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Monetary Policy, Segmentation, and the Term Structure
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Abstract

We develop a segmented markets model which rationalizes the effects of monetary policy on the term structure of interest rates. When arbitrageurs’ portfolio features positive duration, an unexpected rise in the short rate lowers their wealth and raises term premia. A calibration to the U.S. economy accounts for the transmission of monetary shocks to long rates. We discuss the additional implications of our framework for state-dependence in policy transmission, the volatility and slope of the yield curve, and trends in term premia accompanying trends in the natural rate.

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1 Introduction

The effect of a change in short rates on long rates is central to the monetary transmission mechanism. It determines how monetary policy affects mortgage rates, corporate borrowing rates, and other determinants of aggregate demand. Long rates reflect the expected path of short rates plus term premia. There is accumulating empirical evidence that contractionary monetary policy raises long rates by more than can be accounted for by the change in the expected path of short rates.\(^1\) This implies that contractionary monetary policy operates in part by raising term premia.

This evidence poses a challenge to existing models of monetary transmission and the term structure. Representative agent models typically imply that monetary policy shocks have negligible effects on the price and quantity of interest rate and inflation risks. Market segmentation opens the door for transitory shocks to have more substantial effects on term premia if they have relatively large effects on the subset of agents pricing long-term bonds. However, existing models of this kind, most notably those in the preferred habitat tradition, counterfactually imply that a monetary tightening lowers term premia, as the associated rise in long yields causes habitat investors to borrow less long-term and thus exposes arbitrageurs to less risk.

In this paper, we propose a model which rationalizes the effects of monetary policy shocks on the term structure of interest rates. We build on the preferred habitat tradition by studying an environment in which habitat investors and arbitrageurs trade bonds of various maturities. We integrate this with the intermediary asset pricing tradition by studying an environment in which arbitrageur wealth is an endogenous state variable governing the price of risk. When arbitrageurs’ portfolio features positive duration, an unexpected rise in the short rate lowers their wealth and raises term premia. Quantitatively, a calibration matching the portfolio duration of arbitrageurs in the data rationalizes the responses of the yield curve to monetary shocks. The endogenous price of risk further implies state-dependent effects of conventional and unconventional policies; generates endogenous price volatility which accounts for a sizable fraction of the unconditional slope of the yield curve; and helps to explain trends in term premia in recent years owing to trends in the natural rate.

Our model integrates elements of the preferred habitat and intermediary asset pric-

\(^1\)See, e.g., Cochrane and Piazzesi (2002), Gertler and Karadi (2015), Gilchrist, Lopez-Salido, and Zakrajsek (2015), Hanson and Stein (2015), Abrahams, Adrian, Crump, Moench, and Yu (2016), and Hanson, Lucca, and Wright (2021).
ing traditions. As in existing preferred habitat models, habitat investors elastically demand bonds of each maturity. This class of investors captures the government issuing debt securities net of central bank purchases, households borrowing in mortgages, and other investors who do not actively trade across maturities to maximize risk-adjusted returns. Overlapping generations of arbitrageurs, capturing financial institutions such as broker/dealers and hedge funds, trade across maturities to maximize risk-adjusted returns. Time is continuous and there are two driving forces: the short rate and the level of habitat demand across maturities. Unlike existing preferred habitat models, arbitrageurs have CRRA (rather than CARA) preferences, and are characterized by perpetual youth (rather than living only instantaneously). The wealth of arbitrageurs is thus an endogenous state variable relevant for risk pricing, as in the intermediary asset pricing tradition.

We first study a simplified version of this environment which allows us to analytically characterize our main results. In the simplified environment, time is discrete and only one- and two-period bonds are traded. If arbitrageurs die after one period and thus their endowment is exogenous, we recover the existing result from preferred habitat models that an unexpected rise in the short rate lowers the term premium on two-period bonds: the associated increase in the two-period yield causes habitat investors to borrow less at this maturity and thus means arbitrageurs are exposed to less interest rate risk. When arbitrageurs live for more than one period, the revaluation of arbitrageurs’ wealth also determines the response of the term premium to a short rate shock. In particular, if arbitrageurs’ portfolio features positive duration — in this simple setting, if they are long two-period bonds — an unexpected rise in the short rate lowers their wealth. If this force is sufficiently strong relative to the demand elasticity of habitat investors, the term premium rises.

We then numerically quantify these mechanisms in the full, continuous-time model. When arbitrageur wealth is endogenous in the ways described above, bond prices no longer take an exponentially affine structure, and the model does not admit a closed form solution. We can nonetheless describe the equilibrium in terms of a system of four partial differential equations: equilibrium in the bond market implied by arbitrageurs’ optimization and market clearing; the endogenous evolution of arbitrageur wealth; and the exogenous evolutions of the short rate and habitat demand. We solve this system numerically using the Feynman-Kac formula and Monte Carlo simulation. We expect our code can be useful to other researchers who wish to study the yield curve in an
environment with heterogeneous agents and an endogenous price of risk.

We confront the model with estimates of the yield curve responses to monetary policy shocks. In the data, we isolate monetary policy shocks from other shocks by using the high-frequency response of futures prices around FOMC announcements as an instrumental variable. Our baseline estimates imply that a policy-induced rise in the one-year real yield by 1pp raises the 20-year real forward rate by 0.39pp; more generally, the shock raises long-dated real forward rates by statistically and economically significant amounts. This finding is robust to a variety of specifications, including alternative measures of monetary policy shocks and samples which exclude the worst months of the financial crisis. It implies that a monetary tightening raises term premia (and an easing lowers term premia), as any reasonable estimate of nominal rigidity requires that the expected real interest rate must be essentially unchanged several years after a monetary shock. Our primary quantitative question of interest is whether our model can account for this evidence.

We discipline the model to match novel evidence on the duration of arbitrageurs. Following much of the literature, we associate these arbitrageurs with broker/dealers and hedge funds which trade actively across financial instruments with differing maturities. We employ two complementary approaches to measure their aggregate duration. The first combines evidence on the average duration of individual assets such as Treasuries, mortgage-backed securities, and corporate equities with the portfolio holdings of broker/dealers and hedge funds in these asset classes. The second estimates the response of primary dealers’ equity prices in tight windows around FOMC announcements. Both approaches imply that these arbitrageurs have an aggregate duration between roughly 10 and 30. We further validate these measures by documenting that in times when arbitrageur duration is estimated to be high, the effects of monetary shocks on long-dated forwards are amplified.

Calibrated to match this evidence on arbitrageur duration, our model can account for much of the responses of long-dated real forward rates to monetary shocks in the data. In particular, in our baseline calibration with arbitrageur duration at the midpoint of our estimated range in the data, a monetary tightening which raises the one-year real yield by 1pp raises the 20-year real forward rate by 0.17pp, as compared to 0.39pp in the data. At the lower end of our estimated range of arbitrageur duration, the response of the 20-year real forward rate remains around 0.10pp. When habitat demand is less elastic, the model can account for even more of the observed response
in the data. The overreaction of forward rates vis-à-vis the expectations hypothesis is reversed in a counterfactual economy with exogenous arbitrageur wealth, consistent with our analytical results. We further find that the effects of monetary shocks are state-dependent as in the data: in periods when arbitrageurs’ duration is high, the effects on forward rates are amplified.

The endogenous price of risk via arbitrageur wealth has several additional implications beyond the response to monetary shocks. First, it implies state-dependent effects of other shocks. For instance, we simulate the Federal Reserve’s March 18, 2009 announcement that it would purchase long-term Treasuries and increase the size of its agency debt and mortgage-backed security purchases. We find that real yields and forward rates above 10 years maturity would have fallen by roughly 20-30% less if arbitrageur wealth was initially at its average level instead of depressed by a third. Second, the model clarifies that fluctuations in arbitrageur wealth account for roughly a third of the average slope of the yield curve, because they generate endogenous and stochastic volatility in bond prices. Finally, the revaluation of arbitrageur wealth can help account for trends in term premia in recent years via trends in the natural rate of interest. A fall in the natural rate recapitalizes arbitrageurs with positive duration much like a monetary easing. Quantitatively, the model implies that a roughly 2ppp cumulative decline in the steady-state short rate from 2004 to 2016 accounts for 30% of the decline in the 5-year forward, 5-year term premium over this same period.

In the post-pandemic period, yield curve models indicate that long yields have risen in part because of a higher real term premium. At the same time, U.S. monetary policy has tightened and there is evidence of an increase in the U.S. natural rate. Our framework provides a way to relate these developments, though we leave a quantitative exploration of the recent increase in the term premium to future work.

Related literature Our paper builds on preferred habitat models of the term structure of interest rates. The preferred habitat view was proposed by Culbertson (1957) and Modigliani and Sutch (1966) and formalized by the seminal work of Vayanos and Vila (2021). A growing theoretical literature has used this framework to study the implications for corporate finance (Greenwood, Hanson, and Stein (2010)), government debt policy (Guibaud, Nosbusch, and Vayanos (2013)), exchange rates (Gourinchas, Ray, and Vayanos (2022) and Greenwood, Hanson, Stein, and Sunderam (2023)), and the real economy (Ray (2021) and Droste, Gorodnichenko, and Ray (2023)). An enor-
mous empirical literature has drawn on this framework to inform analyses of unconventional monetary policies. In the existing framework, the effects of the key driving force (the short rate) are counterfactual. We enrich this framework to match evidence on the response to such shocks by allowing the wealth of arbitrageurs to be an endogenous state variable relevant for risk pricing.

In doing so, our paper builds on the literature linking changes in intermediary net worth with asset prices. This is at the core of the intermediary asset pricing tradition in finance (He and Krishnamurthy (2013) and Brunnermeier and Sannikov (2014)) as well as the financial accelerator tradition in macroeconomics (Bernanke, Gertler, and Gilchrist (1999)). Our contribution is to embed this insight into a leading model of the term structure of interest rates.\(^2\) The recent analyses of Haddad and Sraer (2020), He, Nagel, and Song (2022), and Schneider (2023) similarly apply insights from intermediary asset pricing models to the term structure, though their focus differs from ours on monetary transmission.

Our emphasis on the wealth revaluation channel in accounting for the term premium effects of monetary shocks contrasts with alternative explanations focused instead on habitat demand or changing policy rules.\(^3\) Hanson (2014), Hanson and Stein (2015), and Hanson et al. (2021) propose models in which habitat investors have upward-sloping demand for long-term bonds in response to short rate shocks, perhaps due to mortgage refinancing, “reaching for yield”, or duration matching of life insurance companies and pension funds.\(^4\) Bianchi, Lettau, and Ludvigson (2021), Bauer, Pflueger, and Sunderam (2023), and Bianchi, Ludvigson, and Ma (2024) propose models in which investors learn about changing policy rules and thus macroeconomic comovements around monetary announcements. Our model is complementary with these mechanisms, and indeed our quantitative results require that they also exist to fully account for the yield curve responses to monetary shocks. Our mechanism is nonetheless distinct — for instance, it predicts that the effects of monetary shocks on term premia vary with arbitrageurs’ duration, which we find indeed is the case in the data.

Our paper is finally part of a broader agenda studying links between macroeconomic

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\(^2\)In their empirical analysis of government bond supply and excess returns, Greenwood and Vayanos (2014) anticipate that if arbitrageurs’ coefficient of absolute risk aversion is a declining function of their wealth, changes in their wealth will have effects on term premia. Our paper formalizes this idea and traces out its theoretical and quantitative implications.

\(^3\)There is an additional mechanism which may be complementary to these, namely that in the presence of a lower bound on the nominal interest rate, a monetary easing lowers the amount of future interest rate risk and thus term premia, and vice-versa for a tightening. See, for instance, King (2019).

\(^4\)See also Malkhozov, Mueller, Vedolin, and Venter (2016) and Domanski, Shin, and Sushko (2017).
shocks, the wealth distribution, and the price of risk in heterogeneous agent models. Alvarez, Atkeson, and Kehoe (2002, 2009) study monetary economies with segmented financial markets in which monetary shocks change the price of risk. Kekre and Lenel (2022) build on these insights in a conventional New Keynesian model enriched with agents having heterogeneous risk-bearing capacity. They find that a monetary easing lowers the risk premium on capital by redistributing wealth to agents who wish to invest more of their marginal wealth in capital. The present paper shows that a similar mechanism is at work for the term premium in a preferred habitat environment.\footnote{We conjecture that introducing heterogeneity in risk aversion into representative agent models in which aggregate comovements deliver a positive term premium, as in Piazzesi and Schneider (2007), Rudebusch and Swanson (2012), and Campbell, Pflueger, and Viceira (2020), would lead to similar results. With a positive price on term risk, relatively risk tolerant agents would endogenously be more exposed to it, implying a redistribution of wealth which affects the price of risk on impact of policy shocks. One important difference in the preferred habitat environment is that it does not rely on aggregate comovements generating a positive term premium, and thus implies that this mechanism remains operative even if, as in recent years, aggregate comovements may have flipped signs.}

While we do not extend the model to feature a New Keynesian production block, we expect that the effects of policy shocks on the term premium would imply that monetary policy is more potent in affecting the real economy to the extent that aggregate demand is rising in the amount habitat investors borrow long-term.\footnote{See Caballero and Simsek (2020) for recent work linking risk premia, aggregate demand, and output in the New Keynesian environment. See Caramp and Silva (2023) for recent work linking term premia and aggregate demand in such an environment in particular.}

**Outline** In section 2 we outline the model environment. In section 3 we characterize our main results analytically in a simple version of this environment. In section 4 we estimate the effects of policy shocks on the yield curve and measure arbitrageurs’ duration in the data. In section 5 we calibrate the full model and assess its ability to rationalize the data. Finally, in section 6 we conclude.

## 2 Model

In this section we outline our model of the term structure of interest rates. The model integrates features of the preferred habitat and intermediary asset pricing traditions.

**Timing and assets** Time $t$ is continuous. At time $t$ there is a continuum of zero coupon bonds with maturities $\tau \in (0, \infty)$. A bond trading at $t$ with maturity $\tau$ pays
one unit of the numeraire at \( t + \tau \) and its price is \( P_t^{(\tau)} \). The instantaneous return on holding such a bond is \( dP_t^{(\tau)}/P_t^{(\tau)} \). The yield of the bond is given by

\[
y_t^{(\tau)} = -\frac{\log \left( P_t^{(\tau)} \right)}{\tau}
\]

and the short rate \( r_t \) is the limit of the yield as \( \tau \) goes to zero.

**Decision problems** There are two types of agents: habitat investors and arbitrageurs. The former captures investors such as the government issuing debt securities net of central bank purchases and households borrowing in long-term mortgages, while the latter captures financial institutions such as broker/dealers and hedge funds which trade across maturities to maximize risk-adjusted returns.

In aggregate habitat investors hold positions

\[
Z_t^{(\tau)} = -\alpha(\tau) \log \left( P_t^{(\tau)} \right) - \theta_t(\tau)
\]

at each maturity \( \tau \in (0, \infty) \), where a positive position implies that these investors are saving in this security. The parameter \( \alpha(\tau) \) controls the elasticity of demand to price. \( \theta_t(\tau) \) controls the level of habitat demand and is given by

\[
\theta_t(\tau) = \theta_0(\tau) + \theta_1(\tau) \beta_t,
\]

where \( \beta_t \) is a demand factor, the parameter \( \theta_1(\tau) \) controls the loading of demand on that factor, and the parameter \( \theta_0(\tau) \) controls the time-invariant level of demand.

Arbitrageurs trade at all maturities as well as at the short rate \( r_t \) with the central bank.\(^7\) Arbitrageurs are born and die at rate \( \xi \), discount the future at rate \( \rho \), and have separable CRRA preferences over consumption upon death with risk aversion

\(^7\)The statement that arbitrageurs trade at the short rate \( r_t \) with the central bank encodes our assumption that the short rate is exogenous, as in existing preferred habitat models. In other words, the key assumption is that the central bank adjusts its borrowing/lending at the short rate to clear the market at that rate, so we do not specify the market clearing condition at that rate. In continuous time, this amounts to a discontinuity in habitat demand (1) at \( \tau = 0 \) to implement a given short rate by market clearing. In the discrete time model studied in the next section, this is reflected in the fact that we do not specify habitat demand for one-period bonds, since the assumption is that the central bank trades to implement the given short rate.
Here we depart from typical preferred habitat models which assume arbitrageurs are alive instantaneously and have CARA preferences over consumption upon death. Using lower case to denote the endowment and choices of an individual arbitrageur with wealth $w_t$, this arbitrageur chooses its sequence of financial portfolios to maximize

$$v_t(w_t) = \max_{\{x_t^{(\tau)}\}} E_t \int_0^\infty \exp(-(\xi + \rho)s)(\xi + \rho) \left(\frac{w_t^{1-\gamma} - 1}{1 - \gamma}\right) ds$$

subject to the budget constraint

$$dw_t = w_tr_t dt + \int_0^\infty x_t^{(\tau)} \left(\frac{dP_t^{(\tau)}}{P_t^{(\tau)}} - r_t dt\right) d\tau,$$

where $x_t^{(\tau)}$ denotes its position in bonds with maturity $\tau$.\(^8\) Using upper case to denote aggregates across arbitrageurs, aggregate arbitrageur wealth thus follows

$$dW_t = W_tr_t dt + \int_0^\infty X_t^{(\tau)} \left(\frac{dP_t^{(\tau)}}{P_t^{(\tau)}} - r_t dt\right) d\tau + \xi (\bar{W} - W_t) dt,$$

where $\bar{W}$ is the exogenous endowment of newborn arbitrageurs. When $\xi \to \infty$, this converges to the constant endowment process in Vayanos and Vila (2021). For finite $\xi$, $W_t$ will be an endogenous state variable of the model as in intermediary asset pricing models such as He and Krishnamurthy (2013) and Brunnermeier and Sannikov (2014) and financial accelerator models such as Bernanke et al. (1999).

**Driving forces** There are two driving forces in this economy: the short rate set by the central bank $r_t$, and the demand factor $\beta_t$. These follow the exogenous processes

$$dr_t = \kappa_r(\bar{r} - r_t) dt + \sigma_r dB_{r,t},$$

\(^8\)Consumption only upon death ensures that, for arbitrary $\gamma$, arbitrageurs choose portfolio shares across assets which are independent of their level of wealth, allowing us to obtain aggregation to a representative arbitrageur. In the usual way, specializing to the case of $\gamma = 1$ (log preferences) would allow us to obtain aggregation despite consumption throughout life, as arbitrageurs would optimally consume at a constant rate out of wealth given their unitary elasticity of substitution.\(^9\)

\(^9\)The death rate $\xi$ acts like a discount rate in arbitrageurs’ decision problem. We nonetheless account for a distinct discount rate $\rho$ because this will control the strength of the intertemporal hedging motive in portfolio choice when $\gamma \neq 1$, as clarified in section 5.

\(^{10}\)To simplify expressions for the equilibrium value function which follows, we include the multiplicative scalar $\xi + \rho$ in the definition of the value function. This is of course without loss of generality.
\[ d\beta_t = -\kappa_\beta \beta_t dt + \sigma_\beta dB_{\beta,t}, \quad (7) \]

where the Brownian motions \( dB_{r,t} \) and \( dB_{\beta,t} \) are independent and \( \kappa_r, \kappa_\beta > 0 \). We assume independent shocks and a single-factor demand structure since our calibration will, for simplicity, focus on this case. We expect our main insights would generalize to correlated shocks and multiple demand factors and leave this for future work.

**Market clearing and equilibrium** Bond markets must clear according to

\[ Z_t^{(\tau)} + X_t^{(\tau)} = 0 \quad (8) \]

for each maturity \( \tau \in (0, \infty) \) at each point in time \( t \). The definition of an equilibrium is standard.

**Interpretation** We interpret the model in real terms. We do this for two reasons. First, focusing on real bonds allows us to study our mechanism focused on interest rate risk in a more parsimonious setting which can abstract away from inflation risk.\(^{11}\) Second, focusing on the real term structure allows us to uncover the effects of monetary shocks on term premia purged from any effects on long-run inflation. In particular, monetary policy shocks may contain news about the long-run inflation target, which in turn will affect long-dated nominal forwards (Gurkaynak, Sack, and Swanson (2005b)). Long-dated real forwards are immune from this issue, and moreover monetary neutrality in the long run implies that expected real interest rates in the distant future should be unaffected by monetary shocks. In both model and data, this allows a tight analysis of the effects of a monetary shock on term premia by studying the response of real forwards on impact of the shock, following Hanson and Stein (2015).

There is a related nuance in how we think of monetary shocks in the model. We view these as inducing a change in the real short rate with some particular speed of mean reversion (and in our quantitative comparison of model impulse responses versus data, the mean reversion will indeed differ from “typical” fluctuations in the real short rate controlled by \( \kappa_r \)). Embedding our model in a fuller New Keynesian framework, the path of the real short rate in response to a monetary shock would reflect the persistence of the underlying shock, the rigidity in prices, and the feedback to the real economy.

\(^{11}\)With that said, our analysis would extend to a setting with inflation risk: a change in arbitrageurs’ wealth would affect their willingness to be exposed to this risk and thus the inflation risk premium.
Our analysis would extend naturally to such an environment conditional on the induced path of the real short rate.

3 Analytical insights

We now study a simplified version of the model which allows us to analytically characterize our main results. When arbitrageur wealth is endogenous and their portfolio features positive duration, an unexpected rise in the short rate lowers their wealth and raises term premia.

3.1 Simplified environment

In this section we assume time is discrete and only two bonds are traded: maturities one and two periods. We further assume log preferences ($\gamma = 1$). This environment captures the essential forces at play in our full model with much simpler mathematics.

We now spell out the details. Arbitrageurs trade in one-period bonds at price $\exp(-r_t)$ set by the central bank and in two-period bonds at price $P_t$, where we now dispense with the notation for maturity $\tau$ since it is unambiguous. Habitat investors hold a position

$$Z_t = -\alpha \log P_t - \theta_t$$

in two-period bonds, as in (1). An arbitrageur with wealth $w_t$ chooses its position in two-period bonds $x_t$ to maximize

$$\max_{\{x_{t+s}\}} E_t \sum_{s=1}^{\infty} \exp(-(\xi + \rho)s)(\xi + \rho) \log w_{t+s}$$

subject to the evolution of wealth

$$w_{t+1} = w_t \exp(r_t) + x_t \left(\frac{\exp(-r_{t+1})}{P_t} - \exp(r_t)\right),$$

the discrete time counterparts to (3)-(4) with $\gamma = 1$. Note that the one-period return on a two-period bond is $\exp(-r_{t+1})/P_t$ because the two-period bond at $t$ becomes a one-period bond at $t+1$, with price $\exp(-r_{t+1})$. Aggregate arbitrageur wealth follows

$$W_{t+1} = \exp(-\xi) \left[W_t \exp(r_t) + X_t \left(\frac{\exp(-r_{t+1})}{P_t} - \exp(r_t)\right)\right] + (1 - \exp(-\xi))\bar{W},$$
the discrete time counterpart to (5). The short rate and habitat demand follow the AR(1) processes

\[ r_{t+1} - \bar{r} = (1 - \kappa_r) (r_t - \bar{r}) + \sigma_r \epsilon_{r,t+1}, \tag{9} \]
\[ \theta_{t+1} - \bar{\theta} = (1 - \kappa_\theta) (\theta_t - \bar{\theta}) + \sigma_\theta \epsilon_{\theta,t+1}, \tag{10} \]

where \( \epsilon_{r,t+1} \) and \( \epsilon_{\theta,t+1} \) are independent standard Normal innovations. \( \kappa_r \in (0, 1) \) and \( \kappa_\theta \in (0, 1) \) can be interpreted as the degree of mean reversion in these driving forces, as in (6) and (7). We dispense with \( \beta_t \) in this section because it is isomorphic to \( \theta_t \) since there is only one long-term bond. Finally, as in (8), bond market clearing requires

\[ X_t + Z_t = 0. \]

### 3.2 Equilibrium

Following standard arguments, each arbitrageur’s value function is characterized by

\[ v_t(w_t) = \log w_t + \log \nu_t, \]

where \( \nu_t \) is common to arbitrageurs and invariant to their individual level of wealth. Arbitrageurs’ optimality condition with respect to \( x_t \) implies

\[ E_t \left( \exp(r_t) + \frac{x_t}{w_t} \left( \frac{\exp(-r_{t+1})}{P_t} - \exp(r_t) \right) \right)^{-1} \left[ \frac{\exp(-r_{t+1})}{P_t} - \exp(r_t) \right] = 0, \tag{11} \]

clarifying that their portfolio share \( \frac{x_t}{w_t} \) is also invariant to wealth. Defining the log one-period holding return on a two-period bond

\[ r_{t+1}^{(2)} \equiv -r_{t+1} - \log P_t \tag{12} \]

and making use of

\[ \frac{x_t}{w_t} = \frac{X_t}{W_t} \tag{13} \]

by aggregation, a second-order Taylor approximation of (11) around \( r_{t+1}^{(2)} = r_t \) implies

\[ E_t r_{t+1}^{(2)} - r_t + \frac{1}{2} \sigma_r^2 \approx \frac{X_t}{W_t} \sigma_r^2. \tag{14} \]
This has an intuitive interpretation. Arbitrageurs require non-zero expected excess returns to compensate them for bearing interest rate risk on two-period bonds. In particular, when \( X_t > 0 \), arbitrageurs are long two-period bonds and thus expected excess returns on two-period bonds must be positive; the opposite is true if \( X_t < 0 \). The higher is arbitrageur wealth \( W_t \), the smaller (in absolute value) expected excess returns must be, because two-period bonds are a smaller share of their wealth and arbitrageurs have CRRA preferences. In the limit \( W_t \to \infty \), arbitrageurs are effectively risk neutral and thus the (local) expectations hypothesis holds.\(^{12}\) The relevance of \( W_t \) in risk pricing is the key distinction between the present model and existing preferred habitat models.

The above condition is the only approximation we use in the rest of this section; all other conditions hold exactly. Combining the above condition with market clearing in two-periods bonds and habitat investors’ demand yields

\[
E_t r_{t+1}^{(2)} - r_t + \frac{1}{2} \sigma_r^2 = \frac{1}{W_t} (\alpha \log P_t + \theta_t) \sigma_r^2. \tag{15}
\]

Combining the evolution of aggregate arbitrageur wealth with market clearing in two-period bonds and habitat investors’ demand yields

\[
W_{t+1} = \exp(\xi) \left[ W_t \exp(r_t) + (\alpha \log P_t + \theta_t) (\exp(r_{t+1}^{(2)}) - \exp(r_t)) \right] + (1 - \exp(-\xi)) W_t. \tag{16}
\]

The dynamical system (9)-(10), (12), and (15)-(16) is thus five equations in five unknowns \( r_{t+1}, \theta_{t+1}, r_{t+1}^{(2)}, P_t, \) and \( W_{t+1} \), given \( r_t, \theta_t, \) and \( W_t \). The rest of this section proceeds through our two main results studying a short rate shock \( \epsilon_{r,t} \).

### 3.3 Effects of short rate shock

We characterize the effects of the shock around the stochastic steady-state, denoted without time subscripts, for expositional simplicity.

Our first result describes the impact effect of the shock on arbitrageur wealth \( W_t \).\(^{13}\)

\(^{12}\)The standard Jensen’s inequality term \( \frac{1}{2} \sigma_r^2 \) implies that the expectations hypothesis does not hold. See Piazzesi (2010) for further discussion of this point.

\(^{13}\)The proofs of this result and the next one are in appendix A.
Proposition 1. The response of arbitrageur wealth to a short rate shock is

\[ d \log W_t = -\exp(-\xi)\omega_r \sigma_r d\epsilon_{r,t}, \]

where \( \omega \) is the duration of arbitrageurs’ wealth and satisfies

\[ \omega \propto \frac{X}{W}. \]

Intuitively, consider an unexpected rise in the short rate. When arbitrageurs’ aggregate wealth is endogenous (finite \( \xi \)), their wealth will be revalued downwards if and only if their portfolio has positive duration at the stochastic steady-state, which amounts in this environment to a positive position in two-period bonds \( X \). When arbitrageurs’ aggregate wealth is exogenous (\( \xi \to \infty \)), this mechanism is shut down.

Our second result describes the impact effect of the shock on the one-period ahead forward rate

\[ f_t \equiv - \log P_t - r_t. \] \hfill (17)

We focus on this anticipating our empirical work studying the impact effect on forward rates, though it is straightforward to characterize the full impulse response of the forward rate or transformations such as bond yields. We obtain:

Proposition 2. The response of the one-period ahead forward rate to a short rate shock is

\[ df_t = \left[ \frac{1 - \kappa_r - \frac{1}{W} \alpha \sigma_r^2}{1 + \frac{1}{W} \alpha \sigma_r^2} + \frac{1}{W} X \sigma_r^2 \right] \sigma_r d\epsilon_{r,t}. \]

Thus, if \( \xi \to \infty \) (exogenous arbitrageur wealth), there is underreaction of the forward rate relative to the expected short rate

\[ |df_t| < (1 - \kappa_r) \sigma_r |d\epsilon_{r,t}| = |dE_t r_{t+1}| \]

if \( \alpha \sigma_r^2 > 0 \). If \( \xi \) is finite (endogenous arbitrageur wealth), there is overreaction of the forward rate relative to the expected short rate

\[ |df_t| > (1 - \kappa_r) \sigma_r |d\epsilon_{r,t}| = |dE_t r_{t+1}| \]

if \( \exp(-\xi)|\omega| \) is sufficiently high relative to \( \alpha \), given \( \sigma_r > 0 \).

Thus, when \( \xi \to \infty \), we recover the effects of short rate shocks in existing preferred
habitat models.\textsuperscript{14} Intuitively, consider an unexpected rise in the short rate. Holding fixed habitat investor borrowing, this raises the two-period bond yield. If habitat investors are price elastic ($\alpha > 0$), this causes them to borrow less in two-period bonds. If arbitrageurs face price risk in these bonds ($\sigma_r > 0$), this lowers the term premium, reflected in underreaction of the forward rate. To summarize: a rise in the short rate lowers the term premium because arbitrageurs must bear less risk.

When arbitrageurs’ wealth is a relevant state variable for risk pricing (finite $\xi$), we can reverse the effects of a short rate shock on the term premium. In particular, if arbitrageurs have positive duration $\omega \propto \frac{X}{W}$, we know from (15) that the steady-state term premium is positive. A fall in their wealth raises their price of bearing interest rate risk. If this force is sufficiently strong relative to the decrease in the quantity of risk they bear — controlled by $\alpha$, as described in the prior paragraph — the term premium will rise. This is reflected in overreaction of the forward rate.\textsuperscript{15}

4 Empirical analysis

Motived by these results, we now estimate the effects of monetary shocks on the yield curve and measure arbitrageurs’ duration. Our core question of interest in the balance of the paper will be whether a calibration of the full model matching arbitrageurs’ duration can account for the effects of monetary shocks along the yield curve.

4.1 Effects of monetary shocks on yield curve

We first study the response of the yield curve to announcements of the Federal Open Market Committee (FOMC).

4.1.1 Approach

Given the one-year yield $y_t^{(1)}$ measured at the end of day $t$ as well as one-year real forward rates $\{f_t^{(\tau-1,\tau)}\}$ paying $\tau \in \{2, \ldots, 20\}$ years from $t$, we estimate the effect of the daily change in $y_t^{(1)}$ on the daily change in $\{f_t^{(\tau-1,\tau)}\}$, instrumenting the former with

\textsuperscript{14}See for instance Proposition 2 in Vayanos and Vila (2021).

\textsuperscript{15}Proposition 2 also implies that if $\omega < 0$ but is sufficiently large in absolute value, there will still be overreaction of the forward rate. This is because the steady-state term premium is negative, and a rise in the short rate will revalue wealth in favor of arbitrageurs. This will make the term premium less negative, and thus cause overreaction of the forward rate. Of course, the more empirically relevant case features $\omega > 0$ and a positive term premium in steady-state, which is why we focus on it.
the change in Fed funds futures in a 30-minute window around the FOMC announce-
ment. By focusing on variation induced by the high-frequency change in Fed funds
futures, we address the point made by Nakamura and Steinsson (2018) that even on
days with FOMC announcements, there is news orthogonal to monetary policy which
simultaneously affects yields and other outcome variables. By nonetheless summarizing
our results in terms of the effect of a daily change in the one-year yield on outcome
variables, we provide estimates which are easy to interpret and compare to the model.\footnote{An alternative approach sometimes considered in the literature is simply an OLS regression of
daily changes in long-dated forwards on daily changes in short-dated yields, an indicator for FOMC
announcement days, and their interaction. The last coefficient is informative of the incremental effects
of monetary shocks. In appendix C.7, we use our model to illustrate why this approach is difficult to
interpret and why using direct measures of monetary shocks from high-frequency data is preferable.}

A long-standing challenge in the identification of monetary policy shocks is that,
even using intraday data, it may be difficult to decouple them from “information
shocks”: information about the state of the economy revealed at the time of FOMC
announcements which is distinct from a shock to the Federal Reserve’s monetary policy
rule. To mitigate the concern that our results may be caused by such shocks, we follow
Jarocinski and Karadi (2020) in focusing on FOMC announcement days in which the
high-frequency change in the S&P 500 and one-year bond yield have opposite signs.
Intuitively, if an increase in the one-year bond yield is due to good news about the
state of the economy, it is more likely to be reflected in an increase in the S&P 500.
Instead, if an increase in the one-year bond yield is due to a monetary policy shock,
it is more likely to be reflected in a fall in the S&P 500 (due to the higher discount
rate and, consistent with Kekre and Lenel (2022) as well as the present paper, a higher
price of risk). We discuss the robustness to using all FOMC announcement days, as
well as a number of other robustness exercises, later in this section.

4.1.2 Data

For high-frequency measures of monetary policy surprises and changes in the S&P
500, we use the data constructed by Jarocinski and Karadi (2020). They measure the
monetary surprise using the three-month ahead Fed funds futures contract. As they
argue, this horizon combines information about near term policy shocks and forward
guidance, useful during times when the zero lower bound was binding.

For data on the yield curve, we use Gurkaynak, Sack, and Wright (2008)’s inter-
polated yield curve on each day to compute yields and forwards at all maturities and
horizons at a daily frequency. We use in particular the updated data maintained by the Federal Reserve. For ease of exposition, we express the effects on all outcome variables relative to a 1pp change in the one-year yield. As previously noted, we focus on the real yield curve since our model is silent about inflation. For completeness, we present empirical estimates using the nominal yield curve in appendix B.2.

We use the January 2004 through December 2016 period for our analysis. While TIPS have been traded since the late 1990s, two- and three-year maturities were only included in Gurkaynak, Sack, and Wright (2006)’s interpolated real yield curve since 2004. We thus begin our sample at this point. We end our sample in 2016 as this is the last year in Jarocinski and Karadi (2020)’s sample.

In robustness exercises described further below, we also use the classification of FOMC announcements of Cieslak and Schrimpf (2019) and the alternative measures of monetary policy surprises constructed by Bauer and Swanson (2023), Nakamura and Steinsson (2018), and Swanson (2021).

4.1.3 Results

Figure 1 plots the baseline estimates and associated 90% confidence intervals. We find that long-dated forward rates respond economically and statistically significantly to a monetary tightening: a monetary-induced increase in the one-year yield by 1pp causes a 0.39pp increase in the one-year forward rate paying 20 years in the future.\textsuperscript{17} The point estimates imply a U-shaped pattern in maturity of the forward rate which is consistent with two effects which move in opposite directions as maturity rises. First, given nominal rigidity, a persistent rise in the nominal interest rate will induce an immediate rise in the real interest rate which dissipates over time. This mechanism is consistent with the fall in the estimated coefficients through 10 years maturity. Second, to the extent a monetary tightening raises term premia, this will be reflected in a rise in forward rates (overreaction of the forward rate, following section 3). This mechanism is consistent with the rise in the estimated coefficients from 10 to 20 years maturity, since longer maturity bonds are exposed to more risk.\textsuperscript{18}

\textsuperscript{17}Appendix B.1 visually depicts the relationship between the change in the real forward rate and the change in the one-year yield induced by the high-frequency monetary surprise, and makes clear that the positive relationship for long-dated forwards is not driven by any one observation.

\textsuperscript{18}We note that the response of the yield curve at maturities as high as 20 years is not based on any extrapolation. Appendix B.3 depicts the time to maturity of all TIPS outstanding over our sample period. As is evident, throughout our sample period there has been at least one outstanding issue with 20 years or more to maturity, and on average almost four such issues at each point in time.
This evidence bridges distinct findings in the literature. Hanson and Stein (2015) estimate that in two-day windows around FOMC announcements, a 1pp increase in the two-year nominal yield is associated with a 0.30pp increase in the 20-year instantaneous real forward, statistically significantly different from zero at all conventional levels (their Table 1). Since estimates of nominal rigidity cannot account for changes in real interest rates this far in the future, they conclude that a monetary tightening raises term premia. Nakamura and Steinsson (2018) argue that using two- or even one-day changes in yields as a measure of monetary policy surprises is misleading, because even on FOMC announcement days most of the variation in yields is induced by non-monetary shocks. We follow Nakamura and Steinsson (2018) in using intraday measures of monetary policy surprises. We find that even using this approach, a monetary tightening economically and statistically significantly raises long-dated forward rates, consistent with the findings of Hanson and Stein (2015).

Guimaraes, Pinter, and Wijnandts (2023) find that the strong response of long-dated real forwards to a monetary tightening is concentrated in times with higher liquidity, as measured using the yield curve noise series of Hu, Pan, and Wang (2013). This further suggests that our results on the response of long-dated real forwards are not driven by relative illiquidity in this market.

Our results may also help make sense of the point estimate in Beechey and Wright (2009) that the five-year ahead, five-year real forward falls upon a tightening (their Table 4). We find that the 10-year forward exhibits the smallest increase upon a tightening, but longer-dated forwards rise more.
Table 1 demonstrates that our results are robust to a number of alternative samples. The first row summarizes the baseline estimates of a monetary tightening on the 5-, 10-, 15-, and 20-year forwards (the same as the relevant points in Figure 1). The next three rows consider all FOMC announcements rather than only those in which the one-year yield and S&P 500 move in opposite directions; drop all announcements between July 2008 and June 2009 to eliminate the most acute phase of the financial crisis; and finally drop all announcements involving any news about asset purchases or non-standard credit operations, as classified by Cieslak and Schrimpf (2019). In all cases, the response of the 20-year forward is economically significant, ranging between 0.27pp and 0.50pp for a 1pp increase in the one-year yield. And in two of these three specifications, the effect is statistically significantly different from zero at a 90% level.

Table 1 also demonstrates that our results are robust to a variety of alternative measures for monetary surprises as an instrument. We use the Fed funds and forward guidance factors estimated by Swanson (2021); the principal component of the change in the first four Eurodollar contracts estimated by Bauer and Swanson (2023); and the policy news shock estimated by Nakamura and Steinsson (2018). We also use the last measure together with a sample restriction that excludes all announcements between July 2008 and June 2009, corresponding most closely to the benchmark specification in Nakamura and Steinsson (2018). In all cases, the response of the 20-year forward is again economically significant, ranging between 0.23pp and 0.41pp for a 1pp increase in the one-year yield. And in all but the last specification, the effect is statistically significantly different from zero at a 90% level.

4.2 Duration of arbitrageurs

We next study the duration of arbitrageurs. Following the literature, our preferred definition of arbitrageurs is broker/dealers and hedge funds, who trade actively across maturities to maximize risk-adjusted returns as in our model. By market clearing, this implies that households, other financial institutions such as pension funds and life insurance companies, non-financial companies, the government (including the Federal

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20Bauer and Swanson (2023) note that this surprise measure, like others in the literature, is predictable with macroeconomic and financial variables known prior to the announcement. As they argue, orthogonalizing the surprise measure is not needed (and leads to a loss of efficiency) when studying the high-frequency effects of monetary policy as we do here. When we orthogonalize our baseline Jarocinski and Karadi (2020) surprise or the Bauer and Swanson (2023) surprise, we still find that a 1pp increase in the one-year yield around FOMC announcements leads to a 0.32pp and 0.22pp increase in the 20-year forward rate, respectively. The standard errors become 0.18pp and 0.12pp, respectively.
Table 1: $\Delta f_t^{(\tau-1,\tau)}$ on $\Delta y_t^{(1)}$, instrumented by high-frequency surprise

<table>
<thead>
<tr>
<th>Specification</th>
<th>$\Delta f_t^{(4,5)}$ ($\tau$)</th>
<th>$\Delta f_t^{(9,10)}$ ($\tau$)</th>
<th>$\Delta f_t^{(14,15)}$ ($\tau$)</th>
<th>$\Delta f_t^{(19,20)}$ ($\tau$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>0.40</td>
<td>0.11</td>
<td>0.25</td>
<td>0.39</td>
</tr>
<tr>
<td></td>
<td>(0.10)</td>
<td>(0.14)</td>
<td>(0.15)</td>
<td>(0.14)</td>
</tr>
<tr>
<td>All FOMC announcements</td>
<td>0.38</td>
<td>0.11</td>
<td>0.13</td>
<td>0.27</td>
</tr>
<tr>
<td></td>
<td>(0.10)</td>
<td>(0.11)</td>
<td>(0.15)</td>
<td>(0.13)</td>
</tr>
<tr>
<td>Excl. 7/08-6/09</td>
<td>0.46</td>
<td>-0.26</td>
<td>0.21</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td>(0.22)</td>
<td>(0.30)</td>
<td>(0.21)</td>
<td>(0.29)</td>
</tr>
<tr>
<td>Excl. announcements with LSAP news</td>
<td>0.28</td>
<td>-0.12</td>
<td>0.07</td>
<td>0.30</td>
</tr>
<tr>
<td></td>
<td>(0.12)</td>
<td>(0.17)</td>
<td>(0.14)</td>
<td>(0.19)</td>
</tr>
<tr>
<td>Swanson (2021) Fed funds IV</td>
<td>0.31</td>
<td>0.15</td>
<td>0.30</td>
<td>0.41</td>
</tr>
<tr>
<td></td>
<td>(0.13)</td>
<td>(0.13)</td>
<td>(0.16)</td>
<td>(0.17)</td>
</tr>
<tr>
<td>Swanson (2021) forward guidance IV</td>
<td>1.05</td>
<td>0.44</td>
<td>0.25</td>
<td>0.23</td>
</tr>
<tr>
<td></td>
<td>(0.23)</td>
<td>(0.13)</td>
<td>(0.13)</td>
<td>(0.12)</td>
</tr>
<tr>
<td>Bauer and Swanson (2023) IV</td>
<td>0.64</td>
<td>0.27</td>
<td>0.17</td>
<td>0.23</td>
</tr>
<tr>
<td></td>
<td>(0.14)</td>
<td>(0.11)</td>
<td>(0.14)</td>
<td>(0.13)</td>
</tr>
<tr>
<td>Nakamura and Steinsson (2018) IV</td>
<td>0.64</td>
<td>0.27</td>
<td>0.35</td>
<td>0.40</td>
</tr>
<tr>
<td></td>
<td>(0.15)</td>
<td>(0.13)</td>
<td>(0.11)</td>
<td>(0.13)</td>
</tr>
<tr>
<td>NS (2018) IV, excl. 7/08-6/09</td>
<td>0.72</td>
<td>-0.07</td>
<td>0.13</td>
<td>0.29</td>
</tr>
<tr>
<td></td>
<td>(0.32)</td>
<td>(0.26)</td>
<td>(0.19)</td>
<td>(0.26)</td>
</tr>
</tbody>
</table>

Notes: robust standard errors provided in parenthesis.

Reserve), and the rest of the world are modeled as habitat investors.

4.2.1 Balance sheets and asset class duration

Our first approach is to combine data on the balance sheets of broker/dealers and hedge funds with estimates of duration by asset class. The advantage of this approach is that it allows us to characterize the aggregate duration of this broad group of institutions. The disadvantage is that it assumes that these institutions hold a representative portfolio within each asset class, and it cannot account for the effect of derivative positions on these institutions’ true interest rate exposure.

We use four data sources. We obtain the aggregate balance sheet of broker/dealers from the Financial Accounts, which includes the broker/dealer subsidiaries of commercial banks. We obtain the aggregate balance sheet of hedge funds filing Form PF to

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The approach of combining risk exposures by asset class with positions by asset class follows Begenau, Piazzesi, and Schneider (2015) and Greenwald, Leombroni, Lustig, and Van Nieuwerburgh (2023). We study the duration of broker/dealers and hedge funds, whereas these papers study the duration of commercial banks and households, respectively.
the Securities and Exchange Commission (SEC), summarized in the Enhanced Financial Accounts.\footnote{22} This data is provided since the fourth quarter of 2012 and provides substantially more information about portfolio holdings and leverage in the hedge fund sector than previously available sources such as BarclayHedge and Lipper TASS. We obtain the effective duration of the U.S. Treasury, U.S. mortgage-backed security, and U.S. corporate bond indices computed by Bloomberg.\footnote{23} These duration measures account for the optionality embedded in the latter two classes of securities, such as the ability to prepay. Finally, we obtain valuation ratios on the S&P 500 available from Robert Shiller’s website to compute the duration of equities.

Given this data, we proceed in three steps. Each quarter, we first compute the net positions of each set of financial institutions in each asset class. The sum of these positions is wealth.\footnote{24} The first three columns of Table 2 summarize their individual and aggregate balance sheets in the fourth quarter of 2012. As is evident, these institutions hold a levered position in cash, Treasuries, corporate and foreign bonds, other debt securities (primarily agency/GSE-backed securities), and corporate equities, financed by repurchase agreements and other short-term loans (primarily secured borrowing of hedge funds from prime brokerages).

We next combine this data with estimates of duration by asset class. The last column of Table 2 summarizes this in the fourth quarter of 2012. We assume that Treasuries, corporate and foreign bonds, and other debt securities have the effective duration of the Bloomberg Treasury, corporate bond, and mortgage-backed security indices, respectively. We assume that cash, deposits, and money market fund shares have an average duration of one quarter, repo and other short-term loans have an average duration of one month, and loans have a duration of five years. We use the price-dividend ratio on the S&P 500 together with the Gordon growth formula to compute the duration of equities, following the approach of Greenwald et al. (2023) and further described in appendix B.4.

We finally compute wealth-weighted aggregate duration using the last two columns

\footnote{22}Hedge funds must file Form PF if they are registered or are required to register with the SEC, manage private funds, and have at least $150 million in such assets under management. Importantly, this includes hedge funds both domiciled in the U.S. and abroad (such as the Cayman Islands).

\footnote{23}These were previously the Barclays indices, and prior to that the Lehman Brothers indices. These are among the most widely used bond indices in the literature.

\footnote{24}In the Financial Accounts (for broker/dealers), wealth is total financial assets less liabilities, less FDI and miscellaneous assets less liabilities. Since the latter largely correspond to transactions with holding and parent companies, this means we measure wealth at the level of the broker/dealer subsidiary itself. In the Form PF filings (for hedge funds), wealth is net asset value.
<table>
<thead>
<tr>
<th>Balance sheet ($bn)</th>
<th>Broker/dealers</th>
<th>Hedge funds</th>
<th>Sum</th>
<th>Duration (years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash, deposits, MMFs</td>
<td>128</td>
<td>553</td>
<td>681</td>
<td>0.25</td>
</tr>
<tr>
<td>Repo and other short-term loans*</td>
<td>-448</td>
<td>-1,231</td>
<td>-1,679</td>
<td>0.083</td>
</tr>
<tr>
<td>Treasuries</td>
<td>185</td>
<td>654</td>
<td>839</td>
<td>5.4</td>
</tr>
<tr>
<td>Corporate and foreign bonds</td>
<td>40</td>
<td>994</td>
<td>1,034</td>
<td>7.2</td>
</tr>
<tr>
<td>Other debt securities†</td>
<td>302</td>
<td>61</td>
<td>363</td>
<td>3.2</td>
</tr>
<tr>
<td>Loans‡</td>
<td>-35</td>
<td>133</td>
<td>99</td>
<td>5</td>
</tr>
<tr>
<td>Corporate equities</td>
<td>127</td>
<td>1,148</td>
<td>1,275</td>
<td>46.5</td>
</tr>
<tr>
<td>Wealth§</td>
<td>299</td>
<td>2,313</td>
<td>2,612</td>
<td>27.9</td>
</tr>
<tr>
<td>Only fixed income</td>
<td>172</td>
<td>1,164</td>
<td>1,336</td>
<td>10.1</td>
</tr>
</tbody>
</table>

Table 2: duration of arbitrageurs in Q4 2012

Notes: see text for data sources and definitions of wealth-weighted duration.

* Includes cash and margin accounts of households at broker/dealers, clearing funds and receivables/payables among broker/dealers (including securities lending), and hedge fund loan liabilities (largely secured borrowing from prime brokerages).
† Includes open market paper, municipal securities, and agency/GSE-backed securities.
‡ Includes broker/dealer loans to non-financial corporates, depository institution loans to broker/dealers not elsewhere counted, and loan assets of hedge funds.
§ For broker/dealers, equals financial assets less liabilities, less FDI and miscellaneous line items (largely transactions with holding companies and parents). For hedge funds, equals net asset value less miscellaneous line items.

of Table 2. In the fourth quarter of 2012, this implies arbitrageurs’ duration is 27.9. Since the duration of equities plays an important role in driving this number up, we also consider the possibility that equity and fixed income arbitrageurs are segmented, in which case we eliminate equities from our calculation and focus on fixed income duration alone. This implies arbitrageurs’ duration is 10.1. Repeating this process for each quarter through 2016 (given our maintained sample period of interest) and averaging over time, arbitrageurs’ duration is between 9.5 (fixed income alone) and 30.9 (also including equities). One way to make sense of these estimates is that the duration of ultimate investments of these arbitrageurs is around 5 to 10 years, and their leverage in these investments is around 2 to 3.25,26

25This interpretation also helps makes sense of the high duration of equities, which are themselves levered claims on long duration assets.
26Drechsler, Savov, and Schabl (2021) provide evidence that commercial banks are not much exposed to interest rate risk because deposits, which constitute an important component of their liabilities, pay sticky interest rates, like long duration assets. We note that for the broker/dealers and hedge funds which are our focus, this is less relevant because deposits are not an aggregate source of funding (in fact, these sectors are net long cash, deposits, and money market fund shares, as demonstrated in
4.2.2 High-frequency response of dealer equity prices

Our second approach is to measure the high-frequency response of primary dealers’ stock prices around FOMC announcements, paralleling our analysis of the yield curve. The advantage of this approach is that it captures the realized exposure to a macroeconomic risk factor without the assumptions required in the prior subsection. The disadvantage is that it is restricted to publicly traded primary dealers, as hedge funds are not publicly traded, and may reflect the economic exposure of other parts of dealers’ holding companies rather than the dealer subsidiary itself.\footnote{As noted by He, Kelly, and Manela (2017), the last point may not be a concern if internal capital markets are frictionless, in which case it is more relevant to measure the holding companies’ exposure.}

To construct the high-frequency response of dealers’ stock prices, we use the list of primary dealers provided by the Federal Reserve and intraday quotes using TAQ.\footnote{The list of dealers for which we have stock market data is provided in appendix B.5. While we focus on data between 2004 and 2016 to be consistent with our analysis of the TIPS yield curve, we find that all of our results regarding the high-frequency response of dealers’ stock prices are robust to beginning the sample in 1993, when the TAQ data becomes available.} For each publicly traded and active dealer around an FOMC announcement, we measure the closest prices of transactions 10 minutes prior to the FOMC announcement and 20 minutes after the FOMC announcement.\footnote{For FOMC announcements occurring outside NYSE trading hours, we use the preceding closing price and following opening price, following Gorodnichenko and Weber (2016).} We then aggregate the change in dealer prices in this 30-minute window, weighting by dealers’ market capitalizations at the end of the previous trading day from CRSP.

We find that a surprise monetary tightening generates an economically and statistically significant fall in dealer equity prices in this 30-minute window. In our baseline specification reported in the first row of Table 3, a 1\textit{pp} increase in the one-year yield induced by a monetary tightening causes a 9.8\textit{pp} decline in dealer equity prices.\footnote{The change in the one-year yield is still the one-day change, as throughout this section.} \footnote{Appendix B.6 depicts this relationship. The appendix further demonstrates that it is important to focus on the response of dealer equities in the 30-minute window around FOMC announcements to have enough power to detect these effects. While the one-day response of dealer prices is comparable to that obtained in the 30-minute window, it is not statistically significantly different from zero. This was not the case for our estimated effects on the yield curve. This makes sense because equity prices are more volatile than forward rates and thus the signal to noise ratio is lower.} The fall in dealer equity prices is in fact 3.6\textit{pp} more than the fall in the broader S&P 500, though we only mention this for additional context; the absolute change in dealer wealth, not the relative change, is relevant for our model. The remaining rows of Table 3 demonstrate that across the same alternative samples and measures of monetary sur-
<table>
<thead>
<tr>
<th>Specification</th>
<th>30-minute change in dealer equity prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>-9.8</td>
</tr>
<tr>
<td></td>
<td>(3.2)</td>
</tr>
<tr>
<td>All FOMC announcements</td>
<td>-1.4</td>
</tr>
<tr>
<td></td>
<td>(4.5)</td>
</tr>
<tr>
<td>Excl. 7/08-6/09</td>
<td>-19.4</td>
</tr>
<tr>
<td></td>
<td>(11.2)</td>
</tr>
<tr>
<td>Excl. announcements with LSAP news</td>
<td>-12.4</td>
</tr>
<tr>
<td></td>
<td>(5.8)</td>
</tr>
<tr>
<td>Swanson (2021) Fed funds IV</td>
<td>-10.0</td>
</tr>
<tr>
<td></td>
<td>(3.6)</td>
</tr>
<tr>
<td>Swanson (2021) forward guidance IV</td>
<td>-11.4</td>
</tr>
<tr>
<td></td>
<td>(5.0)</td>
</tr>
<tr>
<td>Bauer and Swanson (2023) IV</td>
<td>-7.7</td>
</tr>
<tr>
<td></td>
<td>(3.2)</td>
</tr>
<tr>
<td>Nakamura and Steinsson (2018) IV</td>
<td>-12.2</td>
</tr>
<tr>
<td></td>
<td>(4.0)</td>
</tr>
<tr>
<td>NS (2018) IV, excl. 7/08-6/09</td>
<td>-24.2</td>
</tr>
<tr>
<td></td>
<td>(10.0)</td>
</tr>
</tbody>
</table>

Table 3: change in dealer prices on $\Delta y_t^{(1)}$, instrumented by high-frequency surprise

Notes: robust standard errors provided in parenthesis.

prises as in Table 1, dealer equity prices fall by 1.4pp – 24.2pp in response to a 1pp rise in the one-year yield. In all but the second specification, the response is statistically significantly different from zero at a 90% level.

Our earlier estimates of duration imply that a 1pp permanent rise in the real short rate would lower arbitrageurs’ wealth by 9.5 – 30.9pp. This is larger than but consistent with our estimates of the high-frequency response of dealers’ stock prices to a monetary shock, keeping in mind that a monetary tightening implies only a transitory increase in the real short rate, and the response of equity prices to a monetary shock should therefore be lower than our previous duration estimates.

4.3 Relating yield curve responses and arbitrageur duration

Before turning to the quantitative model, we relate the evidence from the prior two subsections: we ask whether, in the data, the effects of monetary shocks on the yield curve depend on the duration of arbitrageurs. This speaks to a distinctive prediction
of our theory vis-à-vis other explanations focused on habitat demand or changing macroeconomic comovements. In addition to a direct test of the mechanism, this evidence is useful to further evaluate our quantitative model in the next section.\footnote{In appendix B.9 we provide a complementary test of the mechanism: we directly regress the change in forward rates on dealers’ high-frequency change in equity prices, instrumenting the latter with the monetary surprise. Mechanically, the coefficients are those from section 4.1 divided by those from section 4.2.2, so long-dated forwards indeed rise by more when dealers’ stock prices fall by more. Less obviously, we find that the results at the 20-year maturity are statistically significantly different from zero in our baseline specification and four of the alternative specifications.}

A challenge is that the hedge fund data used in the prior subsection is not available prior to 2012, the period over which we observe most of the variation in monetary shocks. We nonetheless consider three alternative proxies for arbitrageur duration available over the entire sample period. In each case, focusing on the 20-year forward rate for parsimony, we regress the one-day change in the forward rate on the one-day change in the one-year yield, the proxy for arbitrageur duration measured prior to the monetary surprise, and their interaction. We use as instruments the monetary surprise and its interaction with the proxy for arbitrageur duration. Appendix B.7 plots the three proxies and demonstrates that they track each other quite well.

We first use estimates of the term premium itself as a proxy for arbitrageur duration. As implied by (14) and its analog in the full model, the risk premium on long-term bonds will be higher when arbitrageurs have higher duration and thus bear more interest rate risk. We use estimates of the 5-year forward, 5-year real term premium provided by the Federal Reserve using the methodology of D’Amico, Kim, and Wei (2018).\footnote{Similar results are obtained using their estimates of the 10-year real term premium, but with larger standard errors. This is to be expected, since the 5-year forward, 5-year term premium is more variable than the 10-year term premium (the latter being the average of the 5-year term premium and the 5-year forward, 5-year term premium).}

The first column of Table 4 implies that in the baseline specification, a 1\textit{pp} higher term premium would raise the response of the 20-year forward rate to a 100\textit{bp} increase in the one-year yield by 50\textit{bp}. Across specifications, it would raise the response by 8\textit{−}111\textit{bp}. This is economically meaningful, given that the standard deviation of the estimated term premium is 0.4\textit{pp} over the sample period.

We next use a measure of arbitrageur duration for broker/dealers alone. We construct a higher frequency analog of the broker/dealer duration measure from section 4.2.1 using the primary dealer statistics collected and published weekly by the Federal Reserve Bank of New York. In particular, we multiply primary dealers’ net position in Treasury securities, agency/GSE-backed securities and non-agency mortgage-backed

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### Proxy for arb duration

<table>
<thead>
<tr>
<th>Specification</th>
<th>Proxy for arb duration</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5-yr fwd, 5-yr TP</td>
</tr>
<tr>
<td></td>
<td>Log dealer dur.</td>
</tr>
<tr>
<td></td>
<td>−Dealer income gap</td>
</tr>
<tr>
<td>Baseline</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td>(0.25)</td>
</tr>
<tr>
<td>All FOMC announcements</td>
<td>0.11</td>
</tr>
<tr>
<td></td>
<td>(0.43)</td>
</tr>
<tr>
<td>Excl. 7/08-6/09</td>
<td>1.08</td>
</tr>
<tr>
<td></td>
<td>(0.63)</td>
</tr>
<tr>
<td>Excl. announcements with LSAP news</td>
<td>0.43</td>
</tr>
<tr>
<td></td>
<td>(0.38)</td>
</tr>
<tr>
<td>Swanson (2021) Fed funds IV</td>
<td>0.70</td>
</tr>
<tr>
<td></td>
<td>(0.42)</td>
</tr>
<tr>
<td>Swanson (2021) forward guidance IV</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td>(0.20)</td>
</tr>
<tr>
<td>Bauer and Swanson (2023) IV</td>
<td>0.65</td>
</tr>
<tr>
<td></td>
<td>(0.73)</td>
</tr>
<tr>
<td>Nakamura and Steinsson (2018) IV</td>
<td>0.56</td>
</tr>
<tr>
<td></td>
<td>(0.55)</td>
</tr>
<tr>
<td>NS (2018) IV, excl. 7/08-6/09</td>
<td>1.11</td>
</tr>
<tr>
<td></td>
<td>(0.70)</td>
</tr>
</tbody>
</table>

Table 4: $\Delta f_t^{(20)}$ on $\Delta y_t^{(1)}$, duration of arbitrageurs, and interaction, instrumented by high-frequency surprise

Notes: each cell reports the estimated coefficient on the interaction term. Robust standard errors provided in parenthesis.

The second column of Table 4 implies that in the baseline specification, a 1pp year increase in dealer duration would raise the response of the 20-year forward rate to a 100bp increase in the one-year yield by 0.34bp. Across specifications, it would raise the response by 0.05 − 1.08bp. This is

34We cumulate daily returns to obtain an equity index representative of all dealers, not just the publicly traded ones. Thus, while the level of this series is not interpretable, its changes are.

35Du, Hebert, and Li (2022) document that dealers’ switched from a net short to net long position in Treasuries in the global financial crisis. However, because they have had consistently long positions in agency/GSE-backed securities, non-agency mortgage-backed securities, and corporate bonds, we do not find a sign switch in overall duration around the crisis.
also meaningful, given that the standard deviation of this measure of dealer duration is 50pp over the sample period.

We finally use a measure of interest rate sensitivity more broadly studied in the literature, the income gap (see, for instance, Gomez, Landier, Sraer, and Thesmar (2021) and Haddad and Sraer (2020)). The income gap is constructed at the holding company level from schedule HC-H of form FR Y9-C reported to the Federal Reserve, which directly asks about banks’ sensitivity to interest rates. The income gap for a dealer is the dollar amount of assets that mature or reprice within one year less the dollar amount of liabilities that mature or reprice within one year, relative to total assets. It is thus negatively related to duration. The aggregate dealer income gap is the asset-weighted average across dealers. The last column of Table 4 implies that in the baseline specification, a 1pp lower income gap would raise the response of the 20-year forward rate to a 100bp increase in the one-year yield by 2.0bp. Across specifications, it would raise the response by 1.4 − 16.2bp. This is again meaningful, given that the standard deviation of the aggregate income gap is 8pp over our sample period.

In appendix B.8 we instead use the high-frequency change in primary dealers’ equity prices on the left-hand side of these regressions. An additional virtue of the income gap data in particular is that it is available at the individual dealer level. We show that among dealers, those with a lower income gap (higher duration) experience a larger fall in their stock prices upon a monetary tightening.\footnote{Haddad and Sraer (2020) document an interesting puzzle in the relationship between commercial bank equity prices, bond returns, and income gaps without conditioning on monetary shocks: they show that when the aggregate income gap of commercial banks is low, bank stock prices do not have a more positive relationship with excess long-term bond returns. We demonstrate in appendix B.8 that, when focusing on dealers, this puzzle also disappears.}

5 Quantitative analysis

We now assess the ability of our full model to rationalize the effects of monetary policy on the yield curve. Calibrated to match the evidence on arbitrageur duration, it can account for much of the responses of long-dated real forward rates in the data. We quantify the additional implications of our model for state-dependence in policy transmission, the volatility and slope of the yield curve, and trends in term premia accompanying trends in the natural rate.
5.1 Equilibrium and solution

We first summarize the equilibrium conditions of the full model environment described in section 2 and the computational algorithm we use to solve it.

**Equilibrium** As derived formally in appendix C.1, arbitrageurs’ first-order conditions for the problem (3)-(4) imply that

\[
E_t \left( \frac{dP_t^{(\tau)}}{P_t^{(\tau)}} \right) - r_t dt = \\
\frac{\gamma}{W_t} \int_0^\infty X_t^{(s)} \text{Cov}_t \left( \frac{dP_t^{(\tau)}}{P_t^{(\tau)}}, \frac{dP_t^{(s)}}{P_t^{(s)}} \right) ds - (1 - \gamma) \text{Cov}_t \left( \frac{dP_t^{(\tau)}}{P_t^{(\tau)}}, \frac{dv_t}{v_t} \right),
\]

(18)

where \(v_t\) defines the marginal value of wealth in the value function

\[
v_t(w_t) = \left( \frac{\nu_t w_t}{1 - \gamma} \right)^{1 - \gamma} - 1
\]

solving (3). Generalizing (14) in the simple model, (18) says that arbitrageurs require non-zero expected excess returns on a bond of maturity \(\tau\) to compensate them for bearing price risk on that bond. Their exposure to a bond with maturity \(\tau\) depends in part on the covariance of returns on that bond with all other bonds of maturity \(s \in (0, \infty)\) and the arbitrageurs’ position in those bonds \(\{X_t^{(s)}\}_{s=0}^\infty\). Away from log preferences (\(\gamma \neq 1\)), arbitrageurs’ required risk compensation also reflects a standard intertemporal hedging motive: they require a lower expected excess return on a bond if it pays well when the (instantaneous change in the) marginal value of wealth \((1 - \gamma) d\nu_t\) is positive. When arbitrageurs’ discount rate \(\rho \to \infty\), appendix C.1 proves that the marginal value of wealth \(\nu_t \to 1\), so that the intertemporal hedging motive vanishes. We focus on this case for simplicity. This allows us to continue focusing on endogenous wealth \(W_t\) in risk pricing as the only departure from existing preferred habitat models.

Substituting habitat demand (1) and market clearing (8) into (18) in the \(\rho \to \infty\) limit in which the intertemporal hedging motive drops out, we obtain

\[
E_t \left( \frac{dP_t^{(\tau)}}{P_t^{(\tau)}} \right) - r_t dt = 
\]
Substituting habitat demand and market clearing in arbitrageurs’ aggregate evolution of wealth (5), we obtain

\[
\gamma \int_0^\infty \left( \alpha_1(\tau_1) \log \left( P^{(\tau_1)}_t \right) + \theta_0(\tau_1) + \theta_1(\tau_1) \beta_t \right) Cov_t \left( \frac{dP^{(\tau_1)}_t}{P^{(\tau_1)}_t}, \frac{dP^{(s)}_t}{P^{(s)}_t} \right) ds. \tag{19}
\]

These equilibrium conditions parallel (15) and (16) in the simple model. Together with the driving forces (6)-(7), this characterizes the equilibrium.

**Solution** In a large class of term structure models, including existing models in the preferred habitat tradition, bond prices are exponentially affine in the model’s state variables. The dependence of the price of risk on arbitrageurs’ wealth in our setting implies that bond prices are no longer exponentially affine in this way.

We therefore characterize bond prices as a general function of the three state variables \( r_t, \beta_t \) and \( W_t \)

\[
P^{(\tau)}_t \equiv P^{(\tau)}(r_t, \beta_t, W_t). \tag{21}
\]

Writing the evolution of wealth as

\[
dW_t = \omega(r_t, \beta_t, W_t) dt + \eta_r(r_t, \beta_t, W_t) dB_{r, t} + \eta_\beta(r_t, \beta_t, W_t) dB_{\beta, t} \tag{22}
\]

for some functions \( \omega, \eta_r, \) and \( \eta_\beta, \) we can use (6), (7), and (22) together with Ito’s Lemma to write (19) as a partial differential equation (PDE) relating partial derivatives of \( \{P^{(\tau)}\}_{\tau=0}^\infty \) and the state variables \( r_t, \beta_t, \) and \( W_t. \) Given conjectures for the functions \( \omega, \eta_r, \) and \( \eta_\beta, \) the Feynman-Kac formula implies a solution \( P^{(\tau)} \) which we numerically solve using Monte Carlo simulation. We then use (20) to characterize the implied evolution of \( W_t \) and iterate over our guesses for the functions \( \omega, \eta_r, \) and \( \eta_\beta \) until (22) is consistent with (20). Further details on the algorithm are in appendix C.2.
5.2 Calibration

We assume an exponential form for the price elasticity, intercept, and slope of habitat demand by maturity:

\[
\alpha(\tau) = \alpha \exp^{-\tau}, \\
\theta_0(\tau) = \theta_0 \exp^{-\tau}, \\
\theta_1(\tau) = \theta_1 \exp^{-\tau},
\]

for \( \tau \leq 30 \), and \( \alpha(\tau) = \theta_0(\tau) = \theta_1(\tau) = 0 \) for \( \tau > 30 \). When comparing the model to the data we focus on the term structure through 20 years, since TIPS are relatively illiquid with maturity greater than this. Since only the product \( \theta_1 \sigma_\beta \) matters for the equilibrium dynamics, we normalize \( \theta_1 = 1 \). Since \( \{\bar{W}, \theta_0, \sigma_\beta, \alpha\} \) can each be scaled without changing the state-contingent path of prices or returns, we normalize \( \theta_0 = 1 \).

The calibration of remaining moments is summarized in Table 5. We calibrate the model to match three sets of moments: unconditional moments of the yield curve, the evidence on arbitrageur duration assembled in section 4, and the yield curve responses to quantitative easing studied widely in the literature. We reiterate that our calibration focuses on the real yield curve, since our model is silent about inflation.

We first set a subset of parameters to match unconditional moments of the yield curve.\(^{38}\) We set the average level of the short rate \( \bar{r} \) to match the average one-year yield of 0.06%. We set arbitrageur risk aversion \( \gamma \) to match the yield curve slope \( y_t(20) - y_t(1) \). We set the volatility \( \sigma_r \) and mean reversion \( \kappa_r \) of short rate shocks to match the monthly volatility of the 20-year yield in levels and changes. Finally, we set the volatility \( \sigma_\beta \) of demand shocks to match the relationship between the slope of the yield curve and excess returns on 10-year bonds over the next year, denoted \( \beta_{FB}^{(10)} \) with reference to Fama and Bliss (1987).\(^{39}\) We later discuss why this classic evidence

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\(^{37}\)We could also allow for trade in assets with even greater maturity than 30 years, capturing equity claims. Increasing the maximal duration of traded assets does not meaningfully affect our results, conditional on calibrating parameters to match our targeted moments.

\(^{38}\)All moments in the data are computed over the same January 2004 through December 2016 period studied in section 4. In the usual way, all parameters jointly determine the moments we target. For each parameter, we describe the moment we primarily target by varying that parameter.

\(^{39}\)Formally, in both data and model we estimate the specification

\[
r_t^{(10)} - y_t^{(1)} = \alpha_{FB}^{(10)} + \beta_{FB}^{(10)} (f_t^{(9,10)} - y_t^{(1)}) + \epsilon_{FB,t+1}^{(10)}.
\]

For this moment alone, we report the estimated coefficient in the model estimated on very long samples.
<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
<th>Moment</th>
<th>Target</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Unconditional moments of yield curve</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{r}$ mean short rate</td>
<td>-0.0004</td>
<td>$y^{(1)}_t$</td>
<td>0.06%</td>
<td>0.06%</td>
</tr>
<tr>
<td>$\gamma$ arb. risk aversion</td>
<td>2</td>
<td>$y^{(20)}_t - y^{(1)}_t$</td>
<td>1.54%</td>
<td>1.53%</td>
</tr>
<tr>
<td>$\sigma_r$ std. dev. short rate</td>
<td>0.007</td>
<td>$\sigma(y^{(20)}_t)$</td>
<td>0.74%</td>
<td>0.74%</td>
</tr>
<tr>
<td>$\kappa_r$ mean rev. short rate</td>
<td>0.03</td>
<td>$\sigma(\Delta y^{(20)}_t)$</td>
<td>0.56%</td>
<td>0.57%</td>
</tr>
<tr>
<td>$\sigma_\beta$ std. dev. demand</td>
<td>0.45</td>
<td>$\beta_{FB}^{(10)}$</td>
<td>0.68</td>
<td>0.69</td>
</tr>
<tr>
<td><strong>Duration of arbitrageurs</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{W}$ arb. endowment</td>
<td>0.002</td>
<td>duration</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>$\kappa_\beta$ mean rev. demand</td>
<td>0.08</td>
<td>$\sigma(\log(\text{duration}))$</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td><strong>Yield curve responses to QE announcement on March 18, 2009</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha$ habitat price elast.</td>
<td>5</td>
<td>$df^{(9,10)}_t$</td>
<td>-0.46%</td>
<td>-0.53%</td>
</tr>
<tr>
<td>$\xi$ persistence arb. wealth</td>
<td>0.05</td>
<td>$df^{(19,20)}_t$</td>
<td>-0.25%</td>
<td>-0.25%</td>
</tr>
</tbody>
</table>

Table 5: baseline calibration

Notes: $\Delta$ denotes annual change, $\sigma$ denotes monthly standard deviation, $d$ denotes instantaneous change, and moments without these symbols are simple time-series averages. Model moments are computed by averaging over 3,000 samples of 13 years worth of data, themselves simulated after a burn-in period of 1,200 months.

on return predictability is informative about the magnitude of demand shocks.

We next set parameters to match the novel evidence on arbitrageur duration assembled in section 4. Arbitrageurs’ duration in the model is

$$ (\text{duration})_t \equiv \frac{\int_0^\infty \tau X^{(\tau)}_t d\tau}{\bar{W}_t}. \quad (23) $$

Arbitrageurs’ initial wealth $\bar{W}$ affects their average level of wealth and thus the denominator of (23). We set it to match the midpoint of the range of duration estimated in section 4.2.1, 20. We later explore the sensitivity of our findings to this targeted value.

(of over 3,000 years), rather than over samples of 13 years as in the data. This is because the persistence of the short rate implies that the coefficient would be substantially biased in shorter samples, following Stambaugh (1999). We have verified that adding an additional, highly transitory source of fluctuations to the short rate would eliminate this bias. Our calibrated model is consistent with the interpretation that we are missing such a transitory source of fluctuations in the short rate: the model-generated volatility in the one-year yield is 0.90%, as opposed to 1.66% in the data. We do not include this as an additional driving force to simplify the computation. Since we discipline the current parameters to match second moments in longer yields, adding a transitory source of fluctuations in the short rate should not change the current parameters and thus our results of interest.
The mean reversion $\kappa_\beta$ of demand shocks affects the volatility of arbitrageurs’ assets and thus the volatility in the numerator of (23). We set it to match the volatility of log duration of publicly traded primary dealers discussed in section 4.3.

We lastly set parameters to match evidence on the yield curve responses to quantitative easing (QE) which have been widely studied in the literature. QE is a habitat demand shock in our model since the Federal Reserve is included in the set of habitat investors. The yield curve responses to QE discipline both habitat investors’ price elasticities, controlled by $\alpha$, as well as the speed of mean reversion in arbitrageur wealth, controlled by $\xi$. The intuition for why QE disciplines $\alpha$ can be understood using the simple model of section 3, in which case the response of the two-period bond price to a demand shock is

$$
\frac{d \log P_t}{d \theta_t} = -\frac{1}{\alpha + \frac{w_t}{\sigma^2}}.
$$

(24)

The denominator is the price elasticity of the aggregate demand for two-period bonds; in the usual way, the more elastic it is, the smaller will be the equilibrium price response. Conditional on the price elasticity of arbitrageurs $\frac{w_t}{\sigma^2}$ implied by the other calibration targets above, the equilibrium price response thus disciplines the price elasticity of habitat investors $\alpha$. The intuition for why QE disciplines $\xi$ reflects the fact that — as we later demonstrate — QE revalues wealth and thus arbitrageurs’ risk bearing capacity like a monetary shock. Since $\xi$ governs the speed with which arbitrageurs’ wealth returns to its long-run average, it governs the persistence of term premium responses and thus the shape of the yield curve responses on impact of QE.

We discipline both parameters using the forward rate responses to the March 18, 2009 announcement that the Federal Reserve would begin purchasing Treasuries and expand its purchases of agency/GSE-backed securities and mortgage-backed securities. Appendix C.6 motivates why we focus on this announcement and describes how we translate the announcement into model scale. We simulate it starting from arbitrageur wealth one third below its average value, consistent with the decline in total wealth among broker/dealers and hedge funds between the fourth quarter of 2007 and first quarter of 2009 (also detailed in appendix C.6). We calibrate $\alpha$ and $\xi$ to match the 10- and 20-year forward rate responses to the announcement, respectively. We target the response of long-dated forward rates rather than yields to minimize the role of forward guidance which may have accompanied the announcement. Since some of the announced asset purchases may have been anticipated, this will imply a value for $\alpha$ in particular which is an upper bound, working against our ability to account for the
term premium effects of monetary shocks as demonstrated in our analytical results. Later in this section, we assess the sensitivity of our results to a lower value of $\alpha$.

5.3 Effects of monetary shock

We now turn to the model’s key impulse response: the effects of a monetary shock.

5.3.1 Impulse responses to monetary shock

Figure 2 depicts the impulse responses to a contractionary monetary shock, scaled to generate a 100bp rise in the one-year real yield on impact. As discussed in section 2, we allow a monetary-induced shock to the real short rate to have a different speed of mean reversion $\kappa_m$ than “typical” short rate shocks $\kappa_r$ arising from underlying shocks to preferences or productivity. We set $\kappa_m = 0.25$ so that the response of the two-year forward rate exactly matches that in the data; thus, monetary-induced short rate shocks are less persistent than typical short rate shocks, which we view as sensible.\(^{40}\)

The first row of Figure 2 depicts the short rate, the one-year real yield, and arbitrageur wealth. The second row depicts the 20-year real forward rate, the spread between the 20-year real forward rate and one-year yield, and expected excess returns on the 20-year bond financed by the one-year bond over a one year holding period. The impulse responses are contrasted against those in a counterfactual economy in which $\xi \to \infty$ and thus arbitrageurs’ endowment is constant.\(^{41}\)

The 20-year real forward rate rises in response to the shock, in contrast to the counterfactual model in which arbitrageurs’ endowment is constant. The difference in these responses is driven by the downward revaluation of arbitrageurs’ wealth, which raises their price of bearing risk and raises term premia. Since term premia have risen, future excess returns on the 20-year bond are high — persistently so, reflecting the pattern of arbitrageurs’ wealth.\(^{42}\) The opposite is true in the counterfactual model.

\(^{40}\)Appendix C.5 provides the impulse responses to a typical short rate shock.

\(^{41}\)In this counterfactual economy, we leave all parameters unchanged except $\gamma$, which we recalibrate to match the same level of the yield spread as the baseline calibration (and the data). This ensures that our comparison of risk premium responses across these models does not mechanically reflect differences in the level of the risk premium itself.

\(^{42}\)Notably, since the fall in the short rate is not permanent, the forward spread falls as the yield curve flattens. This implies that, at least around impact of the shock, there is a negative relationship between the slope of the yield curve and subsequent excess returns on long-term bonds. The same is true for typical short rate shocks simulated in appendix C.5. By contrast, habitat demand shocks imply a positive relationship between the slope of the yield curve and subsequent excess returns on long-term bonds. This is why the classic evidence on return predictability of Fama and Bliss (1987)
Figure 2: impulse responses to monetary shock

Notes: monetary shock is a one-time innovation to short rate with mean reversion $\kappa_m = 0.25$ as described in main text. Figure depicts responses to infinitesimal shock, scaled to generate 100bp fall in one-year yield on impact, and $x$-axis denotes number of months since the shock. Responses are averaged (relative to no shock) starting at 100 points drawn from the ergodic distribution of the state space, itself approximated as a simulation over 46,800 months after a burn-in period of 1,200 months.

All of these results are consistent with the analytical results in section 3.

Figure 3 depicts the impact effect of the monetary shock on the forward rate across maturities and compares it to the estimates from Figure 1. The model generates responses within the empirical confidence intervals at all maturities. The model does not generate a $U$-shaped response of forward rates as in our baseline specification in the data; the increase in term premia by maturity is indeed present (evident from the widening gap between the responses of the forward rate and expected spot rate), but in the model it is outweighed by the decline in the expected spot rate by maturity.\textsuperscript{43,44}

\textsuperscript{43} Using that the spread between the forward rate and expected short rate equals the cumulative expected return to a sequence of carry strategies, appendix C.3 demonstrates that it is the persistence of the rise in expected carry trade returns which explains why term premia rise with maturity.

\textsuperscript{44} Appendix C.4 demonstrates that if the monetary shock has a less persistent effect on the real short rate (higher $\kappa_m$), the model generates a $U$-shaped response of forward rates.

\textsuperscript{43,44} Campbell and Shiller (1991) can be used to identify the volatility of demand shocks in Table 5.
5.3.2 Sensitivity to duration and demand elasticity

The empirical evidence uncovered a range of plausible values for arbitrageur duration, and could support lower values of habitat demand elasticities $\alpha(\tau)$. Here we explore the sensitivity of our findings to these key parameters.

We first consider a lower value for arbitrageur duration. We raise $\bar{W}$ so that arbitrageurs’ duration of wealth is 10, at the lower end of the range estimated in section 4.2.1.\(^{45}\) The resulting responses of forward rates around a monetary shock are summarized by the dashed line in Figure 4. Relative to the baseline calibration, the response

\(^{45}\)As in footnote 41, we recalibrate $\gamma$ to target the same yield spread as our baseline calibration, and keep all other parameters fixed at their baseline values.
of the 20-year forward rate to a 100bp increase in the one-year yield is dampened, but remains 9bp. We conclude that even at the lower end of our estimates for arbitrageur duration, the wealth revaluation channel can account for a meaningful share of the overreaction of forward rates to monetary shocks observed in the data.

We next consider lower values for the elasticities of habitat demand $\alpha(\tau)$. In particular, we assume habitat demand is completely inelastic by setting $\alpha = 0$. The resulting responses of forward rates around a monetary shock are summarized by the dotted line in Figure 4. Relative to the baseline calibration, the response of the 20-year forward rate to a 100bp increase in the one-year yield rises to 23bp. Consistent with our analytical results, a lower elasticity of habitat demand dampens the response of the quantity of risk borne by arbitrageurs, amplifying the response of the term premium.

5.3.3 State-dependence

We finally demonstrate that the model generates state-dependent effects of monetary shocks as in the data. The first row of Table 6 reports the 90% confidence intervals

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46 An upward sloping demand of habitat investors for long-term bonds upon a monetary shock, as would be implied by the models in Hanson (2014), Hanson and Stein (2015), and Hanson et al. (2021), is effectively like negative values of $\alpha(\tau)$ (at least for high $\tau$) conditional on a monetary shock.

47 Again, we recalibrate $\gamma$ to match the same yield spread and leave all other parameters unchanged.
under the baseline specification for the coefficients on the interaction terms between the change in the one-year yield and the three proxies for arbitrageurs’ duration (the real term premium, dealers’ duration, and dealers’ income gap) estimated in section 4.3. The second row reports the coefficients on the interaction terms in analogous regressions on model-generated data.48 Higher arbitrageur duration implies a larger response of long-horizon forward rates to a monetary tightening because it implies a larger revaluation of arbitrageur wealth. Quantitatively, the model-implied interaction coefficients are towards the lower ends of the confidence intervals in the data, suggesting that, if anything, the baseline calibration understates the effects of changes in arbitrageur wealth on the term premium.

5.4 Implications beyond monetary shocks

The prior subsection demonstrated that the revaluation of arbitrageur wealth can account for much of the term premium responses to monetary shocks. We now trace out the broader implications of fluctuations in arbitrageur wealth for state-dependence, the slope of the yield curve, and trends in term premia from a declining natural rate.

5.4.1 State-dependent effects of QE

Just as the model implies that the effects of monetary policy along the term structure depend on the level of arbitrageur wealth and thus duration, it implies that the effects

\[ \Delta f_t^{(20)} \text{ on } \Delta y_t^{(1)}, \text{ duration of arbitrageurs, and interaction given monetary shock: model vs. data} \]

Table 6: Notes: empirical estimates correspond to 90% confidence interval from baseline estimates in Table 4. Model moments computed using monetary shocks simulated as in Figure 2.

\[
\begin{array}{ccc}
\text{Data} & [0.09,0.91] & [0.09,0.59] \quad [-0.8,4.8] \\
\text{Model} & 0.19 & 0.16 & 0.4 \\
\end{array}
\]

\[48\text{In the model, the 5-year forward, 5-year term premium is given by } f_t^{(5,10)} = \frac{1}{2} \int_{5}^{10} \mathbb{E}(r_{t+s}) \text{, arbitrageur duration is } \int_0^\infty \tau X_t^{(r)} d\tau / W_t, \text{ and arbitrageurs' income gap is } (W_t - \int_0^\infty X_t^{(r)} d\tau) / A_t, \text{ where total assets are } A_t \equiv \int_{\tau: X_t^{(r)} > 0} X_t^{(r)} d\tau + 1 \left\{ W_t - \int_0^\infty X_t^{(r)} d\tau > 0 \right\} \left( W_t - \int_0^\infty X_t^{(r)} d\tau \right).\]
of other shocks similarly depend on arbitrageur wealth. Here we focus on our QE experiment used to calibrate the model.

As previously noted, we simulate the March 18, 2009 announcement in the model assuming that arbitrageur wealth is initially one third less than its average value, corresponding to the decline in broker/dealer and hedge fund wealth between the fourth quarter of 2007 and first quarter of 2009. Figure 5 compares the yield curve responses of this announcement in the model to an alternative scenario in which arbitrageur wealth is initially at its average value.49

The model implies that the 10- and 20-year real yields and forward rates would have fallen by 20-30% less had broker/dealers and hedge funds not been so poorly capitalized at the time of the announcement. There are two reasons for the amplified yield curve responses when arbitrageurs have lower wealth. First, as is evident from

49The announcement is simulated as an unexpected shock to the path of habitat demand as depicted in the first panel. Appendix C.5 provides the impulse responses to a typical habitat demand shock.
the price effects of QE in the simple model characterized in (24), arbitrageurs have more inelastic demand for longer-term bonds when they have lower wealth, implying larger price responses to changes in the supply they must absorb.\footnote{This result is similar to Proposition 4 in Vayanos and Vila (2021) that changes in supply affect yields only when arbitrageurs are risk averse, and Proposition 4 in Greenwood and Vayanos (2014) that changes in supply have larger effects on expected returns when arbitrageurs are more risk averse. What is novel here is that arbitrageurs’ risk-bearing capacity is endogenous to their level of wealth.} Second, a lower level of arbitrageur wealth, all else equal, implies that they have higher duration. A given increase in bond prices thus generates a larger percentage increase in their wealth, lowering their price of bearing risk and raising bond prices further.\footnote{The endogeneity of arbitrageurs’ wealth also means that their wealth eventually falls relative to its initial value, both because QE reduces the volume of arbitrageurs’ carry trade and its price impact reduces their excess return in doing so. For a similar reason, wealth falls faster when \( W_0 = 0.6W \) than when \( W_0 = W \), since the decline in risk premia and thus carry profits is amplified in the first case.}

5.4.2 Volatility and slope of yield curve

The analysis so far has focused on the role of fluctuations in arbitrageur wealth in shaping the conditional response to monetary and demand shocks. In this subsection we quantify the role of endogenous wealth in the unconditional properties of the term structure more broadly.

Table 7 demonstrates how yield volatilities and the slope of the yield curve change when \( \xi \to \infty \) and thus arbitrageurs’ initial wealth is constant.\footnote{In the latter calibration, we also set \( \bar{W} \) equal to the average value of wealth in the baseline calibration, so that the only difference between the two is in the endogenous volatility of wealth. This contrasts with the \( \xi \to \infty \) calibration depicted in Figure 3, in which (as described in footnote 41) we recalibrate \( \gamma \) (equivalently, \( \bar{W} \), since only the ratio \( \gamma/W \) matters for risk pricing with exogenous wealth) to match the same yield spread and thus average term premium as the baseline model.} As is evident from the first row, yield volatility falls when arbitrageurs’ initial wealth is constant, although the effect is very small. The second row demonstrates that there is a more pronounced effect on stochastic volatility: while the model with endogenous wealth features stochastic volatility because short rate and demand shocks have state-dependent effects on yields as arbitrageur wealth varies, the model with exogenous wealth features constant volatilities. Since bond price volatility in the baseline model is high precisely in times with low wealth and thus high marginal utility for arbitrageurs, the final row demonstrates that it accounts for nearly one third of the unconditional slope of the yield curve. Taken together, we conclude that the endogeneity of arbitrageurs’ wealth plays an important role in shaping the unconditional properties of the term structure.
<table>
<thead>
<tr>
<th>Moment</th>
<th>Model</th>
<th>$\xi \to \infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(y_{t}^{(20)})$</td>
<td>0.74%</td>
<td>0.73%</td>
</tr>
<tr>
<td>$\sigma(\sigma_{t-1}(y_{t}^{(20)}))$</td>
<td>0.23%</td>
<td>0.00%</td>
</tr>
<tr>
<td>$y_{t}^{(20)} - y_{t}^{(1)}$</td>
<td>1.53%</td>
<td>1.00%</td>
</tr>
</tbody>
</table>

Table 7: unconditional moments of long yields

Notes: $\sigma$ denotes monthly standard deviation and last row is simple time-series average. Model moments computed as in Table 5.

5.4.3 Trends in natural rate

While our analysis has focused on the effects of a monetary-induced shock to the short rate, similar mechanisms operate in response to more persistent changes in the short rate. We conclude by quantifying the relationship implied by our model between trends in the natural rate and trends in term premia in recent years.

We can interpret the secular decline in the natural rate in recent years as a sequence of negative shocks to $r_t$ together with a decline in the steady-state $\bar{r}$. Figure 6 considers the following scenario: suppose each month between 2004 and 2016 (our sample period used throughout the paper), there was a negative shock $dB_{r,t}$ and equal unanticipated reduction in $\bar{r}$ so that $r_t$ cumulatively fell by 2.2pp. This is the fall in the natural rate from 2004 to 2016 based on the estimates from the Laubach and Williams (2003) model reported by the Federal Reserve Bank of New York. Figure 6 depicts the model’s implications for arbitrageurs’ wealth and the 5-year forward, 5-year real term premium referenced earlier (similar results are obtained for the 5-year or 10-year real term premia). The model implies a cumulative increase in arbitrageurs’ wealth of roughly 20pp and decline in the term premium of roughly 30bp. The latter can be compared to the 1pp decline in this real term premium over the same period estimated using the methodology of D’Amico et al. (2018) by the Federal Reserve. Following Bauer and Rudebusch (2020), the latter estimate may be overstated because it does not account for a time-varying natural rate. In this case our model accounts for an even larger fraction of the true decline in the real term premium.

Relative to the existing literature, our model thus offers a complementary but

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53 In the long-run, a decline in $r$ by $\Delta$ raises all bond prices. We thus assume that there is also a permanent shock to $\theta_0(\tau)$ of $-\alpha(\tau)\tau\Delta$ such that the ergodic distribution of habitat borrowing in (1) is unchanged in the long run, as well as a permanent shock to $\xi$ of $-\Delta$ and $\bar{W}$ of $\Delta/(\xi-\Delta)$ such that the ergodic distribution of arbitrageur wealth in (5) is unchanged in the long run. This allows us to focus on the transitional dynamics induced by a decline in the natural rate alone.
Figure 6: sequence of negative shocks $dB_{r,t}$ from 2004 to 2016

Notes: each month, $dB_{r,t} = -0.022/156$, so that over the 13 year period $r_t$ falls by 2.2pp. State variables at start of simulation are initialized to their average values in initial equilibrium.

distinct explanation of declining term premia in recent years. Campbell et al. (2020) and Gourio and Ngo (2020) argue that changes in macroeconomic comovements can explain why term premia have fallen in recent years. In particular, these authors argue that because long-term bond prices no longer fall as much (and in fact rise) in bad times, the quantity of risk in long-term bonds has fallen. Our model suggests a complementary explanation focused on the price of risk instead: the decline in the natural rate in recent years has led to a sustained increase in long bond prices, recapitalizing arbitrageurs with positive duration and lowering their price of bearing risk.

6 Conclusion

In this paper, we propose a model which rationalizes the effects of monetary policy shocks on the term structure of interest rates. As in the preferred habitat tradition, habitat investors and arbitrageurs trade bonds of various maturities; as in the intermediary asset pricing tradition, arbitrageur wealth is an endogenous state variable relevant for equilibrium risk pricing. When arbitrageurs’ portfolio features positive duration, an unexpected rise in the short rate lowers their wealth and raises term premia. A calibration matching the duration of broker/dealers and hedge funds in the data rationalizes the identified effects of policy shocks along the yield curve. The revaluation of arbitrageur wealth has additional implications for the state-dependent effects of policy, endogenous price volatility and the average slope of the term structure, and trends in term premia accompanying trends in the natural rate.
Our analysis has stopped short of tracing out the consequences of changes in term premia for the real economy so as to focus on the novel mechanisms in financial markets relative to existing term structure models. Embedding our model in a New Keynesian production economy, we expect that the effects of policy on the price of risk will amplify the real effects of monetary policy, to the extent that aggregate demand is rising in the amount habitat investors borrow long-term. This seems natural if we interpret long-term borrowers as mortgagors or non-financial corporates whose marginal propensity to consume or invest is higher than the owners of financial firms. We view this as among the most interesting applications of our framework in future work.

References


Appendix for Online Publication

A Proofs of analytical results

We first provide proofs of the analytical results in the main text.

A.1 Proposition 1

Proof. Combining (12) and (16), wealth evolves according to

\[ W_t = (1 - \exp(-\xi))W + \exp(-\xi) \exp(r_{t-1}) \left[ W_{t-1} + (\alpha \log P_{t-1} + \theta_{t-1}) (\exp(-r_t - r_{t-1} - \log P_{t-1}) - 1) \right]. \]

Around the stochastic steady-state, this implies

\[
\begin{align*}
    d\log W_t &= \exp(-\xi) \frac{\exp(\bar{r})}{W} \left[ W - X \right] dr_{t-1} + \exp(-\xi) \frac{\exp(\bar{r})}{W} d\log W_{t-1} + \exp(-\xi) \frac{\exp(\bar{r})}{W} \left[ \frac{\exp(-2\bar{r} - \log P)}{W} \right] d\log P_{t-1} + \\
    &\quad \frac{\exp(-\xi) \exp(\bar{r}) (\exp(-2\bar{r} - \log P) - 1)}{W} d\theta_{t-1} - \frac{\exp(-\xi) X \exp(-\bar{r} - \log P)}{W} dr_t + \\
    &\quad \frac{\exp(-\xi) \exp(\bar{r}) \left[ \alpha (\exp(-2\bar{r} - \log P) - 1) - X \exp(-2\bar{r} - \log P) \right]}{W} d\log P_{t-1}.
\end{align*}
\]

The impact response of wealth to a short rate shock follows, with

\[ \omega \equiv \frac{X}{W} \exp(-\bar{r} - \log P) \]

summarizing the duration of arbitrageurs’ wealth. \qed

A.2 Proposition 2

Proof. (12) and (9) imply

\[
\begin{align*}
    E_t r_{t+1}^{(2)} &= -E_t r_{t+1} - \log P_t, \\
    &= -\kappa_r \bar{r} - (1 - \kappa_r) r_t - \log P_t
\end{align*}
\]
Substituting these into (15) yields

\[-\kappa_r \bar{r} - (2 - \kappa_r) r_t - \log P_t + \frac{1}{2} \sigma_r^2 = \frac{\alpha \log P_t + \theta_t}{W_t} \sigma_r^2.\]

This implies that around the stochastic steady-state

\[d \log P_t = -\frac{2 - \kappa_r}{1 + \frac{W}{\bar{W}} \alpha \sigma_r^2} dr_t - \frac{\frac{W}{\bar{W}} \sigma_r^2}{1 + \frac{W}{\bar{W}} \alpha \sigma_r^2} d\theta_t + \frac{X}{W} \sigma_r^2 d \log W_t.\]  

(26)

It follows from (17) that

\[df_t = \frac{1 - \kappa_r - \frac{W}{\bar{W}} \alpha \sigma_r^2}{1 + \frac{W}{\bar{W}} \alpha \sigma_r^2} dr_t + \frac{\frac{W}{\bar{W}} \sigma_r^2}{1 + \frac{W}{\bar{W}} \alpha \sigma_r^2} d\theta_t - \frac{X}{W} \sigma_r^2 d \log W_t.\]  

(27)

The response of the forward rate to a short rate shock follows from Proposition 1. □

B Empirical appendix

We now provide supplementary empirical results accompanying those in section 4.

B.1 Effects of monetary shocks on real forwards

Figure 7 visually depicts the relationship between the change in the 20-year real forward rate and the change in the one-year yield induced by the high-frequency monetary surprise reported in the baseline specification in Table 1. It makes evident that the positive relationship is not driven by any one observation. At the same time, it also makes clear that the distribution of monetary policy surprises is leptokurtic, with large observations in absolute value particularly concentrated around the global financial crisis. One may be worried, then, that our results are driven by anomalies during the most acute phase of this crisis, or reflect news other than conventional monetary policy such as to QE. Table 1 demonstrates that this is in fact not the case: our results are robust to excluding announcements between July 2008 and June 2009, and excluding announcements that contained news about asset purchases or non-standard credit operations.
B.2 Effects of monetary shocks on nominal yield curve

In the main text we study the implications of monetary shocks on real forwards. Here we replicate our analysis using the nominal yield curve. In particular, we regress the change in the one-year nominal yield on our high-frequency monetary policy surprise measure in the first stage, and then regress the change in one-year nominal forward rate paying between 2 and 30 years on the predicted change in the one-year nominal yield in the second stage.\footnote{Whether we use the one-year nominal or real yield in the first stage matters little. Both summarize the stance of monetary policy. What does matter is the outcome variable used in the second stage.}

We use Gurkaynak et al. (2006)'s interpolated nominal yield curve to compute yields and forwards at all maturities and horizons at a daily frequency. We use in particular the updated data maintained by the Federal Reserve. We focus on the same January 2004 through December 2016 period used in our analysis of the real yield curve only to maintain comparability with those results. Data for the nominal yield curve is available earlier and we obtain similar results over the broader sample.

Figure 8 plots the regression coefficients and associated 90\% confidence intervals. Unlike in the case of real forwards, the effect of a monetary tightening on nominal forwards is monotonically declining. Moreover, the effect is economically and statistically significantly negative at long maturities. Table 8 presents the same alternative specifi-
Figure 8: $\Delta f_t^{(\tau-1,\tau)}$, instrumented by high-frequency surprise (nominal)

Notes: at each integer between 2 and 30 on the x-axis, we plot coefficients and 90% confidence interval using $\Delta f_t^{(\tau-1,\tau)}$ as the outcome variable. Confidence interval based on robust standard errors.

cations as in Table 1. As is evident, the same pattern holds across these specifications.

These results are consistent with previous findings in the literature also focused on the nominal yield curve, such as in Gurkaynak et al. (2005b) and Gurkaynak, Sack, and Swanson (2005a). As these papers argue, these patterns are consistent with monetary shocks containing news about the central bank’s long-run inflation target. In particular, if a monetary tightening is associated with news about a lower long-run inflation target, it will lower long-run forward rates. Changes in the long-run inflation target will have no direct effects on long maturity real forwards, underscoring the importance of focusing on the real yield curve to uncover the effects of monetary shocks on term premia.

With that said, the effects of monetary policy shocks on the nominal yield curve are still important for the results in our paper because most of the assets held by arbitrageurs in the data are nominal. Hence, the revaluation of nominal bonds, not real bonds, are likely to drive any changes in arbitrageur wealth. In this context, it is important to note that nominal yields rise on impact of a monetary tightening far out into the yield curve, as shown for our baseline specification in Figure 9, even though long-dated nominal forward rates fall. Similar results are obtained for the alternative specifications described above. We conclude that a monetary tightening will lower the wealth of agents having positive duration in nominal bonds, so long as the duration is
<table>
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<tr>
<th>Specification</th>
<th>$\Delta f_t^{(4,5)}$</th>
<th>$\Delta f_t^{(9,10)}$</th>
<th>$\Delta f_t^{(14,15)}$</th>
<th>$\Delta f_t^{(19,20)}$</th>
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<tbody>
<tr>
<td>Baseline</td>
<td>0.51</td>
<td>-0.09</td>
<td>-0.31</td>
<td>-0.64</td>
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<tr>
<td></td>
<td>(0.47)</td>
<td>(0.41)</td>
<td>(0.31)</td>
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<td>All FOMC announcements</td>
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<td>(0.34)</td>
<td>(0.33)</td>
<td>(0.43)</td>
<td>(0.51)</td>
</tr>
<tr>
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<tr>
<td></td>
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<td>(0.94)</td>
<td>(0.48)</td>
<td>(0.23)</td>
<td>(0.29)</td>
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<td>0.87</td>
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<td></td>
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<td></td>
<td>(0.46)</td>
<td>(0.40)</td>
<td>(0.28)</td>
<td>(0.33)</td>
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<td>NS (2018) IV, excl. 7/08-6/09</td>
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<td>-0.29</td>
<td>-0.37</td>
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<tr>
<td></td>
<td>(0.29)</td>
<td>(0.29)</td>
<td>(0.32)</td>
<td>(0.41)</td>
</tr>
</tbody>
</table>

Table 8: $\Delta f_t^{(\tau-1,\tau)}$ on $\Delta y_t^{(1)}$, instrumented by high-frequency surprise (nominal)

Notes: robust standard errors provided in parenthesis.

not extremely high.

**B.3 TIPS outstanding over time**

We focus in the paper on effects on real forwards paying through 20 years in the future. Here we demonstrate that our analysis does not rely on any extrapolation outside the observed TIPS yield curve.

Figure 10 displays outstanding TIPS issues at each month through 2016 by their remaining time to maturity (updating Figure 1 in Gurkaynak et al. (2008) through more recent data). Importantly, the figure demonstrates that throughout our 2004-2016 sample period, there has been at least one outstanding issue with maturity greater than or equal to 20 years. On average, there have been almost four such issues at each point in time. The figure also demonstrates why we restrict attention to maturities less than or equal to 20 years: throughout much of our sample period, there have not been TIPS outstanding with time to maturity 30 years.
B.4 Estimating equity duration

In the main text we compute arbitrageur duration using the balance sheets of broker/dealers and hedge funds together with estimates of duration by asset class. For corporate equities, we estimate duration using the approach proposed in Greenwald et al. (2023), which uses the Gordon growth formula together with the valuation ratio on the stock market. We motivate this approach here.

Suppose an equity claim trading at price $P$ pays an instantaneous dividend $D$ that grows at constant growth rate $g$. Let $r$ denote the instantaneous and constant short rate. The price of the asset is then

$$P = \int_0^\infty D \exp((g - r)t) dt = \frac{D}{r - g}.$$  

Then the modified duration of the equity claim, the percentage change in the equity price given a 1% permanent change in $r$, is given by

$$\frac{\partial P}{\partial r} \frac{1}{P} = \frac{1}{r - g} \frac{P}{D}.$$  

Thus, we use the time series of the price/dividend ratio for the S&P 500, obtained
Figure 10: remaining time to maturity of TIPS outstanding

Notes: at each point in time, we plot the set of TIPS outstanding by time to maturity. We obtain this data using the auction query on Treasury Direct to search for all TIPS by original issue date and original time to maturity. There were no buybacks of TIPS during this period.

from Robert Shiller’s website, as our estimate of equity duration each quarter.

B.5 List of dealers

Table 9 provides the list of primary dealers whose stock returns and income gaps are studied in the paper. This list is constructed by obtaining the list of past and continuing dealers from the Federal Reserve Bank of New York, and then identifying the subset of these for which we can find the relevant CRSP/TAQ and (of those) FR Y9-C data.

B.6 Effects of monetary shocks on dealer equities

The main text presented the 30-minute response of dealer equities to a monetary surprise in our baseline and alternative specifications. Here we demonstrate that the tight event window is necessary to have enough power to detect these effects, and we visually depict our results in the baseline specification.

The first column of Table 10 reproduces the responses of dealer equities in the 30-minute window around FOMC announcements to a monetary-induced change in the one-year yield. The second column uses instead the one-day change in dealer
<table>
<thead>
<tr>
<th>Dealer</th>
<th>CRSP/TAQ</th>
<th>FR Y9-C</th>
</tr>
</thead>
<tbody>
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<td>Bank of America</td>
<td>BAC 1/2/2004-12/30/2016</td>
<td>1073757 2004Q1-2016Q4</td>
</tr>
<tr>
<td>Barclays</td>
<td>BCS 1/2/2004-12/30/2016</td>
<td>2914521 2004Q4-2010Q3*</td>
</tr>
<tr>
<td>BMO</td>
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<td>1245415 2004Q1-2016Q4</td>
</tr>
<tr>
<td>Bank of Novia Scotia</td>
<td>BNS 1/2/2004-12/30/2016</td>
<td>1238967</td>
</tr>
<tr>
<td>Bear Stearns</td>
<td>BSC 1/2/2004-5/30/2008</td>
<td>1573257</td>
</tr>
<tr>
<td>Citigroup</td>
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<tr>
<td>Credit Suisse</td>
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<td>Prudential</td>
<td>PRU 1/2/2004-12/30/2016</td>
<td>2441728</td>
</tr>
<tr>
<td>RBS</td>
<td>RBS 10/18/2007-12/30/2016</td>
<td>1851106</td>
</tr>
<tr>
<td>RBC</td>
<td>RY 1/2/2004-12/30/2016</td>
<td>3226762 2010Q4-2016Q4*</td>
</tr>
<tr>
<td>TD</td>
<td>TD 1/2/2004-12/30/2016</td>
<td>3606542 2015Q3-2016Q4</td>
</tr>
<tr>
<td>UBS</td>
<td>UBS 1/2/2004-12/30/2016</td>
<td>4846998 2016Q3-2016Q4</td>
</tr>
<tr>
<td>Wells Fargo</td>
<td>WFC 1/2/2004-12/30/2016</td>
<td>1120754 2004Q1-2016Q4</td>
</tr>
<tr>
<td>Zions First National</td>
<td>ZION 1/2/2004-12/30/2016</td>
<td>1027004 2004Q1-2016Q4</td>
</tr>
</tbody>
</table>

Table 9: dealers with stock market and balance sheet data

* Includes a gap in coverage within this range.
Table 10: change in dealer prices on $\Delta y_t^{(1)}$, instrumented by high-frequency surprise

<table>
<thead>
<tr>
<th>Specification</th>
<th>30-minute change</th>
<th>One-day change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>-9.8</td>
<td>-8.0</td>
</tr>
<tr>
<td></td>
<td>(3.2)</td>
<td>(9.9)</td>
</tr>
<tr>
<td>All FOMC announcements</td>
<td>-1.4</td>
<td>-2.7</td>
</tr>
<tr>
<td></td>
<td>(4.5)</td>
<td>(8.2)</td>
</tr>
<tr>
<td>Excl. 7/08-6/09</td>
<td>-19.4</td>
<td>-2.6</td>
</tr>
<tr>
<td></td>
<td>(11.2)</td>
<td>(20.0)</td>
</tr>
<tr>
<td>Excl. announcements with LSAP news</td>
<td>-12.4</td>
<td>4.0</td>
</tr>
<tr>
<td></td>
<td>(5.8)</td>
<td>(10.1)</td>
</tr>
<tr>
<td>Swanson (2021) Fed funds IV</td>
<td>-10.0</td>
<td>-4.2</td>
</tr>
<tr>
<td></td>
<td>(3.6)</td>
<td>(10.5)</td>
</tr>
<tr>
<td>Swanson (2021) forward guidance IV</td>
<td>-11.4</td>
<td>-13.1</td>
</tr>
<tr>
<td></td>
<td>(5.0)</td>
<td>(5.3)</td>
</tr>
<tr>
<td>Bauer and Swanson (2023) IV</td>
<td>-7.7</td>
<td>-14.8</td>
</tr>
<tr>
<td></td>
<td>(3.2)</td>
<td>(6.7)</td>
</tr>
<tr>
<td>Nakamura and Steinsson (2018) IV</td>
<td>-12.2</td>
<td>-21.3</td>
</tr>
<tr>
<td></td>
<td>(4.0)</td>
<td>(6.6)</td>
</tr>
<tr>
<td>NS (2018) IV, excl. 7/08-6/09</td>
<td>-24.2</td>
<td>-20.6</td>
</tr>
<tr>
<td></td>
<td>(10.0)</td>
<td>(17.6)</td>
</tr>
</tbody>
</table>

Notes: robust standard errors provided in parenthesis.

prices obtained using end-of-day prices from CRSP. While the magnitude of the one-day response is comparable to that obtained in the 30-minute window in our baseline specification, it is no longer statistically significantly different from zero. This was not the case for our estimated effects on the yield curve, which detected statistically significant effects using one-day changes in forward rates. This makes sense because equity prices are much more volatile than forward rates and thus the signal to noise ratio is lower. In the alternative specifications, the results using one-day changes in dealer equity prices are sometimes negative and sometimes positive, and in most cases not statistically significantly different from zero, again in contrast to the first column using 30-minute windows. English, den Heuvel, and Zakrajsek (2018) also use tight intraday windows to study the response of commercial bank equity prices to monetary shocks. Our analysis complements theirs but focuses on primary dealers, a subset of intermediaries that are likely to be particularly relevant for risk pricing.

Figure 11 visually depicts the relationship between the change in dealer equity prices
and the change in the one-year yield induced by the high-frequency monetary surprise in the baseline specification. The left panel depicts the tight negative relationship with the high-frequency change in dealer prices. The right panel depicts the much noisier relationship with the one-day change in dealer prices.

\section*{B.7 Measures of duration used to study state-dependence}

In section 4.3 of the main text we use three proxies for arbitrageur duration which can be used to study the state-dependent responses to monetary shocks over the entire sample period. In this appendix we compare these measures of arbitrageur duration to one another.

The first panel of Figure 12 compares the 5-year forward, 5-year real term premium estimated by D’Amico et al. (2018) to our weekly measure of primary dealer duration computed using the Primary Dealer Statistics and market value of dealer equity. As is evident, they comove at low frequencies, rising from 2006 to 2009 and declining thereafter. However, there are some higher frequency divergences between the series particularly evident after 2011, which could be rationalized in a natural extension of our model featuring time variation in the conditional volatilities of the short rate and/or habitat demand.
The second panel of Figure 12 compares this measure of primary dealer duration to their aggregate income gap as measured using the holdings companies’ Form Y9-C filings to the Federal Reserve. As is evident, they comove negatively. This makes sense since the numerator of the income gap is the dollar amount of assets maturing or repricing within a year less the dollar amount of liabilities maturing or repricing within a year, and thus should be inversely related to duration.

B.8 Dealer equity responses and arbitrageur duration

In the main text, we provide a direct test of the model by asking whether, in the data, the effects of monetary shocks on the yield curve depend on the duration of arbitrageurs. Here we instead ask whether the effects of monetary shocks on dealer equity prices depend on arbitrageur duration.

Table 11 presents the analog of Table 4 from the main text, but with the high-frequency change in the dealer equity index on the left-hand side of the second stage.
<table>
<thead>
<tr>
<th>Specification</th>
<th>Proxy for arb duration</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5-yr fwd, 5-yr TP</td>
</tr>
<tr>
<td></td>
<td>Log dealer dur.</td>
</tr>
<tr>
<td></td>
<td>−Dealer income gap</td>
</tr>
<tr>
<td>Baseline</td>
<td>-6.3 (7.8)</td>
</tr>
<tr>
<td></td>
<td>7.3 (5.1)</td>
</tr>
<tr>
<td></td>
<td>-75.3 (37.2)</td>
</tr>
<tr>
<td>All FOMC announcements</td>
<td>1.6 (13.3)</td>
</tr>
<tr>
<td></td>
<td>5.9 (5.8)</td>
</tr>
<tr>
<td></td>
<td>-6.9 (51.8)</td>
</tr>
<tr>
<td>Excl. 7/08-6/09</td>
<td>-33.2 (29.7)</td>
</tr>
<tr>
<td></td>
<td>-9.0 (20.9)</td>
</tr>
<tr>
<td></td>
<td>-257.8 (289.4)</td>
</tr>
<tr>
<td>Excl. announcements with LSAP news</td>
<td>-13.4 (10.4)</td>
</tr>
<tr>
<td></td>
<td>4.3 (8.1)</td>
</tr>
<tr>
<td></td>
<td>-81.6 (51.2)</td>
</tr>
<tr>
<td>Swanson (2021) Fed funds IV</td>
<td>0.6 (9.8)</td>
</tr>
<tr>
<td></td>
<td>8.4 (7.6)</td>
</tr>
<tr>
<td></td>
<td>-74.9 (51.3)</td>
</tr>
<tr>
<td>Swanson (2021) forward guidance IV</td>
<td>-5.0 (8.5)</td>
</tr>
<tr>
<td></td>
<td>6.2 (3.9)</td>
</tr>
<tr>
<td></td>
<td>-132.7 (113.1)</td>
</tr>
<tr>
<td>Bauer and Swanson (2023) IV</td>
<td>-9.5 (8.5)</td>
</tr>
<tr>
<td></td>
<td>6.3 (3.8)</td>
</tr>
<tr>
<td></td>
<td>-51.2 (44.2)</td>
</tr>
<tr>
<td>Nakamura and Steinsson (2018) IV</td>
<td>17.5 (11.2)</td>
</tr>
<tr>
<td></td>
<td>7.0 (3.7)</td>
</tr>
<tr>
<td></td>
<td>-282.2 (257.9)</td>
</tr>
<tr>
<td>NS (2018) IV, excl. 7/08-6/09</td>
<td>1.0 (14.6)</td>
</tr>
<tr>
<td></td>
<td>-34.4 (51.6)</td>
</tr>
<tr>
<td></td>
<td>-296.1 (365.7)</td>
</tr>
</tbody>
</table>

Table 11: 30-minute change in aggregate dealer equity index on $\Delta y_t^{(1)}$, duration of arbitrageurs, and interaction, instrumented by high-frequency surprise

Notes: each cell reports the estimated coefficient on the interaction term. Robust standard errors provided in parenthesis.

Interestingly, the effect of monetary shocks on dealer equity prices do not depend on D’Amico et al. (2018)’s estimated real term premium or our proxy for log dealers’ duration in a consistent way. However, we do see more consistent effects for the income gap. In the baseline specification, in response to a 1 pp increase in the one-year yield, a 1 pp lower dealer income gap amplifies the decline in dealers’ equity prices by 75.3 bp. This is a sizeable effect given the standard deviation of dealers’ income gap of 8 pp over the maintained sample period.

A useful feature of the income gap data is that we can also exploit variation across dealers. Table 12 puts each dealer’s high-frequency stock price change on the left-hand side of the second stage, and each dealer’s income gap on the right-hand side of both stages. As is evident, we continue to find that dealers with lower income gaps exhibit
<table>
<thead>
<tr>
<th>Specification</th>
<th>−Income gap</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>−11.8</td>
</tr>
<tr>
<td></td>
<td>(7.9)</td>
</tr>
<tr>
<td>All FOMC announcements</td>
<td>1.4</td>
</tr>
<tr>
<td></td>
<td>(10.4)</td>
</tr>
<tr>
<td>Excl. 7/08-6/09</td>
<td>−60.8</td>
</tr>
<tr>
<td></td>
<td>(30.4)</td>
</tr>
<tr>
<td>Excl. announcements with LSAP news</td>
<td>−24.6</td>
</tr>
<tr>
<td></td>
<td>(13.4)</td>
</tr>
<tr>
<td>Swanson (2021) Fed funds IV</td>
<td>−5.0</td>
</tr>
<tr>
<td></td>
<td>(9.4)</td>
</tr>
<tr>
<td>Swanson (2021) forward guidance IV</td>
<td>−22.5</td>
</tr>
<tr>
<td></td>
<td>(12.2)</td>
</tr>
<tr>
<td>Bauer and Swanson (2023) IV</td>
<td>−3.8</td>
</tr>
<tr>
<td></td>
<td>(7.0)</td>
</tr>
<tr>
<td>Nakamura and Steinsson (2018) IV</td>
<td>−4.0</td>
</tr>
<tr>
<td></td>
<td>(9.1)</td>
</tr>
<tr>
<td>NS (2018) IV, excl. 7/08-6/09</td>
<td>−22.6</td>
</tr>
<tr>
<td></td>
<td>(22.2)</td>
</tr>
</tbody>
</table>

Table 12: 30-minute change in individual dealer prices on $\Delta y_t^{(1)}$, negative of dealer’s income gap, and interaction, instrumented by high-frequency surprise.

Notes: each cell reports the estimated coefficient on the interaction term. Robust standard errors provided in parenthesis.

larger stock price declines in response to a monetary tightening. In the context of our model, it is furthermore sensible that the estimated effects are smaller in absolute magnitude than using the aggregate income gap in Table 11: when all dealers have a lower income gap, their decline in wealth upon a monetary tightening would feed back to depress long-term bond prices, amplifying the decline in wealth.

Finally, we demonstrate that focusing on dealers helps to resolve an interesting puzzle documented by Haddad and Sraer (2020) in their analysis of bank income gaps without conditioning on monetary shocks. These authors study the average income gap across all banks with more than $1bn in consolidated assets. They find that there is no economically nor statistically significant relationship between the average income gap and the relationship between the excess return on the Fama-French bank industry portfolio and excess return on long-term bonds (see their Table 9). Table 13 runs an analogous regression for primary dealers. We regress the quarterly return on a value-weighted portfolio of publicly-traded dealers on the excess quarterly return to an
Table 13: accounting for quarterly return on dealers

Notes: all specifications use quarterly data over 2004-2016. Robust standard errors in parenthesis.

equal-weighted portfolio of 5- through 10-year bonds (relative to the 3-month Treasury bill rate), the lagged aggregate dealer income gap, and their interaction. We also add the excess S&P 500 return and 3-month Treasury bill return as additional controls. We find that, once we control for the excess S&P 500 return, it is indeed the case that when dealers’ income gap is low, their stock returns rise by economically and statistically significantly more when excess long-term bonds are high. Quantitatively, a 1 pp lower income gap would raise the response of dealers’ return to a 1 pp higher excess long-term bond return by 9.5 bp. This further validates dealers’ income gap as an inverse measure of duration.

B.9 Change in dealer equity in first stage

We finally provide a complementary, direct test of the main mechanism in the model: we directly regress the change in forward rates on dealers’ high-frequency change in equity prices, instrumenting the latter with the monetary surprise. Table 14 summarizes

\footnote{We also obtain this result in a panel regression exploiting variation both within and across dealers, or if we extend the sample period backwards to begin in 1986 as in Haddad and Sraer (2020).}
the results.

Mechanically, the estimated coefficients are those from section 4.1 divided by those from 4.2.2. Thus, we find that long-dated forward rates indeed rise by more when dealers’ stock prices fall by more around monetary announcements. In particular, in our baseline specification, a 1 pp decline in dealers’ stock prices is associated with a 4.0 bp rise in the 20-year forward rate. Less obviously, we find that the results at the 20-year horizon are statistically significantly different from zero in our baseline specification and four of the alternative specifications, despite the noise in equity returns.

### C Quantitative appendix

We now provide supplementary results for our quantitative analysis in section 5.
C.1 Arbitrageurs’ optimality

We first characterize arbitrageurs’ optimality conditions in the full model.

Given an equilibrium pricing function

$$P^{(r)}_t = P^{(r)}(r_t, \beta_t, W_t),$$

Ito’s Lemma implies

$$dP^{(r)}_t = \omega^{(r)}_t P^{(r)}_t \, dt + \eta^{(r)}_{r,t} P^{(r)}_t \, dB_{r,t} + \eta^{(r)}_{\beta,t} P^{(r)}_t \, dB_{\beta,t}$$

for some coefficients $\omega^{(r)}_t$, $\eta^{(r)}_{r,t}$, and $\eta^{(r)}_{\beta,t}$ which we have expressed relative to $P^{(r)}_t$ without loss of generality, and which we denote with $t$ subscripts as shorthand for these being functions of $(r_t, \beta_t, W_t)$ as well. Defining the portfolio shares

$$\chi_t^{(r)} = \frac{x_t^{(r)}}{w_t},$$

we can thus write the arbitrageur problem (3)-(4) as maximizing

$$v(r_t, \beta_t, W_t, w_t) = \max_{\{\chi_t^{(r)}\}} \mathbb{E}_t \int_0^\infty \exp(- (\xi + \rho)s) \left( (\xi + \rho) \left( \frac{w_{t+s}^{1-\gamma} - 1}{1 - \gamma} \right) + \int_0^\infty \chi^{(r)}_t w_t^{\eta^{(r)}_{r,t}} \, d\tau \right) ds$$

subject to

$$dw_t = \left[ w_t r_t + \int_0^\infty \chi^{(r)}_t w_t \left( \omega^{(r)}_t - r_t \right) \, d\tau \right] dt$$

$$+ \left[ \int_0^\infty \chi^{(r)}_t w_t \eta^{(r)}_{r,t} \, d\tau \right] dB_{r,t} + \left[ \int_0^\infty \chi^{(r)}_t w_t \eta^{(r)}_{\beta,t} \, d\tau \right] dB_{\beta,t},$$

and the evolution of aggregates (6), (7), and (22).

The associated Hamilton-Jacobi-Bellman equation is

$$(\xi + \rho)v_t = \kappa_r (\bar{r} - r_t)v_{r,t} - \kappa_{\beta} \beta_v v_{\beta,t} + \omega_t v_{W,t}$$

$$+ \frac{1}{2} \sigma^2 \nu_{rr,t} + \frac{1}{2} \sigma^2 \nu_{\beta \beta,t} + \frac{1}{2} \left( \eta^2_{r,t} + \eta^2_{\beta,t} \right) v_{WW,t} + \sigma_r \eta_{r,t} v_{W,r,t} + \sigma_{\beta} \eta_{\beta,t} v_{W,\beta,t}$$

$$+ \max_{\{\chi_t^{(r)}\}} \left( (\xi + \rho) \left( \frac{w_{t+s}^{1-\gamma} - 1}{1 - \gamma} \right) + \int_0^\infty \chi^{(r)}_t w_t \left( \omega^{(r)}_t - r_t \right) \, d\tau \right) v_{w,t}$$
\[ + \frac{1}{2} \left( \int_{0}^{\infty} \chi_{t}^{(s)} \eta_{r,t}^{(s)} d\tau \right) + \left( \int_{0}^{\infty} \chi_{t}^{(s)} \eta_{\beta,t}^{(s)} d\tau \right) v_{\omega,t} \]

\[ + \left( \int_{0}^{\infty} \chi_{t}^{(s)} \eta_{r,t}^{(s)} d\tau \right) [\sigma_{r} v_{w r,t} + \eta_{r,t} v_{w w,t}] \]

\[ + \left( \int_{0}^{\infty} \chi_{t}^{(s)} \eta_{\beta,t}^{(s)} d\tau \right) [\sigma_{\beta} v_{w \beta,t} + \eta_{\beta,t} v_{w W,t}] \]

where we write \( v_{x,t} \) and \( v_{xx,t} \) to denote the first- and second-order partial derivatives of \( v(r_{t}, \beta_{t}, W_{t}) \) with respect to a generic variable \( x \). The first-order conditions are

\[ w_{t} \left( \omega_{t}^{(r)} - r_{t} \right) v_{w,t} = -w_{t}^{2} \left( \int_{0}^{\infty} \chi_{t}^{(s)} \left[ \eta_{r,t}^{(s)} \eta_{r,t}^{(s)} + \eta_{\beta,t}^{(s)} \eta_{\beta,t}^{(s)} \right] ds \right) v_{ww,t} \]

\[ - w_{t} \eta_{r,t}^{(r)} \left[ \sigma_{r} v_{w r,t} + \eta_{r,t} v_{w W,t} \right] - w_{t} \eta_{\beta,t}^{(r)} \left[ \sigma_{\beta} v_{w \beta,t} + \eta_{\beta,t} v_{w W,t} \right] \]

for each \( \tau \in (0, \infty) \).

Now conjecture that the value function satisfies

\[ v(r_{t}, \beta_{t}, W_{t}, w_{t}) = \frac{(\nu_{t} w_{t})^{1-\gamma} - 1}{1 - \gamma}, \]

where \( \nu_{t} \) does not depend on the arbitrageur’s level of wealth, and the \( t \) subscript again denotes that it is a function of \( (r_{t}, \beta_{t}, W_{t}) \). It follows that

\[ v_{w,t} = \nu_{t}^{1-\gamma} w_{t}^{-\gamma}, \]

\[ v_{ww,t} = -\gamma \nu_{t}^{1-\gamma} w_{t}^{-\gamma-1}, \]

\[ v_{wr,t} = (1 - \gamma) \nu_{t}^{1-\gamma} w_{t}^{-\gamma} \frac{v_{r,t}}{\nu_{t}}, \]

\[ v_{w\beta,t} = (1 - \gamma) \nu_{t}^{1-\gamma} w_{t}^{-\gamma} \frac{v_{\beta,t}}{\nu_{t}}, \]

\[ v_{wW,t} = (1 - \gamma) \nu_{t}^{1-\gamma} w_{t}^{-\gamma} \frac{v_{W,t}}{\nu_{t}}. \]

Substituting into (30), it follows

\[ \omega_{t}^{(r)} - r_{t} = \gamma \int_{0}^{\infty} \chi_{t}^{(s)} \left[ \eta_{r,t}^{(s)} \eta_{r,t}^{(s)} + \eta_{\beta,t}^{(s)} \eta_{\beta,t}^{(s)} \right] ds \]

\[ - (1 - \gamma) \eta_{r,t}^{(r)} \left[ \sigma_{r} v_{r,t}^{(r)} + \eta_{r,t} v_{W,t}^{(r)} \right] \]

\[ + \left( \int_{0}^{\infty} \chi_{t}^{(s)} \eta_{r,t}^{(s)} d\tau \right) [\sigma_{r} v_{w r,t} + \eta_{r,t} v_{w w,t}] \]

\[ + \left( \int_{0}^{\infty} \chi_{t}^{(s)} \eta_{\beta,t}^{(s)} d\tau \right) [\sigma_{\beta} v_{w \beta,t} + \eta_{\beta,t} v_{w W,t}] \]
for each \( \tau \in (0, \infty) \). An implication is that the arbitrageur’s optimal portfolio shares \( \chi_t^{(\tau)} \) do not depend on \( w_t \). Substituting these into (29), on the left-hand side we have

\[
(\xi + \rho)(\nu_t w_t)^{1-\gamma} - 1 \frac{1}{1-\gamma}.
\]

On the right-hand side, using

\[
\begin{align*}
v_{r,t} &= \nu_t^{1-\gamma} w_t^{1-\gamma} \frac{\nu_{r,t}}{\nu_t}, \\
v_{rr,t} &= \nu_t^{1-\gamma} w_t^{1-\gamma} \left(-\gamma \left( \frac{\nu_{r,t}}{\nu_t} \right)^2 + \frac{\nu_{rr,t}}{\nu_t} \right), \\
v_{\beta,t} &= \nu_t^{1-\gamma} w_t^{1-\gamma} \frac{\nu_{\beta,t}}{\nu_t}, \\
v_{\beta\beta,t} &= \nu_t^{1-\gamma} w_t^{1-\gamma} \left(-\gamma \left( \frac{\nu_{\beta,t}}{\nu_t} \right)^2 + \frac{\nu_{\beta\beta,t}}{\nu_t} \right), \\
v_{W,t} &= \nu_t^{1-\gamma} w_t^{1-\gamma} \frac{\nu_{W,t}}{\nu_t}, \\
v_{WW,t} &= \nu_t^{1-\gamma} w_t^{1-\gamma} \left(-\gamma \left( \frac{\nu_{W,t}}{\nu_t} \right)^2 + \frac{\nu_{WW,t}}{\nu_t} \right), \\
v_{Wr,t} &= \nu_t^{1-\gamma} w_t^{1-\gamma} \left(-\gamma \frac{\nu_{W,t} \nu_{r,t}}{\nu_t^2} + \frac{\nu_{Wr,t}}{\nu_t} \right), \\
v_{W\beta,t} &= \nu_t^{1-\gamma} w_t^{1-\gamma} \left(-\gamma \frac{\nu_{W,t} \nu_{\beta,t}}{\nu_t^2} + \frac{\nu_{W\beta,t}}{\nu_t} \right),
\end{align*}
\]

we have

\[
\begin{align*}
\nu_t^{1-\gamma} w_t^{1-\gamma} \left[ \kappa_r (\bar{r} - r_t) \frac{\nu_{r,t}}{\nu_t} - \kappa_{\beta} \beta \frac{\nu_{\beta,t}}{\nu_t} + \omega_t \frac{\nu_{W,t}}{\nu_t} \\
+ \frac{1}{2} \sigma_r^2 \left(-\gamma \left( \frac{\nu_{r,t}}{\nu_t} \right)^2 + \frac{\nu_{rr,t}}{\nu_t} \right) + \frac{1}{2} \sigma_\beta^2 \left(-\gamma \left( \frac{\nu_{\beta,t}}{\nu_t} \right)^2 + \frac{\nu_{\beta\beta,t}}{\nu_t} \right) \\
+ \frac{1}{2} \left( \eta_{r,t}^2 + \eta_{\beta,t}^2 \right) \left(-\gamma \left( \frac{\nu_{W,t}}{\nu_t} \right)^2 + \frac{\nu_{WW,t}}{\nu_t} \right) \\
+ \sigma_r \eta_{r,t} \left(-\gamma \frac{\nu_{W,t} \nu_{r,t}}{\nu_t^2} + \frac{\nu_{Wr,t}}{\nu_t} \right) + \sigma_\beta \eta_{\beta,t} \left(-\gamma \frac{\nu_{W,t} \nu_{\beta,t}}{\nu_t^2} + \frac{\nu_{W\beta,t}}{\nu_t} \right) \right].
\end{align*}
\]
\[
\left.\frac{d}{dt}\right|_t \left( r_t + \int_0^\infty \chi_t^{(\tau)} \left( \omega_t^{(\tau)} - r_t \right) d\tau \right) \\
- \frac{1}{2} \gamma \left( \left[ \int_0^\infty \chi_t^{(\tau)} \eta_t^{(\tau)} d\tau \right]^2 + \left[ \int_0^\infty \chi_t^{(\tau)} \eta_{\beta,t}^{(\tau)} d\tau \right]^2 \right) \\
+ (1 - \gamma) \left[ \int_0^\infty \chi_t^{(\tau)} \eta_{\tau,t}^{(\tau)} d\tau \right] \left[ \sigma_r \nu_{r,t} \nu_t + \eta_{r,t} \nu_{W,t} \mu_t \right] \\
+ (1 - \gamma) \left[ \int_0^\infty \chi_t^{(\tau)} \eta_{\beta,t}^{(\tau)} d\tau \right] \left[ \sigma_{\beta} \nu_{\beta,t} \nu_t + \eta_{\beta,t} \nu_{W,t} \mu_t \right] \\
+ (\xi + \rho) \frac{w_t^{1-\gamma} - 1}{1 - \gamma} \right) = 0.
\]

Since nothing in this partial differential equation depends on \( w_t \), the conjectured form of the value function is satisfied, with \( \nu_t \) solving the above equation.

Finally, since \( \chi_t^{(\tau)} \) does not depend on arbitrageurs' individual wealth, aggregation implies

\[
\chi_t^{(\tau)} = \frac{X_t^{(\tau)}}{W_t},
\]

| 65 |
so that (32) can be written

\[
\omega^{(\tau)}_t - r_t = \frac{\gamma}{W_t} \int_0^\infty X_t^{(s)} \left[ \eta^{(\tau)}_{r,t} \eta^{(s)}_{r,t} ds + \eta^{(\tau)}_{\beta,t} \eta^{(s)}_{\beta,t} ds \right] ds
\]

\[
- (1 - \gamma) \left[ \eta^{(\tau)}_{r,t} \sigma_r \nu_r, t + \eta^{(\tau)}_{\beta,t} \sigma_\beta \nu_\beta, t + \left( \eta^{(\tau)}_{r,t} \eta_{r,t} + \eta^{(\tau)}_{\beta,t} \eta_{\beta,t} \right) \frac{\nu_{W,t}}{\nu_t} \right].
\] (34)

Given (28) together with

\[
d\nu_t = \left[ \kappa_r (\bar{r} - r_t) \nu_{r,t} - \kappa_\beta \beta_t \nu_{\beta,t} + \omega_t \nu_{W,t} + \frac{1}{2} \sigma_r^2 \nu_{rr,t} + \frac{1}{2} \sigma_\beta^2 \nu_{\beta\beta,t} + \frac{1}{2} \left( \eta^2_{r,t} + \eta^2_{\beta,t} \right) \nu_{WW,t} \\
+ \sigma_r \eta_{r,t} \nu_{W,W,t} + \sigma_\beta \eta_{\beta,t} \nu_{W,W,t} \right] dt + \left( \sigma_r \nu_{r,t} + \eta_{r,t} \nu_{W,t} \right) dB_{r,t} + \left( \sigma_\beta \nu_{\beta,t} + \eta_{\beta,t} \nu_{W,t} \right) dB_{\beta,t}
\]

which follows from Ito’s Lemma, (34) can be more intuitively written

\[
E_t \left( \frac{dP^{(\tau)}_t}{P^{(\tau)}_t} \right) - r_t dt = \\
\frac{\gamma}{W_t} \int_0^\infty X_t^{(s)} Cov \left( \frac{dP^{(\tau)}_t}{P^{(\tau)}_t}, \frac{dP^{(s)}_t}{P^{(s)}_t} \right) ds - (1 - \gamma) Cov \left( \frac{dP^{(\tau)}_t}{P^{(\tau)}_t}, \frac{d\nu_t}{\nu_t} \right)
\]

as in the main text. As described in the main text, this generalizes the expression for expected excess returns in Vayanos and Vila (2021) in two ways. First, owing to CRRA preferences, \( W_t \) appears in the denominator of the first term on the right-hand side. Second, when \( \gamma \neq 1 \), arbitrageurs have a standard intertemporal hedging motive that also affects expected excess returns. In particular, arbitrageurs require a lower excess return on a bond of maturity \( \tau \) if it pays well when \( (1 - \gamma)d\nu_t \) and thus the (instantaneously future) marginal utility of wealth is high. The latter motive is eliminated when \( \gamma = 1 \) (log preferences). It is also eliminated when \( \rho \to \infty \), in which case \( \nu_t = 1 \) solves the partial differential equation (33). We focus on the latter case for computational simplicity.

C.2 Solution algorithm

We now provide more details on our computational algorithm.
Given (6), (7), (21), and (22), Ito’s Lemma and \( d\tau = -dt \) implies that

\[
\frac{dP^{(\tau)}_t}{P^{(\tau)}_t} = \frac{1}{P^{(\tau)}_t} \left[ P^{(\tau)}_{r,t} \kappa_r (\bar{r} - r_t) + P^{(\tau)}_{W,t} \omega_t + P^{(\tau)}_{\beta,t} \kappa_{\beta} (\bar{\beta} - \beta_t) - P^{(\tau)}_{r,t} \right]
\]

\[
+ \frac{1}{2} P^{(\tau)}_{rr,t} \sigma_r^2 + \frac{1}{2} P^{(\tau)}_{WW,t} (\eta_r^2 + \eta_{\beta, t}^2) + \frac{1}{2} P^{(\tau)}_{\beta, \beta,t} \sigma_{\beta}^2 + P^{(\tau)}_{rW,t} \sigma_r \eta_{r,t} + P^{(\tau)}_{rW,t} \sigma_{\beta} \eta_{\beta,t} \right] dt
\]

\[
+ \frac{1}{P^{(\tau)}_t} \left( P^{(\tau)}_{r,t} \sigma_r + P^{(\tau)}_{W,t} \eta_{r,t} \right) dB_r + \frac{1}{P^{(\tau)}_t} \left( P^{(\tau)}_{\beta,t} \sigma_{\beta} + P^{(\tau)}_{W,t} \eta_{\beta,t} \right) dB_{\beta,t},
\]

where we again write \( P^{(\tau)}_{r,t} \) and \( P^{(\tau)}_{\beta,t} \) to denote the first- and second-order partial derivatives of \( P^{(\tau)}(r_t, \beta_t, W_t) \) with respect to a generic variable \( x \), and we again write \( \omega_t = \omega(r_t, \beta_t, W_t) \) and analogously for \( \eta_{r,t} \) and \( \eta_{\beta,t} \). It follows that

\[
E_t \left( dP^{(\tau)}_t \right) = \left[ P^{(\tau)}_{r,t} \kappa_r (\bar{r} - r_t) + P^{(\tau)}_{W,t} \omega_t + P^{(\tau)}_{\beta,t} \kappa_{\beta} (\bar{\beta} - \beta_t) - P^{(\tau)}_{r,t} \right]
\]

\[
+ \frac{1}{2} P^{(\tau)}_{rr,t} \sigma_r^2 + \frac{1}{2} P^{(\tau)}_{WW,t} (\eta_r^2 + \eta_{\beta, t}^2) + \frac{1}{2} P^{(\tau)}_{\beta, \beta,t} \sigma_{\beta}^2 + P^{(\tau)}_{rW,t} \sigma_r \eta_{r,t} + P^{(\tau)}_{rW,t} \sigma_{\beta} \eta_{\beta,t} \right] dt
\]

and

\[
Cov_t \left( dP^{(\tau)}_t, dP^{(s)}_t \right) = \left( P^{(\tau)}_{r,t} \sigma_r + P^{(\tau)}_{W,t} \eta_{r,t} \right) \left( P^{(s)}_{r,t} \sigma_r + P^{(s)}_{W,t} \eta_{r,t} \right) dt
\]

\[
+ \left( P^{(\tau)}_{\beta,t} \sigma_{\beta} + P^{(\tau)}_{W,t} \eta_{\beta,t} \right) \left( P^{(s)}_{\beta,t} \sigma_{\beta} + P^{(s)}_{W,t} \eta_{\beta,t} \right) dt.
\]

Plugging both into (19), we obtain the partial differential equation

\[
\left[ P^{(\tau)}_{r,t} \kappa_r (\bar{r} - r_t) + P^{(\tau)}_{W,t} \omega_t + P^{(\tau)}_{\beta,t} \kappa_{\beta} (\bar{\beta} - \beta_t) - P^{(\tau)}_{r,t} \right]
\]

\[
+ \frac{1}{2} P^{(\tau)}_{rr,t} \sigma_r^2 + \frac{1}{2} P^{(\tau)}_{WW,t} (\eta_r^2 + \eta_{\beta, t}^2) + \frac{1}{2} P^{(\tau)}_{\beta, \beta,t} \sigma_{\beta}^2 + P^{(\tau)}_{rW,t} \sigma_r \eta_{r,t} + P^{(\tau)}_{rW,t} \sigma_{\beta} \eta_{\beta,t} \right] dt
\]

\[
= \frac{\gamma}{W_t} \left[ \int_0^\infty \left( \alpha(s) \log \left( P^{(s)}_t \right) + \theta_0(s) \right) \frac{1}{P^{(s)}_t} \left( P^{(s)}_{r,t} \sigma_r + P^{(s)}_{W,t} \eta_{r,t} \right) ds
\]

\[
+ \left( P^{(\tau)}_{\beta,t} \sigma_{\beta} + P^{(\tau)}_{W,t} \eta_{\beta,t} \right) \int_0^\infty \left( \alpha(s) \log \left( P^{(s)}_t \right) + \theta_0(s) \right) \frac{1}{P^{(s)}_t} \left( P^{(s)}_{\beta,t} \sigma_{\beta} + P^{(s)}_{W,t} \eta_{\beta,t} \right) ds
\]

\[
+ \beta_t \left[ \int_0^\infty \theta_1(s) \frac{1}{P^{(s)}_t} \left( P^{(s)}_{r,t} \sigma_r + P^{(s)}_{W,t} \eta_{r,t} \right) ds
\]

\[
+ \left( P^{(\tau)}_{\beta,t} \sigma_{\beta} + P^{(\tau)}_{W,t} \eta_{\beta,t} \right) \int_0^\infty \theta_1(s) \frac{1}{P^{(s)}_t} \left( P^{(s)}_{\beta,t} \sigma_{\beta} + P^{(s)}_{W,t} \eta_{\beta,t} \right) ds \right] \right] dt.
\]
Collecting terms, this can be written

\[
\begin{align*}
P_{r,t}^{(\tau)} \mu_{r,t} + P_{W,t}^{(\tau)} \mu_{W,t} + P_{\beta,t}^{(\tau)} \mu_{\beta,t} - P_{r,t}^{(\tau)} - r_t P_{r,t}^{(\tau)} \\
+ \frac{1}{2} P_{rr,t}^{(\tau)} \sigma_r^2 + \frac{1}{2} P_{WW,t}^{(\tau)} (\eta_{r,t}^2 + \eta_{\beta,t}^2) + \frac{1}{2} P_{\beta \beta,t}^{(\tau)} \sigma_{\beta}^2 + P_{rW,t}^{(\tau)} \sigma_r \eta_{r,t} + P_{\beta W,t}^{(\tau)} \sigma_{\beta} \eta_{\beta,t} = 0.
\end{align*}
\]  

(35)

where

\[
\begin{align*}
\mu_{r,t} &= \kappa_r (\bar{r} - r_t) - \frac{\gamma}{W_t} \sigma_r \eta_{r,t}, \\
\mu_{\beta,t} &= \kappa_\beta (\bar{\beta} - \beta_t) - \frac{\gamma}{W_t} \sigma_{\beta} \eta_{\beta,t}, \\
\mu_{W,t} &= \omega_t - \frac{\gamma}{W_t} (\eta_{r,t}^2 + \eta_{\beta,t}^2),
\end{align*}
\]  

(36)  

(37)  

(38)

anticipating the expressions for \( \eta_{r,t} \) and \( \eta_{\beta,t} \) in (44) and (45) below.

Using the Feynman-Kac formula, the solution to a PDE of the form (35) is

\[
P_t^{(\tau)} = E_t^Q \left[ e^{- \int_0^t r_{t+s} ds} \right]
\]  

(39)

where the stochastic processes of \( r_t, \beta_t \) and \( W_t \) under \( Q \) are given by

\[
\begin{align*}
dr_t &= \mu_r (r_t, \beta_t, W_t) dt + \sigma_r dB_{r,t}, \\
d\beta_t &= \mu_\beta (r_t, \beta_t, W_t) dt + \sigma_{\beta} dB_{\beta,t}, \\
dW_t &= \mu_W (r_t, \beta_t, W_t) dt + \eta_r (r_t, \beta_t, W_t) dB_{r,t} + \eta_\beta (r_t, \beta_t, W_t) dB_{\beta,t}.
\end{align*}
\]  

(40)  

(41)  

(42)

Finally, (20), (22), and the method of undetermined coefficients imply

\[
\begin{align*}
\omega_t &= \xi (\bar{W} - W_t) \\
&+ W_t r_t + \int_0^\infty \left( \alpha(\tau) \log \left( P_t^{(\tau)} \right) + \theta_0(\tau) + \theta_1(\tau) \beta_t \right) \left( \omega_t^{(\tau)} - r_t \right) d\tau, \\
\eta_{r,t} &= \int_0^\infty \left( \alpha(\tau) \log \left( P_t^{(\tau)} \right) + \theta_0(\tau) + \theta_1(\tau) \beta_t \right) \frac{1}{P_t^{(\tau)}} \left( P_t^{(\tau)} \sigma_r + P_{rW,t}^{(\tau)} \eta_{r,t} \right) d\tau, \\
\eta_{\beta,t} &= \int_0^\infty \left( \alpha(\tau) \log \left( P_t^{(\tau)} \right) + \theta_0(\tau) + \theta_1(\tau) \beta_t \right) \frac{1}{P_t^{(\tau)}} \left( P_t^{(\tau)} \sigma_{\beta} + P_{\beta W,t}^{(\tau)} \eta_{\beta,t} \right) d\tau,
\end{align*}
\]  

(43)  

(44)  

(45)

where \( \omega_t^{(\tau)} \) is the expected instantaneous return on a bond of maturity \( \tau \) defined in (28).
This motivates the following computational algorithm:\textsuperscript{56}

1. Create a tensor grid of $r$, $\beta$, and $W$ values.

2. Initialize $\mu_r(r, \beta, W)$, $\mu_\beta(r, \beta, W)$, $\mu_W(r, \beta, W)$, $\omega(r, \beta, W)$, $\eta_r(r, \beta, W)$, and $\eta_\beta(r, \beta, W)$ at each grid point. In the first iteration, set them to zero.

3. Use cubic splines to approximate $\mu_r(r, \beta, W)$, $\mu_\beta(r, \beta, W)$, $\mu_W(r, \beta, W)$, $\omega(r, \beta, W)$, $\eta_r(r, \beta, W)$, and $\eta_\beta(r, \beta, W)$ outside of the grid points.

4. Approximate (39) using Monte Carlo simulation:

   (a) For each grid point simulate $N$ sequences of $r$, $\beta$, and $W$ of length $\bar{\tau}/dt$, where $dt$ is the size of the time steps in the simulation. The stochastic processes are approximated using the Euler-Maruyama method.

   (b) Calculate an approximation of the price of a bond of maturity $\tau$ at each grid point as

   $$
P^{(\tau)}(r, \beta, W) = E^Q_t \left[ e^{-\int_0^\tau r_{t+\alpha} d\alpha} \right] (r, \beta, W) \approx \frac{1}{N} \sum_{n=1}^N e^{-\sum_{i=1}^{\bar{\tau}/dt} r_{n,i}(r, \beta, W) dt}$$

   where $r_{n,i}(r, \beta, W)$ denotes the $i$th observation in the $n$th simulation of the interest rate process starting from state $(r, \beta, W)$ at $t$.

5. Update the values for $\mu_r(r, \beta, W)$, $\mu_\beta(r, \beta, W)$, $\mu_W(r, \beta, W)$, $\omega(r, \beta, W)$, $\eta_r(r, \beta, W)$, and $\eta_\beta(r, \beta, W)$ using (36)-(38) and (43)-(45).

6. If the values in step 5 are sufficiently close to those guessed in step 2, stop. Otherwise, return to step 2 using the values in step 5.

Finally, we note one adjustment we make to the environment to facilitate the solution of the model on wide enough grids for $W$ and $\beta$: we modify the specification of habitat demand (2) so that

$$\theta_t(\tau) = \theta_0(\tau) + \theta_1(\tau) \left( 1 - \frac{1}{1 + bW_t} \right) \beta_t,$$

\textsuperscript{56}Our code is written in Julia and solves the model in 30 minutes on a desktop computer. The code and additional details on the solution algorithm are available on GitHub.
where $b$ is a large scalar. This in turn enters conditions (43)-(45). The factor $1 - \frac{1}{1 + bW_t}$ is monotonic in wealth $W_t$ and equals zero at $W_t = 0$ and one at $W_t \to \infty$. It facilitates stability of the numerical solution for wider grids in $W$ and $\beta$, as it means habitat demand has less volatility when wealth is low (there is no effect on the average level of habitat demand, as $\beta_t$ has zero mean). We set $b = 3/0.07$, where 0.07 is the center of the wealth grid, so that at the grid center this factor already is 0.75 and is very flat in wealth. We also emphasize that this factor pushes against our model’s ability to deliver a rise in the term premium upon a monetary tightening, as it implies that the volatility of demand shocks (slightly) falls when arbitrageurs lose wealth.

We can evaluate the accuracy of our numerical algorithm by considering the case with $\xi \to \infty$. With exogenous wealth, there is a closed form solution for the bond price $P^{(r)}(r, \beta)$ as derived in Vayanos and Vila (2021). Table 15 compares the numerical and closed form solutions in such an environment. We in particular study the parameterization with $\xi \to \infty$ plotted in Figure 2 and subsequent figures, which is our baseline calibration reported in Table 5 but with $\xi \to \infty$ and $\gamma$ recalibrated to approximately match the same yield spread. As is evident from Table 15, the first and second moments of the numerical solution are very close to those from the closed from solution. The same conclusion holds for the entire yield curve at least over the support of $(r, \beta)$ over which the numerical grid is defined, which extends 3 to 5 standard deviations in each variable in each direction.

The factor described in the prior paragraph does not change the fact that the model nests Vayanos and Vila (2021) when $\xi \to \infty$, since of course in that case $1 - \frac{1}{1 + bW_t} = 1 - \frac{1}{1 + b\bar{W}}$ is simply a constant.

Recall that this is different from the $\xi \to \infty$ case reported in Table 7.

<table>
<thead>
<tr>
<th>Moment</th>
<th>Numerical</th>
<th>Closed form</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_t^{(1)}$</td>
<td>0.08%</td>
<td>0.08%</td>
</tr>
<tr>
<td>$y_t^{(10)} - y_t^{(1)}$</td>
<td>0.89%</td>
<td>0.90%</td>
</tr>
<tr>
<td>$y_t^{(20)} - y_t^{(1)}$</td>
<td>1.54%</td>
<td>1.55%</td>
</tr>
<tr>
<td>$\sigma(y_t^{(10)})$</td>
<td>2.26%</td>
<td>2.25%</td>
</tr>
<tr>
<td>$\sigma(y_t^{(20)})$</td>
<td>1.93%</td>
<td>1.93%</td>
</tr>
<tr>
<td>$\beta_{FE}^{(10)}$</td>
<td>0.93</td>
<td>0.93</td>
</tr>
</tbody>
</table>

Table 15: numerical versus closed form solutions given $\xi \to \infty$

Notes: parameters are baseline calibration reported in Table 5 but with $\xi \to \infty$ and $\gamma$ recalibrated to approximately match the same yield spread.
C.3 Decomposing the forward rate responses

We can use the following decomposition to understand more deeply why long-dated real forwards rise upon a monetary tightening in the model. Following Cochrane and Piazzesi (2008), standard identities imply that

\[ f_t^{(\tau-1,\tau)} - y_{t+\tau-1}^{(1)} = \left[ r_t^{(\tau)} - r_{t+1}^{(\tau-1)} \right] + \left[ r_{t+1}^{(\tau-1)} - r_{t+2}^{(\tau-2)} \right] + \ldots + \left[ r_{t+\tau-1}^{(2)} - y_{t+\tau-2}^{(1)} \right], \tag{46} \]

where \( r_t^{(\tau)} \) denotes the log return to purchasing a \( \tau \)-period bond at \( t \) and holding it for one year:

\[ r_t^{(\tau)} \equiv \log P_{t+1}^{(\tau)} - \log P_t^{(\tau)}. \]

The left-hand side of (46) is the forward-spot spread. The right-hand side of (46) reflects the cumulative return to a sequence of carry strategies: purchasing a \( \tau \)-year bond at \( t \) financed by a \( (\tau - 1) \)-year bond, then purchasing a \( (\tau - 1) \)-year bond at \( t+1 \) financed by a \( (\tau - 2) \)-year bond, and so on. Evaluating this identity ex-ante instead of ex-post and taking expectations at \( t \), we have that

\[ f_t^{(\tau-1,\tau)} - E_t y_{t+\tau-1}^{(1)} = \left[ r_t^{(\tau)} - r_{t+1}^{(\tau-1)} \right] + E_t \left[ r_{t+1}^{(\tau-1)} - r_{t+2}^{(\tau-2)} \right] + \ldots + E_t \left[ r_{t+\tau-1}^{(2)} - y_{t+\tau-2}^{(1)} \right]. \tag{47} \]

It follows that the response of the forward rate relative to the expected spot rate — the difference between the red line and black line in Figure 3 — encodes the response of expected excess returns on a sequence of carry trades at future dates.

Figure 13 depicts the response of each of these expected excess returns for \( \tau = \{5, 10, 15, 20\} \)-year bonds corresponding to the baseline results in Figure 3. At \( h = 1 \) (the first year after the monetary tightening), we see that the initial increase in expected carry trade returns is highest for five-year bonds, versus 10-year bonds, and in turn 15- and 20-year bonds. This reflects the fact that average carry trade returns are themselves falling in maturity: 20- and 19-year bonds do not have much different sensitivities to aggregate risk factors, whereas five- and four-year bonds do. However, because longer dated forward rates encode the response of expected carry trade returns over a longer horizon \( h \), the difference in forward rates \( f_t^{(\tau-1,\tau)} \) and expected spot rates \( E_t y_{t+\tau-1}^{(1)} \) generally rises in \( \tau \), even though the initial response in carry trade returns is smaller for bonds with higher \( \tau \).

Thus, the model rationalizes the response of long-dated forward rates to a monetary
\[ \Delta E_t[r_{t+h}(\tau+1-h) - r_{t+h}(\tau-h)] / \Delta y_t^{(1)} \]

Figure 13: decomposing \(d[f_t^{(\tau-1,\tau)} - E_t y_t^{(1)}]/dy_t^{(1)}\) on impact of monetary shock

Notes: as derived in (47), \(\sum_{h=1}^{\tau-1} E_t \left[r_{t+h}^{(\tau+1-h)} - r_{t+h}^{(\tau-h)}\right] = f_t^{(\tau-1,\tau)} - E_t y_t^{(1)}\). Responses simulated as described in Figure 2.

shock through a persistent change in risk premia following the shock.

### C.4 U-shaped response of forward rates to monetary shock

The baseline model does not generate a U-shaped response of forward rates as in our baseline estimates in the data; the increase in term premia by maturity is indeed present, but it is outweighed by the decline in the expected spot rate by maturity. Here we use the model to demonstrate, though, that a U-shaped response as estimated in the data is entirely sensible.

When a monetary shock has a more transitory effect on the short rate (higher \(\kappa_m\)), the model can generate a U-shape in forward rate responses. Figure 14 demonstrates this with \(\kappa_m = 1\), leaving all other parameters unchanged from the main text. At low maturities \(\tau\), the response of forward rates is declining in \(\tau\) because the response of the expected spot rate is declining in \(\tau\). At moderate \(\tau\), the effect on the expected spot rate is already so small that all that remains is the response of term premia, and thus the response of forward rates is rising in \(\tau\) because the response of term premia is rising in \(\tau\). At sufficiently high \(\tau\), the effect on term premia begins to decline in \(\tau\), reflecting the smaller response of long-dated carry trade returns on impact of the
monetary shock, as explained in the last subsection.

C.5 Impulse responses to short rate and habitat demand shocks

Our analysis in the main text and in the prior subsections focuses on the model’s predictions for monetary transmission. Here we focus instead on the impulse responses to “typical” short rate shocks and habitat demand shocks.

Figure 15 presents the responses under the baseline calibration to a short rate shock $dB_{r,0}$, scaled to generate a 100bp rise in the one-year real yield on impact for comparability with the monetary-induced short rate shock in Figure 2. The other panels further depict the same variables as in Figure 2. Qualitatively, the responses are the same as a monetary shock. Quantitatively, the response of the 20-year real forward rate is amplified. This is driven not just by a higher expected future short rate but also by an amplified response of term premia, evident from the larger response of excess returns on the 20-year bond and resulting from the larger revaluation of arbitrageur wealth.

Figure 16 presents the responses under the baseline calibration to a one standard deviation demand shock $dB_{\beta,0}$. The figure demonstrates that habitat demand shocks are effectively term premium shocks. Whether arbitrageurs’ endowment is endogenous
or exogenous, forward rates rise when habitat investors seek to borrow more in long-term bonds and thus arbitrageurs must bear more risk. Since expected short rates are unchanged, this fully reflects a rise in term premia. The rise in term premia manifests as an increase in the slope of the yield curve, as well as high subsequent excess returns on long-term bonds. In the model with endogenous wealth, the rise in long yields revalues wealth against arbitrageurs, further raising expected excess returns on long-term bonds in the short run. However, in the medium run, the increase in the volume of arbitrageurs’ carry trade implies that wealth rises above its initial value, generating an eventual reversal in excess returns. It is for this reason that the response of forward rates on impact of the shock is in fact dampened in the model with endogenous wealth, since — consistent with (47) — these forward rates encode the response of carry trade returns at future dates.
Notes: figure depicts one standard deviation shock, and $x$-axis denotes number of months since the shock. Responses simulated as described in Figure 2.

## C.6 QE in data and model

QE in the model is a habitat demand shock, since we calibrate the model so that the Federal Reserve is among the set of habitat investors. In this appendix we provide the evidence on QE and the mapping between model and data that we use to calibrate the model.

### C.6.1 QE in data

We first describe the evidence on QE, building on a large literature. We focus on the first round of QE from November 2008 through March 2010, and measure both the Federal Reserve’s purchases as well as the announcement effects on the yield curve.

**Purchases** The first round of QE consisted of purchases of Treasuries, agency/GSE-backed debt securities, and mortgage-backed securities (MBS). We measure the purchases of Treasuries and agency/GSE-backed debt securities from the System Open...
Market Account (SOMA) Holdings of Domestic Securities reported by the Federal Reserve Bank of New York. We measure the purchases of MBS from the transaction-level data for the Agency Mortgage-Backed Securities Purchase Program organized by the Federal Reserve Board. All purchases are at the CUSIP level.

We then transform these purchases into a panel dataset of purchases of zero coupon bonds purchased at each maturity $\tau \in (0,30]$ on each date $t$. For Treasuries and agency/GSE-backed debt securities, we observe each security’s coupon, term, and par value purchased. We thus strip the security into its constituent payments at each future date and compute the market values of each stripped payment using the daily nominal yield curve estimated by Gurkaynak et al. (2006). For MBS, we observe each security’s market value purchased. While we also observe the security’s coupon and term, these securities have substantial pre-payment risk and thus their effective duration is much shorter than simple duration. For simplicity we thus assume that all MBS purchased on a given date are zero coupon bonds with maturity equal to the effective duration of the Bloomberg MBS index on that date.

Figure 17 summarizes the Federal Reserve’s purchases over this period in terms of the equivalent purchase of 10-year zero coupon bonds (in terms of having the same duration as the securities purchased). We estimate that the cumulative purchases totaled over $600 billion in 10-year equivalent zero coupon bonds, consistent with other estimates in the literature (e.g., Gagnon, Raskin, Remache, and Sack (2011)).

**Announcement effects on yield curve** We next measure the announcement effects of QE on the TIPS yield curve. We focus on the TIPS yield curve given the paper’s maintained focus on the real term structure. Together with the data on purchases, this will allow us to discipline the price elasticities in the model.

Using Gurkaynak et al. (2008)’s interpolated TIPS yield curve (as throughout our analysis), Table 16 summarizes the two-day changes in the 10- and 20-year yields and one-year forward rates paying in 10 and 20 years on each of the announcement days. We refer the reader to Gagnon et al. (2011) for a description of each announcement.

---

59 Since this dataset only provides the par value of holdings by security, we define purchases as the change in the par value from one week to the next. This dataset also does not distinguish between Treasuries purchased as part of QE and Treasuries purchased as part of more standard open market operations. We (largely) isolate the former by focusing only on Treasury purchases after March 18, 2009 of securities with a remaining time to maturity of at least two years.

60 We follow Krishnamurthy and Vissing-Jorgensen (2011) in using two-day changes, but similar results are obtained using one-day changes.
We follow Krishnamurthy and Vissing-Jorgensen (2011) in distinguishing the first five announcements from the subsequent three, as the former had larger effects. We find that the cumulative declines in the 10- and 20-year TIPS yields on announcement days were 188\text{bp} and 129\text{bp}, respectively. The cumulative declines in the forward rates were 117\text{bp} and 100\text{bp}. The latter is direct evidence of a compression of risk premiums, under the maintained assumption that the announcements contained no news about real interest rates so far into the future.

Our estimates are broadly consistent with those in the literature. Krishnamurthy and Vissing-Jorgensen (2011) estimate a cumulative 187\text{bp} two-day decline in the 10-year TIPS real yield around the first five announcements, almost exactly the same as our estimate. Gagnon et al. (2011) estimate a cumulative 91\text{bp} one-day decline in the 10-year Treasury yield on all announcement days; we estimate a comparable 119\text{bp} one-day TIPS decline (not shown in the table), less than the 188\text{bp} two-day decline as expected if illiquidity during this period caused some delay in price impact. For the March 18, 2009 announcement on which we focus below, D’Amico and King (2013) estimate a one-day decline in the 10-year (20-year) Treasury yield by more than 50\text{bp} (30\text{bp}) using the prices of securities which were eventually purchased, only slightly less
\[ \Delta y_t^{(10)} \quad \Delta y_t^{(20)} \quad \Delta f_t^{(10)} \quad \Delta f_t^{(20)} \]

<table>
<thead>
<tr>
<th>Date</th>
<th>( \Delta y_t^{(10)} )</th>
<th>( \Delta y_t^{(20)} )</th>
<th>( \Delta f_t^{(10)} )</th>
<th>( \Delta f_t^{(20)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>12/1/2008</td>
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<td>-39</td>
<td>-10</td>
<td>-98</td>
</tr>
<tr>
<td>12/16/2008</td>
<td>-58</td>
<td>-39</td>
<td>-31</td>
<td>-17</td>
</tr>
<tr>
<td>1/28/2009</td>
<td>9</td>
<td>14</td>
<td>16</td>
<td>21</td>
</tr>
<tr>
<td>3/18/2009</td>
<td>-61</td>
<td>-41</td>
<td>-46</td>
<td>-25</td>
</tr>
<tr>
<td>Sum to date</td>
<td>-183</td>
<td>-124</td>
<td>-119</td>
<td>-98</td>
</tr>
<tr>
<td>8/12/2009</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>9/23/2009</td>
<td>-1</td>
<td>1</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>11/4/2009</td>
<td>-3</td>
<td>-6</td>
<td>2</td>
<td>-6</td>
</tr>
<tr>
<td>Sum to date</td>
<td>-188</td>
<td>-129</td>
<td>-117</td>
<td>-100</td>
</tr>
</tbody>
</table>

Table 16: announcement effects on real yield curve

Notes: all values in bp. Effect computed as the value at the end of the trading day following the announcement day, less the value at the end of the trading day prior to the announcement day.

than the two-day changes in the interpolated TIPS yield curve which we find.

C.6.2 QE in model

We now describe how we simulate QE in the model.

We focus for concreteness on the March 18, 2009 announcement that the Federal Reserve would begin purchasing up to $300bn of longer-term Treasuries and increase the size of agency/GSE-backed debt and agency MBS purchases up to $200bn and $1.25tn, respectively.\(^{61}\) Prior to this announcement, the Federal Reserve had only announced that it “could” purchase Treasuries without specifying that it would, and it had announced (on November 25, 2008) that it would purchase up to $100bn and $500bn in agency/GSE-backed debt and agency MBS, respectively.

In the model simulation we thus assume that all of the purchases of Treasuries from April 2009 through March 2010 were unanticipated prior to the announcement, and that $100bn and $750bn of the purchases of agency/GSE-backed debt and agency MBS over this period were unanticipated.\(^{62}\) We further assume that, upon announcement,

\(^{61}\)To the extent the subsequent announcements in 2009 had small effects because they were partially anticipated, the primary benefit of focusing on this announcement is that it was the “last” unanticipated announcement in the first round of QE. Hence, we can plausibly simulate a one-time shock, rather than a sequence of unanticipated shocks which is more computationally intensive.

\(^{62}\)Using our security level purchase data, we estimate $132bn purchases of agency/GSE-backed debt and $132bn purchases of agency MBS over this period were unanticipated.


<table>
<thead>
<tr>
<th></th>
<th>Q4 2007</th>
<th>Q1 2009</th>
<th>Percent change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Broker/dealers</td>
<td>285</td>
<td>293</td>
<td>3</td>
</tr>
<tr>
<td>Hedge funds</td>
<td>1,975</td>
<td>973</td>
<td>-51</td>
</tr>
<tr>
<td><strong>Sum</strong></td>
<td>2,260</td>
<td>1,266</td>
<td>-44</td>
</tr>
</tbody>
</table>

Table 17: change in wealth of arbitrageurs during financial crisis

Notes: all values in nominal $bn. Wealth of broker/dealers defined as described in Table 2. Wealth of hedge funds defined as equity capital reported by He et al. (2010), in turn sourced from Barclay Hedge.

agents assume that the assets will roll off the Federal Reserve’s balance sheet simply as they mature. To the extent that some of these purchases were in fact anticipated, or agents expected the assets to be sold off the balance sheet more quickly, it will mean we overestimate $\alpha(\tau)$ in our calibration.

Simulating these asset purchases requires translating them into model scale. We use the following moments. Over the 2012-2016 period overlapping with our maintained sample period of interest, the total assets of broker/dealers and hedge funds as measured using the approach in Table 2 were 29% of annual U.S. GDP.\(^63\) Thus, in the model, we assume that in month 0, agents learn that habitat demand at maturity $\tau$ at time $t$ will fall by an amount

$$\text{purchases}(\tau)_t \frac{\int_0^\infty X(\tau) d\tau}{\text{gdp}_{2007} \cdot 0.29},$$

where $\text{purchases}(\tau)_t$ is the observed value of asset purchases $t$ months after April 2009 in the data, $\text{gdp}_{2007}$ is annual GDP prior to the crisis, and $\int_0^\infty X(\tau) d\tau$ is the average value of arbitrageur assets in the model.

Finally, we must set the initial conditions in the model simulation to associate with March 2009. In particular, we account for the fact that arbitrageurs had lost substantial equity capital by this point in the financial crisis. Table 17 summarizes the decline in wealth among broker/dealers and hedge funds from the fourth quarter of 2007 through the first quarter of 2009. While broker/dealer wealth from the Financial Accounts was debt and $967bn purchases of agency MBS over April 2009 through March 2010. We thus multiply each of the purchases of agency/GSE-backed debt and agency MBS purchases over this period by \((100/132)\) and \((750/967)\), respectively, to obtain the unanticipated component of purchases.

\(^63\)Assets refers to all line items in which arbitrageurs are long, e.g. all line items except repo and other short-term loans. Thus, in Q4 2012 for instance, Table 2 demonstrates that arbitrageur assets were $4.291bn.
essentially flat over this period, hedge fund wealth as measured by He et al. (2010) (their Table 4, in turn from Barclay Hedge) fell substantially.\textsuperscript{64} We estimate that the total wealth of arbitrageurs fell by 44% over this period. To avoid conveying the impression that this number is very precisely estimated, we thus initialize arbitrageur wealth to one third less than its average value.

We set the other state variables, $r$ and $\beta$, to their average values ($\bar{r}$ and zero, respectively). These are only important for our simulation of QE insofar as they govern the magnitude of arbitrageurs’ carry trades and thus the duration of their wealth. By setting $r = \bar{r}$ and $\beta = 0$, the model-implied duration of arbitrageurs is roughly double its average value. This in turn implies term premia roughly double their average values, according well with the data in early 2009 as is evident from the top panel of Figure 12. We thus set $r = \bar{r}$ and $\beta = 0$ for simplicity, without further refinement.

The solid line in Figure 5 in the main text summarizes the resulting paths of habitat demand, the yield curve at selected maturities, and arbitrageur wealth in our simulation of QE. The exogenous component of habitat demand eventually falls by more than 30% given the Federal Reserve’s purchases. The response of the 10- and 20-year real forward rates on impact are 53bp and 25bp, consistent with the observed responses of 46bp and 25bp to the March 18, 2009 announcement (reported in Table 16) because $\alpha$ and $\xi$ were calibrated to match these moments.

C.7 Comparing FOMC days to non-FOMC days

We finally use our model to shed light on the identification of the effects of monetary shocks in the data. Following much of the literature, our maintained approach in section 4 is to use as an instrumental variable the monetary surprise measured using high-frequency (30-minute) changes in futures prices. An alternative approach would be to estimate the relationship between long-term and short-term rates on FOMC versus non-FOMC announcement days, ascribing the difference to monetary policy announcements. In this appendix, we use our model to illustrate the difficulty in interpreting such regressions, and why the use of the high-frequency monetary surprise as an instrumental variable is thus preferable.

We first outline the alternative empirical approach. Consider a regression of one-day changes in real forward rates $\Delta f_t^{(\tau, \tau-1)}$ on one-day changes in the one-year real

\textsuperscript{64}We use this source, rather than the Form PF data used as our primary source for hedge funds in the main text, since the Form PF data is not available prior to 2012.
yield $\Delta y_t^{(1)}$, an indicator for an FOMC announcement occurring on day $t$ $FOMC_t$, and their interaction. The estimates over our maintained sample period of interest, January 2004 through December 2016, for $\tau = \{5, 10, 15, 20\}$ are presented in Table 18. The regressions imply that there is a statistically significant relationship between 20-year real forward rates and the one-year yield even on non-FOMC days, and the point estimate on the interaction coefficient with FOMC days in fact negative, albeit not significantly different from zero.

Our model clarifies why this specification is problematic in inferring the effects of monetary policy shocks on the term structure. Figure 18 summarizes the responses of forward rates across maturities relative to the one-year yield given a monetary shock and demand shock. As is evident, the sensitivity of forward rates to a given change in the one-year yield is higher for demand shocks than monetary shocks; that is, the dashed line in Figure 18 lies above the solid line. This reflects the fact that demand shocks have a small effect on the one-year yield but larger effects on the prices of longer maturity bonds. Similarly, typical short rate shocks with higher persistence than monetary shocks have larger effects on long-dated real forwards than monetary shocks, just via the expectations hypothesis. Since the interaction coefficients in the above regression capture the difference in forward responses between monetary shocks and these other sources of fluctuations in the yield curve (demand shocks and typical short rate shocks), we would expect that the interaction coefficients should be negative.

Table 18: incremental news around FOMC meetings

<table>
<thead>
<tr>
<th></th>
<th>$\Delta f_t^{(4,5)}$</th>
<th>$\Delta f_t^{(9,10)}$</th>
<th>$\Delta f_t^{(14,15)}$</th>
<th>$\Delta f_t^{(19,20)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta y_t^{(1)}$</td>
<td>0.11</td>
<td>-0.01</td>
<td>0.01</td>
<td>0.11</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.02)</td>
<td>(0.02)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>$FOMC_t$</td>
<td>-0.01</td>
<td>-0.00</td>
<td>-0.00</td>
<td>-0.00</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>$\Delta y_t^{(1)} \times FOMC_t$</td>
<td>0.25</td>
<td>0.14</td>
<td>0.00</td>
<td>-0.04</td>
</tr>
<tr>
<td></td>
<td>(0.14)</td>
<td>(0.11)</td>
<td>(0.08)</td>
<td>(0.06)</td>
</tr>
<tr>
<td>$N$</td>
<td>3,252</td>
<td>3,252</td>
<td>3,252</td>
<td>3,252</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.07</td>
<td>0.00</td>
<td>0.00</td>
<td>0.03</td>
</tr>
</tbody>
</table>

Notes: robust standard errors in parenthesis. $FOMC_t = 1$ if there was an FOMC meeting on day $t$.

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$^{65}$Similar results are obtained if we use an indicator variable for all monetary policy events (including speeches, press conferences, and other events) in the database of Cieslak and Schrimpf (2019).
on long-dated bonds. This does not mean, however, that monetary shocks do not affect long-dated real forward rates.

If we observe the underlying shocks \( \{dB_{r,t}\} \), we can directly estimate the conditional response of the yield curve to monetary shocks by using them as instruments. This is precisely what motivates our approach in the main text.