The most severe impacts of the financial crisis of 2007–2009 arose immediately after the failure of Lehman Brothers on September 15, 2008. It is natural to wonder whether the United States should have arranged for an orderly rescue of Lehman as it did for Fannie Mae and Freddie Mac the week before and as it did for AIG, Merrill Lynch, Citigroup, Bank of America, Morgan Stanley, Goldman Sachs, Washington Mutual, and Wachovia as well as many smaller and foreign banks over the next days and weeks. How much capital would have been necessary ex post to arrange such an orderly rescue? Another policy recommendation of the Dodd-Frank Act of 2010 is to facilitate orderly liquidation and/or resolution and require living wills of financial institutions so that no future bailouts will be necessary. Will this work when we need it? There is, however, also a third choice. Rather than discuss whether to rescue or not, it is sensible to regulate ex ante financial institutions whose failure is likely to have major impacts on the financial and real sectors of the economy; for instance, regulate them to reduce their risk, and consequently the probability that taxpayers will face this choice.

Effective and efficient regulation of this type requires identification of systemically important financial institutions (SIFIs). A typical definition has been provided by Federal Reserve Governor Daniel Tarullo:1 “Financial institutions are systemically important if the failure of the firm to meet its obligations to creditors and customers would have significant adverse consequences for the financial system and the broader economy.” This definition is useful because it highlights two important ideas. The first is that the core problem is a firm’s difficulty in performing financial services when it fails, i.e., when its capital falls short. The second is that systemic risk matters only to the extent there is an impact on the broader economy. There is a large theoretical and empirical literature that supports these two ideas (see, for example, Thakor 1996 and Holmstrom and Tirole 1997 on the theoretical side; and Bernanke 1983; Slovin, Sushka, and Polonchek 1993; and Gibson 1997 for empirical observations).

The definition, however, misses a key feature of systemic risk. Systemic risk should not be described in terms of a financial firm’s failure per se but in the context of a firm’s overall contribution to systemwide failure. The intuition is straightforward. When only an individual financial firm’s capital is low, the firm can no longer financially intermediate. This has minimal consequences, though, because other financial firms can fill in the failed firm’s void. When capital is low in the aggregate, however, it is not possible for other financial firms to step into the breach. This breakdown in aggregate financial intermediation is the reason there are severe consequences for the broader economy.

Motivated by this one economic point, it is possible to provide a precise definition of the systemic risk of a financial firm. Acharya et al. (2010c) develop a simple model in which a group of banks set leverage levels and choose asset

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positions in a broader economic environment with systemic risk emerging when aggregate bank capital drops below a given threshold. Within this framework, they show that the systemic risk of a firm is equal to the product of three components:

\[
\text{(1) Real systemic risk of a firm = Real social costs of a crisis per dollar of capital shortage}
\]

\[
\times \text{ Probability of a crisis (i.e., an aggregate capital shortfall)}
\]

\[
\times \text{ Expected capital shortfall of the firm in a crisis}
\]

The focus of this paper is on the third component, namely the expected capital shortfall of a firm in a crisis. Expected capital shortfall captures in a single measure many of the characteristics considered important for systemic risk such as size, leverage, and interconnectedness (e.g., see the 2011 annual report of the Financial Stability Oversight Council (FSOC), formed in the United States following the Dodd-Frank Act of 2010, for the determined regulatory factors for assessing systemic risk of financial firms). All of these characteristics tend to increase a firm’s capital shortfall when there are widespread losses in the financial sector. But a firm’s expected capital shortfall also provides an important addition, most notably the comovement of the financial firm’s assets with the aggregate financial sector in a crisis.

Stress tests are a standard device used to determine the capital that an institution will need to raise if there is a financial crisis. Under the Dodd-Frank Act, the regulators in the United States are required to conduct annual stress tests to assess capital adequacy of financial firms. The expected capital shortfall estimation we describe below can be a useful tool or substitute for such stress tests.

I. The Methodology

In Brownlees and Engle (2011), a model of this form is implemented based on publicly available data in order to determine which institutions are systemically risky, what the cost of a bailout would be, and how this leads naturally to a regulatory strategy. The results of this analysis are updated weekly and posted at http://vlab.stern.nyu.edu/welcome/risk. Results are posted both for approximately 100 US financial firms and for 1,200 global financial firms. Information from this website will be described below.

The method to be described computes SRISK, which is defined as the capital that a firm is expected to need if we have another financial crisis. Symbolically it can be defined as

\[
\text{(2) } \text{SRISK}_{i,t} = E_{t-1} (\text{Capital Shortfall} \mid \text{Crisis}).
\]

This can be estimated with a bivariate daily time series model of equity returns on firm \(i\) and on a broad market index (which could be just the financial sector).

\[
\text{(3) } R_{m,t} = \sigma_{m,t} \varepsilon_{m,t}
\]

\[
R_{i,t} = \sigma_{i,t}(\rho_{i,m,t} \varepsilon_i + \sqrt{1 - \rho_{i,m,t}^2} \xi_i)
\]

\[
(\varepsilon_{m,t}, \xi_{i,t}) \sim F.
\]

In this system, the volatilities are asymmetric GARCH processes and the correlation is estimated by DCC. The joint distribution, \(F\), ensures that the random variables are uncorrelated but not independent. It can be parameterized with a copula or simply treated as an empirical cumulative distribution function (cdf). The parameters of this process can be estimated by QMLE using standard techniques. On the website, recursive estimates are presented so that systemic risk inferences are made at each point using only the information that was available.

To calculate the systemic risk, this system first evaluates the losses that an equity holder would face if there is a future crisis. To do this, the system is simulated for six months into the future many times. This simulation allows volatilities and correlations to change over time and samples from the empirical distribution \(F\) so that empirical tail dependence is maintained. The most pessimistic scenarios for the market return are treated as Crisis scenarios. To be specific, whenever the broad index falls by 40 percent over the next six months, this is viewed as a
crisis. For these scenarios, the expected loss of equity value of firm $i$ is called the Long Run Marginal Expected Shortfall or $LRMES$. This is just the average of the fractional returns of the firm’s equity in the crisis scenarios. In versions of the model where the simulation is not yet implemented, $LRMES$ is approximated as $1-\exp(-18 \times MES)$ where $MES$ is the one day loss expected if market returns are less than $-2$ percent.

The capital shortfall can be directly calculated by recognizing that the book value of debt will be relatively unchanged during this six-month period while equity values fall by $LRMES$. If a prudential capital ratio is considered to be $k$ which we take as 8 percent, then

$$SRISK_{it} = E((k(Debt_i + Equity_i) - Equity_i)|Crisis)$$

$$= kDebt_{it} - (1 - k)(1 - LRMES_{it})$$

$$\times Equity_{it},$$

where $Equity_i$ is the market value of equity today. The contribution to aggregate $SRISK$ by any firm is also tabulated as

$$SRISK\%_{i,t} = \frac{SRISK_{i,t}}{\sum_{j \in J} SRISK_{j,t}},$$

$$J = \{\text{firms with positive } SRISK\}.$$

An enhanced definition of a Crisis will incorporate additional features into the analysis in the future. A crisis can be defined, e.g., by the undercapitalization of the financial sector. Thus if $E$ is the sum of all equity in the financial sector and $D$ is the aggregate book value of debt, then a crisis means that

$$E < \frac{k}{1-k} D.$$  

Thus, $LRMES$ would be evaluated on scenarios satisfying (6). This solution would allow evaluation of the probability of a crisis in equation (1) and implement the leverage externality whereby the systemic risk due to the leverage of each firm will increase with the leverage of other firms. This definition will also make the policy implementation less procyclical.

II. A Market Based Alternative to Basel Risk Weights

A reasonable regulatory requirement might be that $SRISK = 0$. In this case a firm will not need, at least in expectation, to raise capital in a future crisis of the severity assumed. From (4) this implies that

$$Equity_{i,t} \geq \frac{k Debt_{i,t}}{(1-k)(1-LRMES_{i,t})}.$$  

Using numbers from Bank of America in Table 1 below, $LRMES$ is 71 percent and imposing a hard capital requirement of $k = 4$ percent, the firm specific ratio of equity to debt is 0.14 or maximum leverage ratio of debt to equity is 7.1. For Wells Fargo this calculation gives a maximum leverage of 9.6, which is essentially what it has today. Thus, each firm would have individual prudential capital requirements based on the risk profile of its business. Any firm desiring to reduce its capital requirement could delever, derisk, demerge or decline bets that are highly correlated with the broad market.

Basel capital requirements use risk weights to adjust assets against which capital must be held. This is in a sense equivalent to our approach, but with an important difference. Equation (7) can be rewritten in terms of the quasi-assets of the firm by adding market value of equity to book value of debt.

$$Equity_{i,t} \geq \frac{k Assets_{i,t}}{1 - (1-k)LRMES_{i,t}}.$$  

where $k$ is the hard leverage constraint, say 4 percent, and the fraction $(1 - (1-k)LRMES)^{-1}$ can be interpreted as the risk-weight corresponding to our approach.

One can interpret our risk-weight approach as an alternative to the much criticized Basel risk weights. In theory, the risk weight based on the systemic measure $LRMES$ incorporates the risk of the underlying assets. In terms of the underlying intuition, firms with systemically risky assets and leverage will have higher MES and must hold higher amounts of capital. For example, if the expected return in a crisis is $-100$ percent, then the firm would have to be fully capitalized (i.e., no debt). If the expected
return is 0 percent, then the firm would need to hold just 4 percent of capital. During the recent financial crisis, the average return of the 25 percent worst performing bank holding companies was $-87$ percent versus $-17$ percent for the top performing 25 percent. For $k = 4$ percent, this would translate to a 24.27 percent capital requirement for the more systemic firms (as measured by their realized LRME$S$ in the crisis) and just 4.78 percent for the less systemic ones.

III. Alternative Approaches

Two very interesting alternatives will be discussed here. The $CoVaR$ methodology of Adrian and Brunnermeier (2009) can be compared under some simplifying assumptions. Suppose at time $t$, the joint distribution of returns of one firm and the broad market is conditionally normal and expressed as:

$$
\begin{pmatrix}
    R_{i,t} \\
    R_{m,t}
\end{pmatrix} \bigg| F_{t-1} 
\sim N \left( 0, 
    \begin{pmatrix}
        \sigma_{i,t}^2 & \rho_{i,m,t} \sigma_{i,t} \sigma_{m,t} \\
        \rho_{i,m,t} \sigma_{i,t} \sigma_{m,t} & \sigma_{m,t}^2
    \end{pmatrix} \right). 
$$

$VaR(q)$ and $CoVaR(q,p)$ are defined as

$$
P_{t-1}(R_{i,t} < -VaR^p_{i,t}) = q
$$

$$
P_{t-1}(R_{m,t} < -VaR^p_{i,t}) = q
$$

$$
P_{t-1}(R_{m,t} < -CoVaR^p_{i,t} | R_{i,t} = -VaR^p_{i,t}) = q
$$

and the contribution to systemic risk, which measures the increased risk to the system when one firm is at its extreme, is called

$$
\Delta CoVaR_i = CoVaR^P_{i,t} - CoVaR^q_{i,t}. 
$$

Evaluating (11) under (9) yields a simple closed form expression

$$
\Delta CoVaR_i = \rho_{i,m,t} \sigma_{m,t} \Phi^{-1}(q),
$$

where $\Phi^{-1}$ is the inverse function of the standard normal cdf.

Evaluating MES under the same assumptions gives:

$$
MES_{i,t} = E(-R_{i,t} | R_{m,t} < c) = -E(\sigma_{i,t} \rho_{i,m,t} \epsilon_{m,t} | \epsilon_{m,t} < c / \sigma_{m,t}) + 0
$$

$$
= \sigma_{i,t} \rho_{i,m,t} E(\epsilon_{m,t} | \epsilon_{m,t} < c / \sigma_{m,t}).
$$

This expression depends upon firm $i$ through the product of its correlation with the market and its volatility, whereas $CoVaR$ depends only on the correlation. Two firms with the same correlation with the market but different volatilities will be treated the same by $CoVaR$ even if one has volatility nearly zero and is essentially riskless. Adrian and Brunnermeier introduce a second measure called Exposure $CoVaR$, which reverses the conditioning and gets the same result as $MES$. $CoVaR$ measures are also not explicitly sensitive to size or leverage, while other work such as ours is aimed at capturing these features.

This discussion raises the question of causality. Are firms weak because of the crisis, or does the crisis happen because the firms are weak? We would argue that both are true. There cannot be a financial crisis without weak firms. These are jointly endogenous variables with no implication of causality. The research of Billio et al. (2010) presents five different systemic risk measures. The last two are Granger causality tests and therefore address this question of causality. Among these tests, one is linear and the other nonlinear. The nonlinear version includes volatility shocks from one firm or sector to another. The linear test examines whether returns on one firm can predict returns on another firm a month or more in the future. This used to be considered a market efficiency test but here is taken as a sign of a network linkage. The linkages are estimated with bivariate models that partially correct for heteroskedasticity and marketwide shocks. However, it is well known that Granger causality tests cannot be correctly interpreted unless all the shocks are considered at the same time. It is not possible to know whether one firm Granger causes another or whether some third firm causes both. This represents a fundamental difficulty in the interpretation of these results and argues in favor of tests for weakness of indi-
vidual firms as a function of market weakness such as Exposure CoVaR or MES.

IV. US Systemic Risk Measures

At the end of 2011 the ten most systemically risky financial firms in the United States are given in online Appendix Table 1, which shows the SRRISK for these firms as well as the LRMES, Beta, Leverage and MV. Judging from SRRISK, the three top firms have the bulk of the contributions to systemic risk. For Bank of America and Citigroup, this is due to high leverage, and for JP Morgan, it is due to its enormous size (MV).

Online Appendix Table 2 shows the same information two weeks before the Lehman bankruptcy filing. As can be seen, this is a list of the institutions that were either rescued or restructured. All but one of the top ten firms were on the verge of failure within weeks. The interesting observation is that Lehman was number 11. Perhaps it was believed that this institution was not sufficiently systemic to require rescue, or perhaps there was a limit to the resources and Lehman was the next on the list. Nevertheless, the list is a close approximation to the policy decisions that were made at that time.

V. European Systemic Risk Measures

The same set of results is now available for 1,200 global financial institutions. The method is the same, although the econometrics must be adjusted to incorporate nonsynchronous trading in multiple markets. The methodology is based on insights from Scholes and Williams (1977) and more recently by Burns, Engle, and Mezrich (1998). The econometric implementation is based on the Dynamic Conditional Beta model recently proposed in Engle (2011). The top European institutions today are shown in online Appendix Table 3. These institutions represent all the large nations of Europe and tell an important message about the seriousness of the challenge to the European financial system. This list can be compared with the set of 17 European banks considered to be Global SIFIs by the Bank for International Settlements/Financial Stability Board/G-20 in their statement released November 4, 2011. The lists are identical except that the New York University list includes Banco Intessa and BIS includes Dexia (which has dropped down to number 20 for NYU). It took the BIS two years and many meetings to develop this list. The NYU list, ranked by SRRISK%, has already been updated many times since early November. It remains to be seen how the BIS will rank these financial institutions or set capital charges.

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