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**ABSTRACT**

We generalize the closed-economy neoclassical growth model (CNGM) to allow for costly goods trade and capital flows with imperfect substitutability between countries. We develop a tractable, multi-country, quantitative model that matches key features of the observed data (e.g., gravity equations for trade and capital holdings) and is well suited for analyzing counterfactual policies that affect both goods and capital market integration (e.g., U.S.-China decoupling). We show that goods and capital market integration interact in non-trivial ways to shape impulse responses to counterfactual changes in productivity and goods and capital market frictions and the speed of convergence to steady-state.

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# 1 Introduction

The textbook neoclassical growth model remains a key benchmark for thinking about cross-country income dynamics. In the closed-economy version of this model, each country converges along a transition path to its own steady-state level of income per capita, as determined by its production technology and preference parameters. In open economy versions of this model, strong assumptions are typically made about substitutability in goods and capital markets. Often goods are assumed to be homogeneous across countries, or trade between countries is assumed to be costless, whereas conventional quantitative trade models feature both imperfect substitutability across countries and trade frictions. Similarly, capital is often assumed to be homogeneous, which with competitive markets implies perfectly elastic flows of capital between countries to arbitrage away differences in rates of return.

We generalize the closed-economy neoclassical growth model (CNGM) to allow for costly international trade and capital holdings with imperfect substitutability. We develop a tractable, multi-country model that is amenable to quantitative analysis, which simultaneously incorporates international goods trade and capital allocations across countries, as well as intertemporal savings decisions over time. We consider an Armington specification, in which the final consumption index in each country is defined over all countries' goods. Goods can be traded between countries subject to iceberg trade costs. The representative agent in each country chooses how much of the final consumption index to consume and save. Wealth in each country can be invested in any country around the world, subject to capital market frictions and idiosyncratic heterogeneity in investment returns. The model determines both the allocation of wealth at a point in time and the accumulation of wealth over time. As in the CNGM, each country converges to its own steady-state, but this steady-state now depends not only on domestic productivity, but also on foreign productivity and goods and capital market frictions.

With open goods markets, domestic capital accumulation that increases output of a country's good leads to a deterioration in its terms of trade, at a rate that depends on the degree of substitutability in goods markets. With open capital markets, domestic capital accumulation can be achieved through both domestic saving and inflows of foreign capital, where each country faces an upward-sloping supply function for foreign capital, whose slope depends on the degree of substitutability in capital markets. In general, the stock of wealth in each country differs from its stock of capital. Therefore, some countries can specialize as exporters of capital services and importers of goods, while others specialize as exporters of goods and importers of capital services, even in the absence of current account imbalances. With an intertemporal savings decision, some countries can spend more than their income and incur current account deficits, while others spend less than their income and enjoy current account surpluses.

The resulting framework is quantitatively successful in capturing a number of key features of observed international trade and capital holdings. We match the well-known empirical finding that trade flows are well approximated by a gravity equation, such that bilateral trade increases with importer and exporter size, and decreases with bilateral trade frictions. We generate a similar gravity equation for bilateral capital holdings, which again provides a close approximation to the data. More broadly, our framework predicts home bias in international capital allocations, because managing capital is more costly abroad than at home. It is also consistent with a strong positive correlation between domestic saving and investment, because capital is imperfectly substitutable across countries, and foreign investment faces greater capital market frictions. Our model generates deterministic predictions for gross and net capital holdings, with gross flows substantially exceeding net flows, because of idiosyncratic heterogeneity in the returns to investment. Finally, it gives rise to limited capital flows from rich to poor countries, because capital is imperfectly substitutable across countries, and even if poorer countries offer higher rental rates, they can have lower capital productivity or higher capital market frictions.

Incorporating imperfect substitutability and goods and capital market frictions yields new insights for impulse responses to productivity shocks and the speed of convergence to steady-state. In the CNGM, the higher steady-state capital stock implied by a positive productivity shock only can be achieved through domestic wealth accumulation. In contrast, in the conventional open-economy neoclassical growth model, a positive productivity shock induces an immediate reallocation of capital that equalizes the rental rate on capital across all countries. Our framework generates predictions in between these two extremes. The higher steady-state capital stock implied by a positive productivity shock is achieved through a combination of *both* domestic wealth accumulation *and* international capital reallocation. The magnitude of the initial reallocation of capital depends on the degree to which capital is imperfectly substitutable across countries. In the presence of goods and capital market frictions, the real return to investment differs across countries along the transition path to steady-state, such that some countries accumulate wealth more rapidly than others.

We show that goods and capital market integration interact in non-trivial ways to shape the speed of convergence towards steady-state. In the CNGM, the speed of convergence is determined by the degree of diminishing returns to capital accumulation. Opening the CNGM to *only* free trade in goods *raises* the speed of convergence, because each country's good now faces substitutes in world markets. Therefore, increases in output of a country's good lead to larger falls in its price than in the closed economy, which implies a more rapid decline in the real return to investment. Opening the CNGM to *only* free capital flows also *raises* the speed of convergence to steady-state, because each country now competes to attract capital in world markets. Therefore, attracting additional capital to produce more output requires a larger rise in the rental rate than in the

closed economy, which again implies a more rapid decline in the real return to investment. In contrast, opening the CNGM to *both* free trade and free capital *slows* the speed of convergence. The reason is that the opening of trade and capital flows leads to an initial reallocation of capital that equalizes the real return to investment across all countries. After this initial reallocation, there is no correlation between the real return to investment and initial levels of wealth or capital, which implies slower convergence than in the closed economy. All countries accumulate wealth at the same rate, as determined by this common real return to investment, and initial differences in wealth persist forever.

We use our framework to evaluate the impact of counterfactual policies that inherently involve disintegration in both goods and capital markets, such as a decoupling of China and the United States. In the neoclassical growth model with open trade but capital autarky, the conventional static welfare gains from trade integration are magnified by dynamic welfare gains from capital accumulation. The fall in the consumption goods price index from reductions in goods trade frictions raises the real return to investment in each country, which increases the rate of growth along the transition path, and the level of income per capita in steady-state.

In our neoclassical growth model with open trade and capital flows, the static and dynamic effects of trade integration are more subtle. Reductions in trade frictions lead to a reallocation of capital across countries, which affects income and consumption goods price indexes, and hence the static welfare gains from goods trade. This change in consumption goods price indexes in turn feeds back to influence the real return to investment and the dynamic welfare gains from capital accumulation along the transition path to steady-state. Similarly, the static and dynamic effects of capital market frictions depend heavily on goods market openness, highlighting the importance of jointly modelling these two dimensions of international integration.

We show how to quantify our model using readily-available data from national accounts, bilateral trade in goods, and bilateral capital holdings. We suppose that we observe the world economy somewhere along a transition path to an unobserved initial steady-state with time-invariant fundamentals. Given these observed data, we show how to undertake dynamic exact-hat algebra counterfactuals, given only the observed endogenous variables in the data. We also show how to invert the non-linear model to recover the fundamentals that rationalize these observed data as an equilibrium: goods productivity; investment productivity; trade frictions; and capital market frictions. By conditioning on the observed data, we are able to undertake this model inversion without making any assumptions about where the economy is relative to the initial steady-state, or about agents' expectations about future fundamentals.

We linearize the model around the unobserved initial steady-state to derive a closed-form solution for the economy's transition path. We use this linearization to exactly decompose each country's economic growth into the contributions of initial conditions and shocks to domestic

and foreign fundamentals. We also use this linearization to show analytically that the interaction between goods and capital market integration shapes impulse responses to productivity shocks, the impact of counterfactual policies, and the speed of convergence.

Our paper is related to a number of different strands of research. First, we connect with the large literature in macroeconomics on the CNGM following [Ramsey \(1928\)](#), [Solow \(1956\)](#) and [Swan \(1956\)](#). The CNGM's prediction of conditional convergence in income per capita finds strong empirical support in the cross-country growth literature following [Barro \(1991\)](#) and [Mankiw et al. \(1992\)](#). Much of the rapid growth of East Asian countries in recent decades is attributed to its mechanism of factor accumulation in the growth accounting exercise in [Young \(1995\)](#). One quantitative challenge for the CNGM is that empirical estimates of income convergence imply lengthy transitions to steady-state. In specifications with an endogenous savings rate, [King and Rebelo \(1993\)](#) argues that such lengthy transitions require implausibly low intertemporal elasticities of substitution. In response, a number of studies have explored extensions that generate slower convergence, including installation costs in [Rappaport \(2006\)](#), financial frictions in [Barro et al. \(1995\)](#) and multiple sectors in [Buera et al. \(2021\)](#).

A small number of papers have developed versions of the neoclassical growth model with open goods markets, while maintaining the assumption of autarky in capital markets. [Ventura \(1997\)](#) combines the neoclassical growth model with the factor price equalization theorem of the Heckscher-Ohlin model to rationalize both conditional convergence and episodes of rapid growth by developing countries. [Cuñat and Maffezzoli \(2004\)](#) allow for complete specialization and the resulting departures from factor price equalization. [Acemoglu and Ventura \(2002\)](#) show that specialization and trade can generate a stable world income distribution through terms of trade effects, even without diminishing returns in production. Relative to these studies, we generalize the neoclassical growth model to introduce open capital markets with imperfect substitutability, while also allowing for trade in differentiated goods and trade costs, so as to match the observed gravity equation relationships for goods trade and capital flows.

Second, our work is related to research in international trade. We consider the class of constant elasticity trade models, which includes differentiation by country of origin ([Armington 1969](#)), Ricardian technology differences ([Eaton and Kortum 2002](#)) and horizontally-differentiated firm varieties and increasing returns to scale ([Krugman 1980](#) and [Melitz 2003](#) with a Pareto distribution), as examined in [Arkolakis et al. \(2012\)](#). A key implication of these models is that bilateral trade exhibits a gravity equation, as highlighted in [Anderson and van Wincoop \(2003\)](#) and [Head and Mayer \(2014\)](#). Manipulating the conditions for general equilibrium in these static international trade models, [Kleinman et al. \(2023b\)](#) derive sufficient conditions for the impact of foreign productivity shocks on domestic welfare. [Kleinman et al. \(2023a\)](#) introduce capital accumulation into a dynamic model of migration within countries. But capital markets are assumed to be au-

tarkic in each location and a separation is assumed between workers (who live hand to mouth) and capitalists (who can save) in order to tractably model migration.

Much of the quantitative international trade literature assumes exogenous trade imbalances, although [Ju et al. \(2014\)](#), [Reyes-Heroles \(2016\)](#), [Eaton et al. \(2016\)](#) and [Ravikumar et al. \(2019\)](#) endogenize these imbalances following the intertemporal approach of [Obstfeld and Rogoff \(1996\)](#). A related line of research examines the relationship between trade and growth through capital accumulation, including [Anderson et al. \(2015\)](#), [Alvarez \(2017\)](#) and [Mutreja et al. \(2018\)](#). Within this line of research, [Moll \(2008\)](#) introduces bilateral production externalities between countries (e.g., from knowledge spillovers). Relative to these studies, we simultaneously model imperfect substitutability and frictions in goods and capital markets at a point in time and consumption-savings decisions over time.

Third, our analysis relates to several lines of research in international finance and macroeconomics. A first group of studies examines the origins of global imbalances, the exorbitant privilege of the United States, and the reasons why capital does not flow from rich to poor countries, including [Lucas \(1990\)](#), [Jin \(2012\)](#), [Gourinchas and Rey \(2007\)](#), [Gourinchas and Jeanne \(2006, 2013\)](#), [Maggiore et al. \(2020\)](#), [Auclert et al. \(2020\)](#), [Coppola et al. \(2021\)](#), [Davis et al. \(2021\)](#), and [Atkeson et al. \(2022\)](#). A second series of studies examines imperfect substitutability in capital markets, including [Koiijen and Yogo \(2019, 2020\)](#), [Auclert et al. \(2022\)](#) and [Maggiore \(2021\)](#). A third line of work evaluates the international propagation of shocks through goods and capital markets, including [Backus et al. \(1992\)](#), [Kose et al. \(2003\)](#) and [Huo et al. \(2019\)](#).

A fourth body of papers provides evidence that the gravity equation provides a good approximation to international capital flows, as in [Portes and Rey \(2005\)](#). A fifth vein of research explores home bias and the international risk diversification, including [Cole and Obstfeld \(1991\)](#), [Obstfeld \(1994\)](#), [Martin and Rey \(2004, 2006\)](#), [Mendoza et al. \(2009\)](#), [Fitzgerald \(2012\)](#), [Pellegrino et al. \(2021\)](#), [Jiang et al. \(2022\)](#), [Chau \(2022\)](#), [Hu \(2022\)](#) and [Kucheryavy \(2022\)](#). We abstract from international risk diversification by considering an environment with no aggregate uncertainty, in which unanticipated shocks to fundamentals are revealed under perfect foresight. Nevertheless, we show that our framework provides a natural explanation for a gravity equation for capital flows and other features of observed data on bilateral capital holdings.

The remainder of the paper is structured as follows. Section 2 develops our theoretical framework. Section 3 introduces our data and undertakes our quantitative analysis. Section 4 summarizes our conclusions.

## 2 Theoretical Framework

We consider an economy that consists of many countries indexed by  $n \in \{1, \dots, N\}$ . Time is discrete and indexed by  $t \in \{1, \dots, \infty\}$ . Each country supplies a differentiated good that is produced using labor and capital under constant returns to scale. Markets are perfectly competitive. The representative agent in country  $n$  is endowed with a mass  $\ell_n$  of labor.

At the beginning of each period  $t$ , this representative agent inherits a stock of wealth ( $a_{nt}$ ) that can be accumulated using the local consumption good. This stock of wealth ( $a_{nt}$ ) is the aggregation of the wealth invested in each country ( $a_{nit}$ ). These investments are subject to idiosyncratic return shocks and capital market frictions. At the beginning of period  $t$ , wealth is allocated across countries. At the beginning of period  $t + 1$ , investment returns are realized, depreciation occurs, and wealth is again allocated across countries. We assume that agents have perfect foresight for all aggregate variables.

Throughout the paper, we use bold math font to denote a vector (lowercase letters) or matrix (uppercase letters). We summarize the main features of the model's economic environment in Table 1 below. The derivations for all expressions and results in this section are reported in the Online Appendix.

### 2.1 Intertemporal Problem

The representative consumer in each country chooses current consumption and saving to maximize her intertemporal utility. We assume that intertemporal utility takes the constant relative risk aversion (CRRA) form:

$$u_{nt} = \sum_{s=0}^{\infty} \beta^{t+s} \frac{c_{nt+s}^{1-1/\psi}}{1-1/\psi}, \quad (1)$$

where  $\beta$  is the discount rate;  $c_{nt}$  is a consumption index that depends on the consumption of the goods produced by each country; and  $\psi$  is the intertemporal elasticity of substitution.

The representative consumer's period-by-period budget constraint requires that the value of consumption in period  $t$  plus the value of period  $t + 1$  wealth is equal to income from period  $t$  wealth net of depreciation plus labor income:

$$\text{s.t. } p_{nt} c_{nt} + p_{nt} \sum_{i=1}^N a_{nit+1} = \sum_{i=1}^N (p_{nt} (1 - \delta) + v_{nit}) a_{nit} + w_{nt} \ell_n, \quad (2)$$

where  $p_{nt}$  is the price index dual to the consumption index;  $\delta$  is the rate of depreciation;  $v_{nit}$  is the realized return to investment from investor  $n$  in producer  $i$ ; and  $w_{nt}$  is the wage.

If the representative consumer from country  $n$  invests a unit of her consumption bundle in country  $i$  at the beginning of period  $t - 1$ , she receives  $(1 - \delta)$  units back at the beginning of

period  $t$  and a return from the investment of  $v_{nit}$  units of the numeraire. Therefore, her gross nominal return from her investments at the beginning of period  $t - 1$  is:

$$\mathcal{R}_{nt}^{nom} = \frac{p_{nt}(1 - \delta) + v_{nt}}{p_{nt-1}}, \quad (3)$$

where we have defined the average realized return to investment as  $v_{nt} \equiv \sum_{i=1}^N a_{nit}v_{nit}/a_{nt}$ ; we denote total wealth by  $a_{nt} \equiv \sum_{i=1}^N a_{nit}$ ; and we show below that in equilibrium the realized return to investment is the same across all producers  $i$ :  $v_{nit} = v_{nt}$ .

Dividing by the rate of inflation, the gross real return to the investment is:

$$\mathcal{R}_{nt} = \frac{\mathcal{R}_{nt}^{nom}}{p_{nt}/p_{nt-1}} = 1 - \delta + \frac{v_{nt}}{p_{nt}}, \quad (4)$$

and we can re-write the period-by-period budget constraint (2) as:

$$c_{nt} + a_{nt+1} = \mathcal{R}_{nt}a_{nt} + \frac{w_{nt}\ell_n}{p_{nt}}. \quad (5)$$

Using this representation, the consumer's problem can be solved in two stages. First, she chooses how much to consume and save. Second, she chooses how much of her wealth to allocate to each country. From equations (1) and (5), the first of these two decisions for consumption-saving takes the same form as in Angeletos (2007). Therefore, optimal consumption and saving are linear functions of current period wealth:

$$c_{nt} = \varsigma_{nt} \left( \mathcal{R}_{nt}a_{nt} + \frac{w_{nt}\ell_n}{p_{nt}} + h_{nt} \right), \quad (6)$$

where  $h_{nt} \equiv \sum_{s=1}^{\infty} \frac{w_{nt+s}\ell_{nt+s}/p_{nt+s}}{\prod_{u=1}^s \mathcal{R}_{nt+u}}$  is the present discounted value of labor income measured in consumption units, and the saving rate  $(1 - \varsigma_{nt})$  is defined recursively as:

$$\varsigma_{nt}^{-1} = 1 + \beta^\psi \mathcal{R}_{nt+1}^{\psi-1} \varsigma_{nt+1}^{-1}, \quad (7)$$

as shown in Online Appendix C. In the special case of logarithmic utility ( $\psi = 1$ ), consumption and saving are constant functions of current-period wealth ( $\varsigma_{nt} = 1 - \beta$ ), as in Moll (2014).

## 2.2 Intratemporal Wealth Allocation

We now turn to the second wealth allocation decision. We assume that the return to each unit of investment from investor  $n$  is subject to an idiosyncratic shock for each of the possible producers  $i$  to which it can be allocated ( $\varphi_{nit}$ ). We interpret this idiosyncratic shock as capturing all of the unforeseen factors that can affect the returns to debt, equity, portfolio and direct investments,

Table 1: Economic Environment

<b>Production</b>	
Production technology	$y_{it} = z_{it} \left(\frac{\ell_{it}}{\mu}\right)^\mu \left(\frac{k_{it}}{1-\mu}\right)^{1-\mu}$
Bilateral trade frictions	$\tau_{nit} \geq 1$
<b>Intertemporal Preferences and Capital Accumulation</b>	
Utility function	$u_{nt} = \sum_{s=0}^{\infty} \beta^{t+s} \frac{c_{nt+s}^{1-1/\psi}}{1-1/\psi}$
Budget constraint	$p_{nt}c_{nt} + p_{nt}a_{nt+1} = (p_{nt}(1-\delta) + v_{nt})a_{nt} + w_{nt}\ell_n$
Wealth	$a_{nt} = \sum_{i=1}^N a_{nit}$
Investment return	$v_{nt} = \gamma \left[ \sum_{h=1}^N (\eta_{ht}r_{ht}/\kappa_{nht})^\epsilon \right]^{\frac{1}{\epsilon}}$
Capital market frictions	$\kappa_{nit} \geq 1$
<b>Intratemporal Preferences</b>	
Consumption index	$c_{nt} = \left[ \sum_{i=1}^N (c_{nit})^{\frac{\theta}{\theta+1}} \right]^{\frac{\theta+1}{\theta}}$
<b>Goods Market Clearing</b>	
Goods market clearing	$y_{it} = \sum_{n=1}^N c_{nit} + \sum_{n=1}^N g_{nit}$

Note: Preferences, production technology and resource constraints;  $\theta = \sigma - 1 > 0$  is the trade elasticity, as determined by the elasticity of substitution ( $\sigma$ );  $\epsilon$  is the capital elasticity;  $g_{nit}$  denotes the use for investment in country  $n$  of the consumption good produced by country  $i$  at time  $t$ ; and all other variables are defined in the main text.

including search, information and acquisition costs, regulatory and expropriation risk, and productivity shocks.<sup>1</sup> Therefore, each unit of wealth invested becomes  $\varphi_{nit}$  efficiency units of capital for use in production. Investments also face capital market frictions, such that  $\kappa_{nit} \geq 1$  units of assets from investor  $n$  must be invested in producer  $i$  in order for one unit to be available for production, where  $\kappa_{nnt} = 1$  and  $\kappa_{nit} > 1$  for  $n \neq i$ .

The realized rate of return in country  $n$  from investing one unit of wealth in country  $i$  is thus  $\varphi_{nit}r_{it}/\kappa_{nit}$ , where  $r_{it}$  is the rental rate per efficiency unit of capital. We assume that these idiosyncratic shocks to investment returns are drawn independently across investor and producer countries from the following Fréchet distribution:

$$F_{nit}(\varphi) = e^{-(\varphi/\eta_{it})^{-\epsilon}}, \quad \eta_{it} > 0, \quad \epsilon > 1, \quad (8)$$

where the scale parameter ( $\eta_{it}$ ) determines the average return from investments in producer  $i$ , which can depend for example on producer country institutions, such as the protection of property rights. The shape parameter ( $\epsilon$ ) controls the dispersion of these idiosyncratic shocks, and regulates the sensitivity of wealth allocations to relative rates of return.

The first key implication of our extreme value specification for these idiosyncratic shocks is that the share of wealth from investor  $n$  that is invested in producer  $i$  satisfies the following

<sup>1</sup>For evidence on the role of search costs in explaining the observed variation in market shares for mutual funds, see for example [Hortaçsu and Syverson \(2004\)](#).

gravity equation:

$$b_{nit} = \frac{a_{nit}}{a_{nt}} = \frac{(\eta_{it}r_{it}/\kappa_{nit})^\epsilon}{\sum_{h=1}^N (\eta_{ht}r_{ht}/\kappa_{nht})^\epsilon}. \quad (9)$$

Therefore, bilateral capital holdings ( $b_{nit}$ ) are decreasing in bilateral capital frictions ( $\kappa_{nit}$ ). But these bilateral capital holdings ( $b_{nit}$ ) also depend on capital frictions with other locations (“multilateral resistance”), as captured by the denominator. We refer to  $\epsilon$  as the capital elasticity, because it controls the elasticity of capital holdings ( $b_{nit}$ ) to relative rental rates ( $r_{it}$ ), and plays a similar role in capital markets as the trade elasticity ( $\theta$ ) in goods markets.

This specification rationalizes a number of the observed features of international capital holdings that are discussed in [Obstfeld and Rogoff \(2000\)](#). First, it is consistent with empirical findings that international capital holdings are well approximated by a gravity equation (e.g., [Portes and Rey 2005](#)), if bilateral capital frictions ( $\kappa_{nit}$ ) are increasing in the bilateral distance between countries.<sup>2</sup> Second, it is in line with empirical findings of home bias in international capital allocations (e.g., [French and Poterba 1991](#)), because managing capital is more costly abroad than at home ( $\kappa_{nit} > \kappa_{nnt}$  for  $n \neq i$ ). Third, it accounts for a strong positive correlation between domestic saving and investment rates (e.g., [Feldstein and Horioka 1980](#)), because foreign capital is an imperfect substitute for domestic capital ( $0 < \epsilon < 1$ ), and is subject to greater capital market frictions than domestic capital ( $\kappa_{nit} > \kappa_{nnt}$  for  $n \neq i$ ).

Fourth, it generates larger gross capital flows than net capital flows, because each investor country holds a positive amount of capital in each producer country for positive and finite values of capital productivity ( $\eta_{it}$ ) and capital frictions ( $\kappa_{nit}$ ). Fifth, it matches limited capital flows from rich to poor countries (e.g., [Lucas 1990](#)), because capital is imperfectly substitutable across countries, and even if poor countries offer higher rental rates (higher  $r_{it}$ ), they can have lower capital productivity (lower  $\eta_{it}$ ) or higher capital frictions (higher  $\kappa_{nit}$ ).

The second key implication of our extreme value specification for idiosyncratic shocks is that the expected return to investment from investor  $n$  is the same across all producers  $i$ :

$$v_{nit} = v_{nt} = \gamma \left[ \sum_{h=1}^N (\eta_{ht}r_{ht}/\kappa_{nht})^\epsilon \right]^{\frac{1}{\epsilon}}, \quad \gamma \equiv \Gamma \left( \frac{\epsilon - 1}{\epsilon} \right), \quad (10)$$

where  $\Gamma(\cdot)$  is the Gamma function.

Intuitively, producer countries face upward-sloping supply functions for capital, and can differ in terms of their rental rates for capital ( $r_{it}$ ). But producer countries with higher rental rates for capital ( $r_{it}$ ) attract investments with lower realizations for idiosyncratic returns ( $\varphi_{nit}$ ), such that the expected return conditional on investing in a producer country is the same across all possible

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<sup>2</sup>To stay as close to the CNGM as possible, we aggregate all capital holdings (e.g., debt, equity, portfolio and direct) together. Estimating separate gravity equations for both overall capital holdings and portfolio holdings, we find similar elasticities with respect to distance, as shown in Online Appendix [K.2](#).

producers  $i$  for a given investor  $n$  ( $v_{nit} = v_{nt}$  for all  $i$ ). With a continuous measure of units of wealth, this common expected return across producer countries for a given investor country equals the realized return. As a result, there is no aggregate uncertainty in the model, and all location characteristics (including  $\eta_{it}$  and  $\kappa_{nit}$ ) are revealed under perfect foresight.

Although the expected return to investment is the same across producer countries for a given investor, it can differ across these investor countries ( $v_{nt} \neq v_{it}$  for  $n \neq i$ ), if some of them have lower capital market frictions to producer countries than others ( $\kappa_{nht} \neq \kappa_{iht}$  for  $n \neq i$ ). Consequentially, the real return to investment ( $R_{nt}$ ) can differ across countries along the transition path, though we show below that in steady-state, it is equalized across countries ( $R_{nt}^* = R_{it}^*$ ), where we denote the steady-state value of variables with an asterisk.

Finally, we can solve explicitly for average efficiency units for investments from investor  $n$  in producer  $i$  conditional on investment occurring ( $\bar{\varphi}_{nit}$ ), which is monotonically decreasing in the share of wealth from investor  $n$  invested in producer  $i$  ( $b_{nit}$ ):

$$\bar{\varphi}_{nit} = \gamma \eta_{it} b_{nit}^{-\frac{1}{\epsilon}}. \quad (11)$$

Therefore, the third implication of this specification is that it delivers a downward-sloping marginal efficiency of investment schedule, as in Keynes (1935). Each investor  $n$  experiences diminishing marginal returns from allocating a larger share of its investments to a given producer  $i$  (larger  $b_{nit}$ ), where the rate of these diminishing returns is determined by the dispersion of the idiosyncratic shocks ( $\epsilon$ ).

Using the above expression for average efficiency units from equation (11), payments for capital used in production can be either written in terms of efficiency-adjusted capital ( $k_{it}$ ) or in terms of unadjusted wealth ( $a_{nit}$ ):

$$r_{it} k_{it} = \sum_{n=1}^N v_{nt} a_{nit}, \quad k_{it} = \sum_{n=1}^N \bar{\varphi}_{nit} a_{nit}. \quad (12)$$

### 2.3 Consumption, Production and Trade

The consumption index ( $c_{nt}$ ) takes the same form as in the Armington model of international trade and is defined over consumption of the varieties produced by each country  $i$  ( $c_{nit}$ ):

$$c_{nt} = \left[ \sum_{i=1}^N (c_{nit})^{\frac{\theta}{\theta+1}} \right]^{\frac{\theta+1}{\theta}}, \quad \theta = \sigma - 1, \quad \sigma > 1, \quad (13)$$

where  $\theta = \sigma - 1$  is the trade elasticity and  $\sigma > 1$  is the elasticity of substitution between varieties.

Using the properties of CES demand, the share of importer  $n$ 's expenditure on exporter  $i$  takes the conventional form:

$$s_{nit} = \frac{p_{nit}^{-\theta}}{\sum_{h=1}^N p_{nht}^{-\theta}}. \quad (14)$$

Each country's variety is produced using labor and capital according to a constant returns to scale Cobb-Douglas production technology. Production occurs under conditions of perfect competition. Varieties can be traded between locations subject to iceberg variable trade costs, where  $\tau_{nit} \geq 1$  units of a variety must be shipped from country  $i$  in order for one unit to arrive in country  $n$ , where  $\tau_{nnt} = 1$  and  $\tau_{nit} > 1$  for  $n \neq i$ .

Profit maximization and zero profits imply that the price to a consumer in country  $n$  of sourcing the variety supplied by country  $i$  is given by:

$$p_{nit} = \frac{\tau_{nit} w_{it}^{\mu_i} r_{it}^{1-\mu_i}}{z_{it}}, \quad 0 < \mu_i < 1, \quad (15)$$

where  $w_{it}$  is the wage;  $r_{it}$  corresponds to the rental rate per effective unit of capital; and  $z_{it}$  denotes country productivity.

Substituting the equilibrium pricing rule (15) into the CES expenditure share (14), the model also rationalizes empirical findings that bilateral international trade is well approximated by a gravity equation. Therefore, bilateral trade flows are decreasing in bilateral trade frictions, and increasing in measures of multilateral resistance.

The price index ( $p_{nt}$ ) dual to the consumption index ( $c_{nt}$ ) is given by:

$$p_{nt} = \left[ \sum_{i=1}^N p_{nit}^{-\theta} \right]^{-\frac{1}{\theta}}. \quad (16)$$

Applying Shephard's Lemma to the unit cost function, total payments for the capital used in country  $i$  are proportional to the total wage bill in that country:

$$\sum_{n=1}^N v_{nt} a_{nit} = r_{it} k_{it} = \frac{1 - \mu_i}{\mu_i} w_{it} \ell_{it}. \quad (17)$$

## 2.4 Market Clearing

Goods market clearing requires that payments to the factors of production used in a country equal expenditure on the goods produced by it:

$$\left( w_{it} \ell_{it} + \sum_{h=1}^N v_{hit} a_{hit} \right) = \sum_{n=1}^N s_{nit} [p_{nt} c_{nt} + p_{nt} a_{nt+1} - p_{nt} (1 - \delta) a_{nt}], \quad (18)$$

where the term inside the square brackets on the right-hand side is total expenditure on the consumption good in market  $n$  at time  $t$  for both consumption and net investment.

Using the period-by-period budget constraint (2) and our expression for factor payments in equation (17) above, we can rewrite this equality between income and expenditure as follows:

$$w_{it}\ell_{it} = \mu_i \sum_{n=1}^N s_{nit} [v_{nt}a_{nt} + w_{nt}\ell_n]. \quad (19)$$

We choose world GDP as our numeraire, such that:

$$1 = \sum_{i=1}^N \left( w_{it}\ell_i + \sum_{n=1}^N v_{nit}a_{nit} \right) = \sum_{i=1}^N \frac{1}{\mu_i} w_{it}\ell_i. \quad (20)$$

In Online Appendix D, we show that the conventional balance of payments accounting identities hold in the model, such that the current account corresponds to the sum of the trade balance and net investment income, and is equal to minus the financial account.

## 2.5 General Equilibrium

Given the wealth state variables  $\{a_{nt}\}_{n=1}^N$ , the equilibrium endogenous variables in the static trade and cross-country capital allocation bloc of the model  $\{w_{nt}, r_{nt}, s_{nt}, v_{nt}, b_{nt}\}_{n=1}^N$  are determined as the solution to the following system of equations:

$$s_{nit} = \frac{(\tau_{nit} w_{it}^{\mu_i} r_{it}^{1-\mu_i} / z_{it})^{-\theta}}{\sum_{h=1}^N (\tau_{nht} w_{ht}^{\mu_h} r_{ht}^{1-\mu_h} / z_{ht})^{-\theta}}, \quad (21)$$

$$w_{it}\ell_i = \mu_i \sum_{n=1}^N s_{nit} (v_{nt}a_{nt} + w_{nt}\ell_n), \quad (22)$$

$$b_{nit} = \frac{(\eta_{it} r_{it} / \kappa_{nit})^\epsilon}{\sum_{h=1}^N (\eta_{ht} r_{ht} / \kappa_{nht})^\epsilon}, \quad (23)$$

$$v_{nt} = \gamma \left[ \sum_{h=1}^N (\eta_{ht} r_{ht} / \kappa_{nht})^\epsilon \right]^{1/\epsilon}, \quad (24)$$

$$\sum_{n=1}^N v_{nt} b_{nit} a_{nt} = \frac{1 - \mu_i}{\mu_i} w_{it}\ell_i, \quad (25)$$

along with the choice of numeraire:

$$\sum_{i=1}^N \frac{1}{\mu_i} w_{it}\ell_i = 1. \quad (26)$$

The evolution of the state variables  $\{a_{nt}\}_{n=1}^N$  over time is determined by optimal consumption-saving decisions according the following dynamic bloc of equations:

$$a_{nt+1} = (1 - \varsigma_{nt}) \left( \mathcal{R}_{nt} a_{nt} + \frac{w_{nt}\ell_n}{p_{nt}} + h_{nt} \right) - h_{nt}, \quad (27)$$

$$h_{nt} \equiv \sum_{s=1}^{\infty} \frac{w_{nt+s} \ell_{nt+s} / p_{nt+s}}{\prod_{u=1}^s \mathcal{R}_{nt+u}}, \quad (28)$$

$$p_{nt} \equiv \left[ \sum_{i=1}^N (\tau_{nit} w_{it}^{\mu_i} r_{it}^{1-\mu_i} / z_{it})^{-\theta} \right]^{-1/\theta}, \quad (29)$$

where

$$\mathcal{R}_{nt} = 1 - \delta + v_{nt} / p_{nt}, \quad (30)$$

and  $\varsigma_{nt}$  is defined recursively as

$$\varsigma_{nt}^{-1} = 1 + \beta^{\psi} \mathcal{R}_{nt+1}^{\psi-1} \varsigma_{nt+1}^{-1}. \quad (31)$$

## 2.6 Trade and Capital Share Matrices

We now introduce the trade and capital share matrices, and the labor and capital income vectors, which we use to characterize the evolution of the world income distribution over time. To reduce notational clutter, we suppress the time subscript throughout this subsection.

Let  $\mathbf{S}$  be the  $N \times N$  matrix with the  $ni$ -th element equal to importer  $n$ 's expenditure on exporter  $i$  ( $S_{ni} \equiv [s_{ni}]$ ). Let  $\mathbf{T}$  be the  $N \times N$  matrix with the  $in$ -th element equal to the fraction of income that exporter  $i$  derives from selling to importer  $n$  ( $T_{in} \equiv \frac{s_{ni}(v_n a_n + w_n \ell_n)}{\sum_{h=1}^N s_{hi}(v_h a_h + w_h \ell_h)}$ ). We refer to  $\mathbf{S}$  as the *expenditure share* matrix and to  $\mathbf{T}$  as the *income share* matrix. Intuitively,  $S_{ni}$  captures the importance of  $i$  as a supplier to location  $n$ , and  $T_{in}$  captures the importance of  $n$  as a buyer for country  $i$ . Note the order of subscripts: in matrix  $\mathbf{S}$ , rows are buyers and columns are suppliers, whereas in matrix  $\mathbf{T}$ , rows are suppliers and columns are buyers.

Similarly, let  $\mathbf{B}$  be the  $N \times N$  matrix with the  $ni$ -th element equal to the share of investor country  $n$ 's wealth allocated to producer  $i$  ( $B_{ni} \equiv [b_{ni}]$ ). Let  $\mathbf{X}$  be the  $N \times N$  matrix with the  $in$ -th element equal to the share of capital income in producer  $i$  paid to investor  $n$  ( $X_{in} \equiv \frac{v_n b_{ni} a_n}{\sum_{h=1}^N v_h b_{hi} a_h}$ ). We refer to  $\mathbf{B}$  as the *portfolio share* matrix and to  $\mathbf{X}$  as the *payment share* matrix. Intuitively,  $B_{ni}$  captures the importance of  $i$  as a producer for capital investments from investor  $n$ , and  $X_{in}$  captures the importance of  $n$  as an investor of capital investments to producer  $i$ . Again note the order of subscripts: in matrix  $\mathbf{B}$ , rows are investors and columns are producers, whereas in matrix  $\mathbf{X}$ , rows are producers and columns are investors.<sup>3</sup>

Finally, let  $\mathbf{q}$  be the  $N \times 1$  vector of labor income with the  $n$ -th element equal to the labor income of country  $n$  ( $q_n \equiv w_n \ell_n$ ), and let  $\boldsymbol{\zeta}$  be the  $N \times 1$  vector of capital income with the  $n$ -th element equal to the capital income of country  $n$  ( $\zeta_n \equiv v_n a_n$ ).

<sup>3</sup>For theoretical completeness, we maintain two assumptions on these matrices, which are satisfied empirically in all years of our data. First, we assume that the  $\mathbf{S}$  and  $\mathbf{B}$  matrices are irreducible, such that all locations are connected directly or indirectly by trade flows and capital holdings: For any  $i, n$ , there exists  $k$  such that  $[\mathbf{S}^k]_{in} > 0$  and  $[\mathbf{B}^k]_{in} > 0$ . Second, we assume that each location consumes a positive amount of domestic goods and allocates a positive share of capital domestically: For all  $i$ ,  $\mathbf{S}_{ii} > 0$  and  $\mathbf{B}_{ii} > 0$ .

## 2.7 Steady-state Equilibrium

The steady-state equilibrium of the model is characterized by time-invariant values of the state variables  $\{a_n^*\}_{n=1}^N$  and the other endogenous variables of the model  $\{w_n^*, r_n^*, s_{ni}^*, v_{nt}^*, b_{ni}^*\}_{n=1}^N$ , given time-invariant values of country fundamentals  $\{\ell_n, z_n, \eta_n\}_{n=1}^N$  and  $\{\tau_{ni}, \kappa_{ni}\}_{n,i=1}^N$ , where recall that we denote the steady-state values of endogenous variables by an asterisk.

Given constant population in each country ( $\ell_n$ ), diminishing marginal physical productivity of capital in the production technology implies a steady-state level of wealth ( $a_n^*$ ), as in the traditional Solow-Swan Model. Unlike that Solow-Swan model, the saving rate here is endogenously determined as the solution to a forward-looking consumption-saving problem. As a result, the steady-state gross real return to investment ( $\mathcal{R}_n^*$ ) and the steady-state saving rate ( $\varsigma_n^*$ ) are inversely related to discount factor ( $\beta$ ):

$$\mathcal{R}_n^* = \frac{1}{\beta}, \quad \varsigma_n^* = 1 - \beta. \quad (32)$$

This common steady-state value of the gross real return to investment ( $\mathcal{R}_n^*$ ) implies that the steady-state realized real return to investment ( $v_n^*/p_n^*$ ) is the same across all countries:

$$\frac{v_n^*}{p_n^*} = \beta^{-1} - 1 + \delta. \quad (33)$$

## 2.8 Transition Dynamics

As in the conventional closed-economy neoclassical growth model, our open-economy framework features conditional convergence in income per capita, in the sense that each country converges to its own steady-state level of income per capita. In contrast to this conventional framework, each country's steady-state level of income per capita and its growth rate along the transition path are influenced by fundamentals in other countries, where these fundamentals comprise trade frictions ( $\tau_{ni}$ ), capital market frictions ( $\kappa_{ni}$ ), goods productivity ( $z_i$ ), and capital productivity ( $\eta_i$ ).

In Subsection 2.8.1, we show that we can solve for the economy's dynamic response to an anticipated sequence of changes in fundamentals in the non-linear model using dynamic exact-hat algebra techniques. In Subsection 2.8.2, we linearize the model's general equilibrium conditions to obtain a closed-form solution for the economy's transition path in response to these changes in fundamentals. In Subsection 2.8.3, we use this linearization to quantify the contributions of convergence and changes in fundamentals to the evolution of the world income distribution. In Subsection 2.8.4, we undertake a spectral analysis to characterize analytically the speed of convergence to steady-state and the evolution of the economy's state variables along the transition path. Finally, in Subsection 2.8.5, we use our closed-form solution to analyze the role of goods and capital market integration in shaping the speed of convergence.

### 2.8.1 Dynamic Exact-Hat Algebra

We suppose that we observe the world economy somewhere along the transition path towards an unobserved steady state. Given the initial observed endogenous variables of the model, we show that we are able to solve for the economy's transition path in time differences ( $\dot{x}_{it+1} = x_{it+1}/x_{it}$ ) for any anticipated convergent sequence of future changes in fundamentals, without having to solve for the initial level of fundamentals.

**Proposition 1. *Dynamic Exact Hat Algebra.*** *Given observed initial populations  $\{\ell_{i0}\}_{i=1}^N$ , an initial observed allocation of the economy,  $(\{a_{i0}\}_{i=1}^N, \{a_{i1}\}_{i=1}^N, \{S_{ni0}\}_{n,i=1}^N, \{T_{ni0}\}_{n,i=1}^N, \{B_{ni0}\}_{n,i=1}^N, \{X_{ni0}\}_{n,i=1}^N)$ , and a convergent sequence of future changes in fundamentals under perfect foresight:*

$$\left\{ \left\{ \dot{z}_{it} \right\}_{i=1}^N, \left\{ \dot{\eta}_{it} \right\}_{i=1}^N, \left\{ \dot{\tau}_{ijt} \right\}_{i,j=1}^N, \left\{ \dot{\kappa}_{ijt} \right\}_{i,j=1}^N \right\}_{t=1}^{\infty},$$

*the solution for the sequence of changes in the model's endogenous variables does not require information on the level of fundamentals:*

$$\left\{ \left\{ z_{it} \right\}_{i=1}^N, \left\{ \eta_{it} \right\}_{i=1}^N, \left\{ \tau_{ijt} \right\}_{i,j=1}^N, \left\{ \kappa_{ijt} \right\}_{i,j=1}^N \right\}_{t=1}^{\infty}.$$

*Proof.* See Online Appendix G. □

Intuitively, we use the initial observed endogenous variables and the equilibrium conditions of the model to control for the unobserved initial level of fundamentals. Applying this proposition, we can employ dynamic exact-hat algebra methods to solve for the unobserved initial steady state in the absence of any further changes in fundamentals. We can also use this approach to solve counterfactuals for the transition path of the global economy in response to assumed sequences of future changes in fundamentals.

In addition to these dynamic exact-hat algebra results in Proposition 1, we can invert the model to solve for the unobserved changes in goods productivity, capital productivity, trade frictions and capital market frictions that are implied by the observed changes of the endogenous variables of the model under perfect foresight, as shown in Online Appendix H. Importantly, we can undertake this model inversion along the transition path without making assumptions about the precise sequence of future fundamentals, because the observed changes in wealth capture agents' expectations about this sequence of future fundamentals.

### 2.8.2 Linearization

We now linearize the model to characterize analytically the speed of convergence and the evolution of the state variables along the transition path to steady state. We suppose that we observe

population ( $\ell$ ), the wealth state variable ( $\mathbf{a}_t$ ) for time  $t = 0$  and  $t = 1$ , and the trade and capital share matrices ( $\mathbf{S}, \mathbf{T}, \mathbf{B}, \mathbf{X}$ ) of the economy at time  $t = 0$ . The economy need not be in steady-state at  $t = 0$ , but we assume that it is on a convergence path towards a steady-state with constant fundamentals ( $\mathbf{z}, \boldsymbol{\eta}, \boldsymbol{\tau}, \boldsymbol{\kappa}$ ). We refer to the steady-state implied by these initial fundamentals as the *initial steady-state*. We use a tilde above a variable to denote a log deviation from this initial steady-state (e.g.,  $\tilde{a}_{it+1} = \ln a_{it+1} - \ln a_i^*$ ).

We begin by totally differentiating the conditions for general equilibrium around this unobserved initial steady-state, holding constant countries' labor endowments. We thus obtain a system of linear equations that fully characterizes the economy's transition path up to first-order, as reported in Online Appendix I.1. We next show that this system of linearized equations can be reduced to a second-order difference equation in the wealth state variables ( $\tilde{\mathbf{a}}_t$ ) and changes to fundamentals. For expositional convenience, we focus here on the simplest form of changes in fundamentals, such that agents at time  $t = 0$  learn about a one-time permanent shock to fundamentals from time  $t = 1$  onwards. However, analogous results hold in the linearized model for any expected convergent sequence of future shocks to fundamentals under perfect foresight, and for the case in which agents observe an initial shock to fundamentals and form rational expectations about future shocks based on a known stochastic process for fundamentals.

We define measures of incoming and outgoing shocks to trade and capital frictions, which aggregate bilateral changes across partner countries, using initial trade and capital share weights:  $\tilde{\tau}_{nt}^{in} \equiv \sum_{i=1}^N S_{nit} \tilde{\tau}_{nit}$ ,  $\tilde{\tau}_{it}^{out} \equiv \sum_{n=1}^N T_{int} \tilde{\tau}_{nit}$ ,  $\tilde{\kappa}_{nt}^{out} \equiv \sum_{i=1}^N B_{nit} \tilde{\kappa}_{nit}$ , and  $\tilde{\kappa}_{it}^{in} \equiv \sum_{n=1}^N X_{int} \tilde{\kappa}_{nit}$ . Using these definitions, we have the following result.

**Proposition 2. State Variables.** *Suppose that the economy at time  $t = 0$  is on a convergence path toward an initial steady state with constant fundamentals ( $\mathbf{z}, \boldsymbol{\eta}, \boldsymbol{\tau}, \boldsymbol{\kappa}$ ). At time  $t = 0$ , agents learn about one-time, permanent shocks to fundamentals ( $\tilde{\mathbf{f}} \equiv [\tilde{\mathbf{z}} \ \tilde{\boldsymbol{\eta}} \ \tilde{\boldsymbol{\kappa}}^{in} \ \tilde{\boldsymbol{\kappa}}^{out} \ \tilde{\boldsymbol{\tau}}^{in} \ \tilde{\boldsymbol{\tau}}^{out}]'$ ) from time  $t = 1$  onwards. The evolution of the economy's wealth state variables from time  $t = 1$  onwards satisfies the following second-order difference equation:*

$$\Psi \tilde{\mathbf{a}}_{t+2} = \Gamma \tilde{\mathbf{a}}_{t+1} + \Theta \tilde{\mathbf{a}}_t + \Pi \tilde{\mathbf{f}}, \quad (34)$$

where the matrices ( $\Psi, \Gamma, \Theta, \Pi$ ) are functions of the trade and capital share matrices ( $\mathbf{S}, \mathbf{T}, \mathbf{B}, \mathbf{X}$ ) and model parameters ( $\psi, \theta, \beta, \epsilon, \mu_i$ ), as defined in Online Appendix I.2.2.

*Proof.* See Online Appendix I.2.2. □

We solve this matrix system of equations using the method of undetermined coefficients following Uhlig (1999) to obtain a closed-form solution for the evolution of the state variables  $\{\tilde{\mathbf{a}}_t\}_{t=1}^{\infty}$  in terms of an impact matrix ( $\mathbf{Q}$ ), which captures the initial impact of the fundamental shocks, and a transition matrix ( $\mathbf{P}$ ), which governs the updating of the state variables over time.

**Proposition 3. Transition Matrix.** Suppose that the economy at time  $t = 0$  is on a convergence path toward an initial steady state with constant fundamentals  $(z, \eta, \tau, \kappa)$ . At time  $t = 0$ , agents learn about one-time, permanent shocks to fundamentals  $(\tilde{\mathbf{f}} \equiv [\tilde{z} \ \tilde{\eta} \ \tilde{\kappa}^{in} \ \tilde{\kappa}^{out} \ \tilde{\tau}^{in} \ \tilde{\tau}^{out}]')$  from time  $t = 1$  onwards. There exists a  $N \times N$  transition matrix  $(\mathbf{P})$  and a  $N \times 6N$  impact matrix  $(\mathbf{R})$  such that the second-order difference equation system in (34) has the closed-form solution:

$$\tilde{\mathbf{a}}_t = \mathbf{P}\tilde{\mathbf{a}}_{t-1} + \mathbf{R}\tilde{\mathbf{f}}. \quad (35)$$

The transition matrix  $\mathbf{P}$  satisfies:

$$\mathbf{P} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^{-1},$$

where  $\mathbf{\Lambda}$  is a diagonal matrix of  $N$  stable eigenvalues  $\{\lambda_k\}_{k=1}^N$  and  $\mathbf{U}$  is a matrix stacking the corresponding  $N$  eigenvectors  $\{\mathbf{u}_k\}_{k=1}^N$ . The impact matrix  $(\mathbf{R})$  is given by:

$$\mathbf{R} = (\mathbf{\Psi}\mathbf{P} + \mathbf{\Psi} - \mathbf{\Gamma})^{-1} \mathbf{\Pi},$$

where  $(\mathbf{\Psi}, \mathbf{\Gamma}, \mathbf{\Theta}, \mathbf{\Pi})$  are the matrices from the system of second-order difference equations (34).

*Proof.* See Online Appendix I.3. □

The solutions for these impact and transition matrices  $(\mathbf{R}, \mathbf{P})$  depend only on the trade and capital share matrices  $(\mathbf{S}, \mathbf{T}, \mathbf{B}, \mathbf{X})$  and parameters  $(\psi, \theta, \beta, \epsilon, \mu_i)$ . Given this closed-form solution for the wealth state variables  $\{\tilde{\mathbf{a}}_t\}_{t=1}^\infty$ , we can recover all other endogenous variables (including capital  $\{\tilde{\mathbf{k}}_t\}_{t=1}^\infty$ ) as linear functions of these state variables, as shown in Online Appendix I.2.

### 2.8.3 Convergence Dynamics Versus Fundamental Shocks

Using Proposition 3, the transition path of the economy's state variables can be additively decomposed into the contributions of convergence dynamics given initial conditions and fundamental shocks. Applying equation (35) across time periods, we obtain:

$$\ln \mathbf{a}_t - \ln \mathbf{a}_{-1} = \underbrace{\sum_{s=0}^t \mathbf{P}^s (\ln \mathbf{a}_0 - \ln \mathbf{a}_{-1})}_{\text{convergence given initial fundamentals}} + \underbrace{\sum_{s=0}^{t-1} \mathbf{P}^s \mathbf{R}\tilde{\mathbf{f}}}_{\text{dynamics from fundamental shocks}} \quad \text{for all } t \geq 1. \quad (36)$$

In the absence of shocks to fundamentals  $(\tilde{\mathbf{f}} = \mathbf{0})$ , the second term on the right-hand side of equation (36) is zero. In this case, the evolution of the state variables is shaped solely by convergence dynamics given initial conditions, and converges over time to:

$$\ln \mathbf{a}_{\text{initial}}^* = \lim_{t \rightarrow \infty} \ln \mathbf{a}_t = \ln \mathbf{a}_{-1} + (\mathbf{I} - \mathbf{P})^{-1} (\ln \mathbf{a}_0 - \ln \mathbf{a}_{-1}), \quad (37)$$

where  $(\mathbf{I} - \mathbf{P})^{-1} = \sum_{s=0}^{\infty} \mathbf{P}^s$  is well-defined under the condition that the spectral radius of  $\mathbf{P}$  is smaller than one.

In contrast, if the economy is initially in a steady-state at time 0, the first term on the right-hand side of equation (36) is zero. In this case, the transition path of the state variables is solely driven by the second term for fundamental shocks, and follows:

$$\tilde{\mathbf{a}}_t = \ln \mathbf{a}_t - \ln \mathbf{a}_0 = \sum_{s=0}^{t-1} \mathbf{P}^s \mathbf{R} \tilde{\mathbf{f}} = (\mathbf{I} - \mathbf{P}^t) (\mathbf{I} - \mathbf{P})^{-1} \mathbf{R} \tilde{\mathbf{f}} \quad \text{for all } t \geq 1. \quad (38)$$

In period  $t = 1$  when the shocks occur, the response of the state variables is  $\tilde{\mathbf{a}}_1 = \mathbf{R} \tilde{\mathbf{f}}$ . Taking the limit as  $t \rightarrow \infty$  in equation (38), the comparative steady-state response is:

$$\lim_{t \rightarrow \infty} \tilde{\mathbf{a}}_t = \ln \mathbf{a}_{\text{new}}^* - \ln \mathbf{a}_{\text{initial}}^* = (\mathbf{I} - \mathbf{P})^{-1} \mathbf{R} \tilde{\mathbf{f}}. \quad (39)$$

A key implication of this additive separability in equation (36) is that we can examine the economy's dynamic response to fundamental shocks separately from its convergence towards an initial steady-state with unchanged fundamentals. Therefore, without loss of generality, we focus in the remainder of this section on an economy that is initially in steady-state.

#### 2.8.4 Spectral Analysis of the Transition Matrix $\mathbf{P}$

We now provide a further analytical characterization of the economy's dynamic response to fundamental shocks using a spectral analysis of the transition matrix. We show that the speed of convergence to steady-state and the evolution of the wealth state variables along the transition path can be written solely in terms of the eigenvalues and eigenvectors of this transition matrix. Since all other endogenous variables are linear functions of the wealth state variables, we also obtain impulse responses of each of the endogenous variables with respect to shocks to productivity and goods and capital market frictions.

**Eigendecomposition of the Transition Matrix** We begin by using the eigendecomposition of the transition matrix,  $\mathbf{P} \equiv \mathbf{U} \mathbf{\Lambda} \mathbf{V}$ , where  $\mathbf{\Lambda}$  is a diagonal matrix of eigenvalues arranged in decreasing order by absolute values, and  $\mathbf{V} = \mathbf{U}^{-1}$ . For each eigenvalue  $\lambda_h$ , the  $h$ -th column of  $\mathbf{U}$  ( $\mathbf{u}_h$ ) and the  $h$ -th row of  $\mathbf{V}$  ( $\mathbf{v}'_h$ ) are the corresponding right- and left-eigenvectors of  $\mathbf{P}$ , respectively, such that

$$\lambda_h \mathbf{u}_h = \mathbf{P} \mathbf{u}_h, \quad \lambda_h \mathbf{v}'_h = \mathbf{v}'_h \mathbf{P}.$$

That is,  $\mathbf{u}_h$  ( $\mathbf{v}'_h$ ) is the vector that, when left-multiplied (right-multiplied) by  $\mathbf{P}$ , is proportional to itself but scaled by the corresponding eigenvalue  $\lambda_h$ .<sup>4</sup> We refer to  $\mathbf{u}_h$  simply as eigenvectors.

<sup>4</sup>Note that  $\mathbf{P}$  need not be symmetric. This eigendecomposition exists if the transition matrix has distinct eigenvalues. We construct the right-eigenvectors such that the 2-norm of  $\mathbf{u}_h$  is equal to 1 for all  $h$ , where note that  $\mathbf{v}'_i \mathbf{u}_h = 1$  for  $i = h$  and  $\mathbf{v}'_i \mathbf{u}_h = 0$  otherwise.

Both  $\{\mathbf{u}_h\}$  and  $\{\mathbf{v}'_h\}$  are bases that span the  $N$ -dimensional state space.

**Eigen-shock** We next introduce a particular type of shock to fundamentals that proves useful for characterizing the model's transition dynamics. We define an *eigen-shock* as a shock to fundamentals ( $\tilde{\mathbf{f}}_{(h)}$ ) for which the initial impact of the shock on the state variables ( $\mathbf{R}\tilde{\mathbf{f}}_{(h)}$ ) coincides with a real eigenvector of the transition matrix ( $\mathbf{u}_h$ ). With  $N$  state variables ( $\tilde{\mathbf{a}}$ ) and  $6 \times N$  fundamental shocks ( $\tilde{\mathbf{z}}, \tilde{\boldsymbol{\eta}}, \tilde{\boldsymbol{\kappa}}^{in}, \tilde{\boldsymbol{\kappa}}^{out}, \tilde{\boldsymbol{\tau}}^{in}, \tilde{\boldsymbol{\tau}}^{out}$ ), the space of fundamental shocks is of higher dimension than the space of state variables. Therefore, many fundamental shocks generate identical time paths for the state variables. In fact, for any fundamental shock vector ( $\tilde{\mathbf{f}}$ ), there exists a productivity shock vector ( $\tilde{\mathbf{z}}$ ) that generates the same time path of the state variables. For expositional simplicity, we define the eigen-shocks in terms of shocks to productivity ( $\tilde{\mathbf{z}}$ ), setting all other shocks equal to zero.<sup>5</sup> Consequentially, the impact of the eigen-shocks  $\{\tilde{\mathbf{f}}_{(h)}\}_{h=1}^N$  form a basis that spans the  $N$ -dimensional state space. Each eigenvector of  $\mathbf{P}$  ( $\mathbf{u}_h$ ) has a corresponding eigen-shock for which  $\mathbf{R}\tilde{\mathbf{f}}_{(h)} = \mathbf{u}_h$ .

In general, there is no reason why any vector of empirical shocks to fundamentals across countries should correspond to an eigen-shock. But we can use these eigen-shocks to characterize the impact of any empirical shock using the following two properties. First, we can solve for these eigen-shocks from the observed data, because the impact matrix ( $\mathbf{R}$ ) and the transition matrix ( $\mathbf{P}$ ) depend solely on our observed trade and capital share matrices ( $\mathbf{S}, \mathbf{T}, \mathbf{B}, \mathbf{X}$ ) and the structural parameters of the model  $\{\psi, \theta, \beta, \epsilon, \mu_i\}$ . Second, the initial and dynamic impact on the state variables from any vector of empirical shocks to fundamentals ( $\tilde{\mathbf{f}}$ ) can be equivalently expressed as a linear combination of the impact from eigen-shocks ( $\tilde{\mathbf{f}}_{(h)}$ ), where the weights or loadings in this linear combination can be recovered from a linear projection (regression) of the initial impact from the observed shocks ( $\mathbf{R}\tilde{\mathbf{f}}$ ) on the initial impact from the eigen-shocks ( $\mathbf{R}\tilde{\mathbf{f}}_{(h)}$ ). Using this property, the transition path of the state variables in response to any vector of empirical shocks to fundamentals can be expressed solely in terms of the eigenvalues and eigenvectors of the transition matrix, as summarized in the following proposition.

**Proposition 4. Spectral Analysis.** *Consider an economy that is initially in steady-state at time  $t = 0$  when agents learn about one-time, permanent shocks to fundamentals ( $\tilde{\mathbf{f}} \equiv [\tilde{\mathbf{z}} \ \tilde{\boldsymbol{\eta}} \ \tilde{\boldsymbol{\kappa}}^{in} \ \tilde{\boldsymbol{\kappa}}^{out} \ \tilde{\boldsymbol{\tau}}^{in} \ \tilde{\boldsymbol{\tau}}^{out}]'$ ) from time  $t = 1$  onwards. The transition path of the state variables ( $\mathbf{a}_t$ ) can be written as a linear combination of the eigenvalues ( $\lambda_h$ ) and eigenvectors ( $\mathbf{u}_h$ ) of the*

<sup>5</sup>Recall from Proposition 2 that the dimension of the state space is  $N$ , whereas the dimension of the fundamental shocks is  $6 \times N$ , because  $\tilde{\mathbf{f}}$  includes shocks to goods and capital productivities ( $\tilde{\mathbf{z}}, \tilde{\boldsymbol{\eta}}$ ) and aggregations of bilateral shocks to trade and capital frictions ( $\tilde{\boldsymbol{\kappa}}^{in}, \tilde{\boldsymbol{\kappa}}^{out}, \tilde{\boldsymbol{\tau}}^{in}, \tilde{\boldsymbol{\tau}}^{out}$ ). Therefore, defining our eigen-shocks in terms of productivity shocks ( $\tilde{\mathbf{z}}$ ) ensures that each eigenvector is associated with a unique eigen-shock (up to scale).

transition matrix:

$$\tilde{\mathbf{a}}_t = \sum_{s=0}^{t-1} \mathbf{P}^s \mathbf{R} \tilde{\mathbf{f}} = \sum_{h=1}^{2N} \frac{1 - \lambda_h^t}{1 - \lambda_h} \mathbf{u}_h \mathbf{v}_h' \mathbf{R} \tilde{\mathbf{f}} = \sum_{h=2}^{2N} \frac{1 - \lambda_h^t}{1 - \lambda_h} \mathbf{u}_h \varrho_h, \quad (40)$$

where the weights in this linear combination ( $\varrho_h$ ) can be recovered as the coefficients in a linear projection (regression) of the initial impact from the observed shocks ( $\mathbf{R} \tilde{\mathbf{f}}$ ) on the initial impact from the eigen-shocks ( $\mathbf{R} \tilde{\mathbf{f}}_{(h)}$ ).

*Proof.* The proposition follows from the eigendecomposition of the transition matrix:  $\mathbf{P} \equiv \mathbf{U} \Lambda \mathbf{V}$ , as shown in Online Appendix I.4.  $\square$

Another important property of an eigen-shock is that the speed of convergence to steady-state, as measured by the half-life of convergence to steady-state, depends solely on the associated eigenvalue of the transition matrix, as summarized in the following proposition.

**Proposition 5. Speed of Convergence.** *Consider an economy that is initially in steady-state at time  $t = 0$  when agents learn about one-time, permanent shocks to fundamentals ( $\tilde{\mathbf{f}} \equiv [\tilde{\mathbf{z}} \ \tilde{\boldsymbol{\eta}} \ \tilde{\boldsymbol{\kappa}}^{in} \ \tilde{\boldsymbol{\kappa}}^{out} \ \tilde{\boldsymbol{\tau}}^{in} \ \tilde{\boldsymbol{\tau}}^{out}]'$ ) from time  $t = 1$  onwards. Suppose that these shocks are an eigen-shock ( $\tilde{\mathbf{f}}_{(h)}$ ), for which the initial impact on the state variables at time  $t = 1$  coincides with a real eigenvector ( $\mathbf{u}_h$ ) of the transition matrix ( $\mathbf{P}$ ):  $\mathbf{R} \tilde{\mathbf{f}}_{(h)} = \mathbf{u}_h$ . The transition path of the state variables ( $\mathbf{a}_t$ ) in response to such an eigen-shock ( $\tilde{\mathbf{f}}_{(h)}$ ) is :*

$$\tilde{\mathbf{a}}_t = \sum_{j=2}^{2N} \frac{1 - \lambda_j^t}{1 - \lambda_j} \mathbf{u}_j \mathbf{v}_j' \mathbf{u}_h = \frac{1 - \lambda_h^t}{1 - \lambda_h} \mathbf{u}_h \quad \implies \quad \ln \mathbf{a}_{t+1} - \ln \mathbf{a}_t = \lambda_h^t \mathbf{u}_h,$$

and the half-life of convergence to steady-state is given by:

$$t_h^{(1/2)}(\tilde{\mathbf{f}}) = - \left\lceil \frac{\ln 2}{\ln \lambda_h} \right\rceil,$$

for all state variables  $h = 2, \dots, 2N$ , where  $\tilde{a}_{i\infty} = a_{i,new}^* - a_{i,initial}^*$ , and  $\lceil \cdot \rceil$  is the ceiling function.

*Proof.* The proposition follows from the eigendecomposition of the transition matrix ( $\mathbf{P} \equiv \mathbf{U} \Lambda \mathbf{V}$ ), for the case of an eigen-shock in which the initial impact of the shocks to fundamentals on the state variables at time  $t = 1$  coincides with a real eigenvector ( $\mathbf{R} \tilde{\mathbf{f}}_{(h)} = \mathbf{u}_h$ ) of the transition matrix ( $\mathbf{P}$ ), as shown in Online Appendix I.5.  $\square$

We focus on the speed of convergence for wealth in Proposition 5, because the log deviations in wealth from the initial steady-state ( $\tilde{\mathbf{a}}$ ) are the state variables of our dynamical system. Nevertheless, since the log deviations in all other endogenous variables from steady-state (including capital,  $\tilde{\mathbf{k}}$ ) are linear functions of these log deviations in wealth, these other endogenous variables have the same convergence properties as wealth.

From Proposition 5, the impact of an eigen-shock ( $\tilde{\mathbf{f}}_{(h)}$ ) on the state variables in each time period is always proportional to the corresponding eigenvector ( $\mathbf{u}_h$ ), and decays exponentially at a rate determined by the associated eigenvalue ( $\lambda_h$ ), as the economy converges to the new steady-state.<sup>6</sup> These eigenvalues fully summarize the economy's speed of convergence in response to eigen-shocks, even in our setting with many asymmetric countries and a rich geography of trade and capital market frictions.

In general, each eigen-shock ( $\tilde{\mathbf{f}}_{(h)}$ ) has a different speed of convergence (as captured by the associated eigenvalue  $\lambda_h$ ), which reflects the fact that the speed of convergence to steady-state does not only depend on the structural parameters of the model ( $\psi, \theta, \beta, \epsilon, \mu_i$ ), but also on the incidence of the fundamental shock on the state variables in each country (as captured by  $\mathbf{u}_h = \mathbf{R}\tilde{\mathbf{f}}_{(h)}$ ). From Proposition 4, any empirical shock ( $\tilde{\mathbf{f}}$ ) can be expressed as a linear combination of the eigen-shocks. Therefore, the speed of convergence also varies across these empirical shocks with their incidence on the state variables in each country, reflecting the extent to which they load on eigen-shocks with slow versus fast convergence.

### 2.8.5 Goods and Capital Market Integration and Convergence

We now use our analytical results for the economy's transition path to examine the role of goods and capital market integration in determining the speed of convergence in response to fundamental shocks. To simplify the exposition, we begin by considering the special case of the model with a separation between (i) workers, who earn wage income and live hand to mouth, and (ii) capitalists, who have log utility and make forward-looking consumption-saving decisions. We later generalize our analysis to a representative agent and CRRA preferences.

In this special case of a separation between workers and capitalists, capitalists with log utility consume a constant fraction ( $1 - \beta$ ) of their capital wealth every period. Therefore, the evolution of the log deviations in the wealth state variables from steady-state simplifies as follows:

$$\tilde{a}_{nt+1} - \tilde{a}_{nt} = (1 - \beta + \beta\delta)(\tilde{v}_{nt} - \tilde{p}_{nt}), \quad (41)$$

where the derivations for this subsection are reported in Online Appendix J.

A common measure of the speed of convergence is the slope coefficient from a regression of log changes on log initial levels of a variable, as in a conventional  $\beta$ -convergence regression from the growth literature. From equation (41), this measure of the speed of convergence for log deviations in wealth depends on the covariance between the log deviation in the real return to

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<sup>6</sup>In general, these eigenvectors and eigenvalues can be complex-valued. If the initial impact is the real part of a complex eigenvector  $\mathbf{u}_h$  ( $\mathbf{R}\tilde{\mathbf{f}} = \text{Re}(\mathbf{u}_h)$ ), then  $\ln \mathbf{a}_{t+1} - \ln \mathbf{a}_t = \text{Re}(\lambda_h^t \mathbf{u}_h) \neq \text{Re}(\lambda_h) \cdot \text{Re}(\lambda_h^{t-1} \mathbf{u}_h)$ . That is, the impact no longer decays at a constant rate  $\lambda_h$ . Instead, the complex eigenvalues introduce oscillatory motion as the dynamical system converges to the new steady-state. In our empirical application, the imaginary components of  $\mathbf{P}$ 's eigenvalues are small, implying that oscillatory effects are small relative to the effects that decay exponentially.

investment ( $\tilde{v}_{nt} - \tilde{p}_{nt}$ ) and the log deviation in the initial level of wealth ( $\tilde{a}_{nt}$ ):

$$\frac{\text{Cov}(\tilde{a}_{nt+1} - \tilde{a}_{nt}, \tilde{a}_{nt})}{\text{Var}(\tilde{a}_{nt})} = (1 - \beta + \beta\delta) \frac{\text{Cov}(\tilde{v}_{nt} - \tilde{p}_{nt}, \tilde{a}_{nt})}{\text{Var}(\tilde{a}_{nt})}. \quad (42)$$

These log deviations in the real return to investment ( $\tilde{v}_{nt} - \tilde{p}_{nt}$ ) depend on both capital market integration (through the nominal return to investment ( $\tilde{v}_{nt}$ )) and goods market integration (through the consumption price index ( $\tilde{p}_{nt}$ )). The log deviations in the nominal return to investment ( $\tilde{v}_{nt}$ ) in turn depend on log deviations in rental rates ( $\tilde{r}_{nt}$ ). Using the first-order condition for cost minimization in production, and assuming a common labor share ( $\mu$ ) across countries and a constant labor endowment in each country ( $\ell_n$ ), we have the following relationship between log deviations in the rental rate ( $\tilde{r}_{nt}$ ) and log deviations in the capital stock ( $\tilde{k}_{nt}$ ):

$$\tilde{r}_{nt} = \tilde{p}_{nnt} - \mu\tilde{k}_{nt}, \quad (43)$$

where  $\tilde{p}_{nnt}$  is the log deviation in the local price of a country's own good from steady-state (recall  $\tau_{nnt} = 1$ ), and in general differs from the log deviation in the consumption price index ( $\tilde{p}_{nt}$ ) that is a CES aggregate of the goods produced by all countries.

To provide economic intuition for the impact of goods and capital market integration on the speed of convergence, we evaluate this measure of the speed of convergence (42) for the limiting cases of completely open and completely closed goods and capital markets.

**CNGM (Trade and Capital Autarky)** Under capital autarky ( $\kappa_{nit} \rightarrow \infty$  for  $n \neq i$ ), each country's wealth equals its capital stock ( $\tilde{a}_{nt} = \tilde{k}_{nt}$ ), and the nominal return to investment equals the domestic rental rate ( $\tilde{v}_{nt} = \tilde{r}_{nt}$ ). Under trade autarky ( $\tau_{nit} \rightarrow \infty$  for  $n \neq i$ ), the consumption price index equals the local price of a country's own good ( $\tilde{p}_{nt} = \tilde{p}_{nnt}$ ). Using these results in equations (41)-(43), we find that with a Cobb-Douglas production technology and a common labor share ( $\mu$ ), the speed of convergence to steady-state depends solely on this labor share:

$$\frac{\text{Cov}(\tilde{v}_{nt} - \tilde{p}_{nt}, \tilde{a}_{nt})}{\text{Var}(\tilde{a}_{nt})} = -\mu. \quad (44)$$

Intuitively, there is diminishing marginal physical productivity of capital in the production technology. The larger the labor share ( $\mu$ ), the stronger these diminishing marginal returns to capital, and the faster the rate of convergence in capital and wealth towards steady-state.

**Free Trade and Capital Autarky** Under capital autarky ( $\kappa_{nit} \rightarrow \infty$  for  $n \neq i$ ), each country's wealth equals its capital stock ( $\tilde{a}_{nt} = \tilde{k}_{nt}$ ), and the nominal return to investment equals the domestic rental rate ( $\tilde{v}_{nt} = \tilde{r}_{nt}$ ). With free trade ( $\tau_{nit} = 1$  for all  $n, i$ ), the consumption price index takes the same value across all countries ( $\tilde{p}_{nt} = \tilde{p}_t$  for all  $n$ ). But the local price of a

country's own good can differ from the consumption price index ( $\tilde{p}_{nnt} \neq \tilde{p}_{nt}$ ), because countries' goods are imperfect substitutes ( $1 < \sigma < \infty$ ). Using these results in equations (41)-(43), free trade in goods alone implies *faster* convergence than in the CNGM:

$$\frac{\text{Cov}(\tilde{v}_{nt} - \tilde{p}_{nt}, \tilde{a}_{nt})}{\text{Var}(\tilde{a}_{nt})} = -\frac{1}{\sigma} (1 - \mu) - \mu. \quad (45)$$

Intuitively, with autarkic capital markets, wealth accumulation in a given country expands its capital stock, which raises output of its good. With free trade in goods, in order for consumers worldwide to demand more of this good instead of the substitutes produced by other countries, the price of this good must fall. Therefore, wealth accumulation not only leads to a decline in the marginal physical product of capital as in the closed economy (captured by  $\mu$ ), but also leads to a fall in the price of a country's good (with an elasticity determined by  $\sigma$ ), which implies a larger decline in the value marginal product of capital, and faster convergence to steady-state.

**Trade Autarky and Free Capital** Under trade autarky ( $\tau_{nit} \rightarrow \infty$  for  $n \neq i$ ), the consumption price index equals the price of a country's domestic good ( $\tilde{p}_{nt} = \tilde{p}_{nnt}$ ). Under free capital ( $\kappa_{nit} = 1$  for all  $n, i$ ), the nominal return to investment takes the same value across all countries ( $\tilde{v}_{nt} = \tilde{v}_t$  for all  $n$ ). But the domestic capital stock can differ from domestic wealth ( $\tilde{k}_{nt} \neq \tilde{a}_{nt}$ ), and the domestic rental rate can differ from the nominal return to investment ( $\tilde{r}_{nt} \neq \tilde{v}_{nt}$ ), because of imperfect substitutability of capital between countries ( $1 < \epsilon < \infty$ ). Using these results in equations (41)-(43), free capital flows alone imply *faster* convergence than in the CNGM:

$$\frac{\text{Cov}(\tilde{v}_{nt} - \tilde{p}_{nt}, \tilde{a}_{nt})}{\text{Var}(\tilde{a}_{nt})} = -\frac{1}{\epsilon} (1 - \mu) - \mu. \quad (46)$$

Intuitively, with free capital flows, capital reallocates across countries to equalize the nominal return to investment for a given world stock of wealth. Nevertheless, countries accumulate wealth at different rates, because of differences in the real consumption price index under trade autarky, which lead to differences in the real rate of return to investment. Under free capital flows, wealth accumulation in a given country expands investment at home and abroad, which raises the country's income from these investments. Under trade autarky, this increased country income is spent on domestic goods, which bids up the domestic factor prices, where the elasticity of the domestic rental rate with respect to this expenditure depends on  $\epsilon$ . Higher domestic factor prices raise the price of the domestic good, and hence the domestic consumption price index, which reduces the real return to investment, and speeds up convergence to steady-state.

**Free Trade and Free Capital** Under free trade ( $\tau_{nit} = 1$  for all  $n, i$ ), the consumption price index takes the same value across all countries ( $\tilde{p}_{nt} = \tilde{p}_t$  for all  $n$ ). Under free capital flows

( $\kappa_{nit} = 1$  for all  $n, i$ ), the nominal return to investment takes the same value across all countries ( $\tilde{v}_{nt} = \tilde{v}_t$  for all  $n$ ). Using these results in equations (41)-(43), the real return to investment takes the same value across all countries ( $\tilde{v}_{nt} - \tilde{p}_{nt} = \tilde{v}_t - \tilde{p}_t$  for all  $n$ ), and hence is uncorrelated with the initial level of wealth in each country ( $\tilde{a}_{nt}$ ). Therefore, free trade and free capital together imply *slower* convergence to steady-state than in the CNGM:

$$\frac{\text{Cov}(\tilde{v}_{nt} - \tilde{p}_{nt}, \tilde{a}_{nt})}{\text{Var}(\tilde{a}_{nt})} = 0. \quad (47)$$

Intuitively, with free trade and free capital, movements of goods and capital between countries equalize the real return to investment for a given world stock of wealth. Wealth accumulation in each country only affects this common real return to investment ( $\tilde{v}_{nt} - \tilde{p}_{nt} = \tilde{v}_t - \tilde{p}_t$ ) through the world stock of wealth. Therefore, each country accumulates wealth at the same rate, as determined by this common real return to investment, and initial differences in wealth persist forever, as the world economy gradually converges to the world steady-state level of wealth.

For expositional simplicity, we have examined the speed of convergence in this section by comparing autarky and frictionless trade and capital flows for the special case of a separation of workers and capitalists with logarithmic utility. The following proposition extends these results for a representative agent and CRRA utility.

**Proposition 6. Goods and Capital Market Integration.** *The speed of convergence to steady-state is **faster** than in the closed-economy neoclassical growth model (CNGM) with **either** (i) free trade and capital autarky **or** (ii) trade autarky and free capital. This speed of convergence is **slower** than in the CNGM with (iii) **both** free trade and free capital.*

*Proof.* See Online Appendix J. □

More generally, for intermediate levels of trade and capital market frictions in between autarky and free trade, we again find that reductions in *both* goods *and* capital market frictions slow convergence, whereas reductions in *either* goods *or* capital market frictions *alone* accelerate convergence.<sup>7</sup> The intuition is that reductions in both frictions lead to a reallocation of capital across countries that reduces differences in the real return to investment, and hence leads to slower convergence to steady-state after this reallocation. In contrast, when *either* goods *or* capital market frictions *alone* are reduced, this closure of gaps in the real return to investment does not occur, and the real return to investment becomes more sensitive to local wealth, implying faster convergence.

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<sup>7</sup>Proposition 7 in Online Appendix J.2.1 further extends the results in this section by considering this case of intermediate levels of trade and capital market frictions, and evaluating the impact of marginal changes in these frictions on the speed of convergence.

## 2.9 Three-Country Example

We now illustrate these general results using a simple example. Consider a world with three economies—call them US, EU, and China—and a common labor share across countries. The first two economies (US and EU) are relatively more integrated in terms of both trade and capital flows; the third economy, China, is relatively more remote. Suppose the trade expenditure share and capital allocation share matrices ( $\mathbf{S}$  and  $\mathbf{B}$ ) take the following form:

$$\mathbf{S} = \mathbf{B} = \begin{bmatrix} .8 - x & .1 + x & .1 \\ .1 + x & .8 - x & .1 \\ .1 & .1 & .8 \end{bmatrix}, \quad x \in [0, 0.35]$$

That is, 80% of China's expenditure and capital wealth are allocated domestically, with the remaining 20% equally split between the US and the EU. The US and the EU are symmetric; they each allocate  $10\% + x$  expenditure and capital wealth to each other,  $80\% - x$  domestically, and the remaining 10% to China. The value of  $x$  captures the degree of integration between the EU and the US: when  $x = 0$ , all three economies are symmetric; when  $x = 0.35$ , there is free trade and free capital flows between the US and the EU.

The three eigenvectors in this example take the following forms:

$$\mathbf{u}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{u}_2 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \quad \mathbf{u}_3 = \begin{bmatrix} -\gamma \\ -\gamma \\ 1 \end{bmatrix}.$$

The associated eigenshocks, when expressed in terms of shocks to productivities, take the following forms:

$$\tilde{\mathbf{f}}_1 \equiv \begin{bmatrix} \tilde{z}_1 \\ \tilde{z}_2 \\ \tilde{z}_3 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}, \quad \tilde{\mathbf{f}}_2 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \quad \tilde{\mathbf{f}}_3 = \begin{bmatrix} \xi \\ \xi \\ -1 \end{bmatrix}.$$

The associated eigenvalues, as well as the value of  $\gamma$  and  $\xi$  (in  $\mathbf{u}_3$  and  $\tilde{\mathbf{f}}_3$ ), all depend on the model parameters  $\{\beta, \psi, \delta, \theta, \epsilon\}$ , the labor share  $\mu$ , and the degree of integration between the US and the EU,  $x$ .

We now interpret these eigencomponents. The first component captures a global productivity shock that is common to all countries in the world. If the world economy was originally in a steady-state, the shock  $\tilde{\mathbf{f}}_1$  would induce capital dynamics around the globe in a homogeneous manner—capital wealth would evolve proportionally across countries, with the share of global capital wealth belonging to each country staying constant along the entire transition path. In other words, the shock  $\tilde{\mathbf{f}}_1$  does not lead to reallocation of capital across countries; instead, the total global capital wealth behaves as in the CNGM, with the associated eigenvalue coinciding with that of the CNGM.

The second eigen-shock  $\tilde{f}_2$  captures rising dispersion in productivities of the two well-integrated economies, while holding economic fundamentals in the more remote country (China) constant. This shock induces dynamic reallocation of wealth within the integrated block, while leaving wealth in China unchanged along the entire transition path.

Finally, the third eigen-shock  $\tilde{f}_3$  captures rising dispersion in productivities and the resulting reallocation of wealth between the integrated block (US and EU) and China; the relative wealth between the US and EU remains unchanged along the entire transition path following this shock.

The eigenvalues associated with the second and third components capture the rate at which these shocks affect the distribution of capital across countries. Numerically, we find that  $|\lambda_2| > |\lambda_3|$ , meaning the reallocation within the integrated block (i.e., between US and EU) is slower than the reallocation between the integrated block and China. Moreover, we find that  $|\lambda_2|$  is increasing in  $x$ , meaning that the reallocation of wealth between the US and the EU gets slower the more integrated they are. The reason is again that greater integration leads to a larger initial reallocation of wealth, which weakens the correlation between the real return to investment and the initial level of wealth, and hence leads to slower convergence towards steady-state.

### 3 Quantitative Analysis

We now examine the quantitative implications of costly goods trade and capital flows with imperfect substitutability. In Subsection 3.1, we discuss our data sources and the parameterization of our model. In Subsection 3.3, we confirm that the gravity equation provides a good approximation to trade in goods and capital holdings. In Subsection 3.4, we compare impulse responses to productivity shocks in our framework with those in the special cases of the conventional closed and open-economy neoclassical growth models. In Subsection 3.5, we compare the speed of convergence to steady-state in our model and the CNGM. In Subsection 3.6, we undertake counterfactuals for a decoupling of China and the United States in goods and capital markets.

#### 3.1 Data

We quantify the model using data on national accounts, bilateral trade, and bilateral investment. Our sample covers 47 countries plus the rest of the world for the period 2013–2017. Online Appendix L provides further details on the construction of the bilateral investment data.

**National Accounts** The data on gross domestic product (GDP), the labor share (i.e., labor compensation as a share of GDP), the capital stock, and the population are from the Penn World Tables (Feenstra et al., 2015). We map the model to these data as  $\mu_{nt}$  for the labor share,  $k_{nt}$  for the capital stock, and  $\ell_{nt}$  for the population. We construct the wage  $w_{it}$  as the labor compensation divided

by population. We construct the capital payment  $r_{it}k_{it}$  as GDP minus the labor compensation. Then the rental rate of capital  $r_{it}$  is the capital payment divided by the capital stock.

**Bilateral Trade** The data on bilateral trade are from the Comtrade Database ([United Nations, 2013–2017](#)). Following [Feenstra et al. \(2005\)](#), we use the import data, which are more likely to be accurate than the export data because importers typically levy trade policies. We construct own-country expenditure as the gross output from [Timmer et al. \(2015\)](#) minus total exports. Thus, we have a matrix of bilateral expenditure  $E_{nit}$  by importer  $n$  on goods produced by exporter  $i$ , including own-country expenditure as the diagonal elements. We construct the bilateral expenditure shares of importer  $n$  as  $S_{nit} = E_{nit} / \sum_{h=1}^N E_{nht}$ . We construct the bilateral income shares of exporter  $i$  as  $T_{int} = E_{nit} / \sum_{h=1}^N E_{hit}$ .

To estimate gravity equations for bilateral trade and capital holdings, we use the bilateral distance between countries from the GeoDist Database ([Mayer and Zignago, 2011](#)). We use the simple distance, defined as the weighted distance between the most populous cities.

**Bilateral Investment** We construct a comprehensive measure of bilateral investment as the sum of bilateral portfolio investment (in debt securities, equity securities, and fund shares) and bilateral direct investment. We construct the total amounts outstanding in debt securities, equity securities, and fund shares, based on [OECD \(2013–2017\)](#) for the OECD countries and [Bank for International Settlements \(2013–2017\)](#) and [World Bank \(2013–2017\)](#) for the non-OECD countries. The availability of the data on total amounts outstanding limits our sample to 47 countries. Our data on bilateral portfolio investment are from the Coordinated Portfolio Investment Survey ([International Monetary Fund, 2013–2017](#)). To account for investments through tax havens, we restate both the total amounts outstanding and bilateral portfolio investment from the issuer’s residency to nationality, using the restatement matrices of the Global Capital Allocation Project ([Coppola et al., 2021](#)). We construct own-country portfolio investment as the amount outstanding minus the sum of foreign portfolio investment. Our data on bilateral direct investment, restated from residency to nationality accounting, are from [Damgaard et al. \(2019\)](#). The availability of the data on restated bilateral direct investment limits our sample period to 2013–2017.

We construct bilateral ownership shares as bilateral portfolio and direct investment as a share of the total investment in each producer country. We multiply the capital payment  $r_{it}k_{it}$  by the bilateral ownership shares to construct the capital income  $a_{nit}v_{nit}$  earned by investor  $n$  in producer  $i$ . We then construct capital income earned by investor  $n$  in the rest of the world (ROW) as the residual:  $a_{n,ROW,t}v_{n,ROW,t} = a_{nt}v_{nt} - \sum_{i \neq ROW} a_{nit}v_{nit}$ . Thus, the bilateral capital income sums to total capital income  $a_{nt}v_{nt}$  by investor country and total capital payment  $r_{it}k_{it}$  by producer country. We construct the bilateral investment shares of investor  $n$  as  $B_{nit} = a_{nit}v_{nit} / \sum_{h=1}^N a_{nht}v_{nht}$ . We construct the bilateral capital payment shares of producer  $i$  as  $X_{int} = a_{nit}v_{nit} / \sum_{h=1}^N a_{hit}v_{hit}$ .

## 3.2 Parameterization

To quantify the implications of introducing costly trade and capital flows with imperfect substitutability, we assume standard parameter values from the existing empirical literature. We assume a discount factor equal to  $\beta = 0.95$ ; an intertemporal elasticity of substitution of  $\psi = 0.5$ ; a depreciation rate of  $\delta = 0.05$ ; and we set the labor share ( $\mu_{it}$ ) equal to its empirical value in the data for each country, as discussed above. We assume a trade elasticity of  $\theta = 5$ , which lies in the center of the range from 2-12 considered in [Eaton and Kortum \(2002\)](#), and is the baseline value used in [Costinot and Rodríguez-Clare \(2014\)](#). We assume a capital elasticity of  $\epsilon = 4$ , based on the estimates of [Koijen and Yogo \(2020\)](#).

## 3.3 Gravity in Trade and Capital Holdings

We now confirm that the gravity equation provides a good approximation to observed data on goods trade and capital holdings. We estimate the following gravity equation specification between countries for a single year:

$$Y_{ni} = \vartheta_i^O \vartheta_n^D \text{dist}_{ni}^\delta u_{ni}, \quad (48)$$

where  $Y_{ni}$  is expenditure of importer  $n$  on exporter  $i$  ( $E_{ni}$ ) or the capital holdings of investor  $n$  in producer  $i$  ( $H_{ni}$ );  $\vartheta_i^O$  is an origin fixed effect;  $\vartheta_n^D$  is a destination fixed effect;  $\text{dist}_{ni}$  is bilateral distance; and  $u_{ni}$  is a stochastic error. We report standard errors clustered by origin and destination.

Table 2: Gravity Equation Regressions

	(1)	(2)	(3)	(4)
	Log Trade	Trade	Log Capital	Capital
Log Distance	-1.181 (0.0244)	-0.786 (0.0280)	-1.409 (0.0466)	-0.627 (0.0512)
Estimation	OLS	PPML	OLS	PPML
Origin and Destination Fixed Effects	Yes	Yes	Yes	Yes
Observations	2069	2070	2042	2070
R-squared	0.878		0.821	
Pseudo R-squared		0.911		0.917

Note: Cross-section of origin and destination countries in 2017; all columns include origin and destination fixed effects (FEs); Columns (1)-(2) show results for bilateral trade; Columns (3)-(4) report results for bilateral capital holdings; Columns (1) and (3) estimated in logs using ordinary least squares (OLS); Columns (2) and (4) estimated using the Poisson Pseudo Maximum Likelihood (PPML) estimator; standard errors two-way clustered by origin and destination.

In Column (1) of Table 2, we report the results of taking logs in equation (48) and estimating this gravity equation for international trade using ordinary least squares (OLS) with origin and destination fixed effects. In line with existing evidence, we find a negative and highly significant

relationship between bilateral trade and distance, with an elasticity of around minus one, and a regression R-squared of around 85 percent. We next show that these findings are not sensitive to the dropping of zeros when we take logs. In Column (2), we demonstrate the same pattern of results if we estimate this gravity equation in levels using the Poisson Pseudo Maximum Likelihood (PPML) estimator, as in Santos Silva and Tenreyro (2006) and Head and Mayer (2014). Again we find a negative and highly statistically significant coefficient on bilateral distance that is only marginally smaller than that in Column (1).

In Column (3), we estimate the log linear specification for international capital holdings. Although capital holdings are not subject to transportation costs in the way that goods flows are, we again find a negative and highly statistically significant coefficient on distance, and a regression R-squared of around 80 percent. In Column (4), we show that we find the same pattern of results using the Poisson Pseudo Maximum Likelihood (PPML) estimator.

While Table 2 provides overall evidence on the explanatory power of the gravity equation for trade and capital holdings, it does not reveal the relative importance of bilateral distance and the fixed effects for this explanatory power. To separate out the contribution from bilateral distance, we use the Frisch-Waugh-Lovell Theorem. We run two separate OLS regressions of log values and log distance on origin and destination fixed effects, generate the two residuals, and then regress these two residuals on one another. As shown in Section K.1 of the Online Appendix, we find that bilateral distance has a similar explanatory power for capital holdings as for trade, even after removing the origin and destination fixed effects.

Taken together, these findings provide support for the gravity equation predictions of our model of costly trade and capital flows with imperfect substitutability between countries.

### 3.4 Impulse Responses to Productivity Shocks

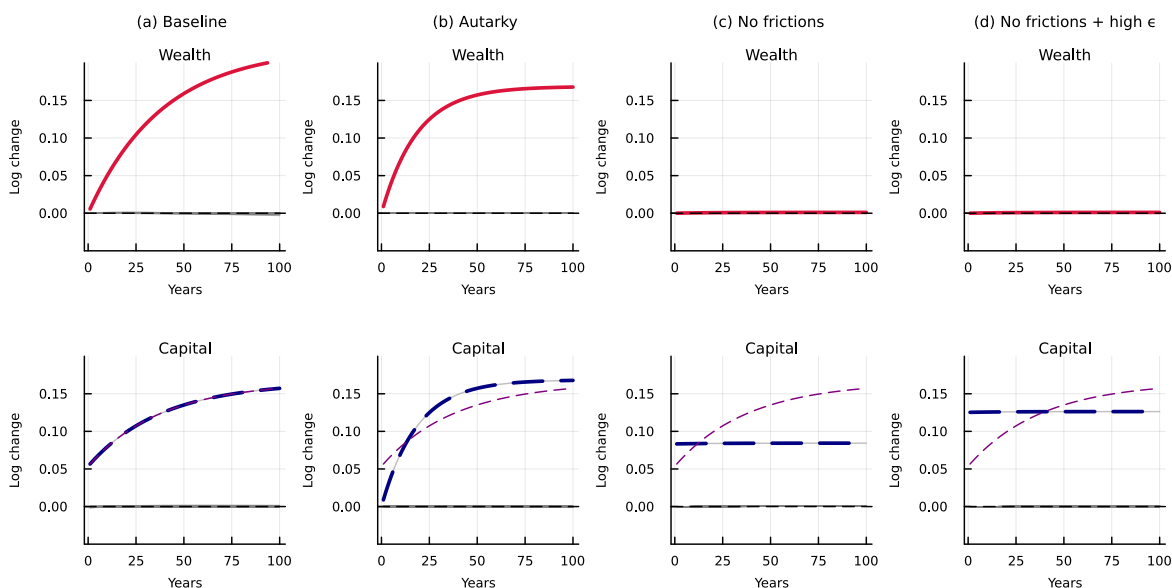
We now use our closed-form solutions for the economy's transition path from Proposition 4 to compare impulse responses to productivity shocks in our framework with those in the conventional closed and open-economy neoclassical growth models.

Figure 1 shows these impulse responses for wealth (top row) and capital (bottom row) with respect to a 10 percent productivity shock in a country that is small relative to global aggregates (Belgium). We display these impulse responses for our baseline model of costly goods trade and capital flows with imperfect substitutability (panel (a)); the CNGM (panel (b)); the special case of our model with no goods or capital market frictions and imperfect capital substitutability:  $\tau_{ni} = \tau_{nn} = \kappa_{ni} = \kappa_{nn} = 1$  and  $\epsilon = 3.15$  (panel (c)); and the conventional open-economy neoclassical growth model (panel (d)), which corresponds to the special case of our model with no goods or capital market frictions ( $\tau_{ni} = \tau_{nn} = \kappa_{ni} = \kappa_{nn} = 1$ ) and perfect capital substitutability ( $\epsilon \rightarrow \infty$ ).

The solid red line shows the transition path for the small country’s wealth (top row), while the dashed blue line shows the transition path for the small country’s capital stock (bottom row). The transition paths for wealth and capital for all other countries are shown by gray lines, which are barely distinguishable from the black horizontal axis, because the country experiencing the productivity shock is small relative to global aggregates. For ease of comparison, we reproduce the transition path for the small country’s capital stock in our baseline model (purple dashed line) across each of the panels in the bottom row.

In the CNGM (panel (b)), the positive productivity shock raises the small country’s steady-state levels of wealth and capital, but has no effect on the steady-state levels of wealth and capital in other countries, because of autarkic goods and capital markets. In the CNGM, domestic capital only can be accumulated through domestic wealth accumulation. Therefore, consumption smoothing implies a gradual accumulation of capital and wealth in the small country along the transition path to steady-state, at a rate determined by diminishing marginal returns to capital in the production technology.

Figure 1: Small Country Productivity Shock



Note: Impulse responses to a 10 percent productivity shock in a small country (Belgium) for our baseline parameter values; first panel from the left (panel (a)) shows impulse responses in our baseline model with costly trade and capital flows ( $\tau_{ni} > 1$  and  $\kappa_{ni} > 1$  for  $n \neq i$ ) and imperfect substitutability ( $0 < \sigma < \infty$  and  $0 < \epsilon < \infty$ ); second panel from the left (panel (b)) shows impulse responses in the CNGM; third panel from the left (panel (c)) shows impulse responses with no trade and capital market frictions ( $\tau_{ni} = \kappa_{ni} = 1$ ) and imperfect substitutability ( $0 < \sigma < \infty$  and  $0 < \epsilon < \infty$ ); fourth panel from the left (panel (d)) shows impulse responses with no trade and capital market frictions ( $\tau_{ni} = \kappa_{ni} = 1$ ) and imperfect substitutability in goods markets ( $0 < \sigma < \infty$ ) but a perfectly elastic supply of capital ( $\epsilon \rightarrow \infty$ ); the red line in the top row shows impulse responses for wealth in Belgium; the dashed blue line in the bottom row shows impulse responses for capital in Belgium; the purple dashed line in the bottom row reproduces the impulse responses for capital in Belgium for our baseline model (panel (a)) in all other panels for ease of comparison.

In the special case of our framework with no frictions in goods and capital markets and im-

perfect capital substitutability (panel (c)), the positive productivity shock leads to an immediate international reallocation of capital. With no frictions in goods and capital markets, the representative agent in each country holds the same global portfolio. Since the country experiencing the productivity shock is small relative to global aggregates, this productivity shock has a negligible effect on this global portfolio, and hence a negligible effect on the steady-state level of wealth in each country (top row). Additionally, the immediate international reallocation of capital implies that the small country converges instantaneously to its new steady-state capital stock (bottom row). In contrast, there is a negligible effect on the steady-state capital stock in all other countries (barely visible gray lines along the horizontal axis), because the country experiencing the productivity shock is small relative to global aggregates.

In the special case of our framework with no frictions in goods and capital markets and perfect capital substitutability (panel (d)), we obtain the limiting case of the conventional open-economy neoclassical growth model. Following the positive productivity shock, the small country again converges instantaneously to its new steady-state capital stock (bottom row). With greater substitutability of capital in panel (d) than in panel (c), there is a larger instantaneous adjustment in the small country's steady-state capital stock.

More generally, in our baseline model with costly goods trade and capital flows and imperfect substitutability (panel (a)), the predicted impacts of the productivity shock lie in between the extremes of the closed and the open-economy neoclassical growth models. On impact, the positive productivity shock raises the rental rate and real return to investment in the small country, which induces a capital flow from other countries, whose magnitude depends on the degree of capital substitutability. This capital inflow dampens the impact of the productivity shock on the rental rate and real return to investment in the small country compared to the CNGM. Nevertheless, with goods and capital market frictions and imperfect substitutability, this initial reallocation of capital does not fully arbitrage away differences in the real return to investment. Therefore, there is a gradual accumulation of wealth and capital in the small country, until in the new steady-state the real return to investment is equalized across all countries. This gradual accumulation of wealth and capital in the small country occurs more slowly than in the CNGM (compare the dashed purple and blue lines in the bottom row), because the initial reallocation of capital dampens the impact of the productivity shock on the rental rate and real return to investment.

While we focus on a small country in Figure 1 to simplify the exposition, we find a similar pattern of results for a large country, as shown in Online Appendix K.3. The key difference is that a productivity shock in a large country affects allocations in other countries. With frictions in goods and capital markets, these effects on other countries are heterogeneous, depending on the network of trade and capital market frictions.

### 3.5 Speed of Convergence

We next use our closed-form solutions for the economy's transition path to analyze the implications of costly trade in goods and capital flows with imperfect substitutability for the speed of convergence to steady-state.

We begin by using our eigendecomposition in Proposition 3 to recover the eigenvectors ( $\mathbf{u}_h$ ) and eigenvalues ( $\lambda_h$ ) of the transition matrix ( $\mathbf{P}$ ) that governs the updating of the wealth state variables over time. Using Proposition 5, we define an eigen-shock as a shock to fundamentals ( $\tilde{\mathbf{f}}_{(h)}$ ) for which the initial impact on the state variables ( $\mathbf{R}\tilde{\mathbf{f}}_{(h)}$ ) coincides with a real eigenvector of this transition matrix ( $\mathbf{u}_h$ ). Each of these eigen-shocks corresponds to a different incidence of fundamental shocks on the wealth state variables ( $\tilde{\mathbf{a}}_t$ ) for each country and is characterized by a speed of convergence that is determined by the corresponding eigenvalue ( $\lambda_h$ ).

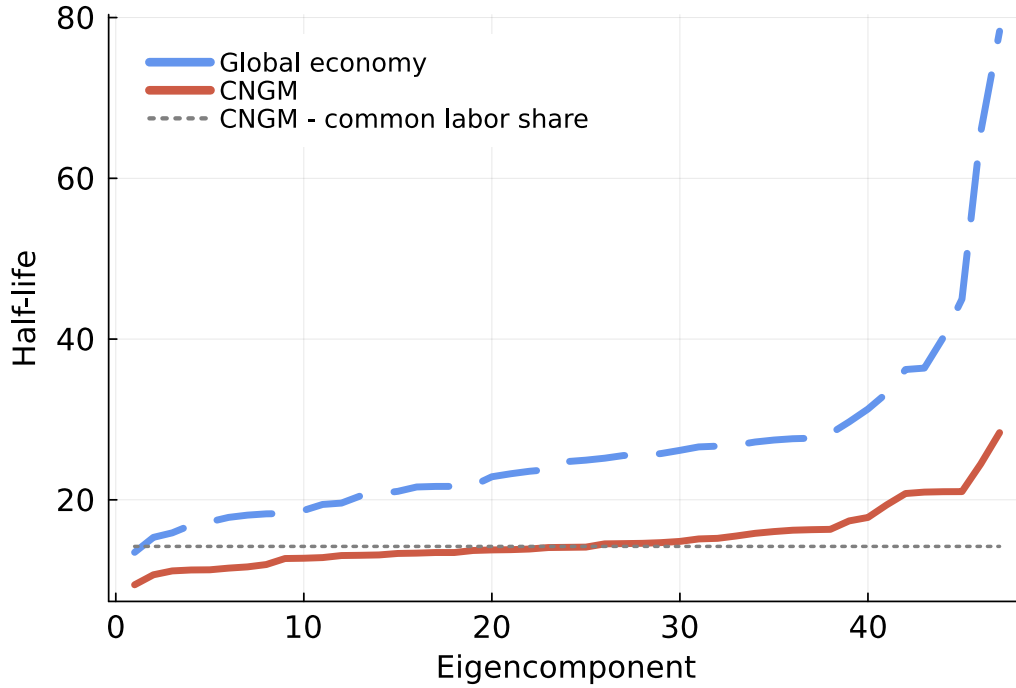
In Figure 2, the long-dashed blue line shows the implied half lives of convergence to steady-state for each eigen-shock using the observed trade and capital share matrices for the year 2017. The vertical axis displays the half life for each eigen-shock, while the horizontal axis sorts these eigen-shocks in terms of increasing half-lives. With open goods and capital markets, these half-lives depend on the entire network of bilateral trade and capital frictions (as captured in the observed trade and capital share matrices) and the parameters of the model.

As a point of comparison, the solid red line displays half-lives of convergence to steady-state for the CNGM, which corresponds to the special case of our model with autarky in both goods and capital markets. In this special case, the half-life of convergence only varies across eigen-shocks, because of differences across countries in the labor share ( $\mu_i$ ). As a further benchmark, the short-dashed black line shows the common half-life of convergence for the CNGM with a common labor share ( $\mu_i = \mu$ ), equal to the average labor share across countries.

We find a substantially slower rate of convergence to steady-state in our neoclassical growth model with open goods and capital markets and imperfect substitutability than in the CNGM. This speed of convergence also displays considerable heterogeneity across the eigen-shocks, ranging from around 15 to 75 years, compared to a range from 10 to 30 years in the CNGM with country-specific labor shares. Since any vector of empirical shocks to fundamentals can be written as a linear combination of the eigen-shocks, these results imply slow rates of convergence to steady-state in response to vectors of empirical shocks. Therefore, our open-economy framework with imperfect substitutability provides a natural approach to addressing the concern that the speed of convergence in the CNGM is too fast relative to empirical transitions for plausible values of the intertemporal elasticity of substitution (as discussed, for example, in King and Rebelo 1993).

In Section K.4 of the Online Appendix, we use our eigendecomposition of the transition matrix ( $\mathbf{P}$ ) to examine how the speed of convergence to steady-state depends on model parameters. We find an intuitive pattern of comparative statics. For example, a higher capital elasticity ( $\epsilon$ )

Figure 2: Half Lives of Convergence to Steady-State for each Eigen-shock



Note: Vertical axis shows half life of convergence to steady-state in years for each eigen-shock; horizontal axis shows the rank of the eigen-shocks in terms of increasing half life in years; long-dashed blue line shows these half lives for our baseline model with costly trade and capital flows and imperfect substitutability between countries for the year 2017; solid red line shows these half lives of convergence for the special case of the closed-economy neoclassical growth model (CNGM) with a country-specific labor share; short-dashed black line shows these half-lives for the special case of the CNGM with a common labor share.

or a higher trade elasticity ( $\theta$ ) imply a longer half-life (slower convergence), because greater substitutability for either capital or goods reduces the absolute value of the covariance between the real return to investment and the initial level of wealth (see equations (45) and (46)). A lower labor share ( $\mu$ ) also implies a longer half-life (slower convergence), because it implies a greater role for wealth accumulation, which again magnifies the impact of fundamental shocks, and hence requires a greater length of time for adjustment to occur.

### 3.6 Counterfactuals for U.S.-China Decoupling

Since our framework matches the observed gravity equation relationships for bilateral trade and capital holdings, and allows for intertemporal consumption-savings decisions, it is particularly well suited for evaluating counterfactual policies that affect bilateral frictions in both goods and capital markets (e.g., U.S.-China decoupling).

We now use our framework to evaluate two counterfactuals for U.S.-China decoupling: (i) a 50 percent increase in bilateral trade frictions alone between China and the United States; (ii) a 50 percent increase in bilateral capital frictions alone between these two countries. We undertake

these counterfactuals for our baseline model using our linearization and the observed trade and capital share matrices ( $\mathbf{S}$ ,  $\mathbf{T}$ ,  $\mathbf{B}$ ,  $\mathbf{X}$ ) for 2017. We assume that agents at time  $t = 0$  learn about a permanent increase in bilateral frictions from time  $t = 1$  onwards. Using Propositions 3 and 4, we solve for the entire transition path of the wealth state variables and all other endogenous variables of the model from time  $t = 1$  onwards.

We also compare the counterfactual predictions of our baseline open economy model to special cases with either capital autarky (and open trade) or trade autarky (and open capital markets), in order to highlight the interaction between goods and capital market integration. When we consider the special case with capital autarky, we replace the observed capital share matrices ( $\mathbf{B}$ ,  $\mathbf{X}$ ) with identity matrices, such that each country only invests domestically. Thus, we make sure to match the observed trade data in both cases, and only vary the degree of capital openness. Similarly, when we consider the special case with trade autarky, we replace the observed trade share matrices ( $\mathbf{S}$ ,  $\mathbf{T}$ ) with identity matrices, such that each country only consumes its own goods, while exactly matching observed data on capital flows.

In Figure 3, we show the results of these four counterfactuals: (i) Higher bilateral trade frictions with open goods markets and capital autarky (panel (a)); (ii) Higher bilateral capital frictions with trade autarky and open capital markets (panel (b)); (iii) Higher bilateral trade frictions with open goods and capital markets (panel (c)); (iv) Higher bilateral capital frictions with open goods and capital markets (panel (d)). In each panel, we show the transition path for consumption in the top row, the transition path for the wealth state variables in the middle row, and the transition path for capital in the bottom row. We show results for China and the United States by the solid red lines labelled with three-letter international standards organization (ISO) country codes (CHN and USA); we show results for all other countries by the solid gray lines; we label the results for the other countries characterized by the largest and smallest changes in a variable with three-letter ISO country codes.

### 3.6.1 Higher Trade Frictions with Open Goods Markets and Capital Autarky

With capital autarky and open goods markets (panel (a)), the financial account and the current account of the balance of payments are necessarily equal to zero, but there can be a trade imbalance that is offset by net investment income from domestic assets.

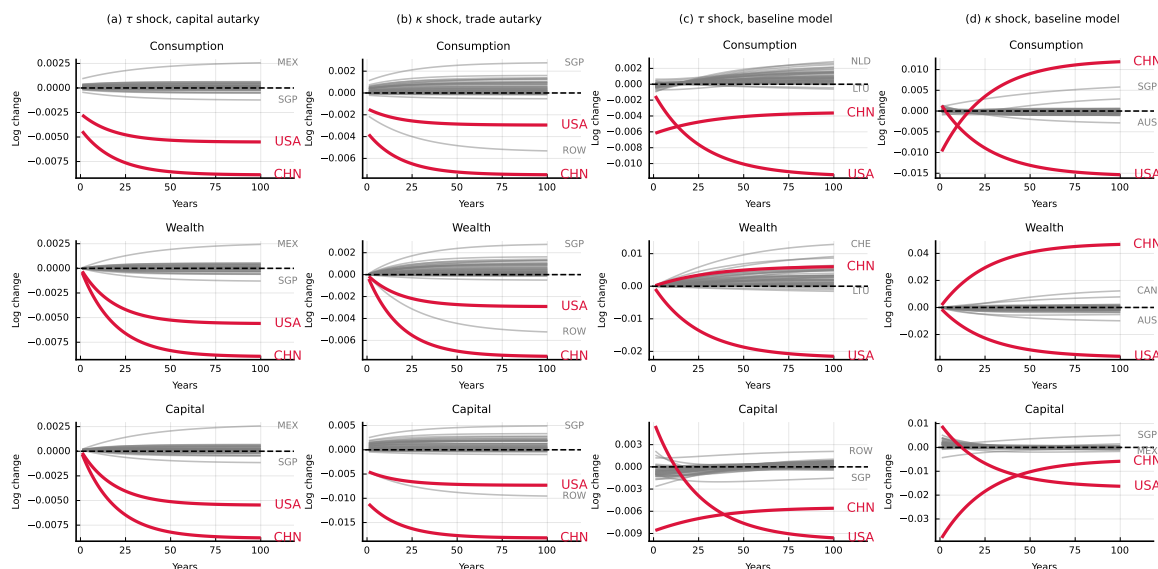
Higher U.S.-China trade frictions lead to an initial drop in consumption in both countries (top row), which captures foregone static welfare gains from trade. This initial drop in consumption is larger for China than for the United States, since the United States is a more central market for China's exports than China is for the United States.

With open goods markets, there are cross-substitution and market size effects on consumption in third countries. On the one hand, the higher cost of Chinese goods in the U.S. market, and

the higher cost of U.S. goods in the Chinese market, raises the demand for other countries' goods. This cross-substitution effect implies that Mexico (MEX) enjoys the largest immediate increase in consumption from higher U.S.-China trade frictions. On the other hand, the reduction in income in China and the United States from higher bilateral trade frictions reduces the demand for other countries' goods. This market-size effect leads Singapore (SGP) to experience the largest immediate reduction in consumption from higher U.S.-China trade frictions.

In addition to conventional static welfare losses, the higher consumption price index in China and United States from higher bilateral trade frictions reduces the real return to investment, which leads to a gradual decumulation of wealth and capital (middle and bottom rows). This decumulation of wealth further reduces consumption in these two countries (top row), and gives rise to dynamic welfare losses, as wealth in these two countries gradually converges to its new lower steady-state level. With open goods markets, third countries can experience either increases or decreases in the real return to investment, depending on the balance of cross-substitution and market size effects. Therefore, Mexico experiences dynamic welfare gains from increased wealth accumulation, while Singapore experiences dynamic welfare losses.

Figure 3: Counterfactuals for an Increase in Bilateral U.S.-China Trade and Capital Frictions



Note: Counterfactuals for permanent increase in bilateral frictions between China and the United States at time  $t = 1$  using our closed-form solution for the economy's transition path. The first and the third columns show counterfactuals for 50% increase in trade frictions, and the second and fourth columns show counterfactuals for 50% increase in capital frictions. The first column considers the special case of the model with international trade but no international capital flows; the second column considers the special case of the model with international capital flows but no international trade; and the last two columns consider our baseline model with both trade and capital flows. Each row shows log deviations from the initial steady-state: the first row shows these deviations for consumption ( $\tilde{c}_{it}$ ); the second row shows these deviations for wealth ( $\tilde{a}_{it}$ ); and the bottom row shows these log deviations for capital ( $\tilde{k}_{it}$ ).

### 3.6.2 Higher Capital Frictions with Trade Autarky and Open Capital Markets

With trade autarky and open capital markets (panel (b)), imports, exports and the trade balance all equal zero, but there can be imbalances in the current and financial accounts, reflecting net investment income from wealth allocated at home and abroad.

Higher U.S.-China capital market frictions lead to an initial drop in consumption in both countries (top row). This initial decline in consumption reflects static welfare losses from capital market disintegration and the resulting international reallocation of capital. The initial fall in consumption is again larger for China than for the U.S., because the U.S. is more important as a capital supplier for China than China is for the U.S.. Higher bilateral capital market frictions between the U.S. and China make these two countries less attractive for capital holdings, which causes a reallocation of capital towards third countries (bottom row), with Singapore (SGP) and the Rest of the World (ROW) experiencing the largest and smallest inflows of capital, respectively (bottom row).

The increase in U.S.-China capital market frictions also reduces the real return to investment in both countries, which leads to a gradual decumulation of wealth and capital (middle and bottom rows). This decumulation of wealth further reduces consumption in both countries (top row), as wealth gradually converges to its new lower steady-state level. Again the effects are larger for China than for the United States. As third countries become more attractive for capital holdings, this increases the real return to investment in those countries, and induces wealth and capital accumulation (middle and bottom rows). Again Singapore (SGP) and the Rest of the World (ROW) experience the largest and smallest increases in the real return to investment and wealth accumulation, respectively.

### 3.6.3 Higher Trade Frictions with Open Goods and Capital Markets

We now examine higher bilateral trade frictions with open goods and capital markets (panel (c)), in which case trade and investment income flows can be imbalanced, and there can be offsetting imbalances in the current and financial accounts.

Higher U.S.-China trade frictions again lead to an initial drop in consumption in both countries from foregone static welfare gains from trade (top row). As for changes in bilateral trade frictions under capital autarky (panel (a)), the initial decline in consumption is larger for China than for the United States, because the United States is a more central market for China's exports than China is for the United States.

However, with open capital markets, changes in bilateral trade frictions now lead to initial reallocation of capital across countries. In particular, China becomes relatively less attractive for capital holdings, because of the decline in demand for its output in U.S. markets, while the United

States becomes more attractive for capital holdings, as it substitutes away from consumption of Chinese goods towards local production. As a result, there is an initial reallocation of capital from China and some third countries towards the United States (bottom row). While the static welfare effects on third countries depended only on the cross-substitution and market size effects under capital autarky (panel (a)), they now also depend on this international reallocation of capital through open capital markets (panel (c)), with the majority of other countries experiencing a reduction in the short-run supply of capital.

In addition to these static welfare effects, higher U.S.-China trade frictions also affect the real return to investment, which has dynamic welfare effects through the accumulation of wealth and capital. Here again we find a starkly different pattern of results from under capital autarky. With open goods and capital markets, the impact on China's real return to investment depends not only on the deterioration in local capital market conditions, but also on the improvement in investment opportunities in other countries. Moreover, the reduction in demand for its goods in the U.S. market lowers China's consumption price index, leading to a cheaper cost of investment goods. Consequentially, its real return on investment rises, which leads to mild accumulation of wealth over time. In contrast, the U.S. experiences an increase in its consumption price index and the cost of investment goods, and a decline in the return to investment in one of its major investment destinations (China), leading to a decumulation of wealth.

Comparing consumption in China and United States in the new steady-state (top row), we find a reversal of fortune between China and the United States over time, with China losing more consumption in the short run, whereas the United States loses more consumption in the long run. This is in contrast to our findings in the case of capital autarky and open goods markets (panel (a)), in which the greater reduction in China's consumption persists over time.

Therefore, we find that the effects of changes in goods market integration depend heavily on the degree of capital market integration, emphasizing the importance of studying these two dimensions of international integration in tandem.

#### **3.6.4 Higher Capital Frictions with Open Goods and Capital Markets**

Finally, we examine higher bilateral capital frictions with open goods and capital markets (panel (d)), in which case trade and investment income flows again can be imbalanced, and there can be offsetting imbalances in the current and financial accounts.

Higher U.S.-China capital market frictions lead to a decline in the supply of capital in China (bottom row), and reallocation of existing capital to the United States and to other countries, stemming from the position of the United States as a major supplier of capital to China. Consequentially, consumption drops in China, while the United States experiences a small increase in consumption (top row). Most third countries also experience an increase in consumption, due to

the greater availability of cheap capital in the short run.

The increase in U.S.-China capital market frictions also increases the real return to investment in China, because of the decline in the supply of capital from the U.S.. In contrast, the real return to investment in the U.S. decreases, because of the reallocation of capital previously invested in China back to the home market. Both effects follow from the position of the United States as a major supplier of capital to China. As a result, China accumulates wealth (middle row), and its consumption gradually increases over time (top row), whereas the opposite occurs in the United States. Moreover, since with open goods and capital markets the United States specializes in exporting capital services, it is more sensitive to rising frictions in capital markets, with this negative income effect further lowering the real return to investment and inducing the decumulation of wealth.

Comparing consumption in China and United States in the new steady-state (upper row), we again find a reversal of fortune. Whereas China is more adversely affected than the U.S., in the short run, it is less adversely affected (and even gains) in the long run. This is in contrast to our findings under goods autarky (second column), in which the adverse effect on China relative to the U.S. persists over time.

Therefore, we find that the effects of changes in capital market integration also depend heavily on the level of goods market integration, again highlighting the importance of simultaneously modelling both these dimensions of international integration.

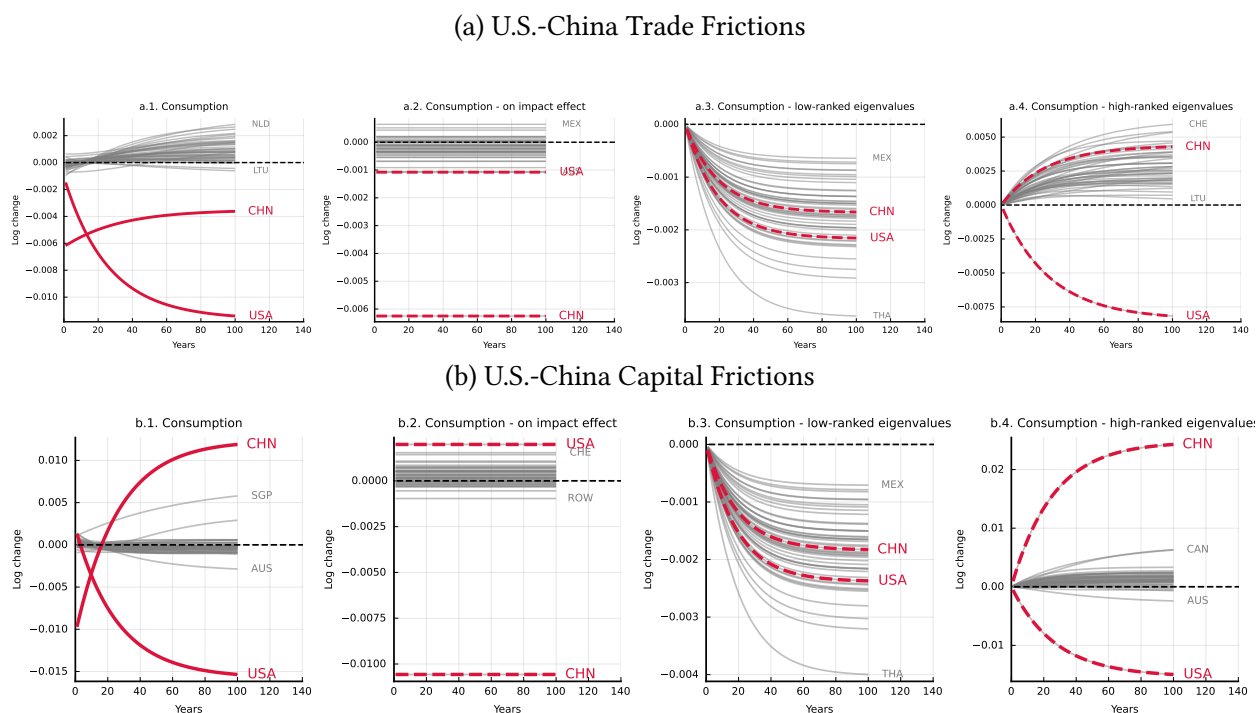
### 3.6.5 Decomposing the Impact of Changes in Goods and Capital Market Frictions

We next use our spectral analysis to further decompose the effects of changes in U.S.-China trade and capital market frictions. We use our result from Proposition 4 that the impact of any shock to fundamentals ( $\tilde{\mathbf{f}}$ ) on the wealth state variables can be expressed as a linear combination of the impacts of the eigen-shocks ( $\tilde{\mathbf{f}}_{(h)}$ ), for which the initial impact on the state variables equals a real eigenvector of the transition matrix ( $\mathbf{R}\tilde{\mathbf{f}}_{(h)} = \mathbf{u}_h$ ). Using this result, we can decompose the overall impact of any fundamental shock on the state variables at each point along the transition path into the contributions of each of the eigen-shocks. Since all of the model's endogenous variables are linear functions of the state variables, we can similarly decompose the impact of the fundamental shock on any endogenous variable at each point along the transition path.

In Figure 4, we implement this decomposition for consumption. The top row shows the effect of changes in bilateral trade frictions, while the bottom row shows the effect of changes in bilateral capital frictions. In Column (1), we show the overall impact on consumption (which replicates the consumption results from the panels (c) and (d) of Figure 3). In Column (2), we show the immediate impact on consumption for the initial values of the state variables, which corresponds to the effect in a static trade or capital allocation model. In Column (3), we show the impact on

consumption for the lowest-ranked eigenshock with the smallest half life (fast convergence). In Column (4), we show the impact on consumption for all higher-ranked eigenshocks with larger half life (slow convergence).

Figure 4: Impulse Responses of Consumption to 50 Percent Increase in Bilateral U.S.-China Trade and Capital Frictions and Their Eigencomponents



Note: Impulse responses of consumption to a permanent 50 percent increase in bilateral U.S.-China trade frictions (top row) and bilateral U.S.-China capital frictions (bottom row); consumption measured as log deviations from the initial steady-state ( $\tilde{c}_{it}$ ); Column (1) shows overall impulse response of consumption; Column (2) shows on impact effect on consumption; Column (3) shows lowest-ranked eigencomponent (fastest convergence); Column (4) shows all other eigencomponents (slower convergence).

Column (2) shows the immediate adjustment of consumption following higher U.S.-China trade frictions (top row) or higher U.S.-China capital market frictions (bottom row). As discussed above, China is particularly negatively affected in both cases, given its position as a major importer of capital from the U.S. and a major exporter of goods to the U.S. Adjustment in other countries reflects on-impact shifts in the terms-of-trade between countries and the supply of the existing stock of global capital.

From Column (3), short-run adjustment is relatively similar in both countries and in third countries, reflecting global capital decumulation in response to increases in trade or capital market frictions. However, from Column (4), long-run adjustment is substantially different, and tends to favor China relative to the United States, especially for capital-market shocks. These patterns of long-run adjustment include a reallocation of wealth away from the U.S. to China and third countries, such as Canada, in order to serve the U.S. market without having to incur the higher

U.S.-China trade frictions, or to supply China with capital, without having to incur the higher U.S.-China capital frictions.

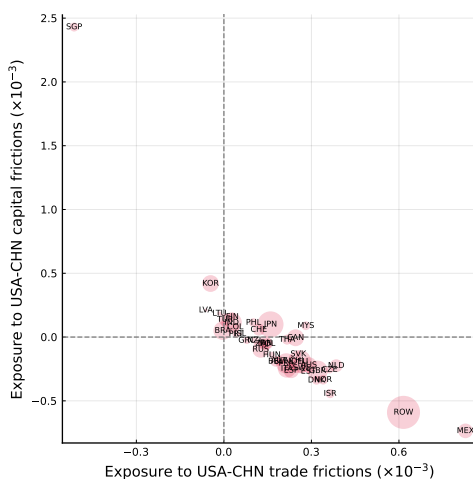
This decomposition highlights the important distinction between the static and dynamic effects of higher trade and capital frictions, and the rich dynamics that occur as the relative importance of eigencomponents with short versus long half lives changes along the transition path towards steady-state.

### 3.6.6 Goods Versus Capital Market Frictions

Finally, we show that changes in U.S.-China trade and capital market frictions tend to have quite different effects on consumption in third countries, because of differences in the initial networks of trade and capital shares and patterns of capital reallocation.

Figure 5 shows the welfare exposure of each third country to changes in U.S.-China goods market frictions (horizontal axis) and U.S.-China capital market frictions (vertical axis). Welfare is measured as the net present value of the discounted stream of utility along the transition path to the new steady-state. Welfare exposure equals the elasticity of welfare with respect to a change in goods or capital market frictions, as computed using our closed-form solutions for the economy's transition path from Proposition 4. The circles for each country are proportional to their GDP and are labelled with their three-letter ISO codes.

Figure 5: Welfare Effects of Rising U.S.-China Trade and Capital Frictions



Note: Welfare changes across countries following a 50 percent increase in bilateral trade frictions between China and the United States (x-axis) and 50 percent increase in bilateral capital frictions between China and the United States (y-axis). Welfare is measured as the net present value of the discounted stream of utility along the transition path to the new steady-state. Welfare exposure equals the elasticity of welfare with respect to a change in goods or capital market frictions, as computed using our closed-form solutions for the economy's transition path from Proposition 4. Results are derived using the baseline version of our model with open goods and capital markets based on data from 2017. Each point represents a country in our data and the size of the circle stands for country GDP. China and the United States are excluded.

Countries with values above [below] zero on the vertical axis gain [lose] from higher U.S.-

China capital frictions, while those with values above [below] zero on the horizontal axis gain [lose] from higher U.S.-China trade frictions. Mexico gains the most from increases in trade frictions between the U.S. and China, through cross-substitution effects in goods markets; and Singapore gains the most from increases in capital frictions between the U.S. and China, through the reallocation of capital to serve the U.S. and Chinese markets without incurring the higher frictions. More generally, we find a negative correlation between welfare exposure to higher trade or capital market frictions between China and the United States.

Therefore, we find that changes in these two different dimensions of international integration can have heterogeneous effects across countries, depending on initial trade and capital shares, and the extent to which countries initially specialize as exporters of capital or exporters of goods. Again these findings highlight the importance of jointly modelling these two forms of international integration, particularly for evaluating counterfactual policies that effect both goods and capital market integration.

## 4 Conclusions

The textbook closed-economy neoclassical growth model (CNGM) remains central to our understanding of cross-country income dynamics. But the open-economy versions of this model make strong assumptions about substitutability in goods and capital markets. We generalize this canonical framework to allow for costly international trade and capital holdings with imperfect substitutability. We develop a tractable, multi-country model that is amenable to quantitative analysis, which simultaneously incorporates international goods trade and capital allocations across countries, as well as intertemporal savings decisions over time.

Our framework captures a number of key features of observed international trade and capital holdings. It generates gravity equations for bilateral trade and capital holdings, because trade and capital frictions are increasing in bilateral distance. It predicts home bias in international capital allocations if managing capital is more costly abroad than at home. It implies a positive correlation between domestic saving and investment, because foreign capital is an imperfect substitute for domestic capital and is subject to greater capital market frictions. It generates gross capital holdings that are substantially larger than net capital holdings, because of idiosyncratic heterogeneity in investment returns. It gives rise to limited capital flows from rich to poor countries, because of imperfect capital substitutability, and even if poor countries offer higher rental rates, they can have lower capital productivity or higher capital market frictions.

Incorporating imperfect substitutability and goods and capital market frictions yields new insights for impulse responses to productivity shocks and the speed of convergence to steady-state. In the CNGM, the higher steady-state capital stock implied by a positive productivity shock only

can be achieved through domestic wealth accumulation. In contrast, in the conventional open-economy neoclassical growth model, a positive productivity shock induces an initial reallocation of capital that equalizes the rental rate on capital across all countries.

Our framework generates predictions in between these two extremes. The higher steady-state capital stock implied by a positive productivity shock is achieved through a combination of *both* domestic wealth accumulation *and* an initial reallocation of capital. This initial reallocation of capital dampens the variation in the real return to investment across countries in response to the productivity shock, thereby implying slower convergence to steady-state. Our open-economy framework thus provides a natural explanation for the finding that empirical estimates of income convergence imply longer transitions than predicted by the CNGM for plausible intertemporal elasticities of substitution.

Since our framework matches the observed gravity equation relationships for bilateral trade and capital holdings, and allows for intertemporal consumption-savings decisions, it is particularly well suited for evaluating counterfactual policies that affect bilateral frictions in both goods and capital markets (e.g., U.S.-China decoupling). Higher bilateral trade frictions give rise to cross-substitution and market size effects in goods markets, as in conventional trade models with capital autarky. However, they also lead to a global reallocation of capital, because they alter the geography of market access between countries. Furthermore, the resulting movements in the real return to investment in each country give rise to a rich pattern of dynamic welfare gains and losses along the global economy's transition path to steady-state.

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