A Cognitive Theory of Reasoning and Choice

Pedro Bordalo, Nicola Gennaioli, Giacomo Lanzani, Andrei Shleifer*

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Abstract

We offer a theory of decisions in which selective attention to the features of the current problem is determined by its categorization in a set of problems the DM solved in the past. Categorization depends on goal-relevant features of the current and past problems, as well as on normatively irrelevant context, such as other goods in the choice set. The model delivers systematic heterogeneity in attention and choice based on past experiences, which persists despite common news, and discontinuous shifts when a bottom-up salient change in problem features or in their description causes re-categorization. The model accounts for major puzzles and framing effects in riskless choice, statistical problems, and lottery choice based on: i) failure of aggregating features that belong to different categories or ii) misdirected attention to features that are relevant in a similar problem but not in the current one.

*The authors are from Oxford Said Business School, Universita Bocconi, Harvard University, and Harvard University, respectively. We thank Ben Enke, Alex Imas, Lawrence Jin, Spencer Kwon, Shengwu Li, Sendhil Mullainathan, Joshua Schwartzstein, Jesse Shapiro, and Tomasz Strzalecki for helpful comments. Steven Ma provided outstanding research assistance. Gennaioli thanks the Italian Ministry of Education for Financial Support (PRIN Grant).

1 Introduction

How do people select which features to attend to when making decisions? Is Donald Trump a hardened criminal or the champion of ordinary Americans? Is the car rental agency Avis, which since 1962 advertised itself with the slogan "We are number two – we try harder" an underdog or a loser? Is buying a stock a long-term investment or a risk? For the last century, economists have traced choice to stable preferences and correct beliefs, but it feels intuitive that in these and other instances many people have similar payoffs or information, and yet make sharply different assessments because they attend to some features of the problem, neglecting others. When a neglected feature becomes prominent due to a salient event, political propaganda, advertising, or social influence, many people refocus on that feature and change decisions.

A similar intuition emerges from research on judgment and decision-making. People disagree in fundamentally similar problems and change their behavior after irrelevant changes in framing (e.g., Kahneman and Tversky [118], Thaler [109]). The behavioral economics of stable biases, such as reference points, self-serving beliefs, social preferences, base rate neglect, or under-reaction explains neither why different people behave differently, nor how choices change due to framing. To understand decisions in the lab and the field, as well as mechanisms of influence – private (advertising) and public (information campaigns) – we must engage with a cognitive mechanism: selective attention to features.

We offer a theory in which attention is driven by a key, hitherto neglected, step: problem recognition. Before evaluating choice options, people categorize a problem in a class of similar problems they solved in the past. This process, which uses both attention and memory, suggests which features are relevant for representing the problem "top down". The problem is solved by aggregating these features, neglecting others. Crucially, however, different categorizations are possible.

In the duck-rabbit illusion, people focus either on the beak and recognize a duck, or on the mouth and recognize a rabbit. Both representations are possible, because each selects i) what to attend to and ii) a confirming category from experience. Similar dilemmas arise in economic problems. Choosing which jam to buy may be categorized as "giving myself a treat," which focuses on consumption pleasure, or as "buying a staple food," which focuses on price. Assessing a political candidate may be categorized as "evaluating skills" which focuses on her performance in different scenarios, or as "evaluating risks" which focuses on the contingencies in which these scenarios may materialize.

In standard theory, these categories are integrated. Our key idea is that integration is difficult because different categories are segregated in memory.[1] We experience the pleasure of the jam at home and the pain of paying at the supermarket, not together. We experience political candidates in office as history unfolds and think about possible contingencies ahead of time, not together. Because these experiences are associated with different contexts, they produce competing categories that utilize a subset of features.

In our model, categorization follows the regularities of human memory: it is stochastic and more likely to retrieve problems that have been more frequently solved in the past or are more similar along features that are attended to. Some such features are goal relevant in the current problem, others are goal relevant in other problems, and still others are purely contextual. Whether the jam is categorized as a treat or a staple depends on the prices and qualities on offer, but also on whether the day of the week, others goods on the shelf, etc., match past experiences of treats versus staples, commanding attention.

[1] See Treisman and Gelade [113] for the seminal study showing that integration of features is significantly more difficult and time-consuming than considering each of the features separately, and Tversky [122] for difficulties in aggregation of spatial knowledge.

We obtain three main results. First, different experiences create systematic heterogeneity in how a problem is represented and solved. Being familiar with the pain of paying, a formerly poor person focuses more on the price of the jam compared to a person with the same income but without the same experiences. Heterogeneity in choices comes from heterogeneity of representations, not preferences. Critically, and unlike in Bayesian learning, different representations focus on and neglect different features, so experiences can produce stable disagreement in assessments, even from common news, as in the confirmation bias (Nickerson [86]) or cognitive dissonance (Festinger [36]).

Second, the model delivers instability due to changes in context that draw attention to a subset of experiences. A special occasion such as a festivity can cue a poor person to exceptionally focus on consumption pleasure and neglect price. In mental accounting (Thaler [109]), a "rainy day" account can induce thrift by serving as a reminder of financial shocks. Unlike in models of noisy perception (Woodford [125], Enke and Graeber [33]), however, instability reflects not i.i.d. noise but re-categorization, which produces large discontinuous shifts systematically associated with context. Like heterogeneity, choice instability comes from changes in representation, and not from preferences.

Third, there is a powerful interaction between top-down attention and bottom-up salience, driven by prominence or contrast of features (e.g., Bordalo, Gennaioli, and Shleifer [17], Bushong, Camerer, and Rangel [21]). A bottom-up salient feature receives a higher weight in decisions, but now it can also cause a switch to a category for which this feature is relevant, causing neglect of previously attended to features. An advertisement showing the jam at a beautiful breakfast table in the countryside prompts retrieval of the "treat" category, boosting quality focus and dampening price focus. This mechanism delivers effects of uninformative persuasion and advertising (e.g., Mullainathan, Schwartzstein, and Shleifer, [83] Mullainathan and Shleifer [84]).

We develop our model using famous puzzles in three domains: riskless choice, statistical problems, and risky choice. Category-driven attention offers a unified mechanism for mental accounting (e.g., sunk cost fallacy and opportunity cost neglect), the Gambler's Fallacy, over and under reaction in statistical problems, as well as the common ratio effect, the certainty effect, and the fourfold pattern in risky choice. Within each domain, the model accounts and yields new predictions for heterogeneity and instability of biases, including based on pure framing. It also connects biases across domains, accounting for the dumbfounding link between the fourfold pattern in risky choice and biased aggregation of riskless features (Oprea [88]). This comes from similarity of representations across domains. It cannot come from risk preferences.

We offer the first cognitive mechanism explaining well-known biases for riskless, statistical, and risky choice domains. Its emphasis on feature neglect relates to bounded rationality (Simon [103]) but not in the sense of computational complexity. Categorization can lead people to confidently make mistakes by either oversimplifying or overcomplicating computations in a problem, due to a focus on the wrong features (Hascher, Imas, Ungeheuer, and Weber [55]).

A very substantial behavioral literature, which we discuss as we apply the model, assumes stable, preference-like, biases, and typically studies decisions in a single domain. This stable bias approach predicts much more uniformity than what is found empirically. It fails to explain the effects of framing, as discussed above, but also why behavior is weakly correlated across choices in a domain, such as demand for different types of insurance or for lotteries in the lab (Barseghyan, Prince, and Teitelbaum [4]). Across domains, loss aversion is assumed to drive both the endowment effect and risk aversion, but empirically these behaviors appear disconnected (Chapman, Dean, Ortoleva, Snowberg, and Camerer [24]). On the other hand, recent work detects a correlation between other biases, both within and across domains, cutting across the standard classification of effects (Stango and Zinman [107], Chapman, Dean, Ortoleva, Snowberg, and Camerer [24], de Clippel, Oprea, and Rozen [30]). Our theory suggests that to study this architecture one should look not for stable choice parameters, but instead measure categories, which can drive different representations in the same domain, and similar representations across domains.

Concretely, our framework suggests a new starting point to think about decisions and anomalies. It calls for measurement of representations, as well as of cognitive functions such as memory and attention that underlie them. Measuring attention to features in a problem, or the reasoning given for a decision helps identify categories. Past experiences, which can be measured, influence the categorization of a problem. Perceived contextual similarity, which can also be measured, connect behavior across different problems. This structure implies that, contrary to the ideal of eliciting "true" preferences under "neutral" conditions, the details of the choice problem matter for choice, because they matter for representations. While neutral conditions may be informative in problems for which people have a frequent, dominant representation, they come at the cost of external validity and predictive power for general situations. In fact, this structure also offers a recipe for producing "biases"from first principles, including new ones: draw attention to a feature that is relevant in a similar problem, albeit not in the current one.

Our approach connects to a broad range of work in psychology, decision theory and behavioral economics. We build on the psychology of attention-based similarity perceptions (Nosofsky [87], Tversky [115],[116]), and of top-down attention, which stresses the role of experiences (Itti and Baldi [61], Awh, Belopolsky, and Theeuwes [2]). Recent economics work studies top down attention as shaped by priors (e.g., Schwartzstein [99], Gagnon-Bartsch, Rabin, and Schwartzstein [42]) and by goal-optimality (e.g., Sims [104], Gabaix [40]). In our paper, attention is driven by selective memory, so it changes with cues, consistent with the evidence in Conlon ([28]).

Our focus on experiences follows work on case-based learning (Schank [98]), formalized in economics by case-based decision theory (Gilboa and Schmeidler [46]), and on habit formation (e.g., Laibson [73]). Mullainathan [82] offers a model in which the category prototype pins down beliefs. Ellis and Masatlioglou [32] axiomatize utility that is stable within but not across categories of choices. Evers, Imas, and Kang [35] present and test a model of hedonic editing based on loss and gain categories.[2]

Unlike in this work, in our model categories determine attention to the different features of choice options, not a specific choice or belief that performed well in the past. In addition, attention and similarity (and hence categorization) are jointly determined by psychological forces, including bottom-up salience. Salant and Rubinstein [95] model choice instability as the result of the use of a different choice function for different frames. We provide a cognitive foundation for the choice function as a result of description and past experiences.

Recent work on memory studies selective information retrieval about probabilities (e.g., Bordalo, Conlon, Gennaioli, Kwon, and Shleifer [19] and Fudenberg, Lanzani, and Strack [39]) and hedonics of goods (Bordalo, Gennaioli, and Shleifer [16]). In our model probabilities and hedonics are known, memory shapes attention to them. Both mechanisms naturally shape decisions, and future work may study them together.

The paper continues as follows. Section 2 presents a general model of top-down attention in riskless choice, statistical problems and risky choice. Section

[2]In psychology, the ALCOVE model by Kruschke ([72], [116]), and ADDCOVE, by Verguts, Ameel, and Storms [123] formalize the assignment of multidimensional stimuli to categories. The key differences with our approach include, among others, that we endogenize similarity to categories shaped by both top-down and bottom-up forces, and that the result of the categorization is not the mere assignement to a group of similar instances, but a way to evaluate the problem that leads to its (possibly wrong) solution.

3 develops the model's general predictions. Section 4 studies applications to mental accounting and judgment biases. Section 5 introduces bottom-up attention and studies puzzles in choice under risk. Section 6 concludes.

2 The Model

Even the simplest problems, such as buying jam, require the DM to evaluate many aspects (taste, price, etc.) and probabilities/risk (is it spoiled?). In reality, only some of these features are attended to and influence choice. We offer a theory of this mechanism based on two building blocks: 1) how attention causes decisions, and 2) attention allocation. Our key innovation is that in 2) attention is shaped "top down" by a simplified representation based on a category of experiences. If the DM thinks about consumption experiences, which often occur removed from paying, she focuses on taste and neglects price. The reverse occurs if she thinks about buying experiences, in which paying is prominent. For many goods, risk is not felt: it is not mentioned and rarely materializes. For others, such as flying, risk is important. So, categorization may also trigger alertness to versus neglect of risks.

For step 1) consider a lottery o delivering a good with hedonics (u_{1s}, u_{2s}), such as quality and price, in state $s = e_{1s} \cap e_{2s}$ defined by events e_{1s} and e_{2s} (e.g., "selection of urn A" and "extraction of a green ball from it"). A DM paying attention $\alpha_x \in (0, 1]$ to feature x then values o as:

$$\sum_s \mathbb{P}\left(e_{1s}\right)^{\alpha_{e_1}} \mathbb{P}\left(e_{2s}|e_{1s}\right)^{\alpha_{e_2}} \cdot \left(\alpha_{u_1} u_{1s} + \alpha_{u_2} u_{2s}\right). \tag{1}$$

Lower attention α_x causes less weighting of probabilities or hedonics, distorting valuation and choice.

Riskless choice entails a sure event s and valuation of hedonics: monetary and non-monetary payoffs. With quality q and price p, Equation (1) yields weighted utility $\alpha_Q q - \alpha_P p$, as in Bordalo, Gennaioli, and Shleifer [15].

A statistical problem entails estimating the probability of event H, to which payoff 1 is attached, with zero payoff outside. If $H =$"green ball g from urn A," Equation (1) becomes $\mathbb{P}(A)^{\alpha_U}\,\mathbb{P}(g|A)^{\alpha_C}$, akin to Grether's [51] formula.

Risky choice entails hedonic and event features. A lottery paying x_g with probability π and $x_b < x_g$ otherwise is valued in Equation (1) as

$$\pi^{\alpha_{e_g}} \cdot \alpha_{u_g} x_g + (1-\pi)^{\alpha_{e_b}} \cdot \alpha_{u_b} x_b,$$

which combines payoff weights in Bordalo, Gennaioli, and Shleifer [14] with probability weights as in Prospect Theory (Kahneman and Tversky [67]).

Compared to previous work, categorization allows attention to depend on non payoff-relevant context features (e.g., an irrelevant good in the choice set, location, etc.), which cue different categories. A beer advertisement showing a party with dancing youngsters can cue "consumption" experiences, causing price neglect and affecting choice when though the beer's attributes are unchanged.

Section 2.1 formalizes step 1 in a general setup embedding Equation (1) and lays out the features of a problem: choice features, whose value is good-specific, and context features, whose value is common across goods. Section 2.2 links features to categories and formalizes step 2: how features cue the DM to match the current problem to a category, shaping attention and choice.

2.1 How Attention Causes Valuation

The primitives of a problem are: i) a menu of options, ii) a set of features, iii) an attention-driven valuation function as in Equation (1), and iv) a decision rule mapping valuations into choice.

Menu of Options. There is a nonempty finite menu of lotteries O. Each lottery $o \in O$ is a finite set of event-payoff combinations, which we call atoms. As in Equation (1), the value of an atom is an attention-based combination of its hedonic and event features and the value of o is the sum of the values of its atoms. Riskless choice and statistical hypotheses are special cases. The valuation of atoms depends on the DM's attention to their features.

Features of Atoms. The features of atom y are collected in the vector

$$y = (u, e) .$$

Subvector u reports *hedonic* features, such as a dollar payoff or the jam's quality and price. The value u_i of hedonic $i \in M_H$ is a real number. There is a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ known to the DM. Subvector e reports *event* features: delivery states for hedonics, e.g., $e_i \in \{urn\ A, urn\ B\}$ or $e_i \in \{jam\ spoiled,\ not\}$, but also events like the jam's brand (e.g., Bonne Maman versus others). Each feature $i \in M_E$ corresponds to a partition of the state space Ω. Event e_i, $i \in M_E$, reports the atom's equivalence class in the partition corresponding to i.

Attention and Valuation. Hedonic and event features can vary across options, so attention to them can shape choice. We collectively call them *choice features*, $i \in M_O = M_H \sqcup M_E$. Attention to them is a vector of weights $\alpha_O \in [0, 1]^{M_O}$. Feature $i \in M_O$ is fully weighted if $\alpha_i = 1$, underweighted if $0 < \alpha_i < 1$, edited out if $\alpha_i = 0$. An edited event is set to $e_i = \Omega$ (so any of its realizations is allowed), and an edited hedonic feature takes a value equal to the feature's average value across atoms, i.e., $u_i = \overline{u}_i = \frac{\sum_{o \in O} \sum_{(u,e) \in O} u_i}{\sum_{o \in O} |o|}$. Let $y(\alpha_O) = (u(\alpha_O), e(\alpha_O))$ be the representation of atom y. It contains the edited version of events $e(\alpha_O)$ and the attention-based perception of hedonics $u(\alpha_O)$, where:

$$u_i(\alpha_O) = \alpha_{O,i} \cdot u_i + (1 - \alpha_{O,i}) \cdot \overline{u}_i, \tag{2}$$

so that limited attention to hedonic $i \in M_H$ shrinks perception toward its average value $\overline{u}_i$ (fully so for $\alpha_{O,i} = 0$, in which case feature i will not impact choice).

The value of representation $y(\alpha_O)$ multiplies its perceived probability and hedonics. Formally:

$$v(y(\alpha_O)) = \left[\prod_{r \in M_E} \mathbb{P}\left(y_r(\alpha_O) \mid \cap_{j<r} y_j(\alpha_O)\right)^{\alpha_r}\right] \cdot \sum_{i \in M_H} u_i(\alpha_O). \tag{3}$$

The first bracket is the perceived probability of $y(\alpha_O)$: the attention-weighted chain product of the probabilities of its events. The product follows a linear order $<$ over event features set by the true sampling process, e.g., first select an urn and then extract a ball from it.[3] The second term is the perceived hedonics in $y(\alpha_O)$. The value of payoff-states in Equation (1) is a special case of Equation (3) with two events and zero average hedonics, $\overline{u}_i = 0$.

As in expected utility, the edited lottery $o(\alpha_O) = \{y(\alpha_O) : y \in o\}$ is valued as the sum of the values of its atoms:

$$v(o(\alpha_O)) = \sum_{y(\alpha_O) \in o(\alpha_O)} v(y(\alpha_O)) \qquad \forall o \in O. \tag{4}$$

Full attention to choice features, $\alpha_O = 1$, yields the rational benchmark. A feature is "relevant" if it affects $v(o(\alpha_O))$ when $\alpha_O = 1$. Whether a feature is relevant or not depends on the problem.

Decision Rule. Given a vector of valuations $v \in \mathbb{R}^O$, selection of an action from the admissible set A is set by a decision map $D : \mathbb{R}^O \to A$. In the case of goods or lotteries, it is to pick the highest valued option, in some statistical

[3]The DM can compute the probabilities of the factors in the product. If the sampling process is not specified, $>$ reflects the DM's subjective beliefs. The order is irrelevant if $\alpha_i \in \{0, 1\}$ for $i \in M_E \cup M_L$.

problems it is to compute the relative value of two lotteries-hypotheses (which yields their relative probability).

While valuation is shaped by the attention α_O to choice features, the problem is categorized based on context features. These are common across goods and allow us to compare the whole problem to problems faced in the past.

Features of Context. Context is partly derived from choice features. For instance the choice set describes whether the problem entails choice among goods versus estimation of different hypotheses, or whether it entails a known or an unknown randomization device. Such context can cue past problems with similar choices and attribute values. But context also includes aspects that are wholly irrelevant for valuation such as time, location, recent events, etc. These features may be spuriously associated with experiences, spontaneously cueing different problems.

Context features $i \in M_K$ are summarized by a vector $\kappa = (\kappa_u, \kappa_e, z)$. Hedonic context κ_u reports, for each hedonic, the set of its possible values in the choice set (the prices of available jams, their qualities, etc.). Event context κ_e reports, for each event, the set of its realizations in the choice set (safe versus spoiled jam, the brands of the jams, etc.). The *situation* $z \in Z$ reports wholly irrelevant non-good specific dimensions of context such as time, location, etc. (e.g. whether the jam is bought during a festivity).[4] Each context feature $i \in M_k$ has an associated distance d_i that measures the perceived dissimilarity between two possible assumed values.

We denote the context of the current problem by κ_t and its generic entry by the dated name of the corresponding feature, i_t for $i \in M_K$. A context feature affects categorization only if the DM attends to it. Vector $\alpha_K \in [0,1]^{M_K}$ reports

[4] Some situation features are also derived from hedonics and events, reporting for instance the average price level (expensive versus cheap goods problem) or the average probabilities of specific events (high versus low risk problem).

attention to each such feature. Denoting overall attention by $\alpha = (\alpha_O, \alpha_K)$ the representation of the current problem is summarized by (α_t, κ_t).

2.2 Categorization and Attention

By choice features and situation, current context κ_t can cue categorization. The DM's database at time $t \in \mathbb{N}$ is partitioned into a set of categories C. Each category c is summarized by the prototypical attention and context vector (α_c, κ_c) experienced in the problems belonging to c, as well as by their temporally discounted frequency $F_c \in \mathbb{R}_+$.[5] Attention to context is binary $\alpha_{c,i} \in \{0, 1\}$ for $i \in M_K$ and identifies which context features are diagnostic of the category (Rosch and Lloyd [93]). To ease notation, κ_c reports only diagnostic features.

We illustrate this formalization using four illustrative categories we consider in the paper. The first two categories capture "riskless" experiences, in which the DM evaluated the hedonics of a good in a specific state. The second two categories capture "statistical" experiences, in which she estimated event probabilities. In more general problems, many more categories would be relevant.

Illustrative Categories. There are two riskless categories: "consuming" and "buying". In consumption, $(\alpha_{con}, \kappa_{con})$, the DM's evaluation of choice features focused on qualities, not prices. This is encoded in the attention subvector $\alpha_{con,O} = (\alpha_{con,i})_{i \in M_O}$. Thus, "consuming" triggers price neglect. The context κ_{con} of these experiences specifies the set of experienced qualities $q \in Q_{con}$ and situations $z \in Z_{con}$ (e.g. being at home, where price is not prominent). The

[5] The category "prototype" can be formalized as having the average attention

$$\alpha_c = \sum_{\tau \in c} \alpha_\tau / |c|$$

and the best compromise context κ_c where for every $i \in M_K$, κ_{ic} minimizes some distance from past contexts $(\kappa_\tau)_{\tau \in c}$. Recency-weighted frequency is $F_c = \sum_{\tau \in c} \delta^{t-\tau}$, $\delta \in (0, 1)$.

diagnosticity of quality and situation context is reported in $\alpha_{K,con} = (\alpha_{con,i})_{i \in M_K}$ by setting $\alpha_{con,i} = 1$ for $i = Q, Z$, and zero otherwise.

In buying experiences, category *buy*, attention to choice features $\alpha_{O,buy}$ focuses on the pain of paying, but at least partly also on the goods' typical quality, else we would not buy. Thus, "buying" triggers price focus. Context consists of the experienced qualities $q \in Q_{buy}$, prices $p \in P_{buy}$, and situations $z \in Z_{buy}$ (e.g. being in a shop or in an online marketplace). These diagnostic features take value 1 in attention to context $\alpha_{buy,K}$.

There are two statistical categories: "simple sampling" and "agnostic inference". Simple sampling, category *ss*, refers to experiences of estimating the probability of a single draw from a known process, e.g., that a fair coin lands h or t. In such problems attention $\alpha_{ss,O}$ focuses on the event corresponding to the name of the hypothesis, h or t. thus, "simple sampling" triggers focus on such single draw events. The diagnostic context features of these experiences include there being a single draw and hypotheses coinciding with its realizations.

Agnostic inference, category *ai*, refers to experiences of judging a data generating process (DGP) based on one or more i.i.d. signals, without having clear prior information about it, e.g. assessing the quality of a restaurant based on a single bad meal. Attention $\alpha_{ai,O}$ focuses on the share of positive signals, not on the prior. Thus, "agnostic inference" causes exclusive focus on such share. Diagnostic context features include having at least two draws (selection of the DGP and signals) and hypotheses coinciding with the possible DGPs (e.g. good or bad restaurant).

These categories capture distinct building blocks in standard choice theory. They thus yield comparative statics based on routinely measured parameters such as the number of draws, the price paid and its spatio/temporal distance from consumption (what Thaler calls "decoupling").[6] Looking ahead, categories

[6] One could also consider a category in which the DM "has no clue" and pays attention

can be elicited by measuring peoples' reasons for their choices and perceived similarity to the current problem.

Like in Mullainathan [82], categories are given. A theory of category formation would enhance predictive power, with respect to both individual heterogeneity (e.g., cross cultural variation in categories) and instability (e.g., how entrepreneurs create new categories). The conclusions outline possible ways forward.

We now show how categorization and attention are jointly determined based on similarity. A real valued function $S\left[\left(\alpha_t, \kappa_t\right), \left(\alpha_c, \kappa_c\right)\right]$ measures the similarity between the current problem $\left(\alpha_t, \kappa_t\right)$ and the prototype $\left(\alpha_c, \kappa_c\right)$ of category c. It decreases if the two disagree in: i) the evaluation of options (i.e. $\alpha_{O,t}$ vs. $\alpha_{O,c}$) and ii) context (i.e., attention $\alpha_{K,t}$ vs. $\alpha_{K,c}$ or values κ_t vs. κ_c). We use the separable form:

$$S\left[\left(\alpha_t, \kappa_t\right), \left(\alpha_c, \kappa_c\right)\right] = \frac{\sum_{i \in M}\left[1 - d\left(\left|\alpha_{t,i} - \alpha_{c,i}\right|\right)\right] + \sum_{i \in M_K}\left[1 - \alpha_{t,i}\alpha_{c,i} d_i\left(\kappa_{it}, \kappa_{ic}\right)\right]}{|M| + |M_K|}, \tag{5}$$

where $d : \mathbb{R}_+ \to \mathbb{R}$ is increasing, strictly convex and twice continuously differentiable with $d'(0) = 0$, $d(1) = 1$. The first sum in (5) measures disagreement in attention $\alpha_{t,i} \neq \alpha_{c,i}$. The second sum measures disagreement in category-diagnostic context.

The DM represents problem κ_t by setting an attention profile that maximizes total (perturbed) similarity with a category in C. This process is formalized in two steps: matching and categorization.

Matching. The DM fits the problem into each $c \in C$ by picking an attention

to nothing, and as a result is indifferent across options. By construction, similarity to this category is constant, so it sets a minimum similarity threshold below which no other category is adopted. Reliance on this category can be traced to reluctance to choose or, when choice is forced, to measured low confidence.

vector $\alpha_t(c)$ that maximizes the total similarity between the problem (α_t, κ_t) and the members of c, summarized by the prototype (α_c, κ_c) and frequency F_c. The maximum total similarity with $c \in C$ is given by:[7]

$$S(t,c) = \max_{\alpha_t \in [0,1]^{M_O \cup M_K}} F_c \cdot S[(\alpha_t, \kappa_t), (\alpha_c, \kappa_c)]. \quad (6)$$

Categorization. Following model assignment tasks in psychology (Mack and Palmeri [80]), the DM chooses the category $c \in C$ maximizing

$$S(t,c) + \epsilon_c,$$

where ϵ_c is an type I extreme-value random shift with scale parameter λ, reflecting random attention to context.

We are often unaware of this top-down process, as when we first see the duck-rabbit and unconsciously select one feature to attend (e.g., the beak) and one category from experience (ducks). Sometimes we are aware of it. In the duck-rabbit, after being told of the ambiguity of the figure, we see different animals on different occasions, but we never fail to see one, nor do we see both at the same time. In real world choices, we often entertain different perspectives, but then select one, which determines our choice.

3 Top down Attention and Choice

We characterize the general implications of our model for categorization, Section 3.1, and choice, Section 3.2.

[7] This yields exactly F_c if $\delta^{t-\tau}$ multiplicatively reduces similarity in Equation ([84]).

3.1 Attention and Categorization

Let $d_{tc}(i) = d_i(\kappa_{t,i}, \kappa_{c,i})$ be the distance between the problem and the category along context feature i. When matching c, the DM tunes attention to this feature to satisfy, in an interior equilibrium, the first order condition:

$$\frac{\partial}{\partial \alpha_{t,i}} d\left(|\alpha_{t,i} - \alpha_{c,i}|\right) + d_{tc}(i) \cdot \mathbb{I}_c(i) = 0, \tag{7}$$

where indicator $\mathbb{I}_c(i)$ is equal to one if i is a diagnostic feature for c and zero otherwise. Adapting attention to the category, i.e. reducing $|\alpha_{t,i} - \alpha_{c,i}|$, tends to increase similarity but may backfire along a discrepant diagnostic feature, $d_{tc}(i) \cdot \mathbb{I}_c(i) > 0$. This yields the following result.

Proposition 1 *When matching the problem with category c, attention to a discrepant diagnostic feature $d_{tc}(i) \cdot \mathbb{I}_c(i) > 0$ is shrunk toward zero, while attention is fully adapted to the category otherwise, $\alpha_{t,i}(c) = \alpha_{c,i}$. Denote by $d(t,c)$ the minimized distance from c. The maximum total similarity satisfies:*

$$\partial S(t,c) / \partial F_c = 1 - d(t,c) > 0, \tag{8}$$

$$\partial S(t,c) / \partial d_{c,i}(t) \propto -F_c \cdot \alpha_{t,i}(c) \leq 0. \tag{9}$$

To best match the problem with a category, the DM's attention aligns as much as possible with α_c but tends to neglect discrepant diagnostic context. When choosing a jam, a DM matching a "consuming" category adapts to it by focusing on taste and neglecting price. The DM however edits out the current "supermarket" location, which is diagnostic of "buying". Editing discrepant context has similarity cost $d(1) = 1$.

High past use F_c in the database fosters matching with c, including by editing a discrepant feature with it. This produces a form of confirmation bias, as we discuss later. Higher contextual discrepancy $d_{tc}(i)$ reduces similarity for each DM, producing instability. Categorization then works as follows.

Proposition 2 *Categorization in context κ_t satisfies*

$$\Pr(c\,|t) = \frac{\exp(\lambda \cdot F_c \cdot (1 - d(t,c)))}{\sum_{c' \in C} \exp(\lambda \cdot F_{c'} \cdot (1 - d(t,c')))}. \tag{10}$$

Problem t is more likely to be categorized in c when:

i) category c was used more frequently (and recently), $\partial \Pr(c\,|t)/\partial F_c > 0$.

ii) category c is less dissimilar to current context κ_t, $\partial \Pr(c\,|t)/\partial d(t,c) < 0$.

The DM is more likely to rely on a category that she more frequently used in the past or that is more congruent with current context κ_t. Frequency and context often promote fitting categories and good decisions. When the DM is highly trained on a well demarcated problem such as the probability that a fair coin lands h, the problem description fosters perfect recognition, prompting a correct answer of 50%.

In other cases, frequent experiences with c or spurious context may block a more fitting c', causing attention distortions and error. Repeated discussions about political corruption in Washington, and a high frequency of contexts where honesty is judged, may cause voters to neglect competence. A sporting event or a referendum, contexts where national pride is cued, may temporarily increase purchases of goods associated with the country's flag, causing neglect of price or quality which are otherwise attended to (Nardotto and Sequeira, [85]).[8]

[8] Psychologists have documented extensively both excessive reliance on a decision logic/model, which is called "overgeneralization", and the failure to apply a known model - even when correct - when context changes, which is called "limited portability of knowledge"; see Bassok [6] for a compelling illustration in math problems. In our model, these forces emerge due to similarity and frequency based retrieval of categories.

3.2 Choice Behavior

As the DM categorizes the problem in c, she adopts the category's attention to choice features $\alpha_{O,c}$. This yields valuation v_{tc} and choice $a_{tc} = D\left(v_{tc}\right) \in A$ (the Appendix relaxes injectivity of a_{tc}). This yields three properties.

Proposition 3 *Attention and choice are stochastic due to categorization,*

$$\Pr\left(a_{tc}\right) = \Pr\left(\alpha_t = \alpha_t(c)\right) = \Pr\left(c|t\right)$$

for all $a_{tc} \in A$. *Furthermore:*

i) Higher F_c *increases* $\Pr\left(a_{tc}\right)$ *and weakly decreases* $\Pr\left(a_{tc'}\right)$ *for* $c' \neq c$.

ii) Higher $d(t,c)$ *decreases* $\Pr\left(a_{tc}\right)$ *and weakly increases* $\Pr\left(a_{tc'}\right)$ *for* $c' \neq c$.

iii) Increasing similarity to c*, decreasing* $d(t,c)$*, boosts* $\Pr\left(a_{tc}\right)$ *more at higher* F_c *if and only if* c *is not dominant, i.e.* $\Pr(c|t) \leq \pi^*$ *for* $\pi^* > 1/2$.

Attention and choice are stochastic due to shock ϵ_c. Stochastic choice is often explained by noisy perception of hedonics or probabilities (Woodford [125], Enke and Graeber [33]). In that approach, noisy signals are used in a Bayesian way, so a person's valuation for an action is distributed around a single mode. Categorization can instead yield multi-modal valuation, with discontinuous shifts across categories. Multi-modality is observed in statistical problems, where the correct answer is continuous in the problem's parameters. In inference, the same person sometimes uses only the base rate, other times only the likelihood (Bordalo, Conlon, Gennaioli, Kwon, and Shleifer [19]). Categorization can produce such systematic and large valuation shifts, noise cannot.

Different experiences create systematic heterogeneity (Point i). A DM who has more frequently or recently used a category c, higher F_c, is more likely to focus on its relevant features and choose a_{tc}. Familiarity with different categories may explain persistent interpersonal differences in attention and judgments in statistical problems despite common information and incentives (Bordalo, Conlon, Gennaioli, Kwon, and Shleifer [19]).

Changes in context deliver instability (Point *ii*). A higher context discrepancy $d(t, c)$ reduces categorization in c, reducing the probability of choosing a_{tc}. Even spurious changes in context can yield this effect. Describing the same inference problem as taxicabs versus balls and urns changes problem representation, namely the focus across available statistics, not information. This yields a theory of framing effects.

Crucially, experiences and context interact (Point *iii*). If category c is not a dominant representation of the current problem, $\Pr(c|t) \leq \pi^*$, context and frequency are complements: familiarity with c boosts $\Pr(a_{tc})$ especially if the context is more similar. This explains mental simulation: non-domain specific experiences affect choice in similar contexts, especially if the current problem is "new".[9] It also means that changes in irrelevant context produce strong framing effects when the DM is "indifferent" between different representations.[10]

If, on the other hand, category c is dominant, then similarity and frequency are substitutes: changing context has a small or no effect for people who often use c. Our model therefore identifies conditions under which framing effects are small. These conditions include those of the rational model (frequent use of a correct representation), but extend to strong familiarity with a certain category, leading to overgeneralization and neglect of informative data, as in

[9] Bordalo, Burro, Coffman, Gennaioli, and Shleifer ([12]) show that past personal financial losses make it easier to imagine a severe cyberattack by helping the DM to imagine how these events may lead to losses. See also Taubinsky, Butera, Saccarola, and Lian [108] on inflation.

[10] Framing effects should also be heterogeneous. There is evidence of this: a risky lottery with a stock market label is less likely to be chosen by people who think stock market participants are greedy (Henkel and Zimpelmann [57]). Describing default on a loan as contrary to sacred texts increases repayment (Bursztyn, Fiorin, Gottlieb, and Kanz [20]), but possibly more so for more religious people. In the original problems the primed category (stocks, religion) is not dominat, but evoking it sways people for whom it is familiar.

cognitive dissonance (Festinger [36]) or the confirmation bias (Nickerson [86]).

4 Top-Down Attention in Famous Puzzles

We illustrate the role of top down attention in consumer choice and statistical problems. Categorization of a choice problem as consuming versus buying explains the high and sometimes excessive price elasticity of poor consumers, as well as various forms of mental accounting. Categorization of statistical problems into simple sampling versus inference reconciles the Gambler's Fallacy, over- and underreaction to data, and their instability.

4.1 Consumer Choice

A consumer values goods g and b with expected qualities q_g and q_b, $q_g \geq q_b$, and prices p_g and p_b, $p_g \geq p_b$. The net utility of these hedonics is $q_l - \eta \cdot p_l$ for $l = g, b$, where $\eta > 0$ maps dollars into utils. Before consumption, quality and market price can be hit by hedonic shocks Δq_l and $-\eta \cdot \Delta p_l$, respectively. We do not formalize the shocks' event features because we study how the consumer represents choice with riskless categories, which neglect events. We study risky choice in Section 5 with bottom-up attention.

The consuming category, $c = con$, has diagnostic context reporting the expected qualities Q_{con} and hedonic shocks ΔQ_{con} evaluated and enjoyed during consumption, as well as the situations Z_{con} in which consumption occurred. Thus, $\kappa_{con} = (Q_{con}, \Delta Q_{con}, Z_{con})$. For choice features, the DM attends to realized qualities while she neglects price features, and sets $\alpha_{Q,con} = \alpha_{\Delta Q,con} = \overline{\alpha} > \alpha_{P,con} = \alpha_{\Delta P,con} = 0$.

Buying, $c = buy$, has diagnostic context $\kappa_{buy} = (Q_{buy}, P_{buy}, Z_{buy})$ consisting of the expected qualities Q_{buy} and prices paid P_{buy} in buying experiences and their situations Z_{buy}. For choice features, the DM attends to paying, to a lesser extent to expected quality, and neglects (low probability) shocks, consistent with *buy* being a riskless category, $\alpha_{P,buy} = \overline{\alpha} > \alpha_{Q,buy} = \underline{\alpha} > \alpha_{\Delta Q,buy} = \alpha_{\Delta P,buy} = 0$.

Current context lists all potentially accessible features: qualities $Q_t = \{q_g, q_b\}$, prices $P_t = \{p_g, p_b\}$, shocks to quality $\Delta Q_t = \{\Delta q_g, \Delta q_b\}$ and to price $\Delta P_t = \{\Delta p_g, \Delta p_b\}$, as well as situation z_t. The distances for diagnostic features of $c = con, buy$ is $d_Q(Q_t, Q_c)$ for quality $d_P(P_t, P_{buy})$ for price and $d_Z(z_t, Z_c)$ for the situation. Here we only consider comparative statics based on situation distance.[11]

4.1.1 Poor Experiences and Price Sensitivity

Shah, Zhao, Mullainathan, and Shafir [102] show that poor people are more likely to deem price as very relevant, and hence to exhibit high price elasticities, across many situations. They argue that such focus on price is the consequence of living on a tight budget and can sometimes leads to mistakes, such as cutting down on important medication whose price increased (Chandra, Flack, Obermeyer [23]). Our model yields this intuition, with new predictions.

Consider a consumer deciding whether to buy g, so $q_b = p_b = 0$. She may think about consuming and focus on quality, or about buying and focus also on price, $C = \{con, buy\}$.[12] Across situations the consumer retrieves buying with probability π_{buy} and consuming otherwise, so quality and price weights are on average $\overline{\alpha}_Q = \overline{\alpha} - (\overline{\alpha} - \underline{\alpha}) \cdot \pi_{buy}$ and $\overline{\alpha}_P = \overline{\alpha} \cdot \pi_{buy}$. Using Propositions 2 and 3 we characterize the effect of poverty on valuation $v(g)$ as a comparative static

[11] A reasonable simplification is to have zero distance along quality and price context, capturing the intuition that a problem can be seen as consumption if there are qualities and as buying if in addition there is also a positive price. More interesting metrics arise along finer buying categories such as buying "high vs low price goods" or along consuming "exceptional vs normal qualities."

[12] In this example, we suppose for simplicity that there are no shocks, $\Delta q_g = \Delta p_g = 0$, so choice is truly riskless. Such shocks play a role in mental accounting, Section 4.1.2.

on frequency: having often been on a tight budget, poor people more often attended to prices, so their F_{buy} is higher. In the Appendix we show:

$$\begin{aligned}\frac{\partial v(g)}{\partial F_{buy}} &\propto \left[(\alpha_{Q,buy}-\overline{\alpha}_Q)\cdot q_g-(\alpha_{P,buy}-\overline{\alpha}_P)\cdot\eta\cdot p_g\right]\cdot\left[1-d\left(t,buy\right)\right] \quad (11)\\ &= -\left[(\overline{\alpha}-\underline{\alpha})\cdot q_b+(1-\underline{\alpha})\cdot\eta\cdot p_b\right]\cdot\pi_{buy}\cdot\left[1-d\left(t,buy\right)\right]<0.\end{aligned}$$

Due to a "mental set" of price-benefit evaluations, a poor consumer is more price sensitive and values a price-quality combination less compared to a richer one. This focus on price does not reflect a higher price distaste η nor optimal attention allocation. It reflects overgeneralization of experiences. This leads to:

1. Mistakes. A poor consumer may forsake valuable expenditures such as health copayments due to her focus on cutting costs and neglect of future benefits, which arises because $\underline{\alpha}<\overline{\alpha}\leq 1$ in (11). A formerly poor consumer may exhibit high price elasticity even if she is no longer poor, namely even though η is small in (11), inconsistent with neoclassical and rational inattention models.[13]

2. Instability. Price focus depends on spurious context. By Proposition 1, $d\left(t,buy\right)$ increases in situation distance $d_Z\left(z_t,Z_{buy}\right)$, which can boost quality focus, reducing price elasticity. The poor can "splurge" on festivals or "treat" goods such as cigarettes (Banerjee and Duflo [3]). These situations are associated with consumption pleasure, which increases $d_Z\left(z_t,Z_{buy}\right)$, in turn increasing $d\left(t,buy\right)$ in (11). Conversely, the poor are more price elastic if costs are monetary rather than in kind: out of pocket costs increase similarity to buying experience, reducing $d_Z\left(z_t,Z_{buy}\right)$ and thus $d\left(t,buy\right)$ in (11). This is in line with the compatibility principle (Tversky, Sattath, and Slovic [120] and Slovic, Griffin, and Tversky [105]). By the same token, a no-longer-poor consumer may

[13] Hoch, Kim, Montgomery, and Rossi [58] show that consumers' price elasticity can be predicted from a range of characteristics beyond current income and wealth.

exhibit high price elasticity for items she used to buy (e.g., clothes), but not on new goods (e.g., i-Phones). The former goods match past experiences of buying, exhibiting lower $d_Z(z_t, Z_{buy})$ and thus lower $d(t, buy)$.

Rick, Cryder, and Loewenstein ([92]) develop a survey measure of thriftiness and show systematic (e.g. age based) heterogeneity in consumers' focus on paying. Wakefield and Inman ([124]) show that consumers' price elasticity is strongly situation-dependent and correlated with the extent to which a good is categorized as "hedonic" (low elasticity) versus "functional" (high elasticity). We offer a testable mechanism for these findings based on interpersonal differences in experiences and changes in context/similarity.

4.1.2 Mental Accounting

People use different accounts to track costs and benefits in different situations, leading to opportunity cost neglect, sunk cost fallacy, non-fungibility of money, etc. Consider the examples below.

Opportunity Cost Neglect. Many years ago a person bought for 20$ a bottle of wine worth 75$ today. The person drinks the wine today. What is the cost she feels? Many answers to this question are zero or 20$ (Thaler [109]). They neglect the opportunity cost of drinking, the 75$ market price.

Sunk Cost Fallacy. A person bought a 20$ ticket to a football game to be played a month later. On the day of the game, there is a severe blizzard. 1) Does the person drive to the game? 2) Would she drive if she was given the ticket for free? Frequent answers are: "yes" to 1) and "no" to 2), which violate revealed preference: if the blizzard is severe enough, it should discourage driving regardless of whether a price had been paid.

Explanations of mental accounting invoke different forces. In Shafir and Thaler [101], opportunity cost neglect is due to forgetting the wine's price, while the sunk cost fallacy is due to diminishing sensitivity (the cost of driving is felt less when combined with the sunk price). It is not obvious, however, how respondents can forget price given that it is described to them and feel diminishing sensitivity given that choice is hypothetical.[14]

In our model, this behavior arises because consumers rely on the consuming versus buying categories to determine relevant (hypothetical or real) features. In both problems, there is utility q_g - from drinking or seeing the game - and the price p_g paid for it. Price and quality "shocks" are also described: the wine capital gain $\Delta p_g > 0$ in one case, the blizzard $\Delta q_g < 0$ in the other. The dollar cost m felt after drinking, and the value v of going to the game, depend on attention $\alpha_O = (\alpha_Q, \alpha_{\Delta Q}, \alpha_P, \alpha_{\Delta P})$:

$$m(drinking(\alpha)) - m(not\ drinking(\alpha)) = \alpha_P \cdot p_g + \alpha_{\Delta P} \cdot \Delta p_g, \quad (12)$$

$$v(driving(\alpha)) - v(not\ driving(\alpha)) = \alpha_Q \cdot q_g + \alpha_{\Delta Q} \cdot \Delta q_g. \quad (13)$$

Full attention $\alpha_O = (1, 1, 1, 1)$ yields rationality. In wine, $\alpha_P = \alpha_{\Delta P} = 1$ recovers in (12) the market price, $p + \Delta p = 75\$$. For the purpose of estimating monetary costs, all that matters is attention to prices, utility q is not relevant. In the football problem, $\alpha_Q = \alpha_{\Delta Q} = 1$ also recovers the rational rule: go to the game if and only if $q + \Delta q > 0$. Now all that matters is attention to quality, the sunk price p is not relevant.

Each vignette creates a context κ_t matching both the diagnostic features of "consuming" (it reports qualities), and of "buying" (it reports also price).

[14] Kőszegi and Matějka [69] offer a rational inattention theory for category-budgets and naive diversification, but cannot explain the sunk cost fallacy or the wine example.

These competing categories prompt evaluations:

$$m(drinking(\alpha)) - m(not\ drinking(\alpha)) = \begin{cases} 0 & if\ \alpha = \alpha_{con} \\ p_g & if\ \alpha = \alpha_{buy} \end{cases}, \tag{14}$$

$$v(driving(\alpha)) - v(not\ driving(\alpha)) = \begin{cases} \overline{\alpha} \cdot (q_g + \Delta q_g) & if\ \alpha = \alpha_{con} \\ \underline{\alpha} \cdot q_g & if\ \alpha = \alpha_{buy} \end{cases} \tag{15}$$

In the wine example, a DM thinking about consuming, α_{con}, focuses on the pleasure of drinking the wine and feels no opportunity cost. A DM thinking about buying, α_{buy}, focuses on the pain of paying and reports p_g, neglecting the capital gain, which is not a standard feature of most purchase decisions.

In the football example, a DM thinking about consuming, α_{con}, focuses on current hedonics, the game q_g and the blizzard Δq_g, making the rational evaluation. A DM thinking about buying, α_{buy}, instead focuses on the pain of paying p_g and the benefit q_g it had secured. This consumer represents her choice as "enjoying a game I paid for". The blizzard shock is neglected because it is not a standard feature of purchase decisions.

In the experiment most people adopt the *con* category in the wine problem, and *buy* in the football one. In both cases, mistakes are not due to stable biases but to "top down" focus on an irrelevant feature: the pleasure of drinking in wine and the sunk price in football. These features draw attention because they are relevant in frequent past problems, but also cause neglect of relevant features of the current one. Proposition 3 yields several comparative statics:

1. Frequency. People who have recently bought lots of wine but have not yet drunk it have high F_{buy}, which favors $c = buy$ and hence the \$20 mode. In football, people who bought season tickets face only one buying decision but many consuming experiences; thus, they have a high F_{con}, which reduces categorization in *buy*, reducing the sunk cost fallacy. A wine trader also has more

frequent buying experiences, so should exhibit less opportunity cost neglect, as in List [77] (and will also have a "sell wine" category, prompting attention to the capital gain).

2. Instability. The different modal categories in the two examples can be explained by the description of each vignette. Describing the wine situation as $z_t =$ "having drunk the wine" promotes similarity to *con* and dissimilarity to *buy*, leading to price neglect.[15] Describing the football situation as $z_t =$ "going versus not going to the game" evokes a buying decision we already made: paying to see a game. By highlighting different features, these descriptions cause the DM to neglect prices in wine and qualities in football. Making the blizzard more salient in the description or making an ex ante plan for bad weather should increase reliance on *cons* and reduce that on *buy*, reducing the fallacy. We study the role of description in categorization in Section 5.

The same mechanism can explain other biases. Consider non-fungibility: transferring money into a category cues buying experiences in that category, promoting in-category spending.[16] Consider account-based commitment (Thaler [109]): setting up a "rainy day" account creates a category of decisions focused on future financial risks, which is retrieved when we consider withdrawing from this account. Such strategies are however on a slippery slope: exceptions can destroy the categories on which commitments are based.

[15] During many consumption experiences, with prices not explicitly mentioned, we often feel no opportunity costs. Frederick, Novemsky, Wang, Dhar, and Nowlis [38] show experimentally that describing the option "not buy" in terms of keeping the money for other purchases substantially decreases the probability of purchase.

[16] A 5$ bonus for drinks at a restaurant is similar to past "drink discount" experiences, in which the consumer focused on whether to buy en extra beer or a higher quality one. This focus on drinks reduces attention to the food spending category (Abeler and Marklein [1]).

4.2 Statistical Problems

Probability judgments in i.i.d. draws and in inference exhibit systematic biases (Benjamin [9]). Bordalo, Conlon, Gennaioli, Kwon, and Shleifer [19] show that multi-modality and instability of attention to features can account for them. Their theory does not however explain why different people systematically attend to different features and make different errors in the same problem and why changes in irrelevant context can create instability. Our theory does.

A coin with probability $\theta \in [0,1]$ of heads is selected according to a prior $\mathbb{P}_0$ in a set of coins Θ, and generates i.i.d. sequences of up to $D \in \mathbb{N}$ binary draws. Hypotheses $o \in O$ are events in the resulting sampling space Ω, whose probability can be estimated in $(\Omega, \mathcal{F}, \mathbb{P})$.[17] In our cases a hypothesis is an elementary event $\omega \in \Omega$, with feature vector:

$$y(\omega) = \left(e_\theta, e_{1|\theta}, ..., e_{D|\theta}, s_{|\theta}\right), \tag{16}$$

where e_θ is the event of selecting coin θ, $e_{j|\theta}$ is the realization of the j-th i.i.d. draw given coin θ, and $s_{|\theta}$ is the event corresponding to the share of positive i.i.d. draws (e.g. heads), again given the selected coin. Statistical categories emerge from experiences with these features.

The context κ_{ss} of simple sampling, $c = ss$, is "the DGP produces a single event" whose probability is given, and "the names of hypotheses correspond to that event". The diagnostic features of ss are thus the number of events N, the event realizations R (a fair coin has $R = \left\{h_{|0.5}, t_{|0.5}\right\}$), and the hypotheses' names V. Then $c = ss$ has context:

$$\kappa_{ss} = (N_{ss} = 1, R_{ss}, V_{ss} = R_{ss}). \tag{17}$$

[17] The option corresponding to $H \subset \Omega$ is a lottery whose atoms specify, for each elementary $\omega \in \Omega$, a payoff of 1\$ if $\omega \in H$ and zero otherwise. Consistent with Savage [97], Section 9, we could allow for an incentive compatible elicitation for multiple events.

Regarding choice features, when evaluating $y(\omega)$ the DM focuses on the names of hypotheses, setting $\alpha_{R,ss} = 1$ for $R = V$ and neglects the rest $\alpha_{ss,i} = 0$ for $i \in M_O \backslash R$. Within ss, this process is both intuitive and correct.[18]

The context of agnostic inference, $c = ai$, is "the DGP has more than one event", "one event is associated with coin selection", "other events are i.i.d. draws", and "the names of hypotheses correspond to the coin selection event". The diagnostic features of ai are thus the number N, the DGP selection event Θ, the i.i.d. draws R, and hypotheses' names V. formally, then, $c = ai$ has context:

$$\kappa_{ai} = \left(N_{ai} \neq \{1\}, \Theta_{ai} \neq \{\theta\}, R_{ai}, V_{ai} = \Theta_{ai}\right), \tag{18}$$

so there is more than one draw and Θ_{ai} is not a singleton. When evaluating $y(\omega)$ the DM focuses on its share of positive signals $s_{|\theta}$ and neglects the rest, including coin selection, setting $\alpha_{ai,s_{|\theta}} = 1$ and $\alpha_{ai,i} = 0$ for $i \in M_O \backslash S_{|\theta}$. With an uninformative prior this process is also intuitive and correct: the share of positive signals is a sufficient statistic for coin type. With informative prior category ai leads to base rate neglect, but since we often have little prior information, the process generally works well.[19]

In our judgment problems, current context κ_t concerns the DGP and hypotheses. DGP context comprises the current number of realizations N_t, the set of possible coin types Θ_t, a sequence of i.i.d. draws $D_{1|\theta}$,..., $D_{N_t-1|\theta}$ and a set of the possible shares $S_{|\theta,t}$ for each coin. Hypotheses V_t reflect events in the DGP features Θ_t or $D_{j|\theta}$. Statistical context consists of potentially relevant events. Mistakes are due to the DM's focus on the wrong events based on past

[18] In the described DGP, ss can entail estimating the probability of the iid draw given the coin, e.g. $V = D_{1|\theta}$ or that of selecting a coin type θ, $V = \Theta$.

[19] A category of lopsided inference, in which the prior is highly skewed and so it receives attention while the signal is neglected, is not necessary in our model. It endogenously emerges from the simple sampling category when statistical contrast is introduced in Section 5.

problems in which these events were relevant.

4.2.1 The Gambler's Fallacy

The DM estimates the relative probability of obtaining sequences $H_1 = hhhhhh$ versus $H_2 = htthht$ from a fair coin. Thus, the diagnostic features of categories ss, ai take values $N_t = 6$, $\Theta_t = \{0.5\}$, $R_t = \left\{h_{|0.5}, t_{|0.5}\right\}$, $V_t = \{H_1, H_2\}$.

Comparing these features to those of simple sampling $c = ss$ in (17), there is a discrepancy along the number of flips $d_N(6, 1) > 0$ and along the names of hypotheses, which do not correspond to any individual realizations in the described DGP, $d_V(V_t, R_t)$, $d_V(V_t, \Theta_t) > 0$.

Comparing the same features to agnostic inference category ai in (18), there is a discrepancy along the set of coin types, because the current one is a singleton, so $d_\Theta(\Theta_t, \Theta) > 0$, and with the name of hypotheses which do not correspond to coin types, $d_V(V_t, \Theta_t) > 0$.

Because the problem is not a perfect match to either category, yet both categories are frequently encountered, some people focus on the fairness of the coin and represent the problem as ss, while others focus on the length of sequences and represent the problem as ai.

A DM relying on ss deems an individual flip relevant, say R_1. Thus, her attention is $\alpha_1 = 1$ and $\alpha_i = 0$ otherwise. By Equation (3):

$$v(H_1(\alpha)) = v(H_2(\alpha)) = \mathbb{P}(h) = 0.5.$$

Hypotheses are deemed equally likely. The DM does not commit the GF nor does she estimate the probability of H_1 and H_2 correctly. She is not "rational". She uses a sampling intuition "with a fair coin any draw is equally likely!"

A DM relying on ai deems only the share of heads to be relevant, $\alpha_{s_{|0.5}} = 1$

and $\alpha_i = 0$ otherwise. By Equation (3),

$$v(H_1(\alpha)) = \mathbb{P}\left(s_{|0.5} = 1\right) = (0.5)^6 \text{ and } v(H_2(\alpha)) = \mathbb{P}\left(s_{|0.5} = 0.5\right) = 5 \cdot (0.5)^4.$$

The DM commits the Gambler's Fallacy, using a "comparative" inference intuition: balanced sequences are much more likely with a fair coin! In general, consistent with the evidence, her estimated probability increases in the size of a hypothesis' share of heads equivalence class.[20]

The GF is thus due to categorization into inference, driven by familiarity with inference and by spurious similarity with it based on having long sequences. Accordingly, as shown in [19] there is less GF when sequences get shorter, $d_N(N_t, 1)$ falls, and when the problem becomes more similar to simple sampling because the name of hypotheses coincides with individual flips, so $d_V(V_t, R_t) = 0$.[21] More generally, our model implies that adding irrelevant context diagnostic of a similar problem should boost attention to a feature that is relevant in the latter problem (and viceversa if such context is removed).

Rabin and Vayanos [89] show that a DM who incorrectly believes in the GF may exhibit a nonmonotonic pattern of under/overreaction depending on whether the DGP is independent or autocorrelated. Their model cannot explain why the same person exhibits the GF on one occasion but not on a normatively equivalent occasion based on changing context.

[20] We could allow for richer sampling categories entailing $r > 1$ iid flips. [19] allow for them in reduced form. These categories would not change our basic results, which rely on the familiarity of inference experiences. r-sampling categories would help produce the well known insensitivity to sample size, in a less crude form than with $r = 1$.

[21] Specifically, when asked to judge *hhhhht* vs. *hhhhhh*, the incidence of the GF is reduced if subjects are told "the first five flips are *hhhhh*" is the last flip h or t?

4.2.2 Biases in Inference

Competition between $c = ss$ and $c = ai$ also delivers multimodality and instability in inference. In balls and urns problems, urn A is selected with probability 25% and the likelihood of drawing a green (versus blue) ball from it is 80%. The composition of urn B is symmetric. People are told: "a drawn ball is green. What is the probability it comes from A versus B?" Current context has $N_t = 2$, $\Theta_t = \{A, B\}$, $R_t = \{g_{|A}, b_{|A}, g_{|B}, b_{|B}\}$, $V_t = \{A, B\}$, where g stands for green and b for blue.

By (16), the urn U-hypothesis is $y_U = (U, g_{|U}, s_U = 1)$. By (3), its value is:

$$v(y_U(\alpha_O)) = \mathbb{P}(U(\alpha_O))^{\alpha_U}\, \mathbb{P}((1|U)\,|U(\alpha_O))^{\alpha_S \cdot \mathbb{I}_S + \alpha_D \cdot (1-\mathbb{I}_S)}, \tag{19}$$

where α_U is attention to urn selection, α_D to the i.i.d. draw, and α_S to the green share. $\mathbb{I}_S = 1$ if $\alpha_S > 0$, zero otherwise.[22] Equation (19) is Grether's [51] formula with coefficients shaped by categorization.

Balls and urns is more similar to inference ai than to ss because there is more than one draw, $d_N(2, N_{ai}) < d_N(2, 1)$, the i.i.d. process is not known, $d_\Theta(\Theta_t, \neq \{\theta\}) < d_\Theta(\Theta_t, \{\theta\})$. Balls and urns is however still quite similar to ss because hypotheses' names coincide with the urn selection event, $d_V(V_t, \Theta_t) = 0$, whose probability is given.

Familiarity with simple sampling, high F_{ss}, prompts some people to rely on it, despite its lower fit compared to ai. These people attend only to urn selection, answering by $v(y_U(\alpha_{ss})) = \mathbb{P}(U)$. They reason "the ball has to come A or B!" They neglect color and report the base rate. People who rely on inference instead attend only to the green share, answering by $v(y_U(\alpha_{ai})) =$

[22] Consistent with the green share being a sufficient summary of iid draw events, the DM's causal model first conditions on it and then on individividual iid draws from U. This assumption is not material for our results.

$\mathbb{P}(1|U) = \mathbb{P}(g_{|U})$. They reason: "$A$ is more green than B!". They neglect the low base rate of A and answer by the likelihood. Categorization yields multimodality in [19].

Experiments show strong instability of estimates under the following taxicab frame: There are two companies, called Green (25% of all cabs) and Blue (75%) according to the color of their cabs. After a hit and run accident, a witness, whose accuracy is 80% for each cab color, reports the cab to be green. What is the probability that the cab is indeed green? In this frame, most people answer "80%" and almost no one answers with the base rate.

In our model this instability arises because in taxicabs the hypotheses' names are the events "accurate" versus "inaccurate" witness reports. Formally, DGP context is the same as under balls and urns but hypotheses' names now correspond to specific i.i.d. draw events in $D_{1|G}$ and $D_{1|B}$. One hypothesis, "the reported color matches the actual color", corresponds to $g_{|G} \in D_{1|G}$, while the other corresponds to "it does not match", $g_{|B} \in D_{1|B}$. By setting names $V_t = \{g_{|G}, g_{|B}\}$ taxicabs increases similarity to and hence categorization in simple sampling ss. This focuses the DM on the i.i.d. draw events, setting $\alpha_D = 1$ and $\alpha_U = \alpha_S = 0$ in (19), yielding the "80%" answer. Bordalo, Conlon, Gennaioli, Kwon, Shleifer [19] indeed show that instability in the taxicab frame obtains almost entirely due to switchers from the base rate to the likelihood.[23]

When the question corresponds to a described feature of the DGP, people anchor to the corresponding statistics based on a simple sampling representation. They then under-react if the statistic is the base rate, over-react if it is the likelihood. This compatibility effect (Tversky, Sattath, and Slovic [120] and

[23] This explanation also accounts for why the same effect is observed when the balls and urns problem is "cabified", namely urn compositions are described in terms of the color match between the urn and the ball, and the hypotheses themselves concern accuracy. The effect is so strong also because the DGP is described in terms of the "match" feature, a result that naturally emerges from bottom up attention.

Slovic, Griffin, and Tversky [105]) is naturally delivered by our model.

5 Top Down and Bottom up Attention

Bottom-up salience is a known driver of choice. Consumers are more sensitive to taxes when these are displayed on the price tag (Chetty, Looney, and Kroft [25]) and prefer goods that are physically present (Bushong, Camerer, and Rangel [21]). Prices, payoffs, or statistics draw attention if they are striking/contrasting (Bordalo, Gennaioli, and Shleifer [14],[15], Koszegi and Szeidl [70]).[24] We next show how two bottom-up forces, sensory prominence and contrast, interact with top-down attention.

Sensory prominence depends on the problem's description. A description is a vector $\alpha_\delta \in [0,1]^{M_O \cup M_K}$ paired with context κ_δ. It is partly set by nature (e.g., "sun" is exogenously part of $\kappa_{\delta,i}$ and visually prominent, high $\alpha_{\delta,i}$), partly by the experimenter or persuader. A shrouded feature has $\alpha_{\delta,i} = 0$ (Gabaix and Laibson [41]). A fully described feature has $\alpha_{\delta,i} = 1$. Described context is all the DM perceives, so $\kappa_t = \kappa_\delta$. Our prior analysis captures descriptions that modify κ_t by adding or shrouding context features. We now consider the role of α_δ for choice features.

Contrast increases in the variability of a choice feature across atoms. High variability draws attention. Let Y be the set of atoms. Contrast of $i \in M_O$ is a real valued function $\sigma_i = \sigma\left[(y_i)_{y \in Y}\right] \geq 1$. A special case is:

$$\sigma_i = 1 + \frac{\sum_{y,y' \in Y : y \neq y'} d_i\left(y_i, y'_i\right)/|Y||Y-1|}{\sum_{y \in Y} d_i\left(y_i, \widetilde{y}\right)/|Y| + \epsilon}, \tag{20}$$

where $\epsilon > 0$ and $\widetilde{y}$ is a reference. For $y_i \in \mathbb{R}$, it nests the exemplar salience

[24] See Lanzani [74] for the axioms underpinning this model.

function in Bordalo, Gennaioli, and Shleifer [15], with $d(y_i, y_i') = |y_i - y_i'|$ and $\widetilde{y} = 0$. For instance, large price variability increases price contrast via the numerator of (20), high average price reduces it via the denominator (capturing Weber-Fechner's Law). Event contrast depends on event probabilities, as in Bordalo, Conlon, Gennaioli, Kwon, and Shleifer [19].

Described contrast of a feature, $\sigma_{i\delta}$, is computed using the described values of i. If the feature is shrouded, $\alpha_{\delta,i} = 0$, its bottom-up contrast is minimal, $\sigma_{i\delta} = 1$. Category contrast of the same feature, σ_{ic}, is computed using the feature's experienced values in c. If i was neglected ($\alpha_{c,i} = 0$), $\sigma_{ic} = 1$. Experiences create a "top down" contrast force. Before flying, we often think about crashes: even if not described, these events are contrasting in $c =$"flying" experiences.[25]

Matching. The DM matches description $(\alpha_\delta, \kappa_t)$ with each category $c \in C$. Equation (6) becomes:

$$S(t, c, \delta|\sigma) = \max_{\alpha_t} \ F_c \cdot S[(\alpha_t, \kappa_t), (\alpha_c, \kappa_c)|\sigma_c] + S[(\alpha_t, \kappa_t), (\alpha_\delta, \kappa_t)|\sigma_\delta]. \quad (21)$$

The DM trades off similarity to category c, the first term, with similarity to the description's prominent features, the second term. Importantly, similarities to the category and the description depend on contrast σ_c and σ_δ, with more contrasting features receiving higher weight in similarity (see, e.g., Nosofsky [87] and Reed [91]). Under a simple multiplicative specification,[26] the interior

[25] The feature values used to compute contrast are encoded in context, κ_t and κ_c, which report the possible values of choice features. Top down contrast can be extended to heterogeneous categories. For each $\tau \in c$, feature κ_{τ,i_K} reports the experienced values of i at τ. If the feature was not attended to, $\alpha_{ic\tau} = 0$, then κ_{τ,i_K} is empty so its contrast in $\tau \in c$ is zero. Else, κ_{τ,i_K} reports the feature's values in Y_τ. The DM computes σ_{ic} as the average $\sum_{\tau \in c} \sigma_{\tau ic}/|c|$ across problems in c.

[26] Contrast weighted similarity is given by:

$$S[(\alpha_t, \kappa_t), (\alpha_x, \kappa_x)] = \frac{\sum_{i \in M} \sigma_{ix} \cdot [1 - d(|\alpha_{it} - \alpha_{ix}|)] + \sum_{i \in M_K} \sigma_{ix} \cdot [1 - \alpha_{it}\alpha_{ic} d_i(\kappa_{it}, \kappa_{ix})]}{|M| + |M_K|},$$

optimal attention satisfies:

$$\frac{\partial}{\partial \alpha_{t,i}} d\left(|\alpha_{t,i}-\alpha_{c,i}|\right) + d_{tc}(i)\cdot \mathbb{I}_c(i) + \frac{\sigma_{i\delta}}{\sigma_{ic}} \cdot \frac{1}{F_c} \cdot \frac{\partial}{\partial \alpha_{t,i}} d\left(|\alpha_{t,i}-\alpha_{\delta,i}|\right) = 0, \quad (22)$$

where $\sigma_{i\delta} = \sigma_{ic} = 1$ for context features (which do not vary across options). Compared to (7), bottom-up salience directly affects attention via $\alpha_{\delta,i}$ in the third term above. The "top down" analysis of Section 2, in which description only affects the context vector κ_t, is a special case for $F_c \to \infty$.

Proposition 4 *When matching c and $(\alpha_\delta, \kappa_t)$, attention $\alpha_{t,i}(c, \delta, \sigma)$ to feature $i \in M_O$ increases in sensory prominence $\alpha_{\delta,i}$ and attention $\alpha_{c,i}$ of category c. The description is more influential, $|\alpha_{t,i} - \alpha_{\delta,i}|$ is lower, when $\frac{\sigma_{i\delta}}{\sigma_{ic}} \cdot \frac{1}{F_c}$ is higher.*

If a feature such as price is visually prominent, high $\alpha_{P,\delta}$, it is more attended to for any matched category c, as in Li and Camerer [75]. The effect is especially strong if current prices are striking, high $\sigma_{P\delta}$, as in Bordalo, Gennaioli, and Shleifer [15],[14]. But top-down factors also matter.[27] If the DM is accustomed to using a quality focused category (e.g. F_{con} is high) price is neglected even if prominently described. Conversely, a poor consumer who often worries about prices, σ_{Pc} is high, attends to them even when shrouded.[28] As shown before, top down attention can produce stability/description invariance.

Description shapes similarity and categorization.

Proposition 5 *Increasing sensory prominence $\alpha_{\delta,i}$ or described contrast, higher $\sigma_{i\delta}$ for $\alpha_{\delta,i} = 1$, increases similarity more for categories in which that feature is*

where $x = \delta, c$, and where we use the convention $\sigma_{ix} = 1$ for $i \in M_K$.

[27] See Kay and Ross [68] for evidence that the sensory stimuli are mediated by categories.

[28] We omit comparative statics with respect to contextual discrepancy d_{ic} because these have the same sign of (8) and (9).

more relevant, $\alpha_{c,i}$ is higher. That is

$$\frac{\partial^2 S(t,c,\delta|\sigma)}{\partial\alpha_{\delta,i}\partial\alpha_{c,i}} \geq 0 \text{ } and \text{ } \left.\frac{\partial^2 S(t,c,\delta|\sigma)}{\partial\sigma_{\delta,i}\partial\alpha_{c,i}}\right|_{\alpha_{\delta,i}=1} \geq 0.$$

Because they attend to features that are prominent and contrasting, categories more consistent with the description are more likely to be selected. A "buying" representation is more consistent with prominent or contrasting described prices. A price-focused advertisement can favor retrieval of this category, hinder retrieval of "consuming", and leads to neglect of quality.

Bottom up attention explains why people tend to focus on what is prominently described, neglecting the rest (What You See is All There Is, Kahneman [62]). At the same time, the model entails a top-down limit to this force. A shrouded feature is not neglected if the DM frequently attends to it (high F_c) or if it is highly contrasting from experience (high $\sigma_{c,i}$). An ad showing a comfortable hotel room can boost our quality focus but does not cause price neglect: hotel prices are frequently attended to and often contrasting.

Contrast introduces goal relevance, a contrasting hedonic is relevant, but not goal optimality in attention (Sims [103], Woodford [125], Gabaix [40]). Indeed, one striking goal - saving money - can distract us from another important goal - buying a good product. Analogously, one may choke under pressure because a highly salient goal such as winning a match point draws attention away from the more relevant features.

The interaction between bottom up and top down attention can throw light on choice under risk, statistical problems, and measured similarity judgments.

5.1 Implications for Lottery Choice

The use of riskless choice and statistical categories sheds light on biases in lottery choice, and unifies them with biases in riskless choice. Experiences with risk can create additional categories, for instance on elation for gains and disappointment/regret for losses, but this analysis is best left for a separate treatment.

The DM chooses between monetary lotteries

$$x = (x_g, x_b; \pi) \textit{ and } w = (w_g, w_b; \beta),$$

which pay their highest prizes x_g and w_g with probabilities π and β, respectively, and pay x_b and w_b otherwise. The lotteries have the same expected value but x is riskier than w, $x_g > w_g$, $x_b \leq w_b$ and $\pi < \beta$. Each lottery has four features: its maximum and minimum payoffs (*hedonic*) and the events in which each is delivered (*event*). The generic atom is $y = (u_g, u_b, e_g, e_b)$. Event feature e_s takes the value s when this state realizes and the neutral value Ω otherwise. The hedonic u_s is equal to the delivered payoff in state $s \in \{g, b\}$, and takes the neutral value 0 otherwise.[29]

When evaluating a lottery, the DM reasons about payoffs and events. Reasoning about payoffs cues experiences of "consuming" similar amounts, and hence category *con* (Section 4.1) Reasoning about events cues experiences of estimating probabilities, and in particular “simple sampling”, category ss (Section 4.2).[30] These categories are ideally integrated, but in reality they compete,

[29] In a more complete formalization, the atom of lottery x also includes the two event features of the alternative lottery w, so that the probability of the atom is computed using the joint probability distribution of payoffs in $(\Omega, \mathcal{F}, \mathbb{P})$. These features are however redundant in our case because the lotteries are independent.

[30] The other categories are less relevant here: there is no price paid as in *buy* and there are neither an unknwon data generating process nor iid draws as in inference.

distorting attention to payoffs and probabilities.

In typical experiments features are prominently described, $\alpha_{\delta,i} = 1$ for all $i \in M_O$. Proposition 4 then implies that, when matching to $c \in \{con, ss\}$, under quadratic distance $d(.)$ attention satisfies:

$$\alpha_{u_s,t}(con) = \frac{\overline{\alpha} + \sigma_{u_s\delta}/F_{con}}{1 + \sigma_{u_s\delta}/F_{con}}, \qquad \alpha_{e_s,t}(con) = \frac{\sigma_{e\delta}/F_{con}}{1 + \sigma_{e\delta}/F_{con}}, \tag{23}$$

$$\alpha_{u_s,t}(ss) = \frac{\sigma_{u_s\delta}/F_{ss}}{1 + \sigma_{u_s\delta}/F_{ss}}, \qquad \alpha_{e_s,t}(ss) = 1, \quad s = g, b. \tag{24}$$

Two properties are noteworthy. Ceteris paribus, the "consuming" category boosts attention to payoffs but dampens sensitivity to probabilities, compared to "simple sampling". Formally, $\alpha_{u_s,t}(con) > \alpha_{u_s,t}(ss)$ and $\alpha_{e_s,t}(con) < \alpha_{e_s,t}(ss)$. Second, increasing payoff or event contrast increases attention to the payoff in state s or to the state's probability, but less so in more frequent categories (see $\sigma_{u_s\delta}/F_c$ and $\sigma_{e\delta}/F_c$). To see the implications of this effect note that, by (20), payoff contrast increases in the payoff difference $|x_s - w_s|$, event contrast increases in the probability difference $|\pi - \omega|$.[31]

By Proposition 5, bottom up attention also shapes categorization. Higher payoff contrast $\sigma_{u_s\delta}$ fosters matching of "consuming" with the description, because in *con* payoffs are relevant. Higher event contrast $\sigma_{e\delta}$ fosters matching of "simple sampling" with the description, because in *ss* probabilities are relevant.

These forces account for a range of puzzles in risky choice.

Common Ratio Effect. Suppose that $x_b = w_b = 0$. It is well known that if $(100, 0; 0.2) \sim (25, 0; 0.8)$, then for many people $(100, 0; 0.02) \succ (25, 0; 0.08)$. This pattern violates expected utility, in which preference rankings are invariant to uniform scaling of probabilities. In our model this effect arises due to contrast.

[31] We implicitly assume that contrast in state s is only computed for features that take a proper value in such state, and not using features that take values in a diferent state.

The DM prefers x to w if and only if:

$$v(x(\alpha_O)) - v(w(\alpha_O)) = \pi^{\alpha_{e,t}} \cdot \alpha_{t,u_g} \cdot (x_g - w_g) + (\pi^{\alpha_{e,t}} - \beta^{\alpha_{e,t}}) \cdot w_g(\alpha_O) \geq 0. \tag{25}$$

Given that $\pi \cdot x_g = \beta \cdot w_b$, the left hand side increases in α_{t,u_g} and decreases in $\alpha_{e,t}$. Thus, higher attention to payoffs α_{t,u_g} boosts risk taking, higher attention to probabilities $\alpha_{e,t}$ boosts risk aversion.

The reduction in probabilities lowers event contrast from $|0.8 - 0.2| = 0.6$ to $|0.08 - 0.02| = 0.06$. This reduces attention to probabilities $\alpha_{e,t}$ in any category, but crucially it also fosters matching to the consuming category, which strongly boosts attention to payoffs α_{t,u_g}. The DM's attention is then drawn away from "peanuts" differences in probabilities; it is drawn instead to the 100\$ versus 25\$ payoff difference, fostering risk taking.[32]

Rubinstein [94] offers an account of the common ratio whereby similarity among (small) probabilities prompts people to choose based on payoffs. We offer a cognitive foundation based on bottom up contrast and top down similarity to past payoff evaluations versus probability estimations. Compared to Bordalo, Gennaioli, and Shleifer [15], we obtain unstable weighting of probabilities versus payoffs and experienced-based heterogeneity.

Suppose now that w is a sure thing $\widetilde{w} \in (0, x_g)$. Hedonic features are whether the lottery pays more or less than $\widetilde{w}$, with corresponding event features. Contrast $\sigma_{u_s\delta}$ of payoff state $s = g, b$ increases in $|x_s - \widetilde{w}|$. Event contrast is minimal, $\sigma_{e\delta} = 1$, because events are now perfectly correlated across x and w.

[32] This may explain why we neglect many risks in everyday life (Gennaioli, Shleifer, and Vishny [43],[44]). Additional context can include the DM's gain/loss state, her wealth, insider/outsider view of risk (Kahneman and Lovallo [64]). For example, when people are prompted to think as a trader, their loss aversion declines (Sokol-Hessner, Hsu, Curley, Delgado, Camerer, and Phelps [106]).

Provided $\alpha_{t,u_g} + \alpha_{t,u_b} > 0$, the DM prefers x to $\widetilde{w}$ if and only if:

$$\widetilde{w} \leq \frac{\alpha_{t,u_g} \cdot \pi^{\alpha_{e,t}}}{\alpha_{t,u_g} \cdot \pi^{\alpha_{e,t}} + \alpha_{t,u_b} \cdot (1-\pi)^{\alpha_{e,t}}} \cdot x_g. \tag{26}$$

Given that $\pi \cdot x_g = \widetilde{w}$, the right hand side increases (boosting risk seeking) when relative attention to the lottery upside payoff $\left(\alpha_{t,u_g}/\alpha_{t,u_b}\right)$ is higher. Risk seeking increases with attention to probability $\alpha_{e,t}$ if and only if the lottery is left skewed, $\pi > 0.5$. The following puzzles arise form these effects.

Framing. Risk attitudes change when different payoffs are more or less visually prominent. Consider two normatively equivalent descriptions of x.

$$\begin{aligned} &\textit{Full Prominence} &: \quad &\textit{win } x_g \textit{ with probability } \pi \textit{ and } 0 \textit{ otherwise}, &(27)\\ &\textit{Shrouded Downside} &: \quad &\textit{win } x_g \textit{ with probability } \pi. &(28)\end{aligned}$$

In (27) both the lottery upside and downside are prominently described, as in (23) and (24). In (28) the lottery downside is described less prominently, i.e., $\alpha_{\delta,u_b} < 1$. This affects equilibrium attention to the downside $\alpha_{t,u_b}(c)$ because, for any matched category c we have:

$$\alpha_{t,u_b}(c) = \frac{\sigma_{\delta,u_b} \cdot \alpha_{\delta,u_b} + F_c \cdot \alpha_{c,u_b}}{\sigma_{\delta,u_b} + F_c}.$$

Although payoffs are constant, lower α_{δ,u_b} reduces attention to the downside α_{t,u_b}, promoting risk taking. This or similar effects are familiar to experimental economists and often treated as unimportant aberrations. In our model these effects arise from bottom up attention, which can shed light on important real world phenomena. Advertising high returns can foster investment and neglect of risk during good times, when the crash event is shrouded, so investors adopt a "consuming" category, Mullainathan and Shleifer [84] and Célérier and Vallée [22].[33]

Discontinuities. Categories can yield aversion to minuscule risks, as in Kahneman and Tversky's [67] "certainty effect". When $\pi = 1$, x is better than w because $x_g > \widetilde{w}$. Categorization in category *con* and choice of x are straightforward: probabilities are not involved.[34]

Adding a small risk ε, so that $\pi = 1 - \varepsilon$, has two effects: i) it creates a downside payoff feature, whose salience increases in contrast $|\widetilde{w} - 0|$, and ii) it creates an event feature, increasing similarity with "simple sampling".

High payoff contrast and small probability contrast favor a payoff evaluation representation.[35] This focuses many DMs on the highly contrasting downside, $\alpha_{t,u_b} > 0$, and leads to insensitivity to probabilities, which is full $\alpha_{e,t} = 0$ if the consuming category if very frequent $F_{con} \to \infty$. High risk aversion follows, even if the risk is small.

Barseghyan, Molinari, O'Donoghue, and Teitelbaum [5] show that discontinuous probability weighting at 0 (as suggested in Kahneman and Tversky [67] but subsequently abandoned in favor of a continuous model) is important to understand insurance demand. Haigh and List [53] document discontinuity also with professional traders.[36] In our model this behavior arises because a strik-

[33] A related treatment varies the description of the sure thing by explicitly mentioning that it never yields a downside (unlike the risky alternative), or by keeping it implicit. We thank Alex Imas for making this point as a discussant.

[34] Arguably, in this case there is neither stochasticity nor heterogeneity in choice. In our model, this occurs provided λ is sufficiently high, or if the attention shock is only relevant when all highly frequent categories exhibit an imperfect match, which seems plausible.

[35] Our model predicts that some people may edit out the small risk, but a few people sticking to *con* and neglecting numerical probabilities are enough to produce a discontinuity.

[36] Another important example of discontinuity arises in situations involving social norms. Gneezy and Rustichini [48] show that a small payment reduce effort in the collection of donations, presumably because the payment is now categorized as a low-salary job. Social norms are also at play in purely strategic situations. The majority of players in the dictator game share some of their endowment, consistent with a categorization in terms of the social

ing downside prompts a payoff evaluation mode. This behavior also illustrate that choice discontinuities can "reveal" categories: either the continued use of a payoff focused category with an added downside as in the current example, or switches between different categories (e.g., describing an irrelevant "coin toss" context can favor the adoption of ss and thus sensitivity to probabilities).

The Fourfold Pattern and "Simplicity Equivalents" Equation (26) also yields the so-called "fourfold pattern" in the gain domain, based on payoff contrast. Given that $x_g = \pi \cdot \widetilde{w}$, upside payoff contrast is $|\widetilde{w}/\pi - \widetilde{w}| = \widetilde{w} \cdot \left(\frac{1-\pi}{\pi}\right)$, downside contrast is $|\widetilde{w} - 0| = \widetilde{w}$. If the lottery is right skewed, $\pi < 0.5$, upside contrast is higher than downside contrast, and vice-versa if $\pi > 0.5$. By (23) and (24), then, in every category the DM focuses more on the upside then on the downside iff $\pi < 0.5$. The DM is then risk seeking for $\pi < 0.5$ and risk averse for $\pi > 0.5$, as in the fourfold pattern.

Contrast of described payoffs reproduces the Bordalo, Gennaioli, and Shleifer [14] payoff-salience mechanism. Importantly, our model adds new explanatory power. By Proposition 5, payoffs contrast favors categorization into category con, leading to probability neglect, $\alpha_e = 0$, which has three implications.

First, the fourfold pattern is now stronger: it can arise even if the lottery upside and downside are equally attended to $\alpha_{t,u_g} = \alpha_{t,u_b}$. Second, context matters. For instance, eliciting certainty equivalents should strengthen the fourfold pattern compared to, say, making binary choices or choosing a "probability equivalent π" to $\widetilde{w}$. Being in the same dollar units as hedonics, certainty equivalents favor payoff evaluation, in line with the compatibility principle.

Third, and key, the "payoff evaluation" mechanism is based a riskless category, so it is a fortiori relevant for riskless domains in which the DM must aggregate payoffs with other features. Consider Oprea's [88] simplicity equiva-

norm of sharing (Krupka and Weber [71]). However, adding the possibility of taking money leads to a discontinuous change in behavior towards no sharing (List [78]).

lents experiment, in which option A consists of 90 boxes with \$25 and 10 boxes with \$0, and option B consists of 100 boxes with \$2.5. The subject is paid the total value of the chosen option divided by 100. The total monetary payment in the two options is equivalent, but subjects exhibit a preference for B.

Options have two features: the payoff in boxes $s = g, b$ and their numerosities n_g and n_b. When evaluating option A versus B, the sharp contrast between A's payoff in its g and b boxes with B's \$2.5 payoff prompts the DM to categorize the problem as "payoff evaluation", or $c = con$. This representation prompts a separate evaluation of different boxes, interfering with attention to the "box type" event, and hence to precise numerosities. From Equation (3), the value of the generic atom of A is

$$v\left(\left(s, x_s\right)\left(\alpha_O\right)\right) = \frac{n_s^{\alpha_{e,t}} \cdot \alpha_{u_s,t} \cdot x_s}{100}, \tag{29}$$

so that option A is preferred to B if an only if:

$$v\left(A\left(\alpha_O\right)\right) \geq v\left(B\left(\alpha_O\right)\right) \Leftrightarrow x \leq \frac{\left(\frac{n_g}{100}\right)^{\alpha_{e,t}} \cdot \alpha_{t,u_g}}{\left(\frac{n_g}{100}\right)^{\alpha_{e,t}} \cdot \alpha_{t,u_g} + \left(\frac{n_b}{100}\right)^{\alpha_{e,t}} \cdot \alpha_{t,u_b}} \cdot x_g. \tag{30}$$

Equation (30) yields, in relative frequencies, the same preferences obtained in Equation (26) with respect to probabilities. Striking payoff differences favor a "payoff evaluation" category, and draw attention away from probabilities or frequencies, unifying risky and riskless settings. In this experiment, the retrieval of the *con* category interferes with the retrieval of the correct "adding up" category. We suspect that many subjects would be able to figure out how to perform the adding up operation if explicitly asked to do so, and that they would then be indifferent between A and B – i.e. complexity is in representation, not in computation. What is also key about Oprea's [88] experiment is that similarity in behavior cannot come from preferences: one choice is risky the other is riskless. It rather comes from similarity in representations, based on the common "payoff evaluation" context feature.

5.2 Bottom up Contrast in Statistical Problems

In Bordalo, Conlon, Gennaioli, Kwon, Shleifer [19], statistical contrast yields: i) stronger GF for longer sequences and ii) stronger base rate neglect in inference when likelihoods are more extreme. We now show that categorization throws new light on them.

A DM judges the likelihood that n draws of a fair coin produce a balanced sequence H_1 versus a full heads sequence H_2. Among the problem's event features, only the contrast of the share of heads varies with n. Using as a proxy for contrast the largest probability difference between any two shares of heads in $(\Omega, \mathcal{F}, \mathbb{P})$ we have:

$$\sigma_{\mathcal{S}} = \frac{\left|\binom{n}{n/2} - 1\right| (0.5)^n}{\binom{n}{n/2}(0.5)^n + (0.5)^n + \epsilon},$$

which indeed increases in n. By Proposition 4, when matching the problem to any category, the DM pays more attention to the share of heads. It feels very striking, and thus attention grabbing, to obtain zero tails in 6 flips. By Proposition 5, this fosters categorization in agnostic inference ai compared to simple sampling ss, causing the GF.

In inference, a DM evaluates the likelihood that a green ball comes from urn A whose base rate is $\pi_A < 0.5$ and whose likelihood of green is $q > 1/2$ or from the symmetric urn B. Here, contrast of the feature "urn selection", $i = U$, increases in $|\pi_A - \pi_B| = 1 - 2\pi_A$. Contrast of the feature "share of green given U" increases in $|q - (1-q)| = 2q - 1$. The more extreme the likelihood, the higher is q, the higher is the contrast of the share of heads. As in the case of the GF, then, categorization in inference ai is more likely, causing a focus on the green share and thus base rate neglect. This is in line with the evidence [19].

Instability in GF and inference are due to the same force: bottom-up salience of the "share of heads", triggered by strong statistical contrast, which causes greater reliance on the inference category.

5.3 Top Down Contrast and Unstable Similarity Judgments

Tversky [116] famously showed that when people rate similarities between countries on a list, they judge Austria and Sweden as more similar when the list includes Hungary and Poland than when it includes Hungary and Norway. He explained this finding by the contrast principle. When Poland is on the list, political differences are contrasting, so Sweden and Austria are deemed similar. When Norway is on the list, geographic differences are contrasting, so Sweden and Austria are deemed dissimilar.

Here contrast arises top down: the only information people are given is country names, but these prompt focus on a feature contrasting among them (similarly to when seeing the "flight" label we think of the "crash" feature).

When assessing the similarity between Austria (A), Sweden (S), Hungary (H) and either Norway or Poland ($X = N, P$), each atom $y \in Y$ lists the features of a country pair. The Austria-Sweden atom (A, S) reports two "hedonic" features: geographical distance $u_G(A, S)$, political distance $u_P(A, S)$. It also reports the country names (event feature). Attention $(\alpha_{G,t}, \alpha_{P,t})$ to hedonics entails estimated distance

$$v_{AS}(\alpha_{G,t}, \alpha_{P,t}) = \alpha_{G,t} \cdot u_G(A, S) + \alpha_{P,t} \cdot u_P(A, S),$$

which is then used as an inverse measure of similarity (embedded in the decision rule $D(.)$). Because Austria and Sweden are intuitively more distant geographically than politically ($u_P(A,S) < u_G(A,S)$), the DM judges them as more similar when she attends to politics compared to geography ($\alpha_{P,t}$ higher relative to $\alpha_{G,t}$).

The DM has experienced two categories: problems $c = G$ in which geographic features are learned or judged, and problems $c = P$ in which political features are learned or judged. The former category attends to geography features while neglecting politics,

$$\alpha_{G,G} = 1 > \alpha_{P,G} = 0,$$

the latter does the reverse

$$\alpha_{G,P} = 0 < \alpha_{P,P} = 1.$$

Top down contrast in $c = G$ occurs only along geography, $\sigma(\kappa_G)$ where $\kappa_G = \{u_G(y)\}_{y \in Y}$ are the distances between the four countries. In $c = P$, on the other hand, it only occurs along politics, $\sigma(\kappa_P)$, with κ_P accordingly defined. The description of the problem makes the country names fully prominent while it shrouds the hedonics.

Critically, top down contrast of different hedonic features changes with the set of country names described. When the DM is presented with names A,S,H,N, variability along geography is high (N, S versus A, H) while variability along politics is low (N,S, A versus H), so $\sigma(\kappa_{G1}) > \sigma(\kappa_{P1})$, where 1 captures the list A,S,H,N. When the DM is presented with names A,S,H,P, variability along geography is low (S versus A, H, P) while variability along politics is high (S, A versus H, P), so $\sigma(\kappa_{G2}) < \sigma(\kappa_{P2})$, where 2 refers to the list A,S,H,P.

Instability in similarity judgments arises because, by point i) in Proposition 6, when the most contrasting category is $c = G$ (country list 1), the DM retrieves this category and focuses on geography, holding A and S dissimilar. When instead the most contrasting category is $c = P$ (country set 2), the DM retrieves this category and focuses on politics, holding A and S similar. The similarity judgment is unstable as documented by Tversky. As in the GF, upon seeing A, S, H, N the DM thinks "what a striking North-East separation between N, S and A, H!". This spontaneous association between the current task and geography does not just increase attention to this feature. It causes neglect of politics, which causes an unstable similarity judgment between A and S, shaped by irrelevant countries in the list.

6 Conclusion

We propose a framework for reasoning and decisions based on problem recognition. Our approach entails heterogeneity and instability of representations based on experiences as well as similarity, including along spurious contextual cues. It produces a unified explanation for "anomalies" across riskless and risky choice, as well as probabilistic judgment, which are inconsistent with neoclassical approaches that assume stability of preferences as well as with behavioral approaches that assume stability of biases.

Our approach implies a new way to study decisions in which one measures *biases and attention to features*. These should be measured across people (accounting for heterogeneity), within individuals (accounting for stochasticity), and across contexts (accounting for instability). This strategy crucially relies on non-choice data such as experiences and similarity, as in Bordalo, Burro, Coffman, Gennaioli, and Shleifer [12], of similarity and attention to features, as in Bordalo, Conlon, Gennaioli, Kwon, Shleifer [19], and on measuring reasons and attention using text analysis (Haaland, Roth, Stantcheva, and Wohlfart [52], Link, Peschl, Roth, and Wohlfart, [76]). Machine learning can help unveil subtle but significant contextual features, as in Ludwig and Mullainathan [79]'s hypothesis generation. Developing systematic approaches and tools to reveal categories, representations, and features from both choice and non-choice data is a major area for future development.

In this paper, we took categories as given, based on intuitive categories from everyday experience. This leaves open the central question of how such categories develop endogenously. Under what circumstances do people develop broad categories (e.g. consumption) versus more specific ones (which pasta for dinner)? How do categories evolve with experiences? Our approach suggests that categories may come from repeated attention to context differences across decisions. Bottom-up attention as formalized in Section 5 may be helpful in thinking about this problem, as is are literatures in psychology and in machine learning concerning processes that synthetize many episodes into lower dimensional models (e.g. Tulving [114]).

Heterogeneity and instability of choices are important in the field. When thinking about redistributive policies, some voters think about fairness, others think about zero-sum transfers from taxpayers (Chinoy, Nunn, Sequeira, and Stantcheva [26]). In politics, framing a salient issue such as immigration as a threat to one's identity can change the lens through which people form opinions in many domains. Similar considerations apply to fairness judgments (Kahneman, Knetsch, and Thaler [63]), taboos (Benabou and Tirole [8]), the distinction between risk and ambiguity (Einhorn and Hogarth [31]), and strategic behavior (e.g. Goke, Weintraub, Mastromonaco, and Seljan [49]).

[P: need to decide whether to add a list of papers and their descriptions, or keep these pointers concise / high level, in which case cut this paragraph] [49] find that prolonged use of a second-price auction causes agents to neglect the number of bidders in a subsequent first price auction, with temporarily higher revenues for the auctioneers. In second-price auctions, in fact, only the private value is relevant, not the number of bidders, and the previous experience with second price auctions causes the neglect of the number of bidders.

By shedding light on the cognitive foundations of heterogeneity and instability, our approach can deliver an economic analysis with greater explanatory and predictive power.

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A Appendix

A.1 Proofs

Proof of Proposition 1. First, we will consider the case when $i \notin M_{Kc}$. By Equation (5), we have that $\alpha_{t,i}$ only affects $S\left[(\alpha_t, \kappa_t), (\alpha_c, \kappa_c)\right]$ additively through the term $-d(|\alpha_{t,i} - \alpha_{c,i}|)$ in the numerator. This is clearly maximized when $\alpha_{t,i} = \alpha_{c,i}$, so DM's attention $\alpha_{t,i}(c) = \alpha_{c,i}$ follows the category.

Next, consider the case where $i \in M_{Kc}$. Because d is strictly convex, $S\left[(\alpha_t, \kappa_t), (\alpha_c, \kappa_c)\right]$ is a strictly concave function of $\alpha_{t,i}$. So it follows that the attention $\alpha_{t,i} \in [0,1]$ that maximizes similarity must satisfy the following first order condition:

$$\frac{\partial S\left[(\alpha_t, \kappa_t), (\alpha_c, \kappa_c)\right]}{\partial \alpha_{t,i}} \begin{cases} = 0 \text{ } and \text{ } \alpha_{t,i} \in [0,1] \\ > 0 \text{ } and \text{ } \alpha_{t,i} = 1 \\ < 0 \text{ } and \text{ } \alpha_{t,i} = 0 \end{cases} . \tag{31}$$

Plugging in $\alpha_{c,i} = 1$, and defining

$$G(\alpha_{t,i}, d_{c,i}) = \frac{\partial}{\partial \alpha_{t,i}} d\left(|\alpha_{t,i} - 1|\right) + d_{c,i}$$

the first order condition simplifies to

$$G(\alpha_{t,i}, d_{c,i}) \begin{cases} = 0 \text{ } and \text{ } \alpha_{t,i} \in [0,1] \\ < 0 \text{ } and \text{ } \alpha_{t,i} = 1 \\ > 0 \text{ } and \text{ } \alpha_{t,i} = 0 \end{cases} .$$

Note that $G(1, d_{c,i}) = \frac{\partial}{\partial \alpha_{t,i}} d\left(0\right) + d_{c,i} \geq d_{c,i} \geq 0$, so the second case for the first order condition can never be satisfied. So if $\alpha(d_{c,i})$ is implicitly defined by

$$\alpha(d_{c,i}) = \begin{cases} 0 & if\ G(0,d_{c,i}) > 0 \\ solution\ to\ G(\alpha(d_{c,i}),d_{c,i}) = 0 & if\ G(0,d_{c,i}) \leq 0 \end{cases}, \tag{32}$$

then this also characterizes the optimal attention $\alpha_{t,i}(c) = \alpha(d_{c,i})$. (Note that if $G(0,d_{c,i}) \leq 0$, then since $G(1,d_{c,i}) \geq 0$, it follows that $G(\alpha(d_{c,i}),d_{c,i}) = 0$ must have some solution for $\alpha(d_{c,i})$ on $[0,1]$.)

Next, we compute $\frac{\partial\alpha(d_{c,i})}{\partial d_{c,i}}$. If $G(0,d_{c,i}) > 0$, then for all $d'_{c,i}$ in a neighborhood of $d_{c,i}$, we still have $G(0,d'_{c,i}) > 0$ and $\alpha(d'_{c,i}) = 0$. So in this case, $\frac{\partial\alpha(d_{c,i})}{\partial d_{c,i}} = 0$. If $G(0,d_{c,i}) \leq 0$, then we can implicitly differentiate $G(\alpha(d_{c,i}),d_{c,i}) = 0$ with respect to $d_{c,i}$ to obtain

$$\begin{aligned} \frac{\partial^2 d(1-\alpha(d_{c,i}))}{\partial\alpha_{t,i}^2}\frac{\partial\alpha(d_{c,i})}{\partial d_{c,i}} + 1 &= 0 \\ \Longrightarrow \quad \frac{\partial\alpha(d_{c,i})}{\partial d_{c,i}} &= -\frac{1}{\frac{\partial^2 d(1-\alpha(d_{c,i}))}{\partial\alpha_{t,i}^2}} < 0, \end{aligned}$$

due to the strict convexity of d. Therefore, $\frac{\partial\alpha(d_{c,i})}{\partial d_{c,i}} \leq 0$ in general.

Observe that

$$\begin{aligned} d(t,c) &= \min_{\alpha_t\in[0,1]^M} \frac{\sum_{i\in M} d\left(|\alpha_{t,i}-\alpha_{c,i}|\right) + \sum_{i\in M_K} \alpha_{t,i}\alpha_{c,i} d_i\left(\kappa_{it},\kappa_{ic}\right)}{|M|+|M_K|} \\ &= 1 - \max_{\alpha_t\in[0,1]^M} S\left[\left(\alpha_t,\kappa_t\right),\left(\alpha_c,\kappa_c\right)\right]. \end{aligned}$$

Then it follows that

$$S(t,c) = F_c \cdot (1 - d(t,c))$$

and (8) follows immediately.

Finally,

$$\begin{aligned}
\frac{\partial S(t,c)}{\partial d_{c,i}} &= F_c \cdot \frac{\partial \max_{\alpha_t \in [0,1]^M} \; S\left[\left(\alpha_t, \kappa_t\right), \left(\alpha_c, \kappa_c\right)\right]}{\partial d_{c,i}} \\
&= F_c \cdot \left[\frac{-\alpha_{t,i}(c)\alpha_{c,i}}{|M| + |M_K|}\right] \\
&\leq 0
\end{aligned}$$

where the second step follows from the envelope theorem (we can ignore the effect of $d_{c,i}$ on $\alpha_t(c)$). ■

Proof of Proposition 2. First, by Equation (10) we calculate

$$\begin{aligned}
& \frac{\partial \Pr\left(c|t\right)}{\partial S(t,c)} \\
&= \frac{\begin{array}{c}\left(\sum_{c' \in C} \exp\left[\lambda \cdot S\left(t,c'\right)\right]\right)\left(\lambda \cdot \exp\left[\lambda \cdot S\left(t,c\right)\right]\right) \\ -\left(\exp\left[\lambda \cdot S\left(t,c\right)\right]\right)\left(\lambda \cdot \exp\left[\lambda \cdot S\left(t,c\right)\right]\right)\end{array}}{\left(\sum_{c' \in C} \exp\left[\lambda \cdot S\left(t,c'\right)\right]\right)^2}, \\
&= \lambda \cdot \left[\frac{\exp\left[\lambda \cdot S\left(t,c\right)\right]}{\sum_{c' \in C} \exp\left[\lambda \cdot S\left(t,c'\right)\right]} - \left(\frac{\exp\left[\lambda \cdot S\left(t,c\right)\right]}{\sum_{c' \in C} \exp\left[\lambda \cdot S\left(t,c'\right)\right]}\right)^2\right] \\
&= \lambda \cdot \left[\Pr\left(c|t\right) - \left(\Pr\left(c|t\right)\right)^2\right] \\
&= \lambda \cdot \Pr\left(c|t\right) \cdot \left[1 - \Pr\left(c|t\right)\right].
\end{aligned}$$

Combining this with Equations (8) and (9) using the chain rule immediately yields the desired result. ■

Proof of Proposition 3. For the proof of the first part of the proposition, we omit the index j. First, note that following Proposition 2,

$$\frac{\partial \Pr(choose\ a_{tc})}{\partial F_c} = \frac{\partial \Pr(c|t)}{\partial F_c} > 0,$$

so choice of a_{tc} is more likely when F_c is higher.

Next, recalling from the proof of Proposition 2 that

$$\frac{\partial \Pr(c|t)}{\partial S(t,c)} = \lambda \cdot \Pr(c|t) \cdot [1 - \Pr(c|t)],$$

and also the fact that $\frac{\partial S(t,c)}{\partial \hat{d}(c)} = -F_c$, we have

$$\frac{\partial \Pr(c|t)}{\partial \hat{d}(c)} = -\lambda \cdot F_c \cdot \Pr(c|t) \cdot [1 - \Pr(c|t)].$$

Then applying Proposition 2 again, we get

$$\begin{aligned}
\frac{\partial^2 \Pr(c|t)}{\partial d(c)\, \partial F_c} &= -\lambda \cdot \Pr(c|t) \cdot [1 - \Pr(c|t)] \\
&\quad + (-\lambda) \cdot F_c \cdot \frac{\partial \Pr(c|t)}{\partial F_c} \cdot [1 - \Pr(c|t)] \\
&\quad + (-\lambda) \cdot F_c \cdot \Pr(c|t) \cdot \left[-\frac{\partial \Pr(c|t)}{\partial F_c} \right] \\
&\propto -\Pr(c|t) \cdot [1 - \Pr(c|t)] \\
&\quad + (-\lambda) \cdot F_c \cdot (1 - d_{tc}) [1 - \Pr(c|t)] \Pr(c|t) \cdot [1 - \Pr(c|t)] \\
&\quad + \lambda \cdot F_c \cdot (1 - d_{tc}) \cdot \Pr(c|t) \Pr(c|t) \cdot [1 - \Pr(c|t)] \\
&= (-1 + \lambda S(t,c) [2 \Pr(c|t) - 1]) \Pr(c|t) \cdot [1 - \Pr(c|t)].
\end{aligned}$$

So

$$\begin{aligned}
&\frac{\partial^2 \Pr(c|t)}{\partial d_{tc} \partial F_c} \leq 0 \\
\iff &\lambda S(t,c) [2 \Pr(c|t) - 1] \leq 1 \\
\iff &\Pr(c|t) \leq \frac{1}{2} + \frac{1}{2\lambda S(t,c)}.
\end{aligned}$$

For the second part of the proposition, we first recall that

$$\Pr(c\,|t,j) = \frac{\exp\left(\lambda \cdot F_c(j) \cdot \left(1 - \hat{d}(c)\right)\right)}{\sum_{c' \in C} \exp\left(\lambda \cdot F_{c'}(j) \cdot \left(1 - \hat{d}(c')\right)\right)}.$$

Observe that $\hat{d}(c)$ is constant across individuals, and only depends on the category c. It is immediately clear that if $F_c(j) = F_c(j')$ for all $c \in C$, then $\Pr(c\,|t,j) = \Pr(c\,|t,j')$ for all $c \in C$. So we just need to prove the converse, which is that if $\sum_{c' \in C} F_{c'}(j) = \sum_{c' \in C} F_{c'}(j')$ and $\Pr(c\,|t,j) = \Pr(c\,|t,j')$ for all $c \in C$, then $F_c(j) = F_c(j')$ for all $c \in C$.

For sake of contradiction, assume that we can select some $c^* \in C$ with $F^j_{c^*} \neq F^{j'}_{c^*}$. Without loss of generality, let $F^j_{c^*} > F^{j'}_{c^*}$. Now select some arbitrary category $c' \in C \setminus \{c^*\}$, and we have

$$\begin{aligned}
\frac{\Pr(c^*\,|t,j)}{\Pr(c'\,|t,j)} &= \frac{\exp\left(\lambda \cdot F_{c^*}(j) \cdot \left(1 - \hat{d}(c^*)\right)\right)}{\exp\left(\lambda \cdot F_{c'}(j) \cdot \left(1 - \hat{d}(c')\right)\right)} \\
&= \exp\left(\lambda\left[F_{c^*}(j) \cdot \left(1 - \hat{d}(c^*)\right) - F_{c'}(j) \cdot \left(1 - \hat{d}(c')\right)\right]\right).
\end{aligned}$$

Therefore

$$\begin{aligned}
&\frac{\Pr(c^*\,|t,j)}{\Pr(c'\,|t,j)} = \frac{\Pr(c^*\,|t,j')}{\Pr(c'\,|t,j')} \\
&\implies F_{c^*}(j) \cdot \left(1 - \hat{d}(c^*)\right) - F_{c'}(j) \cdot \left(1 - \hat{d}(c')\right) \\
&= F_{c^*}(j') \cdot \left(1 - \hat{d}(c^*)\right) - F_{c'}(j') \cdot \left(1 - \hat{d}(c')\right) \\
\implies &\left(F_{c^*}(j) - F_{c^*}(j')\right)\left(1 - \hat{d}(c^*)\right) = \left(F_{c'}(j) - F_{c'}(j')\right)\left(1 - \hat{d}(c')\right)
\end{aligned}$$

so $F_{c^*}(j) > F_{c^*}(j')$ means that $F_{c'}(j) > F_{c'}(j')$ for any arbitrary category

$$c' \in C \setminus \{c^*\}.$$

But this means that it is impossible for $\sum_{c'\in C} F_{c'}(j) = \sum_{c'\in C} F_{c'}(j')$ to hold, so we have a contradiction. ■

Proof of Proposition 4. Because d is strictly convex, $S(t,c,\delta|\sigma)$ is a strictly concave function of $\alpha_{t,i}$. So the optimal attention $\alpha_{t,i}$ satisfies

$$\frac{\partial S(t,c,\delta|\sigma)}{\partial \alpha_{t,i}} \begin{cases} = 0 \; and \; \alpha_{t,i} \in [0,1] \\ > 0 \; and \; \alpha_{t,i} = 1 \\ < 0 \; and \; \alpha_{t,i} = 0 \end{cases}.$$

First let $\sigma^* = \frac{\sigma_{i\delta}}{\sigma_{ic}} \cdot \frac{1}{F_c}$. Then if (based on (22)) we define

$$\begin{aligned} & G(\alpha_{t,i}, \alpha_{c,i}, \alpha_{\delta,i}, \sigma^*) \\ = \; & \frac{\partial}{\partial \alpha_{t,i}} d(|\alpha_{t,i} - \alpha_{c,i}|) + d_{tc}(i) \cdot \alpha_{c,i} + \sigma^* \cdot \frac{\partial}{\partial \alpha_{t,i}} d(|\alpha_{t,i} - \alpha_{\delta,i}|) \\ = \; & d'(|\alpha_{t,i} - \alpha_{c,i}|)\, sign(\alpha_{t,i} - \alpha_{c,i}) + d_{tc}(i) \cdot \alpha_{c,i} \\ & + \sigma^* \cdot d'(|\alpha_{t,i} - \alpha_{\delta,i}|)\, sign(\alpha_{t,i} - \alpha_{\delta,i}), \end{aligned}$$

(where $sign(x)$ is equal to 1 if $x > 0$, -1 if $x < 0$, and 0 if $x = 0$) then the first order condition simplifies to

$$G(\alpha_{t,i}, \alpha_{c,i}, \alpha_{\delta,i}, \sigma^*) \begin{cases} = 0 \; and \; \alpha_{t,i} \in [0,1] \\ < 0 \; and \; \alpha_{t,i} = 1 \\ > 0 \; and \; \alpha_{t,i} = 0 \end{cases}.$$

One can easily see that $G\left(1, \alpha_{c,i}, \alpha_{\delta,i}, \sigma^*\right) \geq 0$ because

$$sign\left(1-\alpha_{c,i}\right), sign\left(1-\alpha_{\delta,i}\right) \geq 0.$$

So if $\alpha\left(\alpha_{c,i}, \alpha_{\delta,i}, \sigma^*\right)$ is implicitly defined by

$$\alpha\left(\alpha_{c,i}, \alpha_{\delta,i}, \sigma^*\right) = \begin{cases} 0 & if\ G\left(0, \alpha_{c,i}, \alpha_{\delta,i}, \sigma^*\right) > 0 \\ solution\ to\ G\left(\alpha\left(\alpha_{c,i}, \alpha_{\delta,i}, \sigma^*\right), \alpha_{c,i}, \alpha_{\delta,i}, \sigma^*\right) = 0 & if\ G\left(0, \alpha_{c,i}, \alpha_{\delta,i}, \sigma^*\right) \leq 0 \end{cases},$$

then this also characterizes the optimal attention $\alpha_{t,i}(c, \delta, \sigma)$.

Now, we can compute $\partial \alpha_{t,i}(c, \delta, \sigma)/\partial \alpha_{c,i} \geq 0$. First, note that the derivative with respect to $\alpha_{c,i}$ only makes sense if i is a choice feature (because context attention is either 0 or 1). In this case, $\mathbb{I}_{Kc}(i) = 0$, and so the continuous differentiability of d implies that $G\left(\alpha_{t,i}, \alpha_{c,i}, \alpha_{\delta,i}, \sigma^*\right)$ is continuous with respect to $\alpha_{c,i}$. If $G\left(0, \alpha_{c,i}, \alpha_{\delta,i}, \sigma^*\right) > 0$, then for all $\alpha_{c,i}$ in a neighborhood of $\alpha_{c,i}$, we still have $G\left(0, \alpha_{c,i}, \alpha_{\delta,i}, \sigma^*\right) > 0$ and $\alpha\left(\alpha_{c,i}, \alpha_{\delta,i}, \sigma^*\right) = 0$. Therefore $\frac{\partial \alpha\left(\alpha_{c,i}, \alpha_{\delta,i}, \sigma^*\right)}{\partial \alpha_{c,i}} = 0$. In the case where $G\left(0, \alpha_{c,i}, \alpha_{\delta,i}, \sigma^*\right) \leq 0$, we can implicitly differentiate $G\left(\alpha\left(\alpha_{c,i}, \alpha_{\delta,i}, \sigma^*\right), \alpha_{c,i}, \alpha_{\delta,i}, \sigma^*\right) = 0$ with respect to $\alpha_{c,i}$ to get

$$\begin{aligned} & \frac{\partial \alpha\left(\alpha_{c,i}, \alpha_{\delta,i}, \sigma^*\right)}{\partial \alpha_{c,i}} \frac{\partial G\left(\alpha\left(\alpha_{c,i}, \alpha_{\delta,i}, \sigma^*\right), \alpha_{c,i}, \alpha_{\delta,i}, \sigma^*\right)}{\partial \alpha_{t,i}} \\ + & \frac{\partial G\left(\alpha\left(\alpha_{c,i}, \alpha_{\delta,i}, \sigma^*\right), \alpha_{c,i}, \alpha_{\delta,i}, \sigma^*\right)}{\partial \alpha_{c,i}} \\ = & \ 0. \end{aligned}$$

We can first compute

$$\begin{aligned}
& \frac{\partial G\left(\alpha\left(\alpha_{c,i},\alpha_{\delta,i},\sigma^*\right),\alpha_{c,i},\alpha_{\delta,i},\sigma^*\right)}{\partial \alpha_{c,i}} \\
= & \frac{\partial^2}{\partial \alpha_{t,i}\partial \alpha_{c,i}} d\left(\left|\alpha\left(\alpha_{c,i},\alpha_{\delta,i},\sigma^*\right)-\alpha_{c,i}\right|\right) \\
= & -d''\left(\left|\alpha\left(\alpha_{c,i},\alpha_{\delta,i},\sigma^*\right)-\alpha_{c,i}\right|\right)<0,
\end{aligned}$$

since d has strictly positive second derivative.

Next, we compute $\frac{\partial \alpha\left(\alpha_{c,i},\alpha_{\delta,i},\sigma^*\right)}{\partial \alpha_{\delta,i}}$. For the same reason as before, if

$$G\left(0,\alpha_{c,i},\alpha_{\delta,i},\sigma^*\right)>0,$$

we immediately get $\frac{\partial \alpha\left(\alpha_{c,i},\alpha_{\delta,i},\sigma^*\right)}{\partial \alpha_{\delta,i}}=0$. If $G\left(0,\alpha_{c,i},\alpha_{\delta,i},\sigma^*\right)\leq 0$, then we can just implicitly differentiate the equation $G\left(\alpha\left(\alpha_{c,i},\alpha_{\delta,i},\sigma^*\right),\alpha_{c,i},\alpha_{\delta,i},\sigma^*\right)=0$ with respect to $\alpha_{\delta,i}$ to get

$$\begin{aligned}
& \frac{\partial \alpha\left(\alpha_{c,i},\alpha_{\delta,i},\sigma^*\right)}{\partial \alpha_{\delta,i}}\frac{\partial G\left(\alpha\left(\alpha_{c,i},\alpha_{\delta,i},\sigma^*\right),\alpha_{c,i},\alpha_{\delta,i},\sigma^*\right)}{\partial \alpha_{t,i}} \\
& +\frac{\partial G\left(\alpha\left(\alpha_{c,i},\alpha_{\delta,i},\sigma^*\right),\alpha_{c,i},\alpha_{\delta,i},\sigma^*\right)}{\partial \alpha_{\delta,i}} \\
= & \ 0.
\end{aligned}$$

We already know $\frac{\partial G\left(\alpha\left(\alpha_{c,i},\alpha_{\delta,i},\sigma^*\right),\alpha_{c,i},\alpha_{\delta,i},\sigma^*\right)}{\partial \alpha_{t,i}}>0$, and we can compute

$$\begin{aligned}
& \frac{\partial G\left(\alpha\left(\alpha_{c,i},\alpha_{\delta,i},\sigma^*\right),\alpha_{c,i},\alpha_{\delta,i},\sigma^*\right)}{\partial \alpha_{\delta,i}} \\
= & \ \sigma^*\cdot\frac{\partial^2}{\partial \alpha_{t,i}\partial \alpha_{\delta,i}} d\left(\left|\alpha\left(\alpha_{c,i},\alpha_{\delta,i},\sigma^*\right)-\alpha_{\delta,i}\right|\right) \\
= & -\sigma^*\cdot d''\left(\left|\alpha\left(\alpha_{c,i},\alpha_{\delta,i},\sigma^*\right)-\alpha_{\delta,i}\right|\right)<0,
\end{aligned}$$

so we have $\frac{\partial\alpha\left(\alpha_{c,i},\alpha_{\delta,i},\sigma^*\right)}{\partial\alpha_{\delta,i}} > 0$, as desired.

For the second part of the proposition, we'll want to compute $\frac{\partial\alpha\left(\alpha_{c,i},\alpha_{\delta,i},\sigma^*\right)}{\partial\sigma^*}$. For the same reason as the previous two parts, if $G\left(0,\alpha_{c,i},\alpha_{\delta,i},\sigma^*\right) > 0$, we immediately get $\frac{\partial\alpha_{t,i}(c,\delta,\sigma)}{\partial\sigma^*} = \frac{\partial\alpha\left(\alpha_{c,i},\alpha_{\delta,i},\sigma^*\right)}{\partial\sigma^*} = 0$, which means that

$$|\alpha_{t,i}(c,\delta,\sigma) - \alpha_{\delta,i}|$$

is (weakly) getting smaller. If $G\left(0,\alpha_{c,i},\alpha_{\delta,i},\sigma^*\right) \leq 0$, then we can just implicitly differentiate the equation $G\left(\alpha\left(\alpha_{c,i},\alpha_{\delta,i},\sigma^*\right),\alpha_{c,i},\alpha_{\delta,i},\sigma^*\right) = 0$ with respect to σ^* to get

$$\begin{aligned}
&\frac{\partial\alpha\left(\alpha_{c,i},\alpha_{\delta,i},\sigma^*\right)}{\partial\sigma^*}\frac{\partial G\left(\alpha\left(\alpha_{c,i},\alpha_{\delta,i},\sigma^*\right),\alpha_{c,i},\alpha_{\delta,i},\sigma^*\right)}{\partial\alpha_{t,i}} \\
&+\frac{\partial G\left(\alpha\left(\alpha_{c,i},\alpha_{\delta,i},\sigma^*\right),\alpha_{c,i},\alpha_{\delta,i},\sigma^*\right)}{\partial\sigma^*} \\
= \;& 0.
\end{aligned}$$

As before, we already know $\frac{\partial G\left(\alpha\left(\alpha_{c,i},\alpha_{\delta,i},\sigma^*\right),\alpha_{c,i},\alpha_{\delta,i},\sigma^*\right)}{\partial\alpha_{t,i}} > 0$, and we can compute

$$\begin{aligned}
&\frac{\partial G\left(\alpha\left(\alpha_{c,i},\alpha_{\delta,i},\sigma^*\right),\alpha_{c,i},\alpha_{\delta,i},\sigma^*\right)}{\partial\sigma^*} \\
= \;& d'\left(\left|\alpha\left(\alpha_{c,i},\alpha_{\delta,i},\sigma^*\right)-\alpha_{\delta,i}\right|\right)\cdot sign\left(\alpha\left(\alpha_{c,i},\alpha_{\delta,i},\sigma^*\right)-\alpha_{\delta,i}\right) \\
\propto \;& sign\left(\alpha\left(\alpha_{c,i},\alpha_{\delta,i},\sigma^*\right)-\alpha_{\delta,i}\right),
\end{aligned}$$

so we finally get that

$$\begin{aligned}
\frac{\partial\alpha_{t,i}(c,\delta,\sigma)}{\partial\sigma^*} &= \frac{\partial\alpha\left(\alpha_{c,i},\alpha_{\delta,i},\sigma^*\right)}{\partial\sigma^*} \propto -sign\left(\alpha\left(\alpha_{c,i},\alpha_{\delta,i},\sigma^*\right)-\alpha_{\delta,i}\right) \\
&= -sign\left(\alpha_{t,i}(c,\delta,\sigma)-\alpha_{\delta,i}\right).
\end{aligned}$$

This is equivalent to $|\alpha_{t,i}(c,\delta,\sigma) - \alpha_{\delta,i}|$ becoming smaller, as desired. ■

Proof of Proposition 5. First, we'll compute $\frac{\partial S(t,c,\delta|\sigma)}{\partial \alpha_{c,i}}$ where i is a choice feature. The envelope theorem says that we only need to evaluate the direct effect of $\alpha_{c,i}$ on $S(t,c,\delta|\sigma)$, so we get

$$\frac{\partial S(t,c,\delta|\sigma)}{\partial \alpha_{c,i}} = -\frac{\frac{\partial}{\partial \alpha_{c,i}} d(|\alpha_{t,i}(c,\delta,\sigma) - \alpha_{c,i}|)}{|M| + |M_K|}.$$

It therefore follows that

$$\begin{aligned}
\frac{\partial^2 S(t,c,\delta|\sigma)}{\partial \alpha_{\delta,i} \partial \alpha_{c,i}} &\propto -\frac{\partial^2 d(|\alpha_{t,i}(c,\delta,\sigma) - \alpha_{c,i}|)}{\partial \alpha_{c,i} \partial \alpha_{t,i}} \frac{\partial \alpha_{t,i}(c,\delta,\sigma)}{\partial \alpha_{\delta,i}} \\
&\propto d''(|\alpha_{t,i}(c,\delta,\sigma) - \alpha_{c,i}|) \frac{\partial \alpha_{t,i}(c,\delta,\sigma)}{\partial \alpha_{\delta,i}} \\
&\propto \frac{\partial \alpha_{t,i}(c,\delta,\sigma)}{\partial \alpha_{\delta,i}} \\
&\geq 0
\end{aligned}$$

where the last step follows from the first part of Proposition 4.

Next, if $\alpha_{\delta,i} = 1$, then

$$\begin{aligned}
\frac{\partial^2 S(t,c,\delta|\sigma)}{\partial \sigma_{\delta,i} \partial \alpha_{c,i}} &\propto -\frac{\partial^2 d(|\alpha_{t,i}(c,\delta,\sigma) - \alpha_{c,i}|)}{\partial \alpha_{c,i} \partial \alpha_{t,i}} \frac{\partial \alpha_{t,i}(c,\delta,\sigma)}{\partial \sigma_{\delta,i}} \\
&\propto \frac{\partial \alpha_{t,i}(c,\delta,\sigma)}{\partial \sigma_{\delta,i}} \\
&\propto -sign\left(\alpha_{t,i}(c,\delta,\sigma) - \alpha_{\delta,i}\right) \\
&\propto -sign\left(\alpha_{t,i}(c,\delta,\sigma) - 1\right) \\
&\geq 0
\end{aligned}$$

where the third step follows from the second part of Proposition 4. ■