\omega_0 \bar{Y}_T / \beta}{\nu^2} e^{2\nu (\hat{r}_t-\hat{n}_t^*)} \text{var}_t (e^{-\nu \Delta \hat{e}_{t+1}}) (\hat{b}_t^* - \hat{n}_t^* - \hat{f}_t^*).
\]

The first-order Taylor expansion of this exact system is:

\[
\hat{e}_t = \hat{q}_t + \hat{x}_t - \hat{z}_t,
\]

\[
\beta \hat{b}_t^* - \bar{b}_{t-1}^* = - \hat{z}_t + O(\nu),
\]

\[
E_t \Delta \hat{z}_{t+1} = \bar{\omega} \text{var}_t (\Delta \hat{e}_{t+1}) (\hat{n}_t^* + \hat{f}_t^* - \hat{b}_t^*) + O(\nu),
\]

while the approximate system for \( \{z_t, e_t, b_t^*\} \) is:

\[
e_t = \hat{q}_t + \hat{x}_t - z_t,
\]

\[
\beta b_t^* - b_{t-1}^* = - z_t,
\]

\[
E_t \Delta z_{t+1} = \bar{\omega} \text{var}_t (\Delta e_{t+1}) (\hat{n}_t^* + \hat{f}_t^* - \hat{b}_t^*),
\]

Therefore, the difference between the exact solution \( \{\hat{z}_t, \hat{e}_t, \hat{b}_t^*\} \) and the approximate solution \( \{z_t, e_t, b_t^*\} \)
where we define $\omega_1 = 0$. Furthermore:

$$\omega_1 = \frac{\beta^2 \omega_0}{\nu^2} e^{\nu \varphi_{t-1} \left( e^{-\nu \Delta e_{t+1}} \right) = \beta^2 \omega_0 \varphi_{t-1} \left( \Delta e_{t+1} \right) + O(\nu) = \beta^2 \omega_0 \delta^2 + O(\nu),$$

and the last equality holds because $\varphi_{t} \left( \Delta e_{t+1} \right) - \varphi_{t} \left( \Delta e_{t+1} \right) = O(\nu^2)$ as $\{\hat{e}_t\} - \{e_t\} = O(\nu)$.

### A2.3 Optimal policies

Set up a Lagrangian for the policy problem (15) for any given path of $\{f_t^*\}$:

$$\ell_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{1}{2} \left( \gamma z_t^2 + (1 - \gamma) x_t^2 \right) - \gamma \lambda_t \left( b_{t-1} - z_t - \beta b_t^* \right) - \gamma \mu_t \left( \mathbb{E}_t \Delta z_{t+1} - \xi \sigma_t^2 (n_t^* + f_t^* - b_t^*) \right) - \delta_t \left( \sigma_t^2 - \mathbb{E}_t (\tilde{q}_{t+1} + x_{t+1} - z_{t+1})^2 + (\mathbb{E}_t (\tilde{q}_{t+1} + x_{t+1} - z_{t+1}))^2 \right) \right],$$

where we substituted in the expression for $e_t = \tilde{q}_t + x_t - z_t$ and replaced $\sigma_t^2 = \varphi_{t} \left( \Delta e_{t+1} \right) = \mathbb{E}_t e_{t+1}^2 - (\mathbb{E}_t e_{t+1})^2$. Note the analogy with the non-linear Lagrangian $L_0$ in Appendix A1; the fact that we have negatives in front of the constraints in the approximate problem reflects the fact that we are minimizing welfare loss, in contrast to maximizing welfare in the exact problem.

The optimality conditions with respect to $\{x_t, z_t, b_t^*, \sigma_t^2\}$ are:

$$0 = (1 - \gamma) x_t + 2 \beta^{-1} \delta_{t-1} (e_t - \mathbb{E}_{t-1} e_t),$$

$$0 = \gamma z_t + \gamma \lambda_t + (\gamma \mu_t - \beta^{-1} \gamma \mu_{t-1}) - 2 \beta^{-1} \delta_{t-1} (e_t - \mathbb{E}_{t-1} e_t),$$

$$0 = \beta (\gamma \lambda_t - \mathbb{E}_t \gamma \lambda_{t+1}) - \gamma \mu_t \omega \sigma_t^2,$$

$$0 = \gamma \mu_t \omega (n_t^* + f_t^* - b_t^*) - \delta_t.$$

where we define $\delta_{-1} = \mu_{-1} = 0$. After simplification:

$$\beta (1 - \gamma) x_t = -2 \delta_{t-1} (e_t - \mathbb{E}_{t-1} e_t),$$

$$\gamma z_t + (1 - \gamma) x_t = -\gamma \lambda_t - \gamma (\mu_t - \beta^{-1} \mu_{t-1}),$$

$$\lambda_t - \mathbb{E}_t \lambda_{t+1} = \beta^{-1} \mu_t \omega \sigma_t^2,$$

$$\delta_t = \gamma \mu_t \omega (n_t^* + f_t^* - b_t^*).$$

These optimality conditions, together with the constraints in the policy problem (A7) characterize the optimal monetary policy $\{x_t\}$ for a given path of FXI $\{f_t^*\}$ and the associated equilibrium allocation.

**Lemma A3** The optimality conditions for the approximate policy problem (A7) correspond to the first-order Taylor expansion (in $\nu$ around $\nu = 0$) of the non-linear optimality conditions for the exact policy problem (A1).

**Proof:** Given Lemma A2, it remains to show that the first-order Taylor expansion of the exact optimality conditions (A3)–(A6) results in the same system of equations as the first order conditions to the
approximate problem (A7) given above.

In addition to the definitions of $\nu$-deviations of variables in Appendix A2, we define the deviations for multipliers $\{M_t, \Lambda_t, D_t\}$ in the Lagrangian $\mathcal{L}_0$ for the exact policy problem (A1'):

$$\Lambda_t = \tilde{\Lambda} e^{\nu \lambda_t}, \quad M_t = \tilde{M} \nu \mu_t, \quad D_t' = \tilde{D}' \delta_t,$$

where $\tilde{\Lambda} = \gamma / \bar{C}_T = \gamma / \bar{Y}_T$, $\tilde{M} = \gamma$, and $\tilde{D}' = \tilde{E}^2$ are proportional scalers. Note that $M_t$ and $D'_t$ are equal to zero in a zero-NFA steady state; furthermore, there is only a zero-order component of $\beta$ which can be verified by generalizing $\omega$ and thereby showing that $d_t \equiv 0$ using our approximation below.\footnote{Note from the solution that $\delta_{t-1}$ is the slope of the policy rule, $\beta(1 - \gamma) x_t = -2 \delta_{t-1} (e_t - \bar{E}_{t-1} e_t)$ and, just like $\omega \tilde{\sigma}_t^2$, it does not scale with $\nu$, while other deviations (in particular, those of $X_t$ and $E_t$) scale proportionally with $\nu$. In other words, the risk premium and the slope of the optimal policy are zero order in $\nu$.}

Consider first the expansion of (A3)–(A6):

$$\Lambda_t \left( e^{\nu \lambda_t} - e^{\nu r^*_t} E_t e^{\nu \lambda_{t+1}} \right) = \tilde{M} \nu \mu_t \frac{\omega_0}{\nu^2} \beta^{-2} e^{2 \nu r_t} \var(t(e^{-\nu \Delta e}) \right),$$

$$\tilde{D}' \delta_t = \tilde{M} \nu \mu_t \frac{\omega_0 / \nu^2}{\nu^2} e^{2 \nu(r_t + \nu \nu^2)} - \nu r^*_t \bar{Y}_T \nu(n^*_t + f^*_t - b^*_t),$$

$$\beta(1 - \gamma)(e^{\nu \lambda_t} - 1) = 2 \tilde{D}' e^{\nu \lambda_t} \frac{1}{\nu^2} \delta_{t-1} \left[ e^{-2 \nu r_t} - e^{-\nu r^*_t} E_t e^{-\nu r^*_t} - e^{-\nu r^*_t} \bar{E}_{t-1} e^{-\nu r^*_t} \left( \bar{E}_{t-1} e^{-2 \nu r_t} - (\bar{E}_{t-1} e^{-\nu r^*_t})^2 \right) \right],$$

$$\gamma(1 - e^{\nu(\lambda_t + \nu \lambda_t)}) - (1 - \gamma)(e^{\nu \lambda_t} - 1) = \tilde{M} \nu \mu_t \left( e^{\nu r^*_t} E_t e^{-\nu r^*_t} \bar{Y}_{t+1} + 2 \omega \sigma_t^2 \beta e^{-\nu r^*_t} \nu(n^*_t + f^*_t - b^*_t) \right)$$

$$- \beta^{-1} \tilde{M} \nu \mu_t \left( e^{\nu r^*_t} e^{-\nu r^*_t} \bar{Y}_{t+1} + 2 \omega \sigma_t^2 \beta e^{-\nu r^*_t} \nu(n^*_t + f^*_t - b^*_t) \right),$$

where we used $\omega = \omega_0 / \nu$ and $\Gamma_t = 1 / C_{Nt} = e^{-\nu r^*_t} / \tilde{\Lambda}$. We take a first order Taylor expansion in $\nu$ around $\nu = 0$:

$$\nu \lambda_t - \nu r^*_t - \tilde{E}_t \nu \lambda_{t+1} + O(\nu^2) = \beta^{-1} \tilde{M} \nu \mu_t (1 + 2 \nu r_t + O(\nu^2)) (\nu r_{t+1} + O(\nu^2)),$$

$$\delta_t = \gamma \mu_t \nu (1 + 2 \nu r_t + O(\nu^2)) (n^*_t + f^*_t - b^*_t),$$

$$\beta(1 - \gamma) x_t = 2 \delta_{t-1} \left[ - \nu (e_t - \bar{E}_{t-1} e_t) + O(\nu^2) - (1 + O(\nu)) O(\nu^2) \right],$$

$$-\gamma \nu (\lambda_t + \nu \lambda_t) + O(\nu) - (1 - \gamma) \nu x_t = \gamma \mu_t \nu (1 + O(\nu)) - \beta^{-1} \gamma \nu \mu_{t-1} (1 + O(\nu)),$$

where we used the definitions of $(\tilde{\Lambda}, \tilde{M}, \tilde{D}')$ and $\bar{\omega} = \omega \tilde{Y}_T / \beta$, and the result in Lemma A2 that $\omega \sigma_t^2 - \bar{\omega} \sigma_t^2 = O(\nu)$ and $\nu \bar{\omega} \sigma_t^2 = O(\nu)$. Dividing all equations (except for the second line) by $\nu$ and grouping together the remaining higher order terms, we obtain:

$$\tilde{\lambda}_t - \tilde{E}_t \lambda_{t+1} = \beta^{-1} \mu_t \tilde{\omega} \sigma_t^2 + O(\nu),$$

$$\tilde{\delta}_t = \gamma \mu_t \tilde{\omega} (n^*_t + f^*_t - b^*_t) + O(\nu),$$

$$\beta(1 - \gamma) x_t = -2 \delta_{t-1} (e_t - \bar{E}_{t-1} e_t) + O(\nu),$$

$$\gamma x_t + (1 - \gamma) x_t = -\gamma \tilde{\lambda}_t - (\gamma \mu_t - \beta^{-1} \gamma \mu_{t-1}) + O(\nu),$$

where $\tilde{\sigma}_t^2 = \var(t(\Delta e_{t+1})$, and we used the optimality condition for the first best tradable consumption,
$r_t^* = E_t \Delta \tilde{c}_{T+1}$, the definition of $z_t = c_{T+1} - \tilde{c}_{T+1}$, and additionally denoted with $\tilde{\lambda}_t = \lambda_t + \tilde{c}_{T+1}$. Dropping the higher order terms $O(\nu)$, this system corresponds to the optimality conditions of the approximate problem. ■

A3 Derivations and Proofs for Section 3

Proof of Theorem 1 Consider the optimality conditions for the approximate policy problem (A7) derived in Appendix A2.3. In particular, the first and the last optimality conditions (with respect to $x_t$ and $\sigma_t^2$) are given by:

$$
\beta(1 - \gamma)x_{t+1} = -2\delta_t(e_t - E_{t-1}e_t),
\delta_t = \gamma \mu_t \tilde{\omega}(n_t^* + f_t^* - b_t^*).
$$

Thus, $\delta_t$ is the optimal monetary policy lean against exchange rate surprises at $t + 1$ and $\mu_t$ is the Lagrange multiplier on $E_t \Delta z_{t+1} = \tilde{\omega} \sigma_t^2(n_t^* + f_t^* - b_t^*)$. If $E_t \Delta z_{t+1} = 0$ in every period, then $\delta_t = \mu_t = 0$, as the policy problem (A7) is effectively unconstrained and $x_t = z_t = 0$ is feasible.

Combining the two conditions to solve out $\delta_t$ yields:

$$
\beta(1 - \gamma)x_{t+1} = -2\gamma \mu_t \tilde{\omega}(n_t^* + f_t^* - b_t^*)(e_{t+1} - E_t e_{t+1}),
$$

which, in particular, implies $E_t x_{t+1} = 0$. ■

Proof of Proposition 1 Consider the approximate policy problem (A7) where the choice of FXI $f_t^*$ is unconstrained, and therefore we have an additional optimality condition:

$$
\mu_t = 0 \quad \text{for all } t.
$$

From the other optimality conditions, we have $x_t = \delta_t = 0$ for all $t \geq 0$, as well as:

$$
E_t \Delta \lambda_{t+1} = -E_t \Delta z_{t+1} = 0,
$$

which together with the budget constraint implies $z_t = b_t^* = 0$ for all $t$ as the unique solution. Consequently, $e_t = \tilde{q}_t$, and $\sigma_t^2 = \text{var}_t(\Delta \tilde{q}_{t+1})$. Finally, $E_t \Delta z_{t+1} = 0$ requires:

$$
\tilde{\omega} \sigma_t(n_t^* + f_t^* - b_t^*) = 0,
$$

and thus generically FXI must satisfy:

$$
f_t^* = b_t^* - n_t^* = -n_t^*.
$$

Note that $f_t^* = -n_t^*$ guarantees $z_t = b_t^* = 0$ as the unique equilibrium, as the non-linear system:

$$
E_t \Delta z_{t+1} = -\tilde{\omega} \sigma_t^2 b_t^*, \quad \sigma_t^2 = \text{var}_t(\tilde{q}_{t+1} - z_{t+1}),
\beta b_t^* - b_{t-1}^* = -z_t
$$
has a unique stable solution $z_t = b_t^* = 0$.

Lastly, consider the discretionary solution with the planner choosing the optimal policy as a function of natural state variables $(b_{t-1}^*, \tilde{q}_t, n_t^*)$. This implies that private agents form their expectations about future policies $z_{t+1} = z(b_{t+1}^*, \tilde{q}_{t+1}, n_{t+1}^*)$ and $x_{t+1} = x(b_{t+1}^*, \tilde{q}_{t+1}, n_{t+1}^*)$. The only way the planner can credibly manipulate the beliefs in period $t$ in the absence of commitment is by changing the future state $b_t^*$. The resulting policy problem corresponds to finding the Markov perfect equilibrium:

$$
V(b^*, \tilde{q}, n^*) = \min_{\{x, z, f, b^*, \sigma^2\}} \frac{1}{2} [\gamma z^2 + (1 - \gamma) x^2] + \beta \mathbb{E} [V(b^*, \tilde{q}', n') | \tilde{q}, n^*] \tag{A8}
$$

subject to $\beta b^* = b^* - z$,

$$
\mathbb{E} [z(b^*, \tilde{q}', n') | \tilde{q}, n^*] = z + \omega \sigma^2 (n^* + f^* - b^*),
\sigma^2 = \text{var}\left(\tilde{q}' + x(b^*, \tilde{q}', n') - z(b^*, \tilde{q}', n') | \tilde{q}, n^*\right),
$$

where functions $x(\cdot)$ and $z(\cdot)$ should be consistent with the solution to this policy problem. Following the primal approach, observe that given a free choice of $f^*$, the latter two constraints do not bind. It follows that the $x(b^*, \tilde{q}, n^*) = 0$ and problem reduces to

$$
V(b^*, \tilde{q}, n^*) = \min_{b^*} \frac{1}{2} (b^* - \beta b^*)^2 + \beta \mathbb{E} V(b^*, \tilde{q}', n').
$$

A combination of the first-order and envelope conditions implies that $z(b^*, \tilde{q}, n^*) = (1 - \beta)b^*$. It follows from $b_{t-1}^* = 0$ that $z_t = b_t^* = 0$ and the discretionary policy implements the same allocation as the optimal policy under commitment. This allocation is also supported by the same prices, FXI and monetary policy.

**Proof of Proposition 2** Consider the case with $\tilde{q}_t \equiv \tilde{q} = 0$ for all $t$, the latter equality without loss of generality given our notation in terms of deviations. Then the equilibrium system becomes:

$$
e_t = x_t - z_t,
\beta b_t^* - b_{t-1}^* = -z_t,
\mathbb{E}_t \Delta z_{t+1} = \bar{\omega} \sigma_t^2 (n_t^* + f_t^* - b_t^*),
\sigma_t^2 = \text{var}_t(\Delta e_{t+1}),
$$

and it is consistent with a $x_t = z_t = b_t^* = \sigma_t^2 = 0$ equilibrium for all $t$ independently of $\{n_t^*, f_t^*\}$. Since this corresponds to the first best, achieving the global minimum of the welfare loss objective, it is the solution to the optimal policy problem (A7). Indeed, with $\lambda_t = \mu_t = \delta_t = 0$ for all $t$, all optimality conditions of (A7) are satisfied, and in particular (16) in Theorem 1 holds irrespective of $\{n_t^*, f_t^*\}$.

In general, when $\tilde{q}_t = 0$ and $x_t = 0$, there exist other equilibria with $\sigma_t^2 > 0$, such that:

$$
\beta b_t^* - b_{t-1}^* = -z_t,
\mathbb{E}_t \Delta z_{t+1} = \bar{\omega} \sigma_t^2 (n_t^* + f_t^* - b_t^*),
\sigma_t^2 = \text{var}_t(z_{t+1}),
$$

and thus output gap targeting, $x_t = 0$, does not guarantee $z_t = b_t^* = 0$ as the unique equilibrium.
In contrast, a policy rule that targets the exchange rate, \( x_t = -\delta (e_t - E_{t-1}e_t) \) with \( \delta \to \infty \), ensures \( x_t = z_t = b_t^* = \sigma_t^2 = 0 \) as the only equilibrium outcome. Indeed, such a rule implies, using \( e_t = x_t - z_t \), that \( x_t = -\delta (z_t - E_{t-1}z_t) = - (z_t - E_{t-1}z_t) \) as \( \delta \to \infty \). This, in turn, means \( e_t = E_{t-1}e_t \) and \( \sigma_t^2 = 0 \), which ensures \( z_t = b_t^* = 0 \), and hence \( x_t = 0 \), irrespectively of \( \{n_t^*, f_t^*\} \). ■

**Proof of Proposition 3** Without commitment, the planner solves problem (A8) with an additional constraint that the path of \( f_t^* \) is exogenous. Given the values of state variables \( (b^*, \tilde{q}, n^*) \), there are three endogenous variables \( (b^*, z, \sigma^2) \) in the three constraints of the problem. It follows that the choice of \( x \) affects the nominal exchange rate \( e = \tilde{q} + x - z \), but not capital flows \( n^* + f^* - b^* \) or UIP deviations \( \mathbb{E}[z(b^*, \tilde{q}, n^*)|\tilde{q}, n^*] - z \). Neither does it change the risk-sharing wedge \( z \) or the continuation value \( V(b^*, \tilde{q}^*, n^*) \), which implies that it is optimal for the monetary policy to set \( x = 0 \). ■

**Proof of Proposition 4** A further unconstrained choice of \( f_t^* \) at any \( t \geq 0 \) additionally results in \( \mu_t = 0 \), which further implies \( \delta_t = 0 \), \( x_t+1 = 0 \), and \( E_t\lambda_{t+1} = \lambda_t \).

Let \( \mu_t := \beta^{-1} \mu_t \) for this proof.

Let \( f_t^* \) be chosen optimal at all \( t \geq 1 \), but fixed at \( f_0^* \) at \( t = 0 \). [This generalizes to any \( t \geq 0 \).]

We then have:

\[
\forall t \geq 1 \Rightarrow x_t = 0 \quad \forall t \neq 1, \quad E_0x_1 = 0 \quad \text{and} \quad (1 - \gamma)x_1 = 2\mu_0 \tilde{\omega}(n_0^* + f_0^* - b_0^*)\tilde{e}_1,
\]

as well as \( E_t\Delta\lambda_{t+1} = 0 \) for all \( t \geq 1 \) and \( \beta b_0^* = -z_0 \). Furthermore, \( \gamma z_t = \lambda_t \) for all \( t \geq 2 \) (since \( x_t = \mu_t = 0 \) for all \( t \geq 1 \)), and therefore:

\[
\forall t \geq 2 : \quad \gamma E_t\Delta z_{t+1} = E_t\Delta \lambda_{t+1} = 0 \quad \Rightarrow \quad f_t^* = b_t^* - n_t^*.
\]

We use \( \gamma z_t + (1 - \gamma)x_t = \lambda_t + \beta \mu_t - \mu_{t-1} \) for \( t = 0, 1 \) together with \( E_0\Delta \lambda_1 = -\tilde{\omega}\sigma_0^2 \mu_0 \) and \( E_0 \Delta z_1 = \tilde{\omega} \sigma_0^2 (n_0^* + f_0^* - b_0^*) \) to solve for:

\[
\mu_0 = -\frac{\gamma \tilde{\omega} \sigma_0^2}{1 + \beta + \tilde{\omega} \sigma_0^2} (n_0^* + f_0^* - b_0^*), \tag{A9}
\]

\[
x_1 = -\frac{2\gamma}{1 - \gamma} \frac{\tilde{\omega}^2 \sigma_0^2}{1 + \beta + \tilde{\omega} \sigma_0^2} (n_0^* + f_0^* - b_0^*)^2 (e_1 - E_0 e_1), \tag{A10}
\]

\[
e_1 - E_0 = \frac{(\tilde{q}_1 - z_1) - E_0 (\tilde{q}_1 - z_1)}{1 + \frac{2\gamma}{1 - \gamma} \frac{\tilde{\omega}^2 \sigma_0^2}{1 + \beta + \tilde{\omega} \sigma_0^2} (n_0^* + f_0^* - b_0^*)^2}, \tag{A11}
\]

where \( \sigma_0^2 = \text{var}_0(e_1) \) and we used \( e_1 - E_0 e_1 = x_1 + (\tilde{q}_1 - z_1) - E_0 (\tilde{q}_1 - z_1) \). ■

To complete characterization, also use \( \gamma z_t + (1 - \gamma)x_t = \lambda_t + \beta \mu_t - \mu_{t-1} \) in difference at \( t = 2 \):

\[
\gamma \Delta z_2 = (1 - \gamma)x_1 + \mu_0 = -\frac{\gamma \tilde{\omega} \sigma_0^2}{1 + \beta + \tilde{\omega} \sigma_0^2} (n_0^* + f_0^* - b_0^*) [1 + 2\tilde{\omega}(n_0^* + f_0^* - b_0^*)(e_1 - E_0 e_1)]
\]

because \( \Delta \lambda_2 = E_1 \Delta \lambda_2 = 0 \) as there is no uncertainty in \( (x_t, z_t, b_t^*) \) after \( t = 1 \).
We solve for $f_t^*$ from:

$$\hat{\sigma}^2_1(n_t^* + f_t^* - b_t^*) = E_t \Delta z_t = \Delta z_t = -\frac{\hat{\omega}^2_0}{1 + \hat{\beta} + \hat{\omega}^2_0} (n_t^* + f_t^* - b_t^*) [1 + 2\hat{\omega}(n_t^* + f_t^* - b_t^*)(e_1 - E_0e_1)],$$

where $\sigma^2 = \text{var}_1(e_2) = \text{var}_1(\hat{q}_2) = \sigma^2_0$.

Note that $\beta b_t^* = b_0^* - z_1 = -\beta^{-1} z_0 - z_1$, and then $\beta b_t^* = b_{t-1}^* - z_2$ for $t \geq 2$.

Finally, we close by solving for $(z_0, z_1, z_2)$ from the intertemporal budget constraint

$$z_0 + \hat{\beta} z_1 + \frac{\hat{\beta}^2}{1 - \hat{\beta}} z_2 = 0, \quad z_t = z_2 \quad \forall t \geq 2,$$

and given solution for $\Delta z_2$ and $E_0 \Delta z_1 = \hat{\omega}^2_0(n_0^* + f_0^* - b_0^*)$, and hence we have:

$$b_0^* = -\beta^{-1} z_0 = E_0[\Delta z_1 + \beta \Delta z_2] = \frac{1 + \hat{\omega}^2_0}{1 + \hat{\beta} + \hat{\omega}^2_0} \hat{\omega}^2_0 (n_0^* + f_0^* - b_0^*) \Rightarrow b_0^* = \frac{(1 + \hat{\omega}^2_0)\hat{\sigma}^2_0}{\beta + (1 + \hat{\omega}^2_0)^2} (n_0^* + f_0^*)$$

and

$$n_0^* + f_0^* - b_0^* = \frac{1 + \hat{\beta} + \hat{\omega}^2_0}{\beta + (1 + \hat{\omega}^2_0)^2} (n_0^* + f_0^*).$$

Then we solve:

$$z_1 = (1 - \hat{\beta}) b_0^* - \beta \Delta z_2 = \frac{\hat{\omega}^2_0 n_0^* + f_0^*}{\beta + (1 + \hat{\omega}^2_0)^2} \left[ (1 - \hat{\beta})(1 + \hat{\omega}^2_0) + \beta \left( 1 + \frac{\hat{\omega}^2_0}{\beta + (1 + \hat{\omega}^2_0)^2} (n_0^* + f_0^* e_1) \right) \right],$$

where $\tilde{e}_1 = e_1 - E_0e_1$, so that:

$$z_1 - E_0z_1 = \frac{2\hat{\beta} \hat{\omega}_0^2 (1 + \hat{\beta} + \hat{\omega}_0^2)}{[\beta + (1 + \hat{\omega}_0^2)^2]^2} (n_0^* + f_0^*)^2 \tilde{e}_1,$n_0^* + f_0^*)^2 \tilde{e}_1,$

$$x_1 = -\frac{2\hat{\omega}^2_0 (1 + \hat{\beta} + \hat{\omega}^2_0)}{1 - \hat{\gamma}} \frac{\hat{\omega}^2_0 (1 + \hat{\beta} + \hat{\omega}^2_0)}{[\beta + (1 + \hat{\omega}^2_0)^2]^2} (n_0^* + f_0^*)^2 \tilde{e}_1,$n_0^* + f_0^*)^2 \tilde{e}_1,$

$$\tilde{e}_1 = \frac{\hat{q}_1 - E_0 \hat{q}_1}{1 + [\gamma + \beta(1 - \gamma)][2 \hat{\omega}^2_0 (1 + \beta + \hat{\omega}^2_0)](1 - \gamma)[\beta + (1 + \hat{\omega}^2_0)^2]^2 (n_0^* + f_0^*)^2}$$

so that $\sigma^2_0$ solves the fixed point:

$$\sigma^2_0 = \left( 1 + [\gamma + \beta(1 - \gamma)] \frac{2\hat{\omega}^2_0 (1 + \hat{\beta} + \hat{\omega}^2_0)}{(1 - \gamma)[\beta + (1 + \hat{\omega}^2_0)^2]^2} (n_0^* + f_0^*)^2 \right)^{-2} \sigma^2_0$$

Optimal local FXI Consider now the case when $f_t^*$ is unconstrained only over some connected interval of time $T = \{t, \ldots, t + k\}$ for $k = 0, 1, \ldots$, and follows some exogenous path otherwise. From FOCs:

$$\forall t \in T : \quad x_{t+1} = \mu_t = \delta_t = E_t \Delta \lambda_{t+1} = 0,$n_0^* + f_0^*)^2 \tilde{e}_1,$

$$\forall t \in T : \quad E_t \Delta z_{t+1} = \frac{1 - \gamma}{\gamma} x_t - (\mu_{t+1} + \beta^{-1} \mu_{t-1}),$$

$$\forall \{t - 1, t\} \in T : \quad E_t \Delta z_{t+1} = -\mu_{t+1},$$

$$\forall \{t - 1, t, t + 1\} \in T : \quad E_t \Delta z_{t+1} = 0 \Rightarrow f_t^* = b_t^* - n_t^*. $$
That is, $\mu_{t-1} = \mu_t = \mu_{t+1} = 0$ is enough for $E_t \Delta z_{t+1} = 0$, and unconstrained FXI at $t - 1$ and $t + 1$ breaks the link of $t$ to past and to future, and hence makes the static choice of $UIP_t = 0$ optimal.

### Derivations for Section 3.3

Iterate forward the risk-sharing condition (13):

$$z_t = E_t \sum_{j=0}^{\infty} \tilde{\omega} \sigma^2_{t+j} n^*_t + E_t z_\infty$$

and note that it also implies:

$$E_t z_{t+i} = z_t + E_t \sum_{j=0}^i \tilde{\omega} \sigma^2_{t+j} n^*_t.$$

Combine this expression with the intertemporal budget constraint $b^*_{t-1} = E_t \sum_{i=0}^{\infty} \beta^i z_{t+i}$ to get

$$b^*_{t-1} = E_t \sum_{i=0}^{\infty} \beta^i \left[ z_t + \sum_{j=0}^i \tilde{\omega} \sigma^2_{t+j} n^*_t \right] = \frac{z_t}{1-\beta} + \tilde{\omega} E_t \sum_{j=0}^{\infty} \beta^j \sigma^2_{t+j} n^*_t,$$

where the last equality uses the fact that shocks are i.i.d. and symmetric around zero and a conjecture that $\sigma^2$ depends on $n^*_t$, but not the sign of $n^*_t$ verified below. It follows

$$z_t = (1-\beta)b^*_{t-1} - \tilde{\omega} \sigma^2_t n^*_t = (1-\beta)b^*_{t-1} + u_t,$$

where shocks $u_t \equiv -\tilde{\omega} \sigma^2_t n^*_t$ are mean-zero and uncorrelated across periods, but are not i.i.d. because of the state-dependent volatility.

Theorem 1 implies that without loss of generality, the optimal policy can be parametrized as

$$x_{t+1} = -\delta_t \left[ \hat{q}_{t+1} - z_{t+1} - E_t (\hat{q}_{t+1} - z_{t+1}) \right].$$

It follows that the conditional exchange rate volatility is given by

$$\sigma^2_t = \text{var}_t(\hat{q}_{t+1} + x_{t+1} - z_{t+1}) = (1-\delta_t)^2 \text{var}_t(\hat{q}_{t+1} - z_{t+1}) = (1-\delta_t)^2 [\sigma^2_q + \sigma^2_{ut}],$$

where $\sigma^2_q \equiv \text{var}(\hat{q}_{t+1}), \sigma^2_{ut} \equiv E_t u^2_{t+1}$ and we used the fact that dynamics of $z_t$ is orthogonal to $\hat{q}_t$.

Moving next to the objective function, notice that $E_{t-1} x_t = 0$ under the optimal policy and

$$E_{t-1} x^2_t = \delta^2_{t-1} \text{var}_{t-1}(\hat{q}_t - z_t) = \delta^2_{t-1} \left[ \sigma^2_q + \sigma^2_{ut-1} \right] = \left( \frac{\delta_{t-1}}{1-\delta_{t-1}} \right)^2 \sigma^2_{t-1}. $$

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It follows from the law of iterated expectation that the total costs of suboptimal output are given by

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t x_t^2 = \mathbb{E} \sum_{t=1}^{\infty} \beta^t \delta^2_{t-1} [\sigma^2_q + \sigma^2_{u_{t-1}}] = \beta \mathbb{E} \sum_{t=0}^{\infty} \beta^t \delta^2_t [\sigma^2_q + \sigma^2_{u_t}],$$

where we used the fact that the optimal policy sets $x_0 = 0$ given no trade-off in the initial period.

To compute the losses from the risk-sharing wedge, note that because $u_t$ are uncorrelated in time, $b_t$ follows a random walk

$$\beta b^*_t = b^*_{t-1} - z_t = b^*_{t-1} - (1 - \beta) b^*_{t-1} - u_t = \beta b^*_{t-1} - u_t$$

and hence, $z_t$ follows ARIMA(0,1,1) with a moving average root equal $1/\beta$:

$$\Delta z_t = -\frac{1 - \beta}{\beta} u_{t-1} + \Delta u_t = u_t - \frac{1}{\beta} u_{t-1}.$$ 

Use the Yule-Walker equations to compute unconditional volatility of $z_t$ without imposing stationarity as the volatility is time dependent:

$$\mathbb{E} z^2_t = \mathbb{E} z_t \left( z_{t-1} + u_t - \frac{1}{\beta} u_{t-1} \right) = \mathbb{E} z_{t-1} + \mathbb{E} u^2_t + \frac{1 - \beta}{\beta^2} \mathbb{E} u^2_{t-1},$$

$$\mathbb{E} z_{t-1} z_t = \mathbb{E} z_{t-1} \left( z_{t-1} + u_t - \frac{1}{\beta} u_{t-1} \right) = \mathbb{E} z^2_{t-1} - \frac{1}{\beta} \mathbb{E} u^2_{t-1}.$$ 

Combine these equations to get a recurrent equation

$$\mathbb{E} z^2_t = \mathbb{E} z^2_{t-1} + \kappa \mathbb{E} u^2_{t-1} + \mathbb{E} u^2_t,$$

where $\kappa \equiv \frac{1 - 2\beta}{\beta^2}$. The welfare losses from the risk-sharing wedge are then given by

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t z^2_t = \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[ z^2_{t-1} + \kappa u^2_{t-1} + u^2_t \right] = \beta \mathbb{E} \sum_{t=0}^{\infty} \beta^t z^2_t + (1 + \beta \kappa) \mathbb{E} \sum_{t=0}^{\infty} \beta^t u^2_t,$$

where we used $z_{-1} = u_{-1} = 0$. Solve this equation and use the law of iterated expectation:

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t z^2_t = \frac{1}{\beta} \mathbb{E} \sum_{t=0}^{\infty} \beta^t u^2_t = \frac{1}{\beta} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \sigma^2_{u_{t-1}},$$

where we take the timeless perspective with the planner choosing state-contingent policies before the realization of shocks in period $t = 0$. This means that $\sigma^2_{u_{-1}} = \mathbb{E} u^2_0$ is well defined.

Combining all pieces together, the planner’s problem reduces to

$$\min_{\{\delta_t, \sigma^2_q, \sigma^2_{u_t}\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[ \beta^2 (1 - \gamma) \delta^2 t \left( \sigma^2_q + \sigma^2_{u_t} \right) + \gamma \sigma^2_{u_{t-1}} \right]$$

s.t. $\sigma^2_{q} = (1-\delta_t)^2 (\sigma^2_q + \sigma^2_{u_t})$

$\sigma^2_{u_t} = \mathbb{E}_t (\omega \sigma^2_{t+1} n^2_{t+1})^2$. 

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Substituting out $\sigma^2_{at} = \frac{\sigma^2_a}{(1-\delta)^2} - \sigma^2_q$, one can rewrite the problem in terms of $\delta_t$ and $\sigma^2_t$:

$$\min_{\{\delta_t, \sigma^2_t\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[ \beta^2(1-\gamma) \left( \frac{\delta_t}{1-\delta_t} \right)^2 \sigma^2_t + \gamma \bar{\omega}^2 (\sigma^2_t n^*_t)^2 \right]$$

s.t. $\frac{\sigma^2_t}{(1-\delta_t)^2} = \sigma^2_q + \bar{\omega}^2 \mathbb{E}_t (\sigma^2_{t+1} n^*_{t+1})^2$,

where the constraint corresponds to equation (17) in Lemma 1. Notice that only the realizations of financial shocks $n^*_t$ matter for the optimization, while the effect of macro shocks is summarized by $\sigma^2_q$.

Also, consistent with the conjecture above, the problem depends on $n^2_t$, but not the sign of the shock.

Denote the Lagrange multiplier with $\lambda_t$ and write down the Lagrangian introducing explicitly the dependence on history of shocks $s^t$:

$$L = \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) \left[ \beta^2(1-\gamma) \left( \frac{\delta(s^t)}{1-\delta(s^t)} \right)^2 \sigma^2(s^t) + \gamma \bar{\omega}^2 (\sigma^2(s^t) n^*(s^t))^2 \right. + \lambda(s^t) \left( \frac{\sigma^2(s^t)}{(1-\delta(s^t))^2} - \sigma^2_q \right) - \bar{\omega}^2 \sum_{s_{t+1}} \frac{n_{s_{t+1}}}{\pi(s^t)} \left( \sigma^2(s_{t+1}) n^*(s_{t+1}) \right)^2 \right]$$

$$= \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) \left[ \beta^2(1-\gamma) \left( \frac{\delta(s^t)}{1-\delta(s^t)} \right)^2 \sigma^2(s^t) + \gamma \bar{\omega}^2 (\sigma^2(s^t) n^*(s^t))^2 \right. + \lambda(s^t) \left( \frac{\sigma^2(s^t)}{(1-\delta(s^t))^2} - \sigma^2_q \right) - \bar{\omega}^2 \sum_{t=1}^{\infty} \sum_{s^t} \beta^{t-1} \pi(s^t) \lambda(s^{t-1}) (\sigma^2(s^t) n^*(s^t))^2 \right].$$

Following the timeless perspective, the asymmetry between the first and other periods can be ignored and the Lagrangian can be rewritten as

$$L \propto \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) \left[ (\beta^2(1-\gamma)\delta^2(s^t) + \lambda(s^t)) \frac{\sigma^2(s^t)}{(1-\delta(s^t))^2} + \bar{\omega}^2 \left( \frac{\gamma - \lambda(s^{t-1})}{\beta} \right) (\sigma^2(s^t) n^*(s^t))^2 \right].$$

The optimality condition with respect to $\delta(s^t)$:

$$\frac{\beta^2(1-\gamma)\delta(s^t)}{(1-\delta(s^t))^2} + \frac{\beta^2(1-\gamma)\delta(s^t) + \lambda(s^t)}{(1-\delta(s^t))^3} = 0$$

can be simplified to get

$$\lambda(s^t) = -\beta^2(1-\gamma)\delta(s^t)(2-\delta(s^t)).$$

Take next the first-order condition with respect to $\sigma^2(s^t)$

$$\frac{\beta^2(1-\gamma)\delta^2(s^t) + \lambda(s^t)}{(1-\delta(s^t))^2} + 2\bar{\omega}^2 \left( \frac{\lambda(s^{t-1})}{\beta} \right) \sigma^2(s^t) (n^*(s^t))^2 = 0$$

and substitute out the Lagrange multiplier suppressing the history dependence:

$$\frac{\delta_t}{1-\delta_t} = \bar{\omega}^2 \left( \frac{\gamma}{1-\gamma} \frac{1}{\beta} + \delta_{t-1} (2-\delta_{t-1}) \right) \sigma^2_t (n^*_t)^2.$$

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This gives the optimality condition (18) in Lemma 1. Expectedly, the optimal policy stabilizes output δt = 0 when there are no intermediary friction \( \bar{\omega} = 0 \), in absence of financial shocks \( n_t^* = 0 \), and when the economy is closed \( \gamma = 0 \).

To solve the system (17)-(18) numerically, notice that the state space is given by two variables \((\delta_{t-1}, n_t^*)\) — a great improvement relative to the original problem — and iterate the policy functions according to the following algorithm:

1. Conjecture initial policy functions \( \sigma_t^2 = \sigma^2(\delta_{t-1}, n_t^*) \) and \( \delta_t = \delta(\delta_{t-1}, n_t^*) \).
2. Compute \( E_t(\sigma_t^2(\delta_t, n_t^*) n_{t+1}^*) = f(\delta_t, n_t^*) \) and \( \sigma_t^2 = \sigma^2(\delta_t, n_t^*) \) using equation (17).
3. Substitute this expression into equation (18) to find \( \delta_{t-1} = \delta_{-1}(\delta_t, n_t^*) \).
4. Invert this function \( \delta_t = \delta(\delta_{t-1}, n_t^*) \) and update \( \sigma_t^2 = \sigma^2(\delta(\delta_{t-1}, n_t^*), n_t^*) \).
5. Iterate until convergence.

A4 Derivations and Proofs for Sections 4 and 5

A4.1 Derivations for Section 4.2

Generalize the equilibrium conditions from the baseline model to include the price of tradables. The household optimality condition for goods

\[
\frac{\gamma}{1-\gamma} \frac{C_{Nid}}{C_{Tit}} = \frac{E_t P_{Tid}}{P_{Nid}}
\]

implies that the nominal exchange rate is given by

\[
e_{it} = c_{Nid} - c_{Tit} - p_{Tid} = \bar{q}_{it} - p_{Tid} + x_{it} - z_{it}.
\]

Linearizing the budget constraint

\[
\frac{B_{it}^*}{R_{it}^*} = B_{it-1}^* + P_{Tid}(Y_{tit} - C_{Tit})
\]

around the zero steady-state positions, we get an expression without valuation effects:

\[
\beta b_{it}^* = b_{it-1}^* - z_{it}.
\]

The household Euler equation combined with the optimal portfolio choice of arbitrageurs implies that the risk-sharing condition is given by

\[
\beta R_{it}^* \frac{C_{Tit}}{C_{Tit+1}^*} \frac{P_{Tid}}{P_{Tid+1}} = 1 + \frac{\omega \sigma_t^2}{R_{it}^*} (B_{it}^* - N_{it}^* - F_{it}).
\]

\[51\]Given that \( \delta_{t-1} \in [0, 1] \), the right hand side of equation (18) is monotonically increasing in \( \delta_{t-1} \) and hence,

\[
\delta_{t-1} = 1 - \sqrt{1 - \left( \frac{\beta \delta_t}{\omega^2} \frac{1}{1-\delta_t} \frac{1}{\sigma_t^2(n_t^*)^2} - \frac{\gamma}{1-\gamma} \frac{1}{\beta} \right)}.
\]
The equilibrium in the U.S. is described by the same conditions, except that \( \mathcal{E}_{it} = 1 \) and \( \sigma_{it}^2 = 0 \). Linearize this condition and integrate it across countries using the fact that

\[
\int C_{T\alpha} \, d\alpha = \int Y_{T\alpha} \, d\alpha \equiv Y_T
\]

to express the log real interest rate as follows

\[
r_t^* = \mathbb{E}_t \Delta y_{Tt+1} - \bar{\omega} \int \sigma_{it}^2 (b_{it}^* - n_{it}^* - f_{it}^*) \, d\alpha.
\]

Substituting this expression back into the risk-sharing condition of an individual economy, we get

\[
\mathbb{E}_t \Delta z_{it+1} = \int \bar{n}_{jt}^* \, d\alpha - \bar{n}_{it}^*, \quad \bar{n}_{it}^* \equiv \bar{\omega} \sigma_{it}^2 (b_{it}^* - n_{it}^* - f_{it}^*).
\]

The derivation of the global planner’s objective function follows the same steps as above. In particular, consider a relaxed problem with the constraints that bind in the steady state:

\[
\max \mathbb{E} \sum_{t=0}^\infty \beta^t \left[ \gamma \log C_{T\alpha} + (1 - \gamma) \left( \log C_{N\alpha} - \frac{C_{N\alpha}}{A_{it}} \right) \right] \, d\alpha
\]

\[
\text{s.t.} \quad B_{it}^* = B_{it-1} + Y_{T\alpha} - C_{T\alpha}, \quad \int B_{it}^* \, d\alpha = 0.
\]

The former constraint limits the number of assets available to share the risk between countries, while the second constraint is equivalent to the resource constraint \( \int C_{T\alpha} \, d\alpha = \int Y_{T\alpha} \, d\alpha \). Writing the Lagrangian and taking the second-order approximation around the efficient allocation, we get quadratic loss function:

\[
\frac{1}{2} \mathbb{E} \sum_{t=0}^\infty \beta^t \left[ \gamma z_{it}^2 + (1 - \gamma) x_{it}^2 \right] \, d\alpha.
\]

While the output gap is defined in the same way as in a non-cooperative case, the risk-sharing wedge is now defined relative to the globally efficient benchmark \( \tilde{c}_{it} \) that satisfies \( \mathbb{E}_t \Delta \tilde{c}_{it+1} = \mathbb{E}_t \Delta y_{Tt+1} \) and the country’s budget constraint.

Combining all pieces together, the global planner’s problem can be written as

\[
\min_{\{x_{it}, z_{it}, b_{it}^*, f_{it}^*, \sigma_{it}^2, \mathcal{T}\}} \frac{1}{2} \mathbb{E} \sum_{t=0}^\infty \beta^t \left[ \gamma z_{it}^2 + (1 - \gamma) x_{it}^2 \right] \, d\alpha
\]

\[
\text{s.t.} \quad \mathbb{E}_t \Delta z_{it+1} = \int \bar{n}_{jt}^* \, d\alpha - \bar{n}_{it}^*, \quad \bar{n}_{it}^* \equiv \bar{\omega} \sigma_{it}^2 (b_{it}^* - n_{it}^* - f_{it}^*),
\]

\[
\beta b_{it}^* = b_{it-1} - z_{it},
\]

\[
\sigma_{it}^2 = \text{var}_t (\tilde{q}_{it} - \mathcal{T} + x_{it} - z_{it}).
\]

The first-best solution is then to close the output gap \( x_{it} = 0 \) with monetary policy and to eliminate the risk-sharing wedge \( f_{it}^* = b_{it}^* - n_{it}^* \) with the FX instruments. It follows that \( r_t^* = \mathbb{E}_t \Delta y_{Tt+1} \) and the efficient consumption of tradables for a given economy is also globally efficient.

Turning next to the second-best policy, the non-cooperative planner takes the world interest rate as given and accommodates global shocks. Instead, the cooperative planner aims to close the risk-sharing wedge by setting \( \bar{n}_{it}^* = \int \bar{n}_{jt}^* \, d\alpha \), i.e. if FX interventions are constrained in other economies and \( \int \bar{n}_{jt}^* \, d\alpha \neq 0 \), it is optimal to deviate from \( \bar{n}_{it}^* = 0 \). In this sense, there are strategic complementarities
in FX interventions across countries under the optimal cooperative policy. Finally, according to the Fischer equation \( r^*_t = i^*_t - \mathbb{E}_t \Delta p_{Tt+1} \), U.S. nominal interest rate \( i^*_t \) affects the price of tradables and the bilateral exchange rates against the dollar \( e_{it} \). Therefore, a partial peg requires depressing the output \( x_{it} \) in response to tightening of U.S. policy that lowers \( p_{Tt} \).

### A4.2 Staggered prices

The derivation of the NKPC and the loss function in the presence of inflation follows the standard steps. Using the property of the model that monetary policy affects exchange rates only via \( \sigma^2_t \), the planner’s problem can be partitioned in two steps. The first one solves for the optimal trade-off between output gap and inflation. Because of the certainty equivalence and only first-period innovations affecting \( \sigma^2_t \), it is sufficient to focus on the following problem:

\[
\min_{\{x_t, \pi_{Nt}\}} \frac{1}{2} \sum_{t=0}^{\infty} \beta^t (x_t^2 + \alpha \pi_{Nt}^2) \quad \text{s.t.} \quad \pi_{Nt} = \kappa x_t + \beta \pi_{Nt+1} + \nu_t, \\
x_0 + \pi_{N0} = m_t.
\]

Taking the first-order conditions, we get

\[
\beta^t x_t = \kappa \lambda_t + \mu_t, \\
\beta^t \alpha \pi_{Nt} = -\lambda_t + \lambda_{t-1} \beta + \mu_t,
\]

where \( \mu_t = 0 \) for \( t > 0 \) and \( \lambda_{-1} = 0 \). It follows that the optimality conditions are

\[
\alpha \kappa \pi_{Nt} = -x_t + x_{t-1}
\]

for \( t \geq 1 \) and

\[
\alpha \kappa \pi_{Nt} = -x_t + (1 + \kappa) \mu_t,
\]

for \( t = 0 \). Substitute the optimality condition into the NKPC, so that dynamics for \( t > 0 \) is given by

\[
\beta x_{t+1} - (1 + \beta + \alpha \kappa^2) x_t + x_{t-1} = \alpha \kappa \nu_t.
\]

This difference equation has two roots \( \lambda_1 > 1 \) and \( \lambda_2 < 1 \)

\[
\lambda_{1,2} = \frac{1}{2\beta} \left[ 1 + \beta + \alpha \kappa^2 \pm \sqrt{(1 + \beta + \alpha \kappa^2)^2 - 4\beta} \right],
\]

and assuming for simplicity that \( \nu_t \) follows an AR(1) process, we get

\[
x_t = \lambda_2 x_{t-1} - \frac{\alpha \kappa}{\beta \lambda_1 - \rho} \nu_t.
\]
This means that one initial condition \( x_0 \) is required. At the same time, the NKPC for the first period together with the initial condition imply that

\[
\alpha \kappa (m_t - x_0) = \alpha \kappa^2 x_0 - \beta \Delta x_1 + \alpha \kappa \varepsilon_{t+0}.
\]

Substitute in expression for \( x_1 \) and solve for

\[
x_0 = \frac{\alpha \kappa}{\alpha \kappa^2 + \alpha \kappa + \beta - \beta \lambda_2} \left[ m_t - \frac{\lambda_1}{\lambda_1 - \rho} \varepsilon_{t+0} \right].
\]

Substituting this result into equation for \( x_t \), we get

\[
x_t = k_{xt}^2 m_t - k_{xt}^2 \varepsilon_{t+0},
\]

\[
\pi_{Nt} = k_{xt}^2 m_t - k_{xt}^2 \varepsilon_{t+0}
\]

for some coefficients \( k \). Substitute this back into the objective function:

\[
\sum_{t=0}^{\infty} \beta^t (x_t^2 + \alpha \pi_{Nt}^2) = \sum_{t=0}^{\infty} \beta^t \left[ (k_{xt}^2 m_t - k_{xt}^2 \varepsilon_{t+0})^2 + \alpha (k_{xt}^2 m_t - k_{xt}^2 \varepsilon_{t+0})^2 \right]
\]

\[
= K_x m_t^2 + K_\nu \varepsilon_{t+0}^2 + K_{xt} \varepsilon_{t+0} m_t m_t = k_1 (m_t - k_2 \varepsilon_{t+0})^2 + k_3 \varepsilon_{t+0}^2.
\]

Going back to the policy in the non-tradable sector, consider whether the price level converges to the initial level in the long run. The optimal policy implements \( \alpha \kappa \pi_{Nt} = -\Delta x_t \) for \( t \geq 1 \), just as in a closed economy. However, in the latter case, this condition holds also for \( t = 0 \) (under timeless perspective), which implies that \( \alpha \kappa \pi_{Nt} = -x_t \) in all periods and given that \( x_t \) is stationary, the price level converges in the long run to the initial level. In contrast, in our model \( \alpha \kappa \pi_{Nt} = -x_t + (x_0 + \alpha \kappa \pi_0) \) and given that \( x_t \to 0 \) in the long run, we get \( p_{Nt} \to \frac{1}{\alpha \kappa} x_0 + \pi_0 \), which is generically not equal zero.
A4.3 Terms of trade

To derive the loss function, follow the same steps as in the baseline model. Write down the Lagrangian of the relaxed problem without nominal or financial frictions:

$$
\mathcal{L} = E \sum_{t=0}^{\infty} \beta^t \left\{ (1 - \gamma) \log C_{Ht} + \gamma \log C_{Ft} - L_t + \lambda_t \left( A_t L_t - C_{Ht} - \gamma P_{Ht}^{*1-\varepsilon} C_{Ht}^* - C_{Ft} - B_{Ht}^* - \frac{B_t}{R_t^*} \right) \mu_t \left[ B_{Ht}^* - 1 + \gamma P_{Ht}^{*1} - \varepsilon \right] C_{Ht}^* \right\}.
$$

Notice that the planner is allowed to set optimal price in foreign market and, in equilibrium, charges a constant markup $\frac{\varepsilon}{\varepsilon - 1}$ over domestic price for the same goods. Take the first-order conditions and solve for the steady-state values of the Lagrange multipliers: $\lambda = 1/A$, $\mu = \left( \frac{\varepsilon}{\varepsilon - 1} \frac{C_{Ft}}{A} \right)^{\frac{\varepsilon - 1}{\varepsilon}} \frac{1}{\varepsilon}$. Using these values and expression (??), derive quadratic loss function:

$$
\mathcal{L} \propto \frac{1}{2} E \sum_{t=0}^{\infty} \beta^t \left\{ (1 - \gamma) c_{Ht}^2 + \gamma c_{Ft}^2 + \gamma (\varepsilon - 1) p_{Ht}^{*2} \right\},
$$

(A12)

where as before, the small letters denote the deviations from the first-best allocation.

PCP When sticky in producer currency, the export price in the currency of destination is equal

$$
P_{Ht}^* = \frac{\varepsilon}{\varepsilon - 1} P_{Ht} = \frac{\varepsilon}{\varepsilon - 1} \frac{1 - \gamma}{\gamma} C_{Ft},
$$

where the latter equality follows from household demand for goods (??). It follows that

$$
p_{Ht}^* = c_{Ft} - c_{Ht}
$$

and it is sufficient to close two gaps in the loss function (A12) to implement efficient allocation. Linearizing the market clearing condition, we get

$$
l_t = (1 - \bar{\gamma}) c_{Ht} - \bar{\gamma} \varepsilon p_{Ht}^*, \quad \bar{\gamma} \equiv \frac{\gamma (\varepsilon - 1)}{\varepsilon - \gamma}
$$

where $\bar{\gamma}$ is the steady-state share of exports in total output. The last two equations can be solved to express $c_{Ht}$ and $p_{Ht}^*$ in terms of the normalized output gap $x_t \equiv \frac{1}{1 + \bar{\gamma} (\varepsilon - 1)} l_t$ and the risk-sharing gap $z_t \equiv \frac{1}{1 + \bar{\gamma} (\varepsilon - 1)} c_{Ft}$:

$$
c_{Ht} = \varepsilon \bar{\gamma} z_t + x_t, \quad p_{Ht}^* = (1 - \bar{\gamma}) z_t - x_t.
$$

Substitute these expressions into the loss function to obtain $\frac{1}{2} E \sum_{t=0}^{\infty} \beta^t \left[ \kappa z_t^2 + x_t^2 \right]$, where $\kappa \equiv \frac{\varepsilon^2}{\varepsilon - \gamma}$.

Linearizing the budget constraint and substituting in expression for $p_{Ht}^*$, we get

$$
\beta b_t^* = b_{t-1}^* + \frac{\varepsilon - 1}{\varepsilon} x_t - z_t,
$$

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where \( b_t^* = \frac{B_t^* - \tilde{B}_t^*}{\varepsilon C_F} \). Normalizing noise trader shocks \( N_t^* \) and FX interventions \( F_t^* \) by \( \frac{1}{\varepsilon C_F} \), we get the risk-sharing condition
\[
\mathbb{E}_t \Delta z_{t+1} = -\tilde{\omega} \sigma^2 (ib_t^* - n_t^* - f_t^*),
\]
where \( \tilde{\omega} = \frac{\omega C_F}{\beta(1+\gamma(\varepsilon-1))} \). As before, the nominal exchange rate is given by
\[
e_t = (c_{Ht} + \tilde{c}_{Ht}) - (c_{Ft} + \tilde{c}_{Ft}) = \tilde{q}_t + x_t - (1-\gamma)z_t.
\]
Combining these conditions, we get the planner’s problem:
\[
\min_{\{x_t, z_t, b_t^*, f_t^*, \sigma_t^2\}} \frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \kappa z_t^2 + x_t^2 \right]
\]
\[
\text{s.t. } \mathbb{E}_t \Delta z_{t+1} = -\tilde{\omega} \sigma_t^2 (ib_t^* - n_t^* - f_t^*),
\]
\[
\beta b_t^* = b_{t-1}^* + \frac{\varepsilon - 1}{\varepsilon} x_t - z_t,
\]
\[
\sigma_t^2 = \text{var}_t(\tilde{q}_{t+1} + x_{t+1} - (1-\gamma)z_{t+1}).
\]
Because \( x_t \) drops from the budget constraint in the first-best allocation, the latter can be implemented under the same conditions as in the baseline model. A sufficient condition for \( \tilde{q}_t = 0 \) is that \( r_t^* = 0 \) and \( a_t = c_t^* \) follow a random walk. Indeed, in this case \( \tilde{c}_{Ft} \) is also a random walk and moves one-to-one with \( a_t \), which gives \( c_{Ht} = a_t \) implies that \( \tilde{q}_t = \tilde{c}_{Ht} - \tilde{c}_{Ft} = 0 \).

**DCP** The dollar pricing implies that \( P_{Ht}^* \) is fixed and therefore,
\[
P_{Ht}^* = -\tilde{p}_{Ht}^* = \tilde{c}_{Ht} - \tilde{c}_{Ft} = \tilde{q}_t.
\]
Define output gap as deviations from the optimal production of locally consumed goods \( x_t = c_{Ht} \) and the risk-sharing wedge as the deviation from the optimal consumption of foreign goods \( z_t = c_{Ft} \) and write the loss function (A12) as
\[
\frac{1}{2} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[ (1-\gamma)x_t^2 + \gamma z_t^2 + \gamma(\varepsilon - 1)\tilde{q}_t^2 \right].
\]
The first-order approximation to the budget constraint is
\[
\beta b_t^* = b_{t-1}^* - (\varepsilon - 1)\tilde{q}_t - z_t,
\]
where \( b_t^* = \frac{B_t^* - \tilde{B}_t^*}{C_F} \). Intuitively, when the natural real exchange rate depreciates, the export price become too high reducing exports relative to the efficient allocation. Normalizing \( N_t^* \) and \( F_t^* \) by \( C_F \) and defining \( \tilde{\omega} = \omega C_F / \beta \), the planner’s problem can be written as
\[
\min_{\{x_t, z_t, b_t^*, f_t^*, \sigma_t^2\}} \frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \kappa z_t^2 + (1-\gamma)x_t^2 + \gamma(\varepsilon - 1)\tilde{q}_t^2 \right]
\]
\[
\text{s.t. } \mathbb{E}_t \Delta z_{t+1} = -\tilde{\omega} \sigma_t^2 (ib_t^* - n_t^* - f_t^*),
\]
\[
\beta b_t^* = b_{t-1}^* - (\varepsilon - 1)\tilde{q}_t - z_t,
\]
\[
\sigma_t^2 = \text{var}_t(\tilde{q}_{t+1} + x_{t+1} - z_{t+1}).
\]
It follows that when \( \tilde{q}_t = 0 \), the first-best allocation with zero losses and \( x_t = z_t = 0 \) is implementable with monetary policy that pegs the nominal exchange rate \( \sigma_t^2 = 0 \). When two policy instruments are available, the risk-sharing condition is not binding and the problem reduces to minimizing the losses subject to the intertemporal budget constraint:

\[
\min_{\{x_t,z_t\}} \frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \gamma z_t^2 + (1 - \gamma) x_t^2 \right]
\]

s.t. \( \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ z_t + (\varepsilon - 1) \tilde{q}_t \right] = 0. \)

Denote the Lagrange multiplier on the budget constraint with \( \mu \) and take the first-order conditions:

\[
\beta^t \gamma z_t = \beta^t \mu, \quad \beta^t (1 - \gamma) x_t = 0.
\]

Therefore, the monetary policy closes the output gap \( x_t = 0 \) and the FX interventions close the UIP gap \( \mathbb{E}_t \Delta z_{t+1} = 0 \) by setting \( f_t^* = \delta b_t^* - n_t^* \).
Budget constraint with transfers  Substitute firm profits $\Pi_t = P_N t Y_{Nt} - W_t L_t$ and household consumption expenditure $P_t C_t = P_N t C_{Nt} + P_T t C_{Tt}$ into the household budget constraint and use market clearing $C_{Nt} = Y_{Nt}$ to obtain:

$$\frac{B_t}{R_t} - B_{t-1} = NX_t + T_t,$$

where $NX_t = P_{Tt} Y_{Tt} - P_{Tt} C_{Tt} = \varepsilon_t (Y_{Tt} - C_{Tt})$. Next combine the household and government budget constraints to obtain:

$$\frac{B_t + F_t}{R_t} + \frac{\varepsilon_t F^*_t}{R^*_t} - B_{t-1} - F_{t-1} - \varepsilon_t F^*_{t-1} = NX_t + \tau \varepsilon_t \pi^*_t.$$

Define $B^*_t$ such that $\frac{B^*_t}{R^*_t} = \frac{F^*_t}{R^*_t} + \frac{B_t + F_t}{\varepsilon_t R_t}$ and use the market clearing $B_t + D_t + N_t + F_t = 0$ and Lemma ?? that $B^*_t = D^*_t + N^*_t + F^*_t$ to rewrite:

$$\frac{\varepsilon_t B^*_t}{R^*_t} - \varepsilon_t B^*_{t-1} + \varepsilon_t (D^*_{t-1} + N^*_{t-1}) + (D_{t-1} + N_{t-1}) = NX_t + \tau \varepsilon_t \pi^*_t.$$

Finally, recall that $\pi^*_t = \bar{R}^*_t D^*_{t-1} + N^*_{t-1} = \left[ 1 - \frac{R_{t-1}}{R^*_t} \right] \varepsilon_t (D^*_{t-1} + N^*_{t-1}).$ Subtract $\varepsilon_t \pi^*_t$ on both sides of the budget of the budget constraint to obtain:

$$\frac{\varepsilon_t B^*_t}{R^*_t} - \varepsilon_t B^*_{t-1} + (D_{t-1} + N_{t-1}) + \frac{R_{t-1}}{R^*_t} \varepsilon_t (D^*_{t-1} + N^*_{t-1}) = NX_t - (1 - \tau) \bar{R}_t \varepsilon_t (D^*_{t-1} + N^*_{t-1}),$$

$$= 0 \text{ as zero capital portfolio at } t - 1$$

Divide through by $\varepsilon_t$, use the fact that $NX_t/\varepsilon_t = Y_{Tt} - C_{Tt}$, and Lemma ?? that $D^*_{t-1} + N^*_{t-1} = B^*_{t-1} - F^*_{t-1}$ to rewrite:

$$\frac{B^*_t}{R^*_t} - B^*_{t-1} = (Y_{Tt} - C_{Tt}) - (1 - \tau) \bar{R}_t \varepsilon_t (B^*_{t-1} - F^*_{t-1}),$$

completing the proof of the lemma. $\blacksquare$
References


