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The price and the quantity of bank money*

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Abstract

Banks create money when they lend. We ask what determines its price and quantity in an economy without fiat money, reserve requirements, or a central bank. We study an overlapping-generations economy in which, by assumption, loans are nominal and settled in deposits, which banks create at no cost. Deposits have value without a bubble, derived from the present value of their repayment services. Their quantity is finite: loan demand limits the deposits created by lending, and net worth limits those created by dividends. Because loans are nominal, price expectations are self-fulfilling. Under aggregate risk, equilibria differ in real allocations and welfare. In equilibria without bubbles, banking implements exactly the allocations of one-period securities, and expectations select among them. The loss from a bad selection is unbounded. A fixed interest rate cannot select among equilibria, but loan contracts indexed to the price level select a unique allocation, that of real bonds.

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1 Introduction

Most of the money in a modern economy consists of bank deposits. Banks create deposits when they lend, and deposits are destroyed when loans are repaid (McLeay et al., 2014; ECB, 2015; Bundesbank, 2017; Nicolaisen, 2017; Doherty et al., 2018; Borio, 2019). Thakor and Yu (2024) estimate that deposits created by bank lending averaged 92% of all deposits in the US between 2001 and 2010. Because most deposits are created by lending, two questions arise: what is a deposit worth, and what limits the quantity of deposits? This paper addresses these questions in the simplest general equilibrium model in which we could pose them.

We do not explain why bank money exists, or why private IOUs do not circulate. That is an important question, studied by a large literature (Kiyotaki and Wright, 1989; Lagos and Wright, 2005; Kocherlakota, 1998; Diamond, 1984; Gorton and Pennacchi, 1990; Cavalcanti and Wallace, 1999; Williamson, 1999; Gu et al., 2013; Kiyotaki and Moore, 2019), and ours is complementary to it. We take as given two features of observed monetary systems: bank loans are nominal, and they are settled in bank deposits. Nobody repays a mortgage with apples. We state these features as an assumption (Assumption 1) and ask what they imply, in a general equilibrium with rational expectations, for the price and the quantity of money. In this respect our approach is close to Starr (1974), in which an obligation denominated in money such as taxes, creates a demand for it, and to the legal-restrictions theory of Wallace (1983). Mitchell-Innes (1914) argued that all money, whether issued by governments or by banks, is credit, and that it is valued because it can be used to discharge debts owed to its issuer, taxes in the case of government money. The difference is that here both the obligation and the stock of money are equilibrium outcomes: borrowers choose how much they owe, and the bank chooses how many deposits to create.

The model is as simple as we could make it. Someone needs to borrow because they have nothing now and will repay later; the simplest such agent is born without an endowment. Someone must produce what this agent consumes, the middle aged, who are the only ones with an endowment. The middle aged need a reason to hold deposits and bank shares beyond repaying their own loans, and the bank needs shareholders to receive its dividends. A third period of life, in which agents consume what they save, supplies both. Agents therefore live three periods in overlapping generations. Borrowers may not survive to repay, so lending is risky. The bank is a firm owned by its shareholders, it creates deposits at no cost when it lends, destroys them when loans are repaid, pays dividends in

deposits, and maximizes its value. There is no fiat money, no reserve requirement, and no central bank.

The price of money.—A deposit is worth what a borrower must give up to obtain it and repay a loan. Middle-aged agents also hold deposits to save, but only because future generations will need deposits to repay their loans. We show that, in the equilibria we study, the value of a deposit is exactly the present value of its repayment services, and that this value is the same whether the deposit repays a loan today or after circulating for any number of periods. Money therefore has value without a bubble, given that loans must be settled in deposits. Without aggregate risk, every equilibrium has this property. With aggregate risk, there are also equilibria in which the bank's stock has a bubble, and we restrict attention to those in which it does not.

The quantity of money.—Although deposits cost nothing to produce and no reserves are required, their equilibrium quantity is finite. Deposits created by lending are limited by the demand for loans, since borrowing more today means consuming less tomorrow. Deposits created by dividends are limited by the bank's net worth, since the bank cannot pay out more than it has. Dividends play a role that is easy to overlook. A bank that lends 100 and is owed 110 creates only 100 deposits when it lends. The interest raises the bank's net worth, and the bank pays it out as a dividend of 10 deposits; the shareholders spend them, and borrowers receive them when they sell their endowment, which is how they pay the interest. How the bank sets dividends does not affect real allocations, but dividends are what supply the deposits that make repayment feasible.

Expectations and welfare.—Because loans are nominal, how much the young borrow depends on the prices they expect when their loans come due, and these prices depend on how much the next generation borrows. With aggregate risk, such expectations are self-fulfilling, and there is a continuum of equilibria that differ in real allocations, not only in the price level. To understand this multiplicity, we solve the same economy with one-period securities instead of a bank. Banking turns out not to be exotic: the consumption allocations of banking equilibria are exactly those that one-period securities can implement. The difference is that complete Arrow securities, or a riskless real bond, implement a single allocation, whereas banking implements all of them, and expectations select which one. This selection matters, because some equilibria are arbitrarily bad. A fixed nominal interest rate cannot select among them, but loan contracts indexed to the price level select a unique allocation, that of real bonds.

The view that banks create money when they lend is sometimes seen as at odds

with standard monetary theory. Our results suggest that it is not. In a standard general equilibrium model with rational expectations, bank money has value without a bubble, its quantity is finite, and banking implements the same allocations as one-period securities. What banking adds is that expectations select among equilibria.

To summarize what is assumed and what is derived: we assume that loans are nominal, that they are settled in deposits, and that only the bank lends; we also restrict attention to equilibria without bubbles. We derive that deposits are worth the present value of their repayment services, that their quantity is finite, that real allocations are indeterminate under aggregate risk, and that banking allocations coincide with those of one-period securities.

The idea that loans create deposits has a long history in economics (Wicksell, 1936; Keynes, 1930; Schumpeter, 1954). Wicksell studied a "pure credit economy," in which all payments are transfers between bank accounts and banks lend whatever is demanded from them. Black (1970) surveyed the answers that had been given for an economy with uncontrolled banking: the price level would rise without limit, money would become worthless, or the price level would be indeterminate while the real side of the economy would be independent of the financial side. Our answers differ from all three. Several recent papers have revived this tradition. Andolfatto (2018) introduces an overlapping generations model that encompasses the loanable-funds and the loans-create-deposits views as special cases; his bank, however, holds government debt as reserves, so its deposits ultimately rest on outside money. Donaldson et al. (2018) explain the historical evolution of banks from warehousing. Jakab and Kumhof (2019) study financing through money creation in a New Keynesian model. Faure and Gersbach (2022) show that, without uncertainty, the money-creation and the loanable-funds approaches to banking yield the same allocation, and indicate that a restricted equivalence holds with aggregate risk when contracts and markets are complete. We study the case of nominal, non-contingent loans, where markets are incomplete, and take trade in one-period securities as the counterpart of loanable funds. Banking then implements exactly the allocations that one-period securities implement (Proposition 3). Benes and Kumhof (2012) study the opposite arrangement, the Chicago Plan, in which deposits must be fully backed by government money. Brunnermeier and Niepelt (2019) take the existence and value of inside money as given and ask when public and private money are equivalent. Bianchi and Bigio (2022) study the transmission of monetary policy through the interbank market. Skeie (2008) studies nominal deposits repaid in inside money with no reserves, but his unit of account is established by a central

bank at the initial date. In our economy, the unit of account is the deposit itself, and the bank's only assets are its loans.

The paper is organized as follows. Section 2 describes the model and defines the equilibrium. Section 3 characterizes the equilibria, compares banking with one-period securities, and studies welfare, policy, and dividends. Section 4 discusses the price and the quantity of money. Section 5 concludes. All proofs are in Appendix A.

2 Model

We employ a stochastic variant of Samuelson (1958). It is a dynamic, pure exchange, OLG economy in which agents live for three periods. They receive an endowment only when middle aged, which creates a need for intergenerational transfers. Our three-period structure deliberately inverts the logic of Samuelson: in his model, the young are endowed and the problem is how to transfer resources to the future; here, the young have nothing and must borrow, while the middle-aged receive an endowment and must save.

2.1 The economy

Time is discrete and extends in both directions. At any date $t \in \mathbb{Z}$ there are three generations: The young, the middle aged, and the old. At any t, a unit mass of young agents is born, and each of them survives into middle age with probability s_{t+1}. Middle-aged agents survive into old age with probability one and receive an endowment of y units of consumption, whereas young and old agents do not. In summary, at any date t there is a mass of size one of young agents, a mass of size s_t of middle-aged agents, a mass of size s_{t-1} of old agents, and $s_t y$ units of consumption.

We assume that s_t is a random variable, drawn independently from a finite set S with full support, probability $\Pr(s_t)$, and mean $\bar{s}$. We define a state as a history of realizations of the survival probability. Thus, a *state* is an element $s = (s_t, s_{t-1}, \dots)$ of $S^{\mathbb{N}}$, where $S^{\mathbb{N}}$ represents the set of all sequences in S. A *node* is a state-date pair $s^t = (s, t)$. Node s^t has a unique immediate predecessor, which we denote $s^t - 1$. The notation $s^{t+j}|s^t$, with $j \geq 1$, indicates that $s^t = s^{t+j} - j$, that is, node s^t is the unique j^{th} predecessor of node s^{t+j}. We write $\mathbb{E}_t[x(s^{t+j})] \equiv \sum_{s^{t+j}|s^t} x(s^{t+j}) \Pr(s^{t+j}|s^t)$ for the expectation conditional on node s^t.

Agents have separable log preferences over consumption. A consumption allocation

$c(s^t) = (c_1(s^t), c_2(s^t), c_3(s^t))$ for young, middle-aged, and old agents is feasible *iff*

$$c_1(s^t) + s_t c_2(s^t) + s_{t-1} c_3(s^t) = s_t y \tag{1}$$

for all s^t.

2.2 Banking

Young agents need to borrow in order to consume. A bank is a public company that produces and destroys deposits, both at zero cost. It lends deposits, and its loans are repaid in deposits. We take the following features of observed monetary systems as given.

Assumption 1. (a) Young agents cannot issue IOUs; they can borrow only from the bank, which can appropriate the endowment of a debtor in arrears. (b) Loans are denominated in deposits and can be settled only in deposits.

We do not explain why these features hold. Part (b) describes how bank loans are settled in practice (McLeay et al., 2014). Our deposits differ from those of Diamond and Dybvig (1983) and Calomiris and Kahn (1991), whose defining feature is that they can be redeemed at par: ours cannot be redeemed at the bank for anything else. We keep the word because they are created by lending, destroyed by repayment, and are the unit of account, and none of our results depend on redeemability. Because there is no conversion right, bank runs are outside the model rather than ruled out by it. Donaldson and Piacentino (2022) show that bank debt that circulates as money can itself be run-prone, while Skeie (2008) shows that, with nominal deposits and no reserves, maturity mismatch alone need not generate runs. We take no position between them.

At each node, the bank loans $L(s^t)$ deposits to borrowers, to whom it charges a nominal gross interest rate $R(s^t)$ due in $t+1$.[1] It also pays out $D(s^t)$ deposits as dividends to its shareholders, and collects $s_t R(s^t - 1)L(s^t - 1)$ deposits from its debtors. Hence, at each node the bank produces $L(s^t) + D(s^t)$ and destroys $s_t R(s^t - 1)L(s^t - 1)$ deposits. The law of motion of deposits is

$$B(s^t) = B(s^t - 1) + \underbrace{L(s^t) + D(s^t)}_{\substack{\text{deposits} \\ \text{produced}}} - \underbrace{s_t R(s^t - 1)L(s^t - 1)}_{\text{deposits destroyed}}. \tag{2}$$

[1] Appendix B below shows that real allocations are the same if the bank charges the state-contingent rate $R(s_{t+1}, s^t) = 1/s_{t+1}$ due in $t+1$.

Assets	*Liabilities*
$s_t R(s^t-1)L(s^t-1)$	$B(s^t-1)$
	$X(s^t)$

Assets	*Liabilities*
$s_t R(s^t-1)L(s^t-1)$	$B(s^t-1)$
$L(s^t)$	$L(s^t)$
	$D(s^t)$
	$X(s^t)-D(s^t)$

Assets	*Liabilities*
$L(s^t)$	$B(s^t)$
	$N(s^t)$

Figure 1: Left: Beginning-of-period bank's balance sheet. Center: Mid-period bank's balance sheet. Right: End-of-period bank's balance sheet.

This equation summarizes the deposit dynamics in our model and is the nominal equivalent to the resource constraint in (1).

The bank earns interest revenue from loan repayments and loses assets from loan defaults due to young agents not surviving to middle age. The net income of the bank is

$$I(s^t) \equiv \underbrace{s_t[R(s^t-1)-1]L(s^t-1)}_{\text{interest revenue}} - \underbrace{(1-s_t)L(s^t-1)}_{\substack{\text{impairment}\\ \text{provision}}}.$$

This quantity represents a flow of profits that the bank may either pay out or retain. Retained earnings add to the end-of-period net worth of the bank, which follows the law of motion

$$N(s^t) = N(s^t-1) + \underbrace{I(s^t) - D(s^t)}_{\substack{\text{retained}\\ \text{earnings}}}. \tag{3}$$

As usual, net worth is defined as assets minus liabilities. Beginning-of-period net worth is $X(s^t) \equiv s_t R(s^t-1)L(s^t-1) - B(s^t-1)$ and end-of-period net worth is $N(s^t) \equiv L(s^t) - B(s^t)$, with the two related through $N(s^t) = X(s^t) - D(s^t)$.

Figure 1 illustrates the changes that take place in the bank's balance sheet within a period. The timeline is as follows:

1. At the beginning of the period (left), the bank holds claims on repayments from its surviving debtors and is liable against its outstanding deposits.

2. In the middle of the period (center), it loans deposits, which represent an asset (the loan) as well as a liability (the deposit). It pays out dividends in deposits, which also represent a liability.

3. At the end of the period (right), once the bank has issued loans, paid out dividends, and collected repayments, its end-of-period net worth is the difference between its assets (loans) and liabilities (deposits outstanding).

We assume that there are many identical banks operating in a perfectly competitive market with free entry and exit. Banks take the interest rate as given and the product is homogeneous; i.e. banks accept other banks' deposits at par. This abstracts from settlement between banks, which Bianchi and Bigio (2022) study explicitly. Henceforth, we will focus on a representative bank, which we will refer to as "the bank".

2.3 Choice problems

2.3.1 Agents

At each node, young agents borrow $L_1(s^t)$ deposits from the bank. They use these deposits to buy $c_1(s^t)$ units of consumption at price $p(s^t)$ from middle-aged agents. Each middle-aged agent consumes $c_2(s^t)$ units and trades the remaining $y - c_2(s^t)$ units with the young and the old for deposits and bank shares. Then he repays $R(s^t - 1)L_1(s^t - 1)$ deposits and saves the rest in $B_2(s^t)$ deposits and $z_2(s^t)$ shares, whose unit price is $q(s^t)$. Every old agent holds $B_2(s^t - 1)$ deposits and collects $D(s^t)z_2(s^t - 1)$ as dividends. He uses his deposits and shares to buy $c_3(s^t)$ units of consumption. As such, the budget constraints of the young, the middle aged, and the old are:

$$\begin{aligned} p(s^t)c_1(s^t) &= L_1(s^t), \\ p(s^t)c_2(s^t) + B_2(s^t) + q(s^t)z_2(s^t) &= p(s^t)y - R(s^t - 1)L_1(s^t - 1), \\ p(s^t)c_3(s^t) &= B_2(s^t - 1) + [D(s^t) + q(s^t)]z_2(s^t - 1). \end{aligned}$$

Old agents consume all their wealth. Middle-aged agents choose a portfolio of deposits and shares given prices, dividends, and their real disposable income $w_2(s^t) \equiv y - p(s^t)^{-1}R(s^t - 1)L_1(s^t - 1)$. Their value function is

$$\begin{aligned} V_2\left(w_2(s^t), s^t\right) = \max_{B_2, z_2} \log\left(w_2(s^t) - \frac{B_2 + q(s^t)z_2}{p(s^t)}\right) \\ + \mathbb{E}_t\left[\log\left(\frac{B_2 + [D(s^{t+1}) + q(s^{t+1})]z_2}{p(s^{t+1})}\right)\right]. \end{aligned}$$

Young agents choose how much to borrow given prices and the interest rate. Their optimization problem is

$$\max_{L_1} \log\frac{L_1}{p(s^t)} + \mathbb{E}_t\left[s_{t+1}V_2\left(y - \frac{R(s^t)L_1}{p(s^{t+1})}, s^{t+1}\right)\right].$$

In terms of consumption, the banking arrangement allows for the middle aged to lend to the young and repay the old for previous loans. In essence, the bank facilitates intergenerational transfers of consumption that are not possible in the absence of IOUs.

2.3.2 The Bank

At each node, the bank chooses how many deposits to produce. Some of these, $L(s^t)$, it loans to the young, and some, $D(s^t)$, it pays out to the old. The bank can produce an unlimited amount of deposits, but it can pay dividends only up to its beginning-of-period net worth:

$$D(s^t) \leq X(s^t). \tag{4}$$

This inequality is equivalent to $N(s^t) \geq 0$, a legal solvency constraint. If the beginning-of-period net worth is negative, which may happen after a low realization of s_t, the bank charges a negative recapitalization dividend to fulfill (4). Negative dividends serve only this purpose: $D(s^t) \geq \min\{X(s^t), 0\}$.

Assumption 2. Shareholders have unlimited liability: whenever $X(s^t) < 0$, they pay the recapitalization dividend $D(s^t) = X(s^t)$.

We assume that the bank maximizes firm value; i.e. cum-dividend stock price $q(s^t) + D(s^t)$, which maximizes shareholder wealth. To implement this program, the bank needs a model—an asset pricing theory—that explains how its loan and dividend policies affect its stock price. The dividend discount model is a natural candidate for such a theory. Adapted to our setting, it says that the price of a stock equals the present value of dividends under the appropriate stochastic discount factor (SDF).

At each node, the bank follows a policy (L, D) that maximizes the present value of dividends, taking as given its beginning-of-period net worth, prices, and a stochastic discount factor m_2. Formally, the bank solves

$$\max_{L,D} \frac{D(s^t)}{p(s^t)} + \mathbb{E}_t \left[\sum_{k=1}^{\infty} \left(\prod_{j=1}^{k} m_2(s^{t+j}) \right) \frac{D(s^{t+k})}{p(s^{t+k})} \right]$$

subject to $\min\{X(s^{t+k}), 0\} \leq D(s^{t+k}) \leq X(s^{t+k})$ at every node $s^{t+k}|s^t$, $k \geq 0$, where net worth evolves according to (3). The stochastic discount factor $m_2(s^{t+1}) = c_2(s^t)/c_3(s^{t+1})$ is the ratio of the marginal utilities of consumption of the old in $t+1$ and the middle aged

in t. This is the appropriate SDF for the bank because only the middle aged buy stocks and only the old receive dividends.

2.4 Equilibrium definition

Three prices are determined in equilibrium: The loan rate $R(s^t)$, the consumption price $p(s^t)$, and the stock price $q(s^t)$. Normalizing the total supply of bank shares to one, the market clearing conditions are as follows:

$$L(s^t) = L_1(s^t), \tag{5}$$

$$B(s^t) = s_t B_2(s^t), \tag{6}$$

$$1 = s_t z_2(s^t). \tag{7}$$

Up to this point, nothing guarantees that there will be enough deposits in circulation to repay all loans. The following condition ensures that there are sufficient deposits in equilibrium:

$$\underbrace{B(s^t - 1) + L(s^t) + D(s^t)}_{\text{deposits in circulation}} \geq \underbrace{s_t R(s^t - 1) L(s^t - 1)}_{\substack{\text{repayments} \\ \text{(deposits destroyed)}}}. \tag{8}$$

Because of (2), this is equivalent to $B(s^t) \geq 0$. This condition is an assumption. It rules out a shortage of deposits that would leave solvent debtors unable to repay. The bank is indifferent among many dividend policies, some of which satisfy this condition (Section 3). The condition therefore selects among optimal choices; it does not force a suboptimal one.[2] Agents, in turn, demand sufficient deposits to repay, because the bank can appropriate the endowment of a debtor in arrears (Assumption 1). Our definition of equilibrium is otherwise standard.

Definition 1. A competitive banking equilibrium is a collection of prices: $R(s^t)$, $p(s^t)$, $q(s^t)$, bank's choices: $L(s^t)$, $D(s^t)$, and agents' choices: $L_1(s^t)$, $B_2(s^t)$, $z_2(s^t)$ such that, for all nodes: (i) Bank's choices are optimal given beginning-of-period net worth, prices, and the stochastic discount factor; (ii) Agents' choices are optimal given prices and dividends; (iii) $B(s^t) \geq 0$ holds; (iv) Markets clear.

[2]The bank can increase the amount of deposits in circulation by paying more dividends to the old.

2.5 The golden rule

We complete our model description by characterizing the golden rule allocation. At any node s^t, the tuple (s_t, s_{t-1}) contains all the necessary information about the physical condition of the economy: The number of agents in each generation and the total endowment available. Denote by φ_1, φ_2, and φ_3 the golden rule consumption functions that maximize expected lifetime utility,

$$\mathbb{E}\left[\log \varphi_1(s_t, s_{t-1}) + s_{t+1}\left\{\log \varphi_2(s_{t+1}, s_t) + \log \varphi_3(s_{t+2}, s_{t+1})\right\}\right], \tag{9}$$

subject to

$$\varphi_1(s_t, s_{t-1}) + s_t\varphi_2(s_t, s_{t-1}) + s_{t-1}\varphi_3(s_t, s_{t-1}) = s_t y$$

for all (s_t, s_{t-1}). Then the following holds.

Lemma 1. *The golden rule allocation is*

$$\varphi_1(s_t, s_{t-1}) = \varphi_2(s_t, s_{t-1}) = \varphi_3(s_t, s_{t-1}) = \frac{s_t y}{1 + s_t + s_{t-1}}. \tag{10}$$

for all (s_t, s_{t-1}).

Lemma 1 formally states that the most preferred allocation of a social planner who puts equal weights on all agents allocates the same level of consumption to each of them.

3 Results

In this section we solve the choice problems of agents and the bank. We then characterize the resulting equilibria, first without aggregate risk and then with aggregate risk. Next, we compare banking with trade in one-period-maturity securities and study welfare and equilibrium selection. We close the section with the bank's dividend policy.

3.1 Choice problems

3.1.1 Agents

The problems of the agents are concave, and the first-order conditions (FOCs) are necessary and sufficient. The FOCs of the middle aged are:

$$1 = \mathbb{E}_t \left[\frac{m_2(s^{t+1})}{\pi(s^{t+1})} \right], \tag{11}$$

$$q(s^t) = \mathbb{E}_t \left[\frac{m_2(s^{t+1})}{\pi(s^{t+1})} \left(D(s^{t+1}) + q(s^{t+1}) \right) \right], \tag{12}$$

where $\pi(s^{t+1}) = p(s^{t+1})/p(s^t)$ is the gross inflation rate. The FOC of the young is

$$\frac{1}{R(s^t)} = \mathbb{E}_t \left[s_{t+1} \frac{m_1(s^{t+1})}{\pi(s^{t+1})} \right], \tag{13}$$

where $m_1(s^{t+1}) = c_1(s^t)/c_2(s^{t+1})$.

3.1.2 The Bank

Because the bank's problem is linear, it has a solution with positive loans only if the loan rate is such that the bank is indifferent to the quantity of loans it issues. To see this, suppose that the policy (L, D), with $L(s^t) > 0$, is optimal at some node s^t and consider the following one-shot deviation: The bank increases $L(s^t)$ by one unit, pays out the proceeds in $t+1$, and holds (L, D) unchanged in all other nodes. The net profit from such a deviation must be zero:

$$\mathbb{E}_t \left[m_2(s^{t+1}) \frac{s_{t+1} R(s^t) - 1}{p(s^{t+1})} \right] = 0.$$

Combine with (11) to get

$$\frac{1}{R(s^t)} = \mathbb{E}_t \left[s_{t+1} \frac{m_2(s^{t+1})}{\pi(s^{t+1})} \right]. \tag{14}$$

Equations (13) and (14) represent the demand and supply of loans. The following lemma formalizes the above argument.

Lemma 2. *If* (11) *holds at* s^t *and the bank's problem at* s^t *has a solution such that* $L(s^t) > 0$, *then* (14) *holds at* s^t.

Because the bank takes the stochastic discount factor as given, the supply of loans is perfectly elastic at the interest rate given by (14). This means that demand alone pins down the quantity of loans in equilibrium. In addition, because the market is perfectly competitive, risk-adjusted expected profits are zero at the interest rate given by (14). This gives rise to the following result.

Lemma 3. *Let $V(X(s^t), s^t)$ be the maximal present value of dividends at s^t given the beginning-of-period net worth $X(s^t)$. If (11) and (14) hold at every node, then*

$$V(X(s^t), s^t) = \frac{X(s^t)}{p(s^t)}. \tag{15}$$

Moreover, a dividend policy that satisfies (4) attains this value if and only if

$$\lim_{k\to\infty} \mathbb{E}_t \left[\left(\prod_{j=1}^{k} m_2(s^{t+j}) \right) \frac{N(s^{t+k})}{p(s^{t+k})} \right] = 0. \tag{16}$$

Lemma 3 simply states that because the present value of all future profits is zero under (14), so is the present value of all future dividends that can be distributed from them. Thus at any node s^t the present value of all dividends is simply the real value of the bank's net worth at the beginning of the period. Condition (16) states that the bank does not retain net worth whose discounted value persists: net worth that is never paid out has no value for shareholders.

3.2 Equilibrium

In the equilibria we study, the nominal stock price $q(s^t)$ is equal to net worth and the nominal loan rate $R(s^t)$ is such that the bank's risk-adjusted expected profits equal zero. The price level $p(s^t)$ is indeterminate, because agents anticipate that the bank will accommodate whatever quantity of deposits they demand in accordance with their inflation expectations: multiplying all nominal variables by the same positive number yields another equilibrium. This indeterminacy is nominal and leaves real allocations unchanged, as in Sargent and Wallace (1975); the determination of the price level with inside money has long been discussed (Fama, 1980; Sargent and Wallace, 1982). With aggregate risk, there is in addition a real indeterminacy, which we discuss after Proposition 2.

For *given* amounts of $L(s^t)$ and $B(s^t - 1) + D(s^t)$ deposits held by the young and the old, the consumption price $p(s^t)$ is determined as follows. On the supply side, both young

and old agents supply their deposits inelastically. On the demand side, middle-aged agents' demand for deposits has an inelastic component (repayment) and an elastic component (saving). The consumption price is the inverse of the present value of a deposit (Section 4).

3.2.1 Without aggregate risk

To fix ideas, we start with the case without aggregate risk, $s_t = \bar{s}$, which is simple enough to be solved by hand.

Agents' problem.—Middle-aged agents consume half of their disposable income, $w_{2t} = y - R_{t-1}L_{1t-1}/p_t$, and save the other half in deposits and shares. Their first-order conditions are

$$\frac{c_{2t}}{c_{3t+1}} = \pi_{t+1}, \qquad q_t = D_{t+1} + q_{t+1}.$$

The first is the Euler equation for deposits, $m_{2t+1} = \pi_{t+1}$; the second states that shares and deposits earn the same return. Young agents borrow

$$L_{1t} = \frac{p_{t+1}}{R_t}\frac{y}{1+2\bar{s}}.$$

Bank's problem.—Without aggregate risk, bank profits are certain, and the bank's problem has a solution only if $R_t = 1/\bar{s}$ (Lemma 2 combined with the Euler equation). At this rate the bank's net income is zero, so beginning-of-period net worth equals end-of-period net worth in the previous period, $X_{t+1} = N_t$, and dividends can only distribute net worth already in the bank:

$$D_{t+1} = N_t - N_{t+1}.$$

By Lemma 3, a dividend policy is optimal if and only if it satisfies (16). Because $m_{2t+1}/\pi_{t+1} = 1$, this condition reads $\lim_{T\to\infty} N_T = 0$: a bank that retains net worth forever does not maximize the present value of its dividends.

Feasibility.—Substituting $R_{t-1} = 1/\bar{s}$ into the loan demand of the previous period gives $w_{2t} = 2\bar{s}y/(1+2\bar{s})$, so the consumption of the middle aged, $c_{2t} = \bar{s}y/(1+2\bar{s})$, is the same at every date and does not depend on prices. Real loans, $L_t/p_t = \pi_{t+1}c_{2t}$, and therefore the consumption of the young, increase with expected inflation, and the old consume what is left. Combining the resource constraint (1) with the Euler equation, the

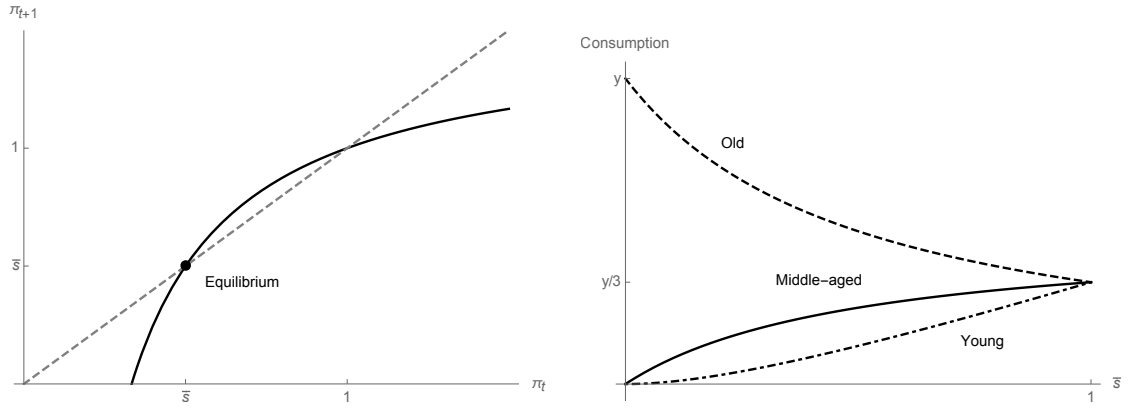

Figure 2: Left: Phase diagram of (17) for $\bar{s} = 0.5$. Right: Equilibrium consumption of the old (dashed line), the middle aged (solid line), and the young (dot-dashed line) for $0 < \bar{s} < 1$.

inflation rate must satisfy the difference equation

$$\pi_{t+1} = 1 + \bar{s} - \frac{\bar{s}}{\pi_t}. \tag{17}$$

Its phase diagram is depicted in Figure 2, left. It has two steady states, $\pi = \bar{s}$ and $\pi = 1$. For all $0 < \pi_t < \bar{s}$, inflation eventually becomes negative, so these paths are not equilibria. For all $\pi_t > \bar{s}$, inflation converges to one.

Market clearing.—Multiply (7) by q_t, add it to (6), use $B_t = L_t - N_t$, and substitute (5) to get:

$$\frac{q_t - N_t}{p_t} = (\bar{s} - \pi_{t+1})\frac{\bar{s}y}{1 + 2\bar{s}}. \tag{18}$$

On every path with $\pi_{t+1} > \bar{s}$, including the steady state $\pi = 1$, the stock would trade below the bank's net worth. Such paths are ruled out by the bank's optimality and free disposal of shares, as we now show. The stock-pricing equation and $D_{t+1} = N_t - N_{t+1}$ imply that the gap between the stock price and net worth is constant over time, $q_t - N_t = q_{t+1} - N_{t+1}$. Because $\lim_{T\to\infty} N_T = 0$ under any optimal dividend policy,

$$q_t - N_t = \lim_{T\to\infty} q_T \;\geq\; 0,$$

where the inequality follows from free disposal of shares, which is consistent with Assumption 2 because, without aggregate risk, $X_{t+1} = N_t \geq 0$, so dividends are never negative and the stock never obliges its holders to pay. By (18), $\pi_{t+1} \leq \bar{s}$. Together with the

previous paragraph, $\pi_t = \bar{s}$ and $q_t = N_t$ for all t. This proves the following result.

Proposition 1. *Assume that $s_t = \bar{s}$. Then, there exists a banking equilibrium, unique up to the price level and dividend policy. In this equilibrium, prices and consumption allocations are: $q_t = N_t$, $R_t = 1/\bar{s}$, $\pi_t = \bar{s}$, and*

$$c_{1t} = \frac{\bar{s}^2 y}{1 + 2\bar{s}}, \quad c_{2t} = \frac{\bar{s} y}{1 + 2\bar{s}}, \quad c_{3t} = \frac{y}{1 + 2\bar{s}}$$

for all t.

Proposition 1 requires no assumption on bubbles beyond free disposal of shares. The stock is priced at its fundamental value because a bank that maximizes the present value of its dividends never retains net worth indefinitely, and the price of a deposit, $1/p_t$, reflects its repayment services alone.

Figure 2, right, shows the consumption of the young, the middle aged, and the old as a function of $\bar{s}$. Only the middle aged consume the golden rule quantity of Lemma 1. The young consume less and the old consume more. In the golden rule allocation, young and middle-aged agents consume the same amount *per capita*. With log utility, for a decentralized equilibrium to produce the same consumption allocations, the savings of a middle-aged agent must equal the consumption of a young agent. However, markets clear only if the *aggregate* savings of the middle aged equal the aggregate consumption of the young. Because there are more young agents than middle-aged agents, the golden rule allocation is not an equilibrium.

The equilibrium is deflationary: the price level falls at the gross rate $\bar{s}$. The sign of inflation in this equilibrium, however, is not a robust prediction of the model, because it depends on preferences. With power utility, the stationary equilibrium is deflationary when relative risk aversion is below two and inflationary when it is above two (Appendix B). What does not depend on preferences is that deposits have a positive value and that their quantity is finite.

3.2.2 With aggregate risk

With aggregate risk, the equilibrium conditions no longer pin down the stock price, and we restrict attention to equilibria in which the stock is priced at its fundamental value.

Definition 2. A *fundamental banking equilibrium* is a banking equilibrium in which the stock price equals end-of-period net worth, $q(s^t) = N(s^t)$, at every node.

Definition 2 makes explicit what the dividend discount model of Section 2 presumes: that, in equilibrium, the stock is priced at the present value of its dividends. In a fundamental banking equilibrium, the cum-dividend stock price, $q(s^t) + D(s^t)$, equals the maximal present value of dividends, $X(s^t)$ (Lemma 3). By Proposition 1, every banking equilibrium without aggregate risk is fundamental.

Agents' problem.—The choices of the agents parallel those in the case without aggregate risk. The middle aged save half of their disposable income, and the young borrow more if they expect a lower real rate:

$$\frac{1}{L_1(s^t)} = \mathbb{E}_t \left[\frac{2s_{t+1}}{\frac{p(s^{t+1})}{R(s^t)} y - L_1(s^t)} \right].$$

Market clearing.—In a fundamental banking equilibrium, $q(s^t) = N(s^t)$, so by (6) and (7) the nominal savings of the middle aged are $B(s^t) + q(s^t) = B(s^t) + N(s^t) = L(s^t)$. Because the middle aged consume as much as they save, their aggregate consumption equals that of the young: $s_t c_2(s^t) = L(s^t)/p(s^t) = c_1(s^t)$. By the definitions of X and N, the nominal wealth of the old at s^{t+1} is

$$B(s^t) + D(s^{t+1}) + q(s^{t+1}) = s_{t+1} R(s^t) L(s^t) + q(s^{t+1}) - N(s^{t+1}),$$

which, because $q(s^{t+1}) = N(s^{t+1})$, equals the repayments of the surviving borrowers. In a fundamental banking equilibrium, the wealth of the old consists exactly of the deposits that borrowers need to repay.

Interest rate.—Write the FOC for deposits (11) in nominal terms,

$$\frac{1}{p(s^t) c_2(s^t)} = \mathbb{E}_t \left[\frac{1}{p(s^{t+1}) c_3(s^{t+1})} \right],$$

and substitute $s_t p(s^t) c_2(s^t) = L(s^t)$ and $s_t p(s^{t+1}) c_3(s^{t+1}) = s_{t+1} R(s^t) L(s^t)$. This gives $R(s^t) = 1/\hat{s}$, where

$$\frac{1}{\hat{s}} \equiv \mathbb{E} \left[\frac{1}{s_{t+1}} \right],$$

and the stochastic discount factor of the middle aged satisfies

$$\frac{m_2(s^{t+1})}{\pi(s^{t+1})} = \frac{p(s^t) c_2(s^t)}{p(s^{t+1}) c_3(s^{t+1})} = \frac{\hat{s}}{s_{t+1}}. \tag{19}$$

By (19), this rate also makes the bank indifferent to the quantity of loans it issues (Lemma 2). The bank charges a premium over the actuarial rate ($R(s^t) - 1/\bar{s} > 0$) because its shareholders are risk-averse.

Inflation and loans.—Let $l(s^t) \equiv L(s^t)/p(s^t)$ denote real loans. Since $c_2(s^t) = w_2(s^t)/2$ and $s_t c_2(s^t) = l(s^t)$, the definition of disposable income gives the inflation rate

$$\pi(s^t) = \frac{s_t}{\hat{s}} \frac{l(s^t - 1)}{s_t y - 2l(s^t)}. \tag{20}$$

Substituting this inflation rate and $R(s^t) = 1/\hat{s}$ into the demand for loans of the young yields the following equilibrium condition for loans:

$$\mathbb{E}_t \left[\frac{s_{t+1}^2}{l(s^{t+1})} \right] = \frac{1 + 2\bar{s}}{y}. \tag{21}$$

Equation (21) must hold at every node. Feasibility requires $c_1(s^t) > 0$ and $c_3(s^t) > 0$, that is, $0 < l(s^t) < s_t y/2$.

Bank's problem.—Given $R(s^t) = 1/\hat{s}$, the bank is indifferent to the quantity of loans (Lemma 2), and, by Lemma 3, a dividend policy is optimal if and only if it satisfies (16). Optimal policies that also satisfy (4) and (8) exist; the proof of Proposition 2 uses one of them, and Section 3.5 discusses dividend policy. With $q = N$, the stock-pricing equation (12) reduces to $N(s^t) = \mathbb{E}_t \left[\frac{\hat{s}}{s_{t+1}} X(s^{t+1}) \right]$, which holds identically because beginning-of-period net worth is end-of-period net worth in the previous period plus the net income from loans, $X(s^{t+1}) = N(s^t) + (s_{t+1}/\hat{s} - 1)L(s^t)$.

The following result summarizes.

Proposition 2. *There exist fundamental banking equilibria. In every fundamental banking equilibrium, prices are:* $q(s^t) = N(s^t)$, $R(s^t) = 1/\hat{s}$, *and*

$$\pi(s^t) = \frac{s_t}{\hat{s}} \frac{l(s^t - 1)}{s_t y - 2l(s^t)},$$

and consumption allocations are:

$$c_1(s^t) = l(s^t), \quad c_2(s^t) = \frac{1}{s_t} l(s^t), \quad c_3(s^t) = \frac{1}{s_{t-1}} \left[s_t y - 2l(s^t) \right],$$

where I fulfills $0 < I(s^t) < s_t y/2$ and (21) at every node. Moreover, any such I characterizes a fundamental banking equilibrium.

Without aggregate risk, (21) pins down the unique equilibrium value $I(s^t) = \frac{\bar{s}^2 y}{1+2\bar{s}}$, as in Proposition 1. With aggregate risk, (21) imposes, at each node, a single restriction on the $|S|$ real loans of the successor nodes, which leaves $|S| - 1$ degrees of freedom. Unlike the nominal indeterminacy discussed above, this indeterminacy is real: different functions I give different consumption allocations. Some of the resulting equilibria depend on the history of the economy: at each node, expectations may coordinate on a different point of the set of successor values allowed by (21). By contrast, in a *stationary* fundamental banking equilibrium, real loans depend on the current state alone, $I(s^t) = I(s_t)$, with a slight abuse of notation.

3.3 Banking and one-period securities

Our banks differ from the standard way of modeling intertemporal trade in overlapping generations economies: borrowers and lenders do not trade claims with each other, but with a bank that creates deposits when it lends. To see how banking compares with the standard approach, we now solve the same economy with one-period-maturity securities instead of banks. The set of securities is arbitrary and includes, as special cases, a complete set of Arrow securities and a riskless bond.

Suppose that there are $J \geq 1$ one-period-maturity securities in zero net supply traded at every node. Security $j = 1, \ldots, J$ pays $d_j(s^t) \geq 0$ units of consumption, with $\sum_{s^{t+1}|s^t} d_j(s^{t+1}) > 0$ for all s^t.

Agents' problem.—The budget constraints of young, middle-aged, and old agents are:

$$
\begin{aligned}
c_1(s^t) + \sum_{j=1}^{J} Q_j(s^t) a_{1j}(s^t) &= 0, \\
c_2(s^t) + \sum_{j=1}^{J} Q_j(s^t) a_{2j}(s^t) &= y + \sum_{j=1}^{J} d_j(s^t) a_{1j}(s^t - 1), \\
c_3(s^t) &= \sum_{j=1}^{J} s_t d_j(s^t) a_{2j}(s^t - 1),
\end{aligned}
$$

where $a_{1j}(s^t)$ and $a_{2j}(s^t)$ are holdings of security j and $Q_j(s^t)$ is its price. The factor s_t in

the budget constraint of the old reflects that only a fraction s_t of the young who issued the securities survive to pay them.

Market clearing and equilibrium.—Markets clear at s^t if

$$a_{1j}(s^t) + s_t a_{2j}(s^t) = 0 \tag{22}$$

for all j; that is, the middle aged buy the securities that the young issue. Then, a *competitive equilibrium with one-period-maturity securities* is a collection of prices and agents' choices such that, for all nodes: (i) Agents' choices are optimal given prices; (ii) Markets clear.

The following result characterizes the relationship between banking and trade in one-period-maturity securities.

Proposition 3. *A consumption allocation is the allocation of an equilibrium with one-period-maturity securities, for some choice of security payoffs, if and only if it is the allocation of a fundamental banking equilibrium. In particular, the allocation of a fundamental banking equilibrium can be implemented with a single one-period-maturity security with payoffs* $d(s^t) = \frac{1}{\hat{s}\pi(s^t)}$, *where* $\pi(s^t)$ *is as in Proposition 2.*

Proposition 3 shows that banking is not an exotic trading arrangement: the consumption allocations that it implements are exactly those that one-period-maturity securities implement.

We now consider two classic examples. In the first example, there is a complete set of one-period-maturity Arrow securities in zero net supply traded at every node. In the second example, there is a one-period-maturity real bond in zero net supply traded at every node that promises a unit of consumption next period in any event.

Corollary 1. *There exists a unique equilibrium with Arrow securities and a unique equilibrium with real bonds. The corresponding consumption allocations are*

$$c_1(s^t) = \frac{s_t^2 y}{1+2s_t}, \quad c_2(s^t) = \frac{s_t y}{1+2s_t}, \quad c_3(s^t) = \frac{1}{s_{t-1}}\frac{s_t y}{1+2s_t}$$

and

$$c_1(s^t) = \frac{s_t \bar{s} y}{1+2\bar{s}}, \quad c_2(s^t) = \frac{\bar{s} y}{1+2\bar{s}}, \quad c_3(s^t) = \frac{1}{s_{t-1}}\frac{s_t y}{1+2\bar{s}}.$$

By Proposition 3, both allocations can be implemented as fundamental banking equilibria. Both are stationary. This is a result, not an assumption: we have not restricted attention to stationary equilibria, and, since Arrow securities complete the markets, it is not an artifact of a restricted set of assets.

The difference between banking and trade in securities is therefore one of selection. Each of the two arrangements of Corollary 1 implements a single allocation. Banking implements all the allocations of Proposition 3, and expectations select which one.

3.4 Welfare and equilibrium selection

With aggregate risk, Proposition 2 delivers a continuum of fundamental banking equilibria, which differ in their real allocations. By Proposition 3, each of them is implemented by a single one-period security whose payoff, the real return on a nominal loan, is set by inflation expectations. Selecting among equilibria thus amounts to selecting the payoff of the security through which the young and the middle aged share aggregate risk. We rank the equilibria by the criterion that defines the golden rule, expected lifetime utility (9). As in Lemma 1, we consider allocations that depend on the current and the previous state. A *Markov* fundamental banking equilibrium is one in which real loans are a function of these two states, $l(s^t) = l(s_t, s_{t-1})$. This class contains the stationary equilibria, and, by Proposition 2, a function l characterizes a Markov fundamental banking equilibrium if and only if $0 < l(s_t, s_{t-1}) < s_t y/2$ and

$$\mathbb{E}_{t-1}\left[\frac{s_t^2}{l(s_t, s_{t-1})}\right] = \frac{1+2\bar{s}}{y} \tag{23}$$

for all $s_{t-1} \in S$, which is (21) at node $s^t - 1$. Substituting the allocation of Proposition 2 into (9), and using that survival probabilities are independent over time, the expected lifetime utility of an equilibrium indexed by l is

$$U(l) = \mathbb{E}\left[(1+s_t)\log l(s_t, s_{t-1}) + s_{t-1}\log\left(s_t y - 2l(s_t, s_{t-1})\right)\right] - 2\,\mathbb{E}\left[s_t \log s_t\right], \tag{24}$$

where the first expectation is taken over the two independent states s_t and s_{t-1}.

Definition 3. A *best banking equilibrium* is a Markov fundamental banking equilibrium whose function l maximizes $U(l)$ subject to (23).

The best banking equilibrium is the most preferred allocation among those that banking

can implement, not among all feasible allocations. By Proposition 2, these allocations satisfy $c_1(s^t) = s_t c_2(s^t)$, whereas the golden rule gives young and middle-aged agents the same consumption *per capita*, and I must satisfy (23). Define

$$I^u(s_t, s_{t-1}) \equiv \frac{(1+s_t)s_t y}{2(1+s_t+s_{t-1})} = \frac{1+s_t}{2}\,\varphi_1(s_t, s_{t-1}),$$

the value of $I(s_t, s_{t-1})$ that maximizes (24) state by state when (23) is ignored, where φ_1 is the golden-rule consumption of the young in (10).

Proposition 4. *A best banking equilibrium exists. Its function I satisfies*

$$\frac{2(1+s_t+s_{t-1})}{s_t^2}\,\frac{I(s_t, s_{t-1})}{s_t y - 2I(s_t, s_{t-1})}\,[I^u(s_t, s_{t-1}) - I(s_t, s_{t-1})] = \lambda(s_{t-1}) \tag{25}$$

for all $s_t, s_{t-1} \in S$, where $\lambda(s_{t-1}) > 0$ is the Lagrange multiplier on (23). In particular, $I(s_t, s_{t-1}) < I^u(s_t, s_{t-1})$ for all $s_t, s_{t-1} \in S$.

Since $\lambda > 0$, the equilibrium condition (23) binds: relative to I^u, it requires lower real loans, and hence lower consumption of the young and the middle aged, in every state. The weight on the consumption of the old changes with s_{t-1}, and so does the best banking equilibrium, which in general is not stationary.

The best banking equilibrium thus corresponds to the security that best shares aggregate risk. It is at least as good as complete Arrow securities and a real bond, whose allocations are Markov fundamental banking equilibria (Corollary 1). Some securities share risk very poorly: as I approaches the boundary of the equilibrium set, the consumption of the old or of the young in some state vanishes, and expected lifetime utility falls without bound. Expectations therefore matter for welfare, and they can matter a great deal.

Example 1. Let $S = \{0.9, 1\}$, with $\Pr(0.9) = \Pr(1) = 0.5$, and $y = 1$. The stationary fundamental banking equilibria are the points of the curve $0.405/I(0.9) + 0.5/I(1) = 2.9$ with $0 < I(0.9) < 0.45$ and $0 < I(1) < 0.5$. The Arrow allocation corresponds to $I(0.9) = 0.2893$ and $I(1) = 0.3333$, the bonds allocation to $I(0.9) = 0.2948$ and $I(1) = 0.3276$, and the best stationary fundamental banking equilibrium to $I(0.9) = 0.2923$ and $I(1) = 0.3302$. Figure 3 shows their welfare, measured as the uniform percentage change in the consumption of all agents in all states that would make the best stationary fundamental banking equilibrium as good as each of them. The Arrow and bonds allocations are within 0.01 percent of the best, whereas welfare falls without bound toward the boundary of the

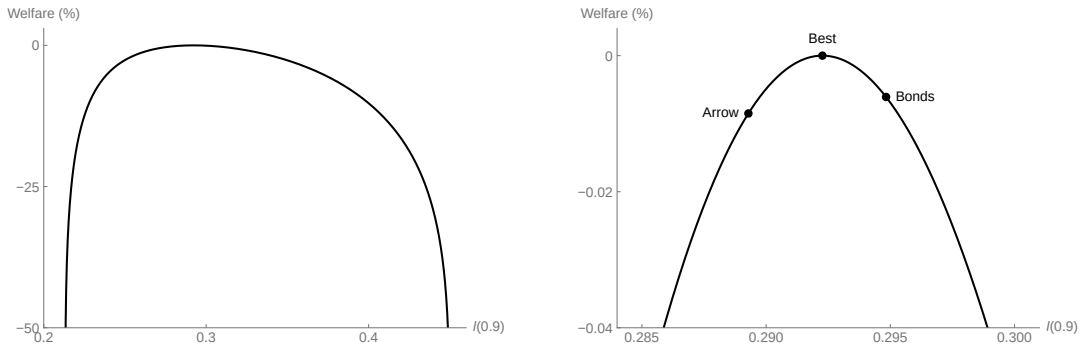

Figure 3: Left: Welfare of the stationary fundamental banking equilibria of Example 1 as a function of $l(0.9)$, relative to the best stationary fundamental banking equilibrium, in percent of consumption. Right: Detail near the maximum, with the Arrow and bonds allocations of Corollary 1.

set. The magnitudes are only illustrative, as the model abstracts from most features of actual economies, but the conclusions above do not depend on the parameters.

Policy.—Which instruments select among the fundamental banking equilibria? A fixed nominal interest rate does not: by Proposition 2, the nominal loan rate is $R(s^t) = 1/\hat{s}$ in every fundamental banking equilibrium, so the equilibria differ in expected inflation, not in the interest rate. Expected inflation determines how the real repayment of a loan, $R(s^t)/\pi(s^{t+1})$ per unit of real loans, varies with next period's state. An instrument that removes this dependence is an indexed loan contract: for each unit of consumption borrowed at node s^t, a young agent owes $\tilde{R}(s^t)$ units of consumption at $t+1$, which he repays in deposits at the price $p(s^{t+1})$. The bank chooses $\tilde{R}(s^t)$, and the rest of the model is unchanged. Such contracts exist: in Chile, for instance, mortgages are usually denominated in an indexed unit of account, the *Unidad de Fomento*, and repaid in pesos (Shiller, 1998).

Proposition 5. *Under indexed loan contracts, there exist fundamental banking equilibria, and all of them have the bonds allocation of Corollary 1.*

An indexed loan is a bond issued by the young, and the proof follows that of Corollary 1. Indexation turns the real indeterminacy of Proposition 2 into a nominal one: the allocation is unique, but the price level is not. This nominal indeterminacy is harmless in the model, but not necessarily in actual economies, where indexed contracts still require a price level

anchored by other means; the *Unidad de Fomento*, for instance, is computed from a consumer price index of goods priced in pesos. Indexation thus rules out the bad equilibria, but the allocation it selects is in general not the best one. The bonds allocation is also the only fundamental banking equilibrium in which inflation $\pi(s^{t+1})$ does not depend on s_{t+1}: anchoring expectations so that inflation does not respond to the current state selects the same allocation.

3.5 Dividends, deposits, and net worth

Equilibrium loans are determined by demand, but dividends are chosen by the bank. The dividend policy adopted by the bank defines its capital structure; that is, the mix of debt (deposits) and equity (net worth). This margin of deposit production turns out to be irrelevant for real allocations because, under rational expectations, for every extra dollar in dividends, the stock price is reduced by exactly one dollar. Since the nominal wealth of the old does not change with the new dividend issue, prices do not respond to dividend policy.

By Lemma 3, a dividend policy is optimal if and only if it does not retain net worth whose discounted value persists, condition (16); this rules out suboptimal earnings retention (DeAngelo and DeAngelo, 2006). Without aggregate risk, this condition is what rules out bubbles (Section 3). It follows that dividend policy is irrelevant for the value of the bank.

Corollary 2 (Miller and Modigliani, 1961)**.** *In every fundamental banking equilibrium, all dividend policies that satisfy* (16) *attain the maximal value in* (15)*.*

The bank is financed with deposits and shares. Deposits are nominally risk-free and do not pay interest. Shares are risky and pay a risk premium.

Proposition 6 (Modigliani and Miller, 1958)**.** *The expected nominal stock return is*

$$\mathbb{E}_t\left[\frac{q(s^{t+1})+D(s^{t+1})}{q(s^t)}\right]=\frac{\bar{s}}{\hat{s}}+\frac{\bar{s}-\hat{s}}{\hat{s}}\frac{B(s^t)}{N(s^t)}$$

in every fundamental banking equilibrium in which $N(s^t)>0$*.*

The cost of equity is larger than one and increases with the bank's leverage ratio. This has no real consequences as agents may revert any bank policy through portfolio re-balancing.

We illustrate dividend policy with a simple example. An equilibrium dividend policy must leave the bank with nonnegative end-of-period net worth (4) and produce enough

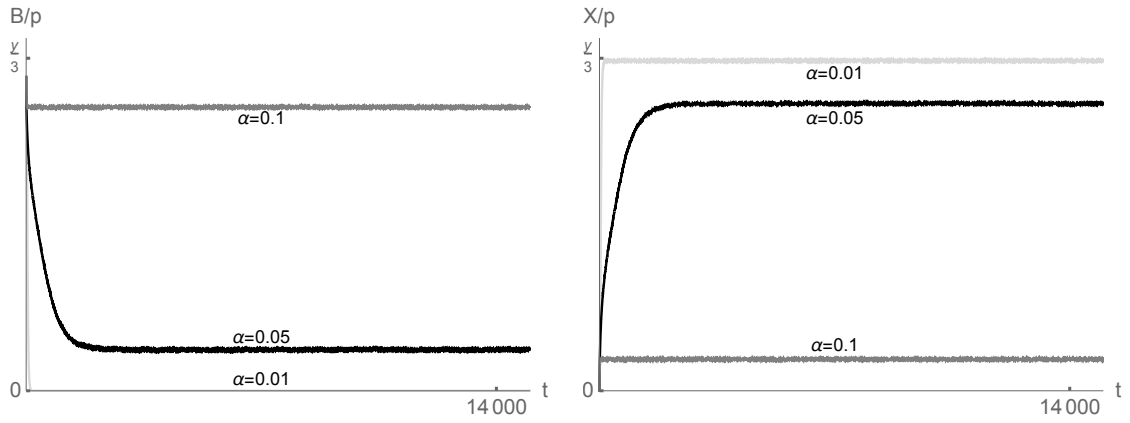

Figure 4: Left: Real deposits for the dividend policies of Example 2 with $\alpha = 0.01$, 0.05, and 0.1. Right: Real beginning-of-period net worth. Averages of 5,000 simulations over 14,000 periods in the best stationary fundamental banking equilibrium of Example 1.

deposits for loans to be repaid (8). The following policy does both and otherwise pays out a constant fraction of net worth.

Example 2. Let $\alpha \in (0, 1)$ and consider the following stationary dividend policy:

$$D(s^t) = \begin{cases} X(s^t) & \text{if } X(s^t) < 0, \\ \alpha X(s^t) & \text{if } 0 \leq X(s^t) < (1-\alpha)^{-1} L(s^t), \\ X(s^t) - L(s^t) & \text{if } X(s^t) \geq (1-\alpha)^{-1} L(s^t). \end{cases}$$

The first case is the recapitalization required when net worth is negative; the third caps end-of-period net worth at $L(s^t)$, so that $B(s^t) \geq 0$. To verify that the policy is optimal, we use stationary fundamental banking equilibria, in which a simple condition suffices: if $\mathbb{E}[l(s_t)/(s_t y - 2l(s_t))] < 1$, the discount factor $\prod_{j=1}^{k} m_2(s^{t+j})$ vanishes in expectation as $k \to \infty$, so every policy that keeps real net worth bounded, such as this one, satisfies (16) and is optimal by Corollary 2. In the best stationary fundamental banking equilibrium of Example 1, this expectation is 0.95. Figure 4 shows deposits and net worth in this equilibrium for $\alpha = 0.01$, 0.05, and 0.1. With low dividends ($\alpha = 0.01$), the bank produces few deposits, accumulates net worth, and almost never needs to be recapitalized. With high dividends ($\alpha = 0.1$), it produces more deposits, holds little net worth, and is recapitalized often. In between ($\alpha = 0.05$), beginning-of-period net worth stays positive most of the time.

4 Discussion

4.1 The price of money

A deposit is a claim that repays debt: once it is used to repay a bank loan, the corresponding part of the debt is extinguished. If the middle aged use a deposit to repay at date t, its real value is the consumption they must forgo to obtain it, $1/p(s^t)$. If instead they save it and the middle aged of some future date $t+k$ use it to repay, its present value at t is

$$\mathbb{E}_t\left[\left(\prod_{j=1}^{k} m_2(s^{t+j})\right)\frac{1}{p(s^{t+k})}\right],$$

where we use the SDF of the middle aged because the decision to save or repay is theirs. In a fundamental banking equilibrium, (19) gives $m_2(s^{t+1})/p(s^{t+1}) = (\hat{s}/s_{t+1})/p(s^t)$, and $\mathbb{E}_t[\hat{s}/s_{t+1}] = 1$, so this present value equals $1/p(s^t)$ for every $k \geq 1$. The value of a deposit is therefore the present value of its repayment services, and it does not depend on how many periods elapse before the deposit is used to repay a loan. That deposits are demanded follows directly from Assumption 1(b); what the calculation shows is that their value is the same whatever the horizon, so no bubble term is needed.

Middle-aged agents hold deposits both to repay and to save, but the second motive is derived from the first: a deposit saved today is worth holding only because future middle-aged agents will need it to repay. This differs from the usual argument for money in overlapping generations economies (Samuelson, 1958), in which each generation values money only because the next will accept it; here the resale value is itself derived from repayment. In the aggregate, the same logic holds: in a fundamental banking equilibrium, the wealth of the old consists exactly of the deposits that borrowers need to repay (Section 3). Money is a means of repayment.

4.2 The quantity of money

Although deposits cost nothing to produce, their equilibrium quantity is finite. In period t, the young hold the $L(s^t)$ deposits created by lending, and the old hold $B(s^t-1)+D(s^t)$ deposits, including those created by dividends. Deposits created by lending are limited by the demand for loans,

$$\frac{1}{l(s^t)} = \mathbb{E}_t\left[\frac{2s_{t+1}}{\pi(s^{t+1})\hat{s}y - l(s^t)}\right],$$

where $I(s^t) = L(s^t)/p(s^t)$: the young must repay out of their future endowment, so borrowing today means consuming less tomorrow. This demand depends on next period's prices, and hence on expectations. Self-fulfilling expectations of this kind are well known in overlapping generations economies (Azariadis, 1981; Cass and Shell, 1983; Grandmont, 1985; Farmer, 1993); what is specific to our setting is that expectations select the real quantity of credit that the bank creates, with welfare consequences (Section 3.4).

The loan contract is nominal and the bank only accepts deposits as repayment, so the real burden of a loan depends on the price level when it comes due. That price level depends, in turn, on how much the next generation borrows: the young spend the deposits they borrow on the endowment of the middle aged, so the more they borrow, the higher the price level. The young in t therefore borrow according to their expectations about the borrowing of the next generation in each state. Suppose they expect it to borrow more when s_{t+1} is high and less when it is low. Then they expect high inflation, and a light real repayment, in good states and the opposite in bad states, and they borrow accordingly. When the next generation borrows as expected, these expectations are fulfilled. Equilibrium imposes a single restriction on the pattern across states, (21): borrowing cannot be higher in every state, so higher borrowing in some states must be matched by lower borrowing in others. With aggregate risk, a single restriction leaves room for many patterns, and expectations select among them; without aggregate risk, it pins down the quantity of credit.

Deposits created by dividends are limited by the bank's net worth, because the bank cannot pay out more than $X(s^t)$ (4). Within this limit, dividends are chosen by the bank. When many borrowers survive, repayments are high, and dividends supply the extra deposits that the middle aged need to repay (8). When few survive, negative dividends destroy the deposits that would otherwise leave the bank with negative net worth. Because $\bar{s} \geq \hat{s}$, the bank is profitable in expectation and may distribute some earnings and retain the rest (Example 2). Dividends thus adjust the quantity of deposits in the economy, a role that is easy to overlook.

There is an optimal real quantity of deposits created by lending, that of the best banking equilibrium, but not of deposits created by dividends, which do not affect real allocations.

5 Conclusion

We take as given two features of observed monetary systems, that bank loans are nominal and that they are settled in bank deposits, and study their implications in an overlapping generations economy in which banks create deposits at no cost, with no fiat money, no reserve requirements, and no central bank. The value of a deposit is the present value of its repayment services, however many periods it circulates before it is used to repay a loan, so deposits can have value without a bubble. Although deposits cost nothing to produce, their equilibrium quantity is finite. Deposits created by lending are limited by the demand for loans, and deposits created by dividends are limited by the requirement that banks' net worth not be negative. Dividends affect neither real allocations nor inflation, which depend only on the quantity of loans, but they supply the deposits that borrowers need to pay interest.

Because loans are nominal, price expectations are self-fulfilling, and with aggregate risk the indeterminacy is real. Banking implements the same consumption allocations as one-period securities, but expectations select among them, and this selection matters for welfare: some equilibria are arbitrarily bad. Indexed loan contracts select a unique allocation, that of real bonds.

Our results have implications for each party in the economy. For borrowers and savers, the real value of a nominal loan or a deposit depends on expectations, not only on fundamentals; indexed loans fix both. For banks, dividend policy does not affect real allocations, but it determines whether there are enough deposits for borrowers to pay interest. For central banks, our results support the emphasis on anchoring expectations: with aggregate risk, expectations determine how inflation responds to shocks and select among equilibria with very different welfare, and fixing the nominal interest rate does not anchor them.

The model has clear limitations, which also suggest extensions. Loans and deposits last one period, so banks do not transform maturities or liquidity. Borrowers default only when they do not survive, so banks neither screen nor monitor them. Deposits cannot be redeemed, so the model is silent on bank runs, and shareholders have unlimited liability, so banks never become insolvent. Relaxing each of these assumptions is a natural next step, as are introducing a fiscal or monetary authority and studying other instruments, such as capital requirements or limits on the quantity of credit, in an environment like ours, where the quantity of money is determined endogenously rather than as a multiple of required

reserves.

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A Proofs

Proof of Lemma 1

Substitute the constraint into the objective function and maximize

$$\sum_{x\in S}\sum_{x'\in S}\log(x'y - x'\varphi_2(x',x) - x\varphi_3(x',x))\Pr(x')\Pr(x) + \sum_{x'\in S}\sum_{x''\in S} x''\log(\varphi_2(x'',x'))\Pr(x'')\Pr(x') + \sum_{x''\in S}\sum_{x'''\in S} x''\log(\varphi_3(x''',x''))\Pr(x''')\Pr(x'').$$

The FOCs of the problem are:

$$\frac{x'}{x'y - x'\varphi_2(x',x) - x\varphi_3(x',x)}\Pr(x')\Pr(x) = \frac{x'}{\varphi_2(x',x)}\Pr(x')\Pr(x),$$
$$\frac{x}{x'y - x'\varphi_2(x',x) - x\varphi_3(x',x)}\Pr(x')\Pr(x) = \frac{x}{\varphi_3(x',x)}\Pr(x')\Pr(x),$$

for all $(x',x)\in S^2$. Therefore, the solution is

$$\varphi_1(x',x) = \frac{x'y}{1+x'+x},\quad \varphi_2(x',x) = \frac{x'y}{1+x'+x},\quad \varphi_3(x',x) = \frac{x'y}{1+x'+x}.$$

Proof of Lemma 2

Our proof is by contradiction. Let $X(s^t)$ be given, and let (L,D) be an optimal policy, with value $V(X(s^t),s^t)$, such that $L(s^t)>0$. Suppose that

$$\frac{1}{R(s^t)} < \mathbb{E}_t\left[s_{t+1}\frac{m_2(s^{t+1})}{\pi(s^{t+1})}\right] \tag{26}$$

and consider the alternative policy $(\hat{L},\hat{D})$ given by:

$$\hat{L}(s^t) = L(s^t) + \varepsilon,$$
$$\hat{D}(s^{t+1}) = D(s^{t+1}) + (s_{t+1}R(s^{t+1}-1)-1)\varepsilon,$$

for all $s^{t+1}|s^t$ and some $\varepsilon>0$, and $(\hat{L},\hat{D}) = (L,D)$ otherwise. Policy $(\hat{L},\hat{D})$ is feasible and has value

$$V(X(s^t),s^t) + \underbrace{\mathbb{E}_t\left[m_2(s^{t+1})\frac{s_{t+1}R(s^{t+1}-1)-1}{\pi(s^{t+1})}\right]}_{>0 \text{ by } (26)} p(s^t)\varepsilon,$$

which contradicts the assumption that (L, D) is optimal. The argument is essentially the same if inequality (26) is reversed.

Proof of Lemma 3

Let $X(s^t)$ be given and consider any feasible policy (L, D). Write D_k, X_k, N_k, and L_k for the values at node s^{t+k}, and let

$$Q_k \equiv \prod_{j=1}^{k} \frac{m_2(s^{t+j})}{\pi(s^{t+j})}, \qquad Q_0 \equiv 1,$$

be the nominal discount factor, which satisfies $\mathbb{E}_{t+k-1}[Q_k] = Q_{k-1}$ by (11). Since $D_k = X_k - N_k$ and, by (3), $X_k = N_{k-1} + I_k$, with net income $I_k = [s_{t+k} R(s^{t+k} - 1) - 1] L_{k-1}$, we may write

$$\sum_{k=0}^{K} Q_k D_k = X_0 + \sum_{k=1}^{K} Q_k I_k + \sum_{k=1}^{K} (Q_k - Q_{k-1}) N_{k-1} - Q_K N_K$$

for all $K \geq 0$. Take expectations conditional on s^t. The third term vanishes because N_{k-1} is known at node s^{t+k-1} and Q is a martingale. The second term vanishes because, by (11) and (14),

$$\mathbb{E}_{t+k-1}\left[\frac{Q_k}{Q_{k-1}} I_k\right] = L_{k-1}\left(R(s^{t+k} - 1)\,\mathbb{E}_{t+k-1}\left[\frac{s_{t+k} m_2(s^{t+k})}{\pi(s^{t+k})}\right] - \mathbb{E}_{t+k-1}\left[\frac{m_2(s^{t+k})}{\pi(s^{t+k})}\right]\right) = 0.$$

Dividing by $p(s^t)$ and using $Q_k / p(s^t) = \left(\prod_{j=1}^{k} m_2(s^{t+j})\right) / p(s^{t+k})$, we obtain

$$\mathbb{E}_t\left[\sum_{k=0}^{K}\left(\prod_{j=1}^{k} m_2(s^{t+j})\right)\frac{D(s^{t+k})}{p(s^{t+k})}\right] = \frac{X(s^t)}{p(s^t)} - \mathbb{E}_t\left[\left(\prod_{j=1}^{K} m_2(s^{t+j})\right)\frac{N(s^{t+K})}{p(s^{t+K})}\right].$$

Because $N \geq 0$ by (4), the right-hand side is at most $X(s^t)/p(s^t)$, with equality for every K under the policy $D = X$, which keeps $N \equiv 0$. Hence $V(X(s^t), s^t) = X(s^t)/p(s^t)$, and a policy attains this value if and only if (16) holds.

Proof of Proposition 2

We break the proof in three parts.

Part 1.—Assume that there exists a fundamental banking equilibrium and show that: (*i*) Real loans $l(s^t) = \frac{L(s^t)}{p(s^t)}$ fulfill $0 < l(s^t) < \frac{s_t y}{2}$ and (21); (*ii*) Consumption fulfills

$c_1(s^t) = s_t c_2(s^t) = I(s^t)$; (*iii*) Prices fulfill $q(s^t) = N(s^t)$, $R(s^t) = 1/\hat{s}$, and (20).

First, $q(s^t) = N(s^t)$ by Definition 2. Middle-aged agents consume one half of their disposable income and save the other half; by (6), (7), $q(s^t) = N(s^t)$, and $N(s^t) = L(s^t) - B(s^t)$, their aggregate nominal savings are $L(s^t)$, so $s_t p(s^t) c_2(s^t) = L(s^t) = p(s^t) c_1(s^t)$. This result together with (1) implies that $0 < I(s^t) < \frac{s_t y}{2}$ (real loans are strictly positive because utility is logarithmic). Second, by (2) and $q = N$, the aggregate nominal wealth of the old at s^{t+1} is $s_{t+1} R(s^t) L(s^t)$, so that $s_t p(s^{t+1}) c_3(s^{t+1}) = s_{t+1} R(s^t) L(s^t)$. Substituting both expressions into (11) gives $R(s^t)\hat{s} = 1$ and (19). Third, combining $c_2(s^t) = w_2(s^t)/2$ with $s_t c_2(s^t) = I(s^t)$ and $R(s^t - 1) = 1/\hat{s}$ gives (20). Plugging this inflation rate into the FOC of young agents and using market-clearing condition (5), we get (21).

Part 2.—Assume that I is a function fulfilling $0 < I(s^t) < \frac{s_t y}{2}$ and (21) and show that there exists a fundamental banking equilibrium in which real loans are $\frac{L(s^t)}{p(s^t)} = I(s^t)$.

Consider the prices $R(s^t) = 1/\hat{s}$, $q(s^t) = 0$, and

$$\frac{p(s^t)}{p(s^t - 1)} = \frac{s_t}{\hat{s}} \frac{I(s^t - 1)}{s_t y - 2I(s^t)},$$

and the quantities $L(s^t) = L_1(s^t) = p(s^t) I(s^t)$, $B(s^t) = s_t B_2(s^t) = p(s^t) I(s^t)$, $z_2(s^t) = 1/s_t$, and $D(s^t) = X(s^t) = \frac{s_t - \hat{s}}{\hat{s}} p(s^t - 1) I(s^t - 1)$. First, these quantities fulfill $B(s^t) \geq 0$, market-clearing conditions (5)–(7), and $q(s^t) = N(s^t) = 0$, so the equilibrium is fundamental. Second, the bank optimizes because (14) is satisfied by (19) and $D(s^t) = X(s^t)$ attains the value in (15). Third, agents optimize because their FOCs are satisfied for the proposed prices and quantities under (21); in particular, the stock-pricing equation (12) holds identically, as shown in the main text.

Part 3.—Prove existence by providing an example. Consider the function

$$I(s_t) = \frac{s_t \bar{s} y}{1 + 2\bar{s}}.$$

Because this function fulfills $0 < I(s_t) < \frac{s_t y}{2}$ and (21), Part 2 implies that there exists a fundamental banking equilibrium, which is stationary.

Proof of Proposition 3

Only if.—We show that any equilibrium consumption allocation with one-period-maturity securities fulfills $0 < c_1(s^t) < s_t y/2$, $c_1(s^t) = s_t c_2(s^t)$, and (21) with $I(s^t) = c_1(s^t)$ at every node. Since feasibility (1) then gives $c_3(s^t) = [s_t y - 2c_1(s^t)]/s_{t-1}$, Proposition 2 implies that it is the allocation of a fundamental banking equilibrium.

Agents' problem.—Let $w_2(s^t)$ denote disposable income,

$$w_2(s^t) = y + \sum_{j=1}^{J} d_j(s^t) a_{1j}(s^t - 1).$$

The FOCs of middle-aged agents are

$$\frac{Q_j(s^t)}{w_2(s^t) - \sum_{j=1}^{J} Q_j(s^t) a_{2j}(s^t)} = \sum_{s^{t+1}|s^t} \frac{d_j(s^{t+1})}{\sum_{j=1}^{J} d_j(s^{t+1}) a_{2j}(s^t)} \Pr(s^{t+1}|s^t)$$

for $j = 1, \ldots, J$. Multiply both sides of each FOC by the corresponding $a_{2j}(s^t)$ and add them all to see that middle-aged agents save (consume) half of their disposable income:

$$\sum_{j=1}^{J} Q_j(s^t) a_{2j}(s^t) = \frac{w_2(s^t)}{2}. \tag{27}$$

The FOCs of the young are

$$\frac{Q_j(s^t)}{\sum_{j=1}^{J} Q_j(s^t) a_{1j}(s^t)} + \sum_{s^{t+1}|s^t} s_{t+1} \frac{2 d_j(s^{t+1})}{y + \sum_{j=1}^{J} d_j(s^{t+1}) a_{1j}(s^t)} \Pr(s^{t+1}|s^t) = 0$$

for $j = 1, \ldots, J$. Multiply both sides of each FOC by the corresponding $a_{1j}(s^t)$ and add them all to see that

$$\sum_{s^{t+1}|s^t} s_{t+1} \frac{y - w_2(s^{t+1})}{w_2(s^{t+1})} \Pr(s^{t+1}|s^t) = \frac{1}{2}. \tag{28}$$

Market clearing.—Multiply both sides of (22) by the corresponding $Q_j(s^t)$ and add them all to write

$$\sum_{j=1}^{J} Q_j(s^t) a_{1j}(s^t) + s_t \sum_{j=1}^{J} Q_j(s^t) a_{2j}(s^t) = 0.$$

Then, use the budget constraints and (27) to obtain $c_1(s^t) = s_t c_2(s^t)$. Combining this with (28), we get that

$$\sum_{s^{t+1}|s^t} \frac{s_{t+1}^2}{c_1(s^{t+1})} \Pr(s^{t+1}|s^t) = \frac{1 + 2\bar{s}}{y},$$

whereas $c_1(s^t) < \frac{s_t y}{2}$ follows from $c_3(s^t) > 0$ and the feasibility constraint (1).

If.—Because $c(s^t)$ is the consumption allocation of a fundamental banking equilibrium,

there exists I fulfilling the conditions of Proposition 2 and such that

$$c_1(s^t) = I(s^t), \quad c_2(s^t) = \frac{1}{s_t} I(s^t), \quad c_3(s^t) = \frac{1}{s_{t-1}}[s_t y - 2I(s^t)].$$

Hence, we may rewrite security payoff $d(s^t) = \frac{1}{s_t}\frac{c_3(s^t)}{c_2(s^t-1)}$ as

$$d(s^t) = \frac{s_t y - 2I(s^t)}{s_t I(s^t - 1)}. \tag{29}$$

With a single one-period-maturity security, the FOCs of young and middle-aged agents are:

$$\sum_{s^{t+1}|s^t} s_{t+1} \frac{d(s^{t+1})a_1(s^t)}{y + d(s^{t+1})a_1(s^t)} \Pr(s^{t+1}|s^t) = -\frac{1}{2},$$

$$a_2(s^t) = \frac{y + d(s^t)a_1(s^t - 1)}{2Q(s^t)},$$

and the market-clearing condition is $a_1(s^t) + s_t a_2(s^t) = 0$. To prove the result, simply check that, given security payoffs (29), market-clearing security holdings $a_1(s^t) = -I(s^t)$ and $a_2(s^t) = \frac{1}{s_t} I(s^t)$, and constant security prices $Q(s^t) = 1$, fulfill the FOCs of the agents.

Proof of Corollary 1

With Arrow securities, the budget constraints of young, middle-aged, and old agents are:

$$c_1(s^t) + \sum_{s_{t+1} \in S} Q(s_{t+1}, s^t) a_1(s_{t+1}, s^t) = 0,$$

$$c_2(s^t) + \sum_{s_{t+1} \in S} Q(s_{t+1}, s^t) a_2(s_{t+1}, s^t) = y + a_1(s_t, s^t - 1),$$

$$c_3(s^t) = s_t a_2(s_t, s^t - 1),$$

where $a_1(s_{t+1}, s^t)$ and $a_2(s_{t+1}, s^t)$ are the holdings of the security that pays one unit of consumption if s_{t+1} realizes and $Q(s_{t+1}, s^t)$ is its price. The demand of Arrow securities by a middle-aged agent is

$$a_2(s_{t+1}, s^t) = \frac{y + a_1(s_t, s^t - 1)}{2Q(s_{t+1}, s^t)} \Pr(s_{t+1}) \tag{30}$$

for all $s_{t+1} \in S$ and the supply of Arrow securities by the young is

$$-a_1(s_{t+1}, s^t) = y - 2\frac{s_{t+1} y}{1 + 2\bar{s}} \Pr(s_{t+1}) \frac{\sum_{x \in S} Q(x, s^t)}{Q(s_{t+1}, s^t)} \tag{31}$$

for all $s_{t+1} \in S$. Use (31) to rewrite (30) as follows:

$$a_2(s_{t+1}, s^t) = \frac{s_t y}{1+2\bar{s}} \Pr(s_t) \frac{\Pr(s_{t+1})}{Q(s_{t+1}, s^t)} \frac{\sum_{x \in S} Q(x, s^t - 1)}{Q(s_t, s^t - 1)}.$$

Given these, market clearing in asset markets, $a_1(s_{t+1}, s^t) + s_t a_2(s_{t+1}, s^t) = 0$, can be written as

$$(1+2\bar{s}) Q(s_{t+1}, s^t) = \Pr(s_{t+1}) \left[2 s_{t+1} \sum_{x \in S} Q(x, s^t) + s_t^2 \Pr(s_t) \frac{\sum_{x \in S} Q(x, s^t - 1)}{Q(s_t, s^t - 1)} \right].$$

Adding over $s_{t+1} \in S$ shows that $\sum_{x \in S} Q(x, s^t)$ equals the last term in brackets. Substituting back, security prices fulfill

$$\frac{Q(s_{t+1}, s^t)}{\sum_{x \in S} Q(x, s^t)} = \frac{1+2s_{t+1}}{1+2\bar{s}} \Pr(s_{t+1})$$

at every node, independently of the history of prices; hence the equilibrium is unique. Substituting these conditions back into the supply and demand functions, we get the following equilibrium quantities:

$$-a_1(s_{t+1}, s^t) = \frac{y}{1+2s_{t+1}}, \quad a_2(s_{t+1}, s^t) = \frac{1}{s_t} \frac{y}{1+2s_{t+1}},$$

as well as the following equilibrium prices:

$$Q(s_{t+1}, s^t) = \frac{s_t^2}{1+2s_t} (1+2s_{t+1}) \Pr(s_{t+1}).$$

Going back to the budget constraints, the equilibrium consumption allocations are:

$$c_1(s^t) = \frac{s_t^2 y}{1+2s_t}, \quad c_2(s^t) = \frac{s_t y}{1+2s_t}, \quad c_3(s^t) = \frac{1}{s_{t-1}} \frac{s_t y}{1+2s_t},$$

as asserted.

With bonds, the payoff is $d(s^t) = 1$, so the disposable income of the middle aged, $w_2(s^{t+1}) = y + a_1(s^t)$, does not depend on s_{t+1}. By the first part of the proof of Proposition 3, $c_1(s^{t+1}) = s_{t+1} c_2(s^{t+1}) = s_{t+1} w_2(s^{t+1})/2$, and (21) with $I = c_1$ becomes $2\bar{s}/w_2(s^{t+1}) = (1+2\bar{s})/y$. Hence $w_2(s^{t+1}) = 2\bar{s}y/(1+2\bar{s})$ at every node, which gives the asserted allocation, with c_3 following from feasibility (1). Conversely, the bond price $Q(s^t) = s_t \bar{s}$ and the holdings $a_1(s^t) = -y/(1+2\bar{s})$ and $a_2(s^t) = y/[s_t(1+2\bar{s})]$ fulfill the FOCs of the agents and market clearing.

Proof of Proposition 4

Because (24) is a sum over $s_{t-1} \in S$ and (23) restricts $I(\cdot, s_{t-1})$ separately for each s_{t-1}, the problem splits into one problem for each $s_{t-1} \in S$:

$$\max_{I(\cdot,s_{t-1})} \mathbb{E}_{t-1}\left[(1+s_t)\log I(s_t,s_{t-1}) + s_{t-1}\log\left(s_t y - 2I(s_t,s_{t-1})\right)\right] \quad \text{subject to (23)}.$$

Fix s_{t-1}. The set of functions $I(\cdot, s_{t-1})$ that satisfy $0 < I(s_t, s_{t-1}) < s_t y/2$ for all $s_t \in S$ and the constraint is nonempty by Proposition 2. On this set the objective is continuous and tends to $-\infty$ at its boundary, so it attains a maximum.

The Lagrangian is

$$\mathcal{L} = \mathbb{E}_{t-1}\left[(1+s_t)\log I(s_t,s_{t-1}) + s_{t-1}\log\left(s_t y - 2I(s_t,s_{t-1})\right)\right] + \lambda(s_{t-1})\left(\mathbb{E}_{t-1}\left[\frac{s_t^2}{I(s_t,s_{t-1})}\right] - \frac{1+2\bar{s}}{y}\right).$$

Since $\Pr(s_t) > 0$, the first-order conditions read

$$\frac{1+s_t}{I(s_t,s_{t-1})} - \frac{2s_{t-1}}{s_t y - 2I(s_t,s_{t-1})} = \lambda(s_{t-1})\frac{s_t^2}{I(s_t,s_{t-1})^2}$$

for all $s_t \in S$. Multiplying by $I(s_t,s_{t-1})^2/s_t^2$ and collecting terms gives (25).

It remains to show that $\lambda(s_{t-1}) > 0$. In (25), the factor multiplying $I^u(s_t,s_{t-1}) - I(s_t,s_{t-1})$ is positive, so $I(s_t,s_{t-1}) \geq I^u(s_t,s_{t-1})$ for all s_t if $\lambda(s_{t-1}) \leq 0$. Since s_t^2/I is decreasing in I, this would imply

$$\mathbb{E}_{t-1}\left[\frac{s_t^2}{I(s_t,s_{t-1})}\right] \leq \mathbb{E}_{t-1}\left[\frac{s_t^2}{I^u(s_t,s_{t-1})}\right] = \frac{2}{y}\left(\bar{s} + s_{t-1}\mathbb{E}\left[\frac{s_t}{1+s_t}\right]\right) < \frac{1+2\bar{s}}{y},$$

where the last inequality holds because $s_t/(1+s_t) \leq 1/2$ and $s_{t-1} \leq 1$, with equality only in the trivial case $S = \{1\}$, which we exclude. This contradicts (23). Hence $\lambda(s_{t-1}) > 0$ and, by (25), $I(s_t,s_{t-1}) < I^u(s_t,s_{t-1})$ for all s_t.

Proof of Proposition 5

Let $l_1(s^t) = L_1(s^t)/p(s^t)$. Under the indexed loan contract, the disposable income of a middle-aged agent at s^{t+1} is $w_2(s^{t+1}) = y - \tilde{R}(s^t)l_1(s^t)$, which does not depend on s_{t+1}. The problem of the middle aged is otherwise unchanged, so $c_2 = w_2/2$, and the FOC of the young reads

$$\frac{1}{l_1(s^t)} = \frac{2\bar{s}\tilde{R}(s^t)}{y - \tilde{R}(s^t)l_1(s^t)}.$$

Hence $\tilde{R}(s^t)l_1(s^t) = y/(1+2\bar{s})$ and $c_2(s^{t+1}) = \bar{s}y/(1+2\bar{s})$ at every node. In a fundamental banking equilibrium, the aggregate savings of the middle aged are $B(s^t) +$

$N(s^t) = L(s^t)$, so $c_1(s^t) = s_t c_2(s^t)$, as in the proof of Proposition 2. This gives the bonds allocation of Corollary 1, with c_3 following from feasibility (1).

For existence, note that, by the one-shot deviation of Lemma 2, the bank is indifferent to the quantity of loans if and only if $\tilde{R}(s^t) = 1/\mathbb{E}_t[s_{t+1} m_2(s^{t+1})]$. Under this condition, the proof of Lemma 3 goes through with net income $[s_{t+1}\tilde{R}(s^t)\pi(s^{t+1}) - 1]L(s^t)$, so the policy $D = X$ is optimal. In the bonds allocation, $m_2(s^{t+1}) = \bar{s}s_t/s_{t+1}$, so this condition gives $\tilde{R}(s^t) = 1/(\bar{s}s_t)$, the same rate implied by the FOC of the young with $l_1(s^t) = s_t \bar{s} y/(1 + 2\bar{s})$. Take any positive inflation rates that satisfy (11), which here reads

$$\bar{s}s_t \, \mathbb{E}_t \left[\frac{1}{s_{t+1}\pi(s^{t+1})} \right] = 1,$$

for instance $\pi(s^{t+1}) = \bar{s}s_t/\hat{s}$, and the quantities of Part 2 of the proof of Proposition 2, with $D = X$, $N = q = 0$, and $B = L \geq 0$. The stock-pricing equation (12) holds because $\mathbb{E}_t[(m_2/\pi)X(s^{t+1})] = \left(\tilde{R}(s^t)\,\mathbb{E}_t[s_{t+1} m_2(s^{t+1})] - 1\right) L(s^t) = 0$. Since (11) is a single restriction on the inflation rates of the successor nodes, the price level is indeterminate.

Proof of Corollary 2

By Proposition 2, (11) and (14) hold at every node, so, by Lemma 3, every dividend policy that satisfies (16) attains $X(s^t)/p(s^t)$, which is the maximal value in (15).

B Power utility and state-contingent rate

Power utility

We show that the sign of inflation in the stationary equilibrium of Proposition 1 depends on preferences. Let agents have a power utility function $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, with $\gamma > 0$ and $\gamma \neq 1$; logarithmic utility is the limit $\gamma \to 1$.

The value function of middle-aged agents is

$$V_2(w_{2t}) = \max_{B_2, z_2} \frac{1}{1-\gamma} \left[\left(w_{2t} - \frac{B_2 + q_t z_2}{p_t} \right)^{1-\gamma} + \left(\frac{B_2 + [D_{t+1} + q_{t+1}] z_2}{p_{t+1}} \right)^{1-\gamma} \right].$$

From the FOCs of this problem:

$$B_{2t} + q_t z_{2t} = \frac{p_t w_{2t}}{1 + \pi_{t+1}^{1/\gamma - 1}},$$
$$q_t = D_{t+1} + q_{t+1}.$$

Combining these with (14), we get that $R_t = 1/\bar{s}$. Young agents solve

$$\max_{L_1} \frac{1}{1-\gamma}\left(\frac{L_1}{p_t}\right)^{1-\gamma} + \bar{s}V_2\left(y - \frac{L_1}{\bar{s}p_{t+1}}\right).$$

By the Envelope Theorem, the loan demand is

$$L_{1t} = \frac{\pi_{t+1}^{1/\gamma}\bar{s}p_t y}{\bar{s} + \pi_{t+1}^{1/\gamma-1} + \bar{s}\pi_{t+2}^{1-1/\gamma}}.$$

Using market-clearing conditions (5)–(7) and $q_t = N_t$ (a fundamental banking equilibrium, Definition 2), we get the following second-order difference equation for inflation:

$$\pi_{t+2} = \left(\frac{\bar{s} + \pi_t^{1/\gamma-1} + \bar{s}\pi_{t+1}^{1-1/\gamma}}{\bar{s}^2\pi_{t+1}^{1-2/\gamma}} - 1 - \frac{1}{\bar{s}}\pi_{t+1}^{1/\gamma-1}\right)^{\gamma/(\gamma-1)}.$$

In a stationary equilibrium, $\pi_t = \pi$ for all t, and the equilibrium conditions reduce to $\bar{s} = \pi^{2/\gamma-1}$. Hence, for every $\gamma \neq 2$, there is a unique stationary fundamental banking equilibrium, with

$$\pi = \bar{s}^{-\gamma/(\gamma-2)},$$

and the implied consumption allocation is strictly positive. The equilibrium is deflationary if $\gamma < 2$ and inflationary if $\gamma > 2$. The stationary solution is singular at $\gamma = 2$, and not only in prices. As $\gamma \to 2^-$ the allocation collapses toward giving all consumption to the old ($c_1, c_2 \to 0$, $c_3 \to y$), and as $\gamma \to 2^+$ toward giving it all to the young ($c_1 \to \bar{s}y$, $c_2, c_3 \to 0$); for $\bar{s} = 0.95$ and $y = 1$, $\gamma = 1.99$ gives $c_3 = 0.994$ and $\gamma = 2.01$ gives $c_1 = 0.945$. The allocation is strictly positive for every $\gamma \neq 2$, but economically meaningful only for γ well away from 2. Away from the singularity the equilibrium converges, as risk aversion grows, to the golden rule.

Lemma 4. *As $\gamma \to \infty$, the stationary fundamental banking equilibrium converges to the golden rule allocation of Lemma 1 at $s_t = s_{t-1} = \bar{s}$: $c_1, c_2, c_3 \to \bar{s}y/(1+2\bar{s})$, and $\pi \to 1/\bar{s}$.*

Proof. By the formula for π, $\pi \to 1/\bar{s}$, so $\pi^{1/\gamma} \to 1$, $\pi^{1/\gamma-1} \to \bar{s}$ and $\pi^{1-1/\gamma} \to 1/\bar{s}$. The stationary form of the young's loan demand gives $c_1 \to \bar{s}y/(\bar{s}+\bar{s}+1) = \bar{s}y/(1+2\bar{s})$. Then $w_2 = y - c_1/(\bar{s}\pi) \to (1+\bar{s})y/(1+2\bar{s})$ and $c_2 = w_2\pi^{1/\gamma-1}/(1+\pi^{1/\gamma-1}) \to \bar{s}y/(1+2\bar{s})$. Feasibility, $c_1 + \bar{s}c_2 + \bar{s}c_3 = \bar{s}y$, then forces $c_3 \to \bar{s}y/(1+2\bar{s})$. □

State-contingent rate

The set of equilibrium allocations is the same if the bank charges at node s^t the state-contingent nominal loan rate $R(s_{t+1}, s^t) = 1/s_{t+1}$ due in $t+1$, which makes its profit

zero in every state. Given this new nominal rate, the real rate remains the same as in Proposition 2 *iff* the new inflation rate is

$$\pi(s^t) = \frac{l(s^t - 1)}{s_t y - 2l(s^t)}. \tag{32}$$

The problem of the middle-aged does not change with the new rates; they still save (consume) half of their disposable income. Hence, the restriction to fundamental banking equilibria is unaffected. The problem of young agents becomes

$$\max_{L_1} \log \frac{L_1}{p(s^t)} + \mathbb{E}_t \left[s_{t+1} V_2 \left(y - \frac{L_1}{s_{t+1} p(s^{t+1})}, s^{t+1} \right) \right].$$

Introducing market-clearing condition (5) into the FOC of this problem, we get that

$$\frac{1}{L(s^t)} = \mathbb{E}_t \left[\frac{2 s_{t+1}}{s_{t+1} p(s^{t+1}) y - L(s^t)} \right].$$

Combining this with (32) takes us back to (21).