

Bond Market Views of the Fed

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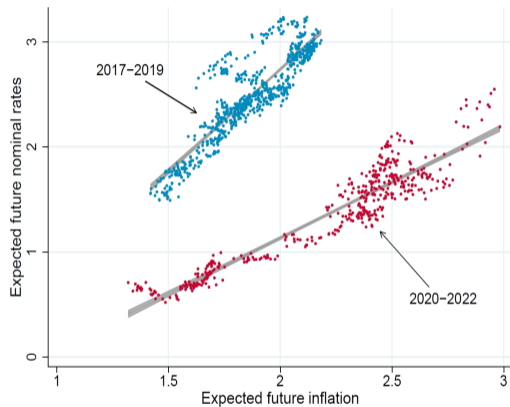
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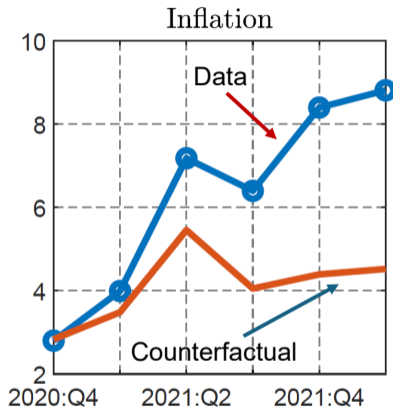
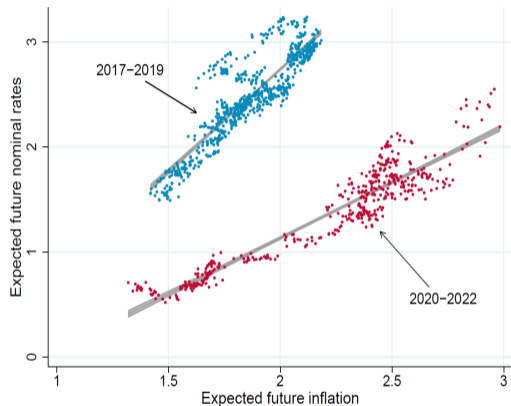
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 - 1 **Did the private sector change its view on the Fed's stance on inflation?**
 - 2 **To what extent this shift contributed to inflation dynamics?**
- This paper: answers these questions in two steps
 - 1 Use **bond market data** to detect **shifts in the Fed's policy**
 - 2 Combine these estimates with a NK model to **measure role of monetary policy**

Two main results



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- Detect a decline in the sensitivity of nominal rates to inflation at [0-5] yrs horizon
- Through the lens of a benchmark NK model, a policy shift of this magnitude has large effects on inflation

Outline

1 Conceptual framework and empirical results

2 Counterfactual analysis

This idea in a nutshell

Suppose the private sector thinks the monetary authority follows a Taylor rule

$$i_t = \rho_i i_{t-1} + (1 - \rho_i) \{i^* + \psi_\pi (\pi_t - \bar{\pi})\} + \varepsilon_t$$

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- **Want:** test for changes in ψ_π and measure expected **duration** of new "regime"
- **Issue:** only few years of realized data (all at the ZLB), endogeneity

This idea in a nutshell

Taking expectations in year k , we have

$$\mathbb{E}_t [i_k - \rho_i i_{k-1}] = c + (1 - \rho_i) \psi_\pi \mathbb{E}_t [\pi_k] + \mathbb{E}_t [\varepsilon_k]$$

- From bond prices we obtain daily signals on $\mathbb{E}_t [i_k]$ and $\mathbb{E}_t [\pi_k]$
- With expectations data vs. actual realizations
 - We have information even if economy is currently at the ZLB
 - We can exploit the **term structure** (vary k) to measure persistence of changes in ψ_π
 - We can address endogeneity (thanks to high-frequency nature of the data)

Empirical specification

Taking first differences wrt t (E.g., $\Delta\mathbb{E}_t[x_k] = \mathbb{E}_t[x_k] - \mathbb{E}_{t-1}[x_k]$), we obtain

$$\Delta\mathbb{E}_t[i_k - \rho_i i_{k-1}] = \psi_\pi \Delta\mathbb{E}_t[(1 - \rho_i)\pi_k] + \Delta\mathbb{E}_t[\varepsilon_{m,k}]$$

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- We assume that forecast revisions of future policy "shocks" are small, $\Delta \mathbb{E}_t[\varepsilon_{m,k}] \approx 0$
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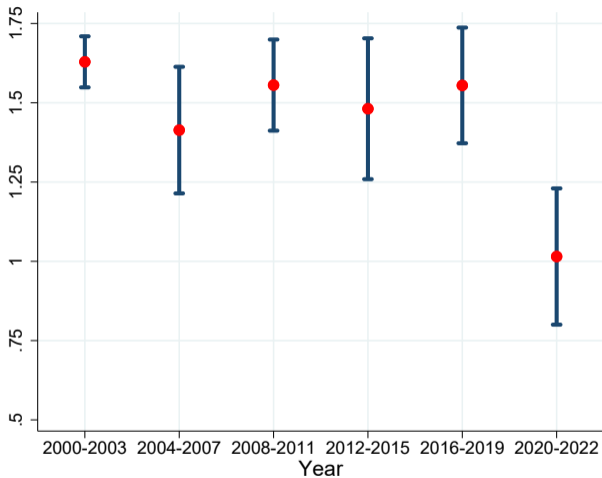
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- Yields on nominal and inflation-protected bonds to compute (risk-neutral) forecast revisions of nominal rates and inflation *on average* over the life of a bond [▶ Data](#)
- Fixing $\rho_i = 0.8$, we **test for the stability of ψ_π** by estimating

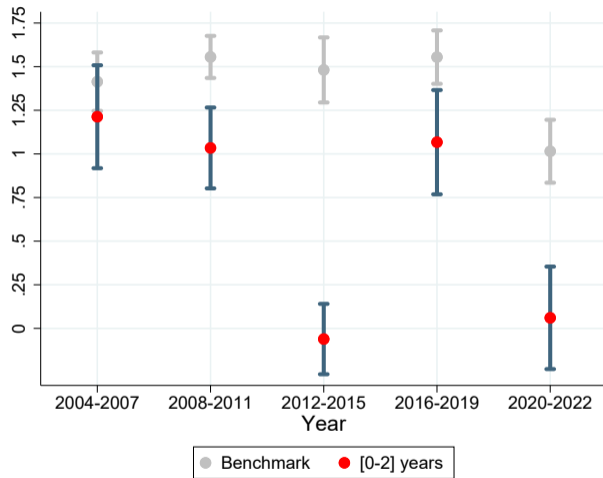
$$\Delta \mathbb{E}_t \left[\bar{i}_{t,k} - \left(\frac{k-1}{k} \right) \rho_i \bar{i}_{t,k-1} \right] = c + \sum_{s=1}^6 \psi_{\pi,s} (D_{t,s} \times \Delta \mathbb{E}_t [(1 - \rho_i)\bar{\pi}_{t,k}]) + e_t,$$

Estimating ψ_π across sub-samples, [0-10] years horizon



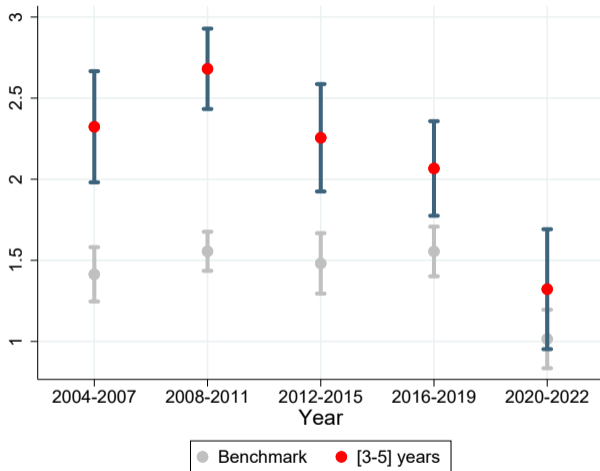
- ψ_π remarkably stable pre 2020, drops significantly after 2020

Estimating ψ_π across sub-samples, [0-2] years horizon



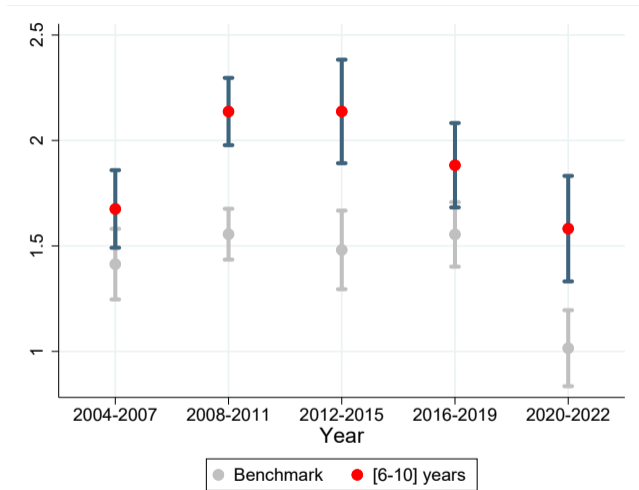
- At the [0-2] horizon, **smaller sensitivity post 2008** (possibly due to ZLB)

Estimating ψ_π across sub-samples, [3-5] years horizon



- Differently from previous years, **marked reduction in sensitivity [3-5] years out**

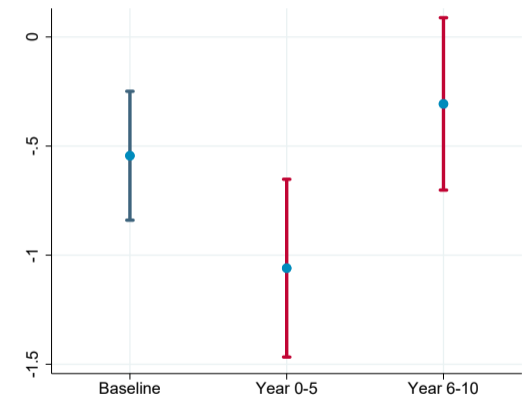
Estimating ψ_π across sub-samples, [6-10] years horizon



- Long-run sensitivity ([6-10] years out) more **"anchored"**

Pre 2020 vs. 2020-2022

$$\Delta \mathbb{E}_t \left[\bar{i}_{t,k} - \rho_i \frac{k-1}{k} \bar{i}_{t,k-1} \right] = c + \psi_\pi \Delta \mathbb{E}_t \left[(1 - \rho_i) \bar{\pi}_{t,k} \right] + d \left(D_t \times \Delta \mathbb{E}_t \left[(1 - \rho_i) \bar{\pi}_{t,k} \right] \right) + e_t,$$



- Significant reduction in interest rate sensitivity, stronger for shorter maturities

Sensitivity analysis

- 1 Presence of output gap leads to biased estimates of ψ_π [▶ Details](#)
 - Fed may seem more Dovish in 2020, but it was just responding to different shocks relative to the past (supply shocks)
 - We repeat the analysis conditioning on the **same type of shock** before and after 2020 (a **monetary shock**)
 - We obtain comparable results
- 2 At the zero lower bound, interest rates less responsive to inflation [▶ Details](#)
 - We control explicitly for the ZLB constraint and find comparable results
- 3 Risk premia and liquidity premia/convenience yields [▶ Details](#)
 - Take out risk premium on treasuries and TIPS
 - Risk-neutral expectations recovered from Swaps

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The economy in one slide

- Households have preferences

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \tilde{\theta}_t \left(\frac{c_t^{1-\sigma}}{1-\sigma} - \chi \frac{l_t^{1+\nu}}{1+\nu} \right) \right]$$

- Competitive final good firms use intermediates to produce final good

$$y_t = \left(\int_0^1 y_{i,t}^{\frac{1}{\mu_t}} di \right)^{\mu_t}$$

- Monopolistic competitive firms use labor to produce intermediate goods, $y_{i,t} = n_{i,t}$. They face quadratic adjustment costs when setting prices, $\frac{\phi}{2} \left(\frac{P_{i,t}}{P_{i,t-1}} \frac{1}{1+\bar{\pi}} - 1 \right)^2$
- Monetary authority follows Taylor rule with Markov switching regimes

$$1 + i_t = (1 + i_{t-1})^{\rho_i} \left\{ \bar{i} \left[\frac{1 + d(\xi_t)\pi_t + [1 - d(\xi_t)]\bar{\pi}_t}{1 + \pi^*} \right]^{\psi_\pi(\xi_t)} \left(\frac{y_t}{\bar{y}_t} \right)^{\psi_y} \right\}^{1-\rho_i} \exp \{ \sigma_m \varepsilon_{m,t} \}$$

where $\xi_t \in \{H(awk), D(ove)\}$ is a two-state Markov chain with transition matrix \mathbf{P} and $\bar{\pi}_t = \frac{1}{N} \sum_{j=0}^N \pi_{t-j}$

Parameters

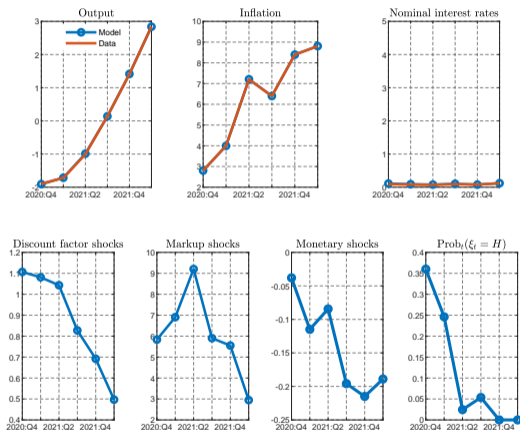
Panel A: Fixed parameters		
	Value	Notes
σ	1.000	Intertemporal elasticity of substitution of 1
ν	1.000	Frish elasticity of 1
χ	0.833	Normalize output to 1 in steady state
$\bar{\mu}$	1.200	20% markup in steady state
π^*	0.005	Inflation target of 2%
β	0.995	Annualized real interest rate of 2% in steady state
N	12.000	3 year horizon when averaging inflation in the D regime
P_{HH}	0.994	40 years expected duration of H regime

Panel B: Estimation of single regime model					
Parameter	Posterior mean	90% interval	Prior distribution	Prior mean	Prior st. dev.
ϕ	58.35	[39.94,75.97]	Gamma	80.00	10.00
$\psi_\pi(H)$	2.52	[2.09,2.95]	Normal	1.50	0.50
ψ_y	0.29	[0.18,0.39]	Normal	1.50	0.50
ρ_i	0.90	[0.87,0.92]	Beta	0.50	0.29
ρ_μ	0.83	[0.73,0.93]	Beta	0.50	0.29
ρ_θ	0.94	[0.92,0.97]	Beta	0.50	0.29
$\sigma_\mu \times 100$	2.67	[1.85,3.48]	InvGamma	1.00	Inf
$\sigma_\theta \times 100$	0.17	[0.14,0.20]	InvGamma	1.00	Inf
$\sigma_m \times 100$	0.18	[0.15,0.20]	InvGamma	1.00	Inf

Panel C: Parameters of Dovish rule		
	Value	Notes
$\psi_\pi(D)$	0.66	Point estimates of d , 1-5 yrs. Data: -1.00, Model: -1.00
P_{DD}	0.83	Point estimates of d , 6-10 yrs. Data: 0.00, Model: -0.03

Filtering

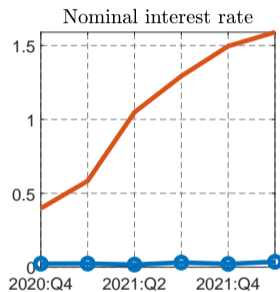
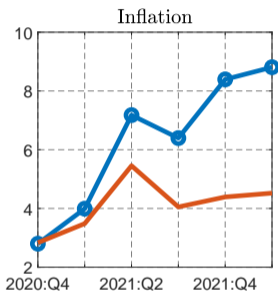
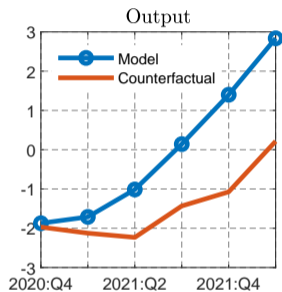
What realization of shocks do we need to fit the data post covid (2020:Q4-2022:Q1)?



- High markup shocks, increasing demand and a more Dovish monetary authority

Role of monetary policy during the pandemic

What if there was no shift in the policy regime post 2020?



- Without a shift in the policy regime, inflation would have peaked at 5%

Why is the change in the monetary rule so consequential?

Let's start from the log-linearized Phillips curve and Euler equation

$$\begin{aligned}\hat{\pi}_t &= \kappa(\hat{y}_t + \mu_t) + \beta\mathbb{E}_t[\hat{\pi}_{t+1}] \\ \hat{y}_t &= -\frac{1}{\sigma}(\hat{r}_t + \hat{\theta}_t) + \mathbb{E}_t[\hat{y}_{t+1}]\end{aligned}$$

Why is the change in the monetary rule so consequential?

Iterating forward these relationships we obtain

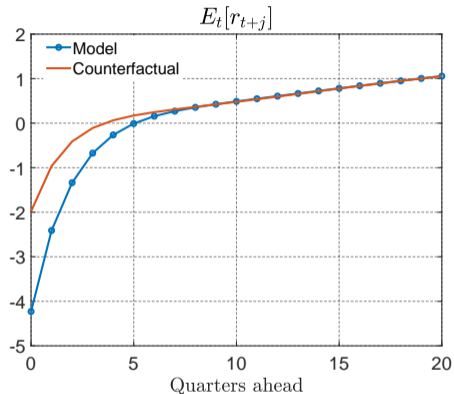
$$\begin{aligned}\hat{\pi}_t &= \kappa \sum_{j=0}^{\infty} \beta^j \mathbb{E}_t[\hat{y}_{t+j}] + \kappa \sum_{j=0}^{\infty} \beta^j \mathbb{E}_t[\hat{\mu}_{t+j}] \\ \hat{y}_t &= -\frac{1}{\sigma} \sum_{j=0}^{\infty} \mathbb{E}_t[\hat{r}_{t+j}] - \frac{1}{\sigma} \sum_{j=0}^{\infty} \mathbb{E}_t[\hat{\theta}_{t+1+j}]\end{aligned}$$

So, the difference between inflation across the two regimes is approximated by

$$\begin{aligned}\hat{\pi}_t^D - \hat{\pi}_t^H &= -\frac{\kappa}{\sigma} \left\{ (r_t^D - r_t^H) + (1 + \beta) [\mathbb{E}_t(r_{t+1} | \xi_t = D) - \mathbb{E}_t(r_{t+1} | \xi_t = H)] \right. \\ &\quad \left. + (1 + \beta + \beta^2) [\mathbb{E}_t(r_{t+2} | \xi_t = D) - \mathbb{E}_t(r_{t+2} | \xi_t = H)] + \dots \right\},\end{aligned}$$

Sensitivity of π_t to $\mathbb{E}_t[r_{t+j}]$ **increasing** in j . Due to "forward guidance puzzle" and forward-looking nature of inflation

Why is the change in the monetary rule so consequential?



- Difference in real rates across regimes not empirically implausible (comparable to the change in TIPS after the lift-off in March 2022)
- Large effects on inflation due to "forward guidance puzzle" (squared)

Conclusion

- Test for perceived shifts in the monetary policy rule using bond market data
- Evidence of a reduction in the sensitivity of nominal interest rates to inflation during the pandemic. Can plausibly be interpreted as a shift in the policy rule
- When coupled with an off-the-shelf NK model, this change in the policy regime has quantitatively important implications for the dynamics of inflation over this episode
- Shift in medium-term expectations, but long-run sensitivity "anchored"
 - Different in emerging markets like Brazil and Turkey
 - **Work in progress:** reputation model to account for cross-country differences in the degree of "anchoring" of long-run expectations

Literature

- Estimation of monetary policy rules: Clarida, Gali and Gertler (2000), De Bortoli, Gali and Gambetti (2020), [Hamilton, Pruitt and Borger \(2011\)](#), [Bauer, Pflueger and Sunderam \(2022\)](#)
 - We exploit high-frequency identification to test for a shift in the monetary policy rule
- High-frequency identification of monetary shocks: Kuttner (2001), Piazzesi and Swanson (2008), Gertler and Karadi (2015), Nakamura and Steinsson (2018), Bauer and Swansson (2023)
 - We use monetary events to identify shifts in the policy rule (rather than effects of shocks)
- Drivers of recent spikes in inflation: Gagliardoni and Gertler (2023), Comin, Johnson and Jones (2023), Ferrante, Graves and Iacoviello (2023), [Doh and Yang \(2023\)](#), Bianchi, Faccini, and Melosi (2023).
 - We detect shift in policy rule and assess the impact on recent inflation dynamics
- Macro effects of regime shifts in monetary policy: Bianchi (2013), Bianchi and Ilut (2017), [Bianchi, Lettau and Ludvigson \(2022\)](#), [Bianchi, Ludvigson and Ma \(2023\)](#)

The data

- Daily data on nominal and real (TIPS) yields on zero-coupon bonds (ZCBs) from Gurkaynak, Sack and Wright (2007, 2008). Main sample: 2000-2022
- Yields on ZCBs maturing in year k are linked to expectations of future short-term rate

$$i_t^{(k)} = \mathbb{E}_t^{\mathcal{Q}} \left[\frac{1}{k} \sum_{i=0}^{k-1} i_{t+i}^{(1)} \right] = \mathbb{E}_t [\bar{i}_k] + \text{term premium}_{k,t}$$

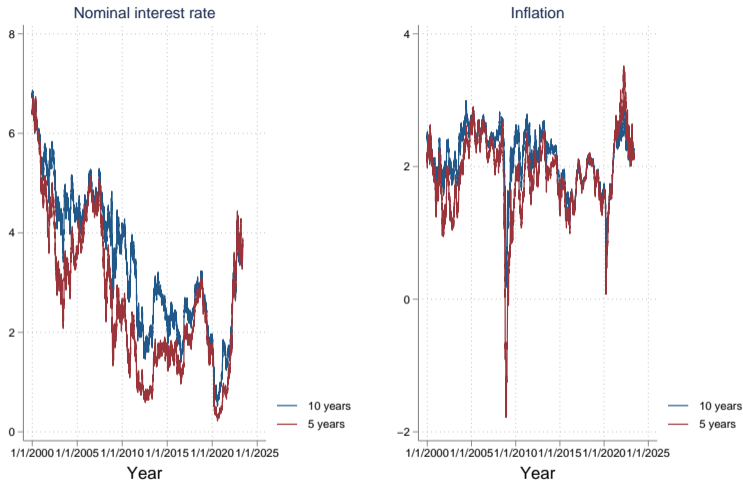
- Inflation compensation are linked to expectations of future inflation

$$IC_t^{(k)} = i_t^{(k)} - r_t^{(k)} = \mathbb{E}_t^{\mathcal{Q}} \left[\frac{1}{k} \sum_{i=1}^k \pi_{t+i} \right] = \mathbb{E}_t [\bar{\pi}_k] + \text{inflation risk premium}_{k,t}$$

- Use different maturities to obtain forward rates. E.g. expected inflation in year k is

$$\mathbb{E}_t^{\mathcal{Q}} [\pi_k] = (k - t) \times \mathbb{E}_t^{\mathcal{Q}} [\bar{\pi}_k] - (k - 1 - t) \times \mathbb{E}_t^{\mathcal{Q}} [\bar{\pi}_{k-1}]$$

The time path of risk-neutral expectations



- Liquidity premium during financial crises (TIPS not as liquid as treasuries). We exclude 2008 and 2020:M1-2020:M6 from the sample

Misspecification bias: Output gap

- Suppose conduct of monetary policy is described by

$$i_k = \rho_i i_{k-1} + (1 - \rho_i) \{i_k^* + \psi_\pi (\pi_k - \bar{\pi}) + \psi_y \tilde{y}_k\} + \varepsilon_k$$

- Taking expectations, first differencing and averaging across k as before

$$\Delta \mathbb{E}_t[\bar{i}_k - \rho_i \bar{i}_{k-1}] = (1 - \rho_i) \psi_\pi \Delta \mathbb{E}_t[\bar{\pi}_k] + (1 - \rho_i) \psi_y \Delta \mathbb{E}_t[\bar{\tilde{y}}_k]$$

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- OLS does not identify ψ_π but

$$\hat{\psi}_\pi^{\text{OLS}} \rightarrow \psi_\pi + \psi_y \frac{\text{Cov}(\Delta \mathbb{E}_t[\bar{\pi}_k], \Delta \mathbb{E}_t[\tilde{y}_k])}{\text{Var}(\Delta \mathbb{E}_t[\bar{\pi}_k])}$$

Detecting a structural break in the policy rule

$$\hat{\psi}_{\pi}^{\text{OLS}} \rightarrow \psi_{\pi} + \psi_y \frac{\text{Cov}(\Delta \mathbb{E}_t[\bar{\pi}_k], \Delta \mathbb{E}_t[\tilde{y}_k])}{\text{Var}(\Delta \mathbb{E}_t[\bar{\pi}_k])}$$

- A reduction in $\hat{\psi}_{\psi}^{\text{OLS}}$ between two sub-samples could signal two things
 - A shift in the policy rule
 - A change in the type of shocks the Fed is facing (E.g. larger **supply shocks** than before)

Detecting a structural break in the policy rule

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- A reduction in $\hat{\psi}_{\psi}^{\text{OLS}}$ between two sub-samples could signal two things
 - A shift in the policy rule
 - A change in the type of shocks the Fed is facing (E.g. larger **supply shocks** than before)
- **Our approach:** test for a break in ψ_{π} **conditioning on the same type of shock**
 - Forecasts updates around "monetary events". Regress $\Delta \mathbb{E}_t^m[\bar{i}_k - \rho \bar{i}_{k-1}]$ on $\Delta \mathbb{E}_t^m[(1 - \rho_i)\bar{\pi}_k]$ and test for a structural break in ψ_{π} in 2020
 - Assumption: **conditional** correlation between inflation and the output gap constant across sub-samples **under the null hypothesis of no shift in policy**

The logic of the test in the 3-equations NK model

The bias in the 3-equations NK model

Consider the log-linearized 3-equations NK model. Then

$$\hat{\psi}_{\pi}^{\text{OLS,m}} \rightarrow \psi_{\pi} + \psi_y \frac{1 - \beta \rho_y}{\kappa},$$

where κ is the slope of the Phillips curve, β is the rate of time preference and ρ_y solves

$$\rho_y = \left[\rho_i + \frac{\sigma \rho_y}{\rho_y - \left[1 - \sigma \kappa \left(\frac{\rho_y}{1 - \beta \rho_y} \right) \right]} (1 - \rho_i) \left(\psi_y + \psi_{\pi} \frac{\kappa}{1 - \beta \rho_y} \right) \right]$$

- Under the null hypothesis of no change in the policy rule, the asymptotic bias of $\hat{\psi}_{\pi}^{\text{OLS,m}}$ is constant across sub-samples as long as (κ, σ, β) are constant
- A reduction in $\hat{\psi}_{\pi}^{\text{OLS,m}}$ across the two sub-samples indicates a reduction in ψ_{π} (ρ_y not sensitive to ψ_{π} in standard calibrations)

Results

$$\Delta \mathbb{E}_t[\bar{i}_k - \rho_i \bar{i}_{k-1}] = a + \psi_\pi \Delta \mathbb{E}_t[(1 - \rho_i) \bar{\pi}_k] + d (\Delta \mathbb{E}_t[(1 - \rho_i) \bar{\pi}_k] \times D_{2020:M8}) + \eta_t$$

	(1) Baseline	(2) Monetary events	(3) Risk premia	(4) ZLB
d (10 year avg)	-0.54*** (0.11)	-0.88** (0.43)	-0.27 (0.19)	-0.35* (0.20)
d (5 year avg, 1-5)	-1.05*** (0.15)	-1.94*** (0.50)	-1.05*** (0.12)	-1.25*** (0.27)
d (5 year avg, 6-10)	-0.13 (0.10)	-0.26 (0.30)	0.16*** (0.04)	0.19 (0.14)
N. obs.	4019	455	4019	4019

- Results hold even when conditioning on the same type of "shock"

Controlling for a binding ZLB

Suppose interest rates follow the process

$$\begin{aligned}\hat{i}_k &= \rho_i i_{k-1} + (1 - \rho_i) \{i_k^* + \psi_\pi(\pi_k - \bar{\pi}) + \psi_y \tilde{y}_t\} + \varepsilon_k \\ i_k &= \max \{ \hat{i}_k, 0 \}\end{aligned}$$

If $\varepsilon_k | \mathcal{I}^t \sim \mathcal{N}(0, \sigma_\varepsilon)$, then we have

$$\mathbb{E}_t[i_k] = \rho_i \mathbb{E}_t[i_{k-1}] + (1 - \rho_i) \{ \mathbb{E}_t[i_k^*] + \psi_\pi \mathbb{E}_t[\pi_k - \bar{\pi}] + \psi_y \mathbb{E}_t[\tilde{y}_t] \} + \underbrace{\frac{\varphi\left(\frac{\mathbb{E}_t[\hat{i}_k]}{\sigma}\right)}{1 - \Phi\left(\frac{\mathbb{E}_t[\hat{i}_k]}{\sigma}\right)} \sigma}_f$$

Approximate the expression around $\mathbb{E}_t[i_{k-1}] = \bar{i}_{k-1}$, $\mathbb{E}_t[i_k^*] = i^*$, $\mathbb{E}_t[\pi_k] = \bar{\pi}$, $\mathbb{E}_t[\tilde{y}_k] = 0$

$$\mathbb{E}_t \left[i_k - \rho_i \left(1 + \frac{1}{\sigma} f' \right) i_{k-1} \right] = (1 - \rho_i) \left(1 + \frac{1}{\sigma} f' \right) \{ \mathbb{E}_t[i_k^*] + \psi_\pi \mathbb{E}_t[\pi_k - \bar{\pi}] + \psi_y \mathbb{E}_t[\tilde{y}_t] \}$$

Controlling for a binding ZLB

For each k and sub-period s , we construct $\{f'_k\}$ by setting:

- $i^* \Rightarrow$ sample average of the Laubach-Williams series in each sub-period s
- $\bar{i}_{k-1} \Rightarrow$ sample average of $\mathbb{E}_t^Q[i_{k-1}]$ in each sub-period s
- We set $\sigma = 0.03$, a fairly conservative value for this exercise

We then perform our analysis using the following equation

$$\Delta \mathbb{E}_t[\bar{i}_k] - \frac{1}{10} \sum_{k=1}^{10} \rho_i \left(1 + \frac{1}{\sigma} f'_k\right) \Delta \mathbb{E}_t[i_{k-1}] = \psi_\pi \frac{1}{10} \sum_{k=2}^{10} \rho_i \left(1 + \frac{1}{\sigma} f'_k\right) \Delta \mathbb{E}_t[(1 - \rho_i)\pi_k] + \eta_t$$

Controlling for risk premia

- Most asset pricing models predict expected inflation, inflation compensation and nominal bond yields to functions of the same underlying factors X_t
- To approximate the relation between expected inflation and these factors, we estimate the following relation

$$SPF_t^{(n)} = \beta_{n,0} + \beta_{n,1}IC_t^{2y} + \beta_{n,2}IC_t^{5y} + \beta_{n,3}IC_t^{10y} + \beta_{n,4}i_t^{2y} + u_t^{(n)}$$

over rolling sub-samples, where $SPF_t^{(n)}$ is the average inflation expectation at horizon n in the Survey of Professional Forecasters. We use the estimated β 's to construct daily inflation expectations

- We use the Fed board term structure model to infer $\mathbb{E}_t[\bar{i}_k]$
- We repeat our analysis with $\{\mathbb{E}_t[\bar{i}_k], \mathbb{E}_t[\bar{\pi}_k]\}$.

Repeating the analysis using swaps

- Nominal and real treasuries may have different liquidity/convenience properties
- We repeat the analysis constructing expected inflation and nominal interest rates using
 - Overnight Index Swaps (OIS) tied to the federal funds rate
 - Inflation-Linked Swaps (ILS)
- Data limitations: need to start in 2005 and focus on the 5 years horizon

