# Technology, Skill and Long Run Growth

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# 1. Introduction

This paper develops a model in which **two factors**—technology and skill—contribute to long run growth.

Long run growth in turn comes in **two forms**—TFP growth and increased variety. Both factors contribute to both forms.

The rate of TFP growth, the common growth rate of technology and skill, is determined mainly by the parameters governing skill accumulation.

Growth in variety depends, roughly, on the **difference** between the parameters governing technology and skill accumulation. .

The main result are conditions for the existence of a balanced growth path (BGP), and simple formulas for TFP growth and growth in variety.

# 1. Introduction

Why is it interesting to include both factors in one model?

Because there is complementarity in the incentives to invest in each.

Without growth in technology (in skill), the incentive to invest in skill (in technology) would decline and, presumably, vanish in the long run.

Long run growth requires investment in both.

Here there is no 'race' between technology and skill:

neither factor has an incentive to get 'ahead' of the other.

# 1. Introduction: related literature

1. Growth based on human capital accumulation
   Uzawa (1965), Romer (1986), Lucas (1988, 2009),
   Lucas & Moll (2014), Perla & Tonetti (2014), and others.
2. Growth based on endogenous technological change
   Romer (1990), Grossman & Helpman (1991), Aghion & Howitt (1992), Stokey (1995), Acemoglu (2002), Klette & Kortum (2004), Luttmer (2007), Atkeson & Burstein (2010), and others.
3. Growth based on learning by doing
   Arrow (1962), Stokey (1987), Young (1991, 1993)
4. Growth in product variety, growth in OLG models, ...

# 1. Introduction: outline

2. Production and prices
3. Dynamics
4. Conditions for balanced growth
5. Balanced growth paths
6. Conclusions

# 2. Production and prices

Continuum of intermediates indexed by technology $x > x_m > 0$,

with density $f(x)$. $N_p$ is the number (mass) of producers.

Continuum of workers indexed by skill $h > h_m > 0$,

with density $\widetilde{g}(x)$. $L_w$ is the number (mass) of workers.

If a firm with tech. $x$ employs $\ell(x)$ workers with human capital $h$,

its output is

$$y(x) = \ell(x)\phi(h, x),$$

where $\phi(h, x)$ is the CES function with elasticity $\eta < 1$, and weights

$\omega$, $1 - \omega$ on skill and technology.

Since $\eta < 1$, the function is log supermodular.

# 2. Production and prices

A single final good, produced competitively with the CRS technology

$$y_F = \left[ N^{1-\nu\rho} \int y(x)^{(\rho-1)/\rho} f(x) dx \right]^{\rho/(\rho-1)},$$

where the elasticity is $\rho > 1$, and $\nu \in (0, 1/\rho]$ measures decreasing returns to increased variety.

The optimal price for each firm is a markup $\rho / (\rho - 1)$ over unit cost.

# 2. Production and prices

A **production equilibrium** consists of a wage function $w(h)$ and functions $h^*(x), \ell(x), p(x), y(x), \pi(x)$ describing the skill allocation, employment, price, output, and profits for each type of firm, and output $y_F$ of the final good. (Price normalization $p_F = 1$.)

In equilibrium

—intermediate firms choose labor quality and price $h^*(x), p(x)$, to maximize profits, and these choices determine $y(x), \ell(x), \pi(x)$;

—final good firms choose intermediates to minimize cost,

—labor markets clear, and $y_F$ is consistent with $y(x)$.

# 2. Production and prices

The production environment is described by the parameters, including

$N_p, L_w$, and the distribution functions (DFs) $F, G$.

LEMMA 1: For any environment, a production equilibrium exists,

and it is unique and efficient.

Production equilibria have useful homogeneity properties for

—changes in $L_w$ or $N_p$, and

—common shifts in the two DFs.

We'll specialize to Pareto distributions to look at these. Define

$$\Omega = \frac{1-\rho\nu}{\rho-1} \in \left[0, \frac{1}{\rho-1}\right).$$

# 2. Production and prices: Pareto DFs

Suppose $F$, $G$ are Pareto distributions, with parameters $(\alpha, x_m)$, $(\gamma, h_m)$,

where $\alpha, \gamma > 1$, and

$$-1 < \alpha - \gamma < \rho - 1,$$

and their locations are 'aligned,'

$$h_m = a_h x_m,$$

where $a_h$ is a constant.

Then the production equilibrium has a simple form.

The shape parameters $\alpha, \gamma$ can differ, but not by too much.

PROP. 4: For Pareto distributions as above, the skill allocation is linear, and the wage, price, output, employment, and profit functions are isoelastic,

$$
\begin{aligned}
h^*(x) &= a_h x, \\
w(h) &= w_0 \quad N_p^{\Omega} \quad x_m \quad (h/x_m)^{1-\varepsilon}, \\
p(x) &= p_0 \quad N_p^{\Omega} \quad (x/x_m)^{-\varepsilon}, \\
y(x) &= y_0 \quad L_w N_p^{-1} \quad x_m \quad (x/x_m)^{\rho\varepsilon}, \\
\ell(x) &= \ell_0 \quad L_w N_p^{-1} \quad (x/x_m)^{\rho\varepsilon-1}, \\
\pi(x) &= \pi_0 \quad L_w N_p^{\Omega-1} \quad x_m \quad (x/x_m)^{(\rho-1)\varepsilon}, \\
y_F &= y_{F0} \quad L_w N_p^{\Omega} \quad x_m,
\end{aligned}
$$

where $w_0, p_0, y_0, \ell_0, \pi_0, y_{F0}$, are constants, and

$$\varepsilon \equiv \frac{1}{\rho}(1+\alpha-\gamma) \in (0,1).$$

# 3. Dynamics

The investment technology for each factor is taken from Perla and Tonetti (2014).

It is a simple and flexible model based on imitation:

- simple because the investment decision is 0-1.
- flexible because it allows any profit function.

It puts a strong restriction (Pareto) on the shape of the distributions of skill and technology.

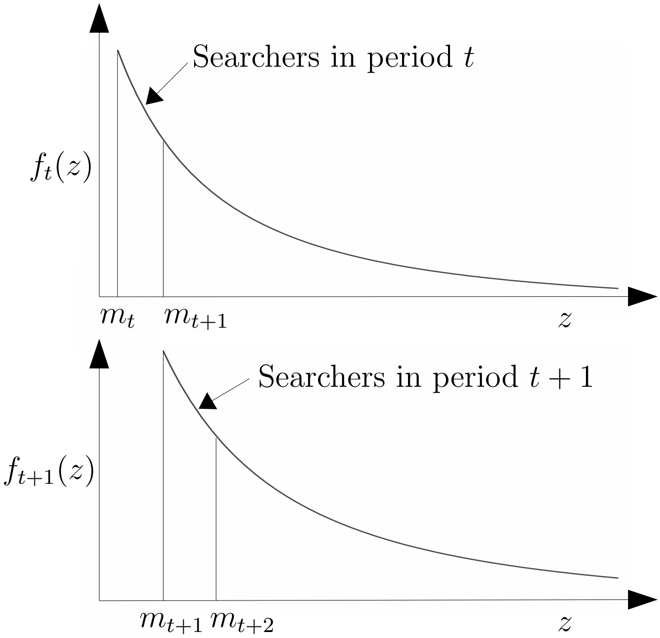


Figure 1: Evolution of the Productivity PDF

# 3. Dynamics

Firms/labor force members exit exogenously at the fixed rates $\delta_x, \delta_h > 0$.

The technology/skill of producers/workers grow at the fixed rates $\mu_x, \mu_h$.

A producer/worker can at any time abandon its current technology/skill and attempt to acquire a new one.

Call this process innovation/retooling.

The only cost of process innovation/retooling is an opportunity cost: the firm/worker cannot produce while investing.

Success is stochastic, with fixed hazard rates $\lambda_x, \lambda_h$

Conditional on success, the process innovator/retooler receives a technology/skill that is a random draw from those of current producers/workers.

# 3. Dynamics

Hence producers/workers switch to investing if and only if their technology/skill lies below (endogenously determined) thresholds.

Population grows at the fixed rate $\upsilon$.

New entrants to the labor force join the pool of retoolers.

On a BGP, firms enter at a constant (endogenously determined) rate $n$.

Entering firms make costly investments.

Call this product innovation.

# 3. Dynamics

An entrant chooses its hazard rate $\lambda_e$ for success and pays a flow cost $i_e(\lambda_e)$, scaled by the average profits of current producers.

After successful innovation, the entrant must also pay a one-time sunk cost to begin production.

The sunk cost $s_e > 0$ is also scaled by the average profits of current producers.

A free entry condition determines the number of firms on a BGP.

We will skip directly to the conditions for balanced growth.

# 4. Conditions for balanced growth

On a BGP

—the DFs for $X$ and $H$ shift at a common, constant rate $g$.

—the number of firms grows at a constant rate $n$.

Average profits per firm, the average wage, and final output grow at rates

$$\begin{aligned} g_\pi &= g + \upsilon + (\Omega - 1)\, n, \\ g_w &= g + \Omega n, \\ g_Y &= g + \upsilon + \Omega n. \end{aligned}$$

Hence the thresholds $X_m, H_m$ grow at rate $g$,

—value functions for firms shift at rate $g_\pi$,

—value functions for workers shift at rate $g_w$.

# 4. Conditions for balanced growth

Let $v_f(x)$ and $v_{f0}$ denote the normalized values for producers and process innovators.

For Pareto distributions, the normalized Bellman equations are

$$\begin{aligned}(r+\delta_x-g_\pi)\,v_f(x) &= \pi_1 x^{1-\zeta}+(\mu_x-g)\,xv_f'(x), \qquad x\geq x_m,\\ (r+\delta_x-g_\pi)\,v_{f0} &= \lambda_x\left\{\mathrm{E}_F\left[v_f(x)\right]-v_{f0}\right\}.\end{aligned}$$

Value matching (VM) provides a boundary condition for $v_f$, and optimization—smooth pasting (SP)—relates $x_m$ and $v_{f0}$.

The normalized DF has $\mathrm{E}_F(x)=1$, which determines $x_m$, so SP determines $v_{f0}$.

$v_{f0}$ factors out of the VM condition, giving an equation in $g, n$,

$$r+\delta_x-g_\pi=\frac{-(1-\zeta)\,R_x\lambda_x}{(\alpha+\zeta-1)\,(-R_x+\alpha)}.$$

# 4. Conditions for balanced growth

Let $v_e$ denote the normalized value of a product innovator.

The entering firm chooses a hazard rate $\lambda_e$.

The cost function $i_e$ is strictly strictly convex, with $i_e(0) > 0,\ i_e'(0) = 0$.

The normalized Bellman equation for an entrant is

$$(r + \delta_x - g_\pi)\, v_e = \max_{\lambda_e \geq 0} \lambda_e \left\{ \mathrm{E}_F \left[ v_f(x) \right] - v_e \right\} - i_e(\lambda_e) \mathrm{E}_F \left[ \pi(x) \right],$$

and the optimal $\lambda_e$ satisfies

$$i_e'(\lambda_e) \mathrm{E}_F \left[ \pi(x) \right] = \mathrm{E}_F \left[ v_f(x) \right] - v_e.$$

# 4. Conditions for balanced growth

The entrant also pays the sunk cost $s_e$, scaled by the average profits of producers, so the free entry condition is

$$\begin{aligned} v_e &= s_e \mathrm{E}_F \left[ \pi(x) \right] \\ &= s_e L_{w0} N_{p0}^{\Omega-1} \pi_0 \mathrm{E}_F \left[ x^{1-\zeta} \right]. \end{aligned}$$

Given $L_{w0}$, this condition determines the required value $N_{p0}$ for a BGP.

# 4. Conditions for balanced growth

The Bellman equations for workers and retoolers are

$$\begin{aligned}(r+\delta_h-g_w)\,v_w(h) &= w_0 h^{1-\varepsilon}+(\mu_h-g)\,hv_w'(h)\\ (r+\delta_h-g_w)\,v_{w0} &= \lambda_h\left\{\mathrm{E}_G\left[v_w(h)\right]-v_{w0}\right\}.\end{aligned}$$

$h_m$ is determined by the normalization, so smooth pasting determines $v_{w0}$.

$v_{w0}$ factors out of the VM condition, giving another equation in $g, n$,

$$r+\delta_h-g_w=\frac{-(1-\varepsilon)\,R_h\lambda_h}{(\gamma+\varepsilon-1)\,(-R_h+\gamma)}.$$

# 4. Conditions for balanced growth: flows among types

The entry rate for firms is the sum of the exit and growth rates,

$$E = (\delta_x + n) N.$$

The laws of motion for $N_p, N_i$ and $N_e$ then imply that

$$\begin{aligned} \frac{N_i}{N_p} &= \frac{\alpha (g - \mu_x)}{n + \delta_x + \lambda_x}, \\ \frac{N_e}{N_p} &= \frac{1}{\lambda_e} \left[ n + \delta_x + \alpha (g - \mu_x) - \lambda_x \frac{N_{i0}}{N_{p0}} \right]. \end{aligned}$$

New entrants to the labor force arrive at the rate $(\delta_h + v) L$.

The laws of motion for $L_w, L_i$ imply that

$$\frac{L_i}{L_w} = \frac{1}{\lambda_h} \left[ v + \delta_h + \gamma (g - \mu_h) \right].$$

The DFs keep their Pareto forms, shifting at rate $g$.

# 4. Conditions for balanced growth

There is a unit mass of identical, infinitely lived, dynastic households.

Each household comprises a representative cross-section of the population, so each grows at rate $\upsilon \geq 0$.

It has wage and profit income, used for consumption and to finance the entry costs of new firms.

Optimal investment decisions are already built in.

All household members share equally in consumption.

The household has preferences

$$U = \int_0^\infty L_0 e^{\upsilon t} e^{-\hat{r} t} \frac{1}{1-\theta} c(t)^{1-\theta} dt.$$

# 4. Conditions for balanced growth

Per capita consumption on the BGP grows at the rate $g_w$, so the interest rate is

$$r = \hat{r} + \theta g_w .$$

Market clearing determines $c_0$, the initial value for consumption.

There are five (5) inequality restrictions for a BGP:

$$\begin{aligned} n + \delta_x &> 0, \qquad g > \mu_x, \qquad g > \mu_h. \\ r &> g_Y, \qquad c_0 > 0, \end{aligned}$$

—positive entry, process innovation and retooling;

—finite PDV of aggregate income;

—positive consumption.

# 5. Balanced growth paths

Existence of a BGP and determination of $g, n$.

PROPOSITION 5: For initial distributions $\hat{F}_0, \hat{G}_0$ that are Pareto, with

—shape parameters $\alpha, \gamma > 1$ satisfying $\varepsilon \equiv (1 + \alpha - \gamma) / \rho \in (0, 1)$;

—location parameters satisfying $H_{m0} / X_{m0} = a_h$;

—and for $\theta < 1$, assume in addition $\rho > \gamma / (\gamma - 1)$;

the pair of equations in $g, n$, has a unique solution.

If the 5 inequalities hold, and the initial ratios $N_{i0} / N_{p0}$, $L_{i0} / L_{w0}$ and $N_{p0}^{1-\Omega} / L_{w0}$ satisfy ..., then the economy has an equilibrium that is a BGP.

# 5. Balanced growth paths

The growth rates $g, n$, are determined by the two VM conditions,

$$
\begin{aligned}
g &= \frac{1}{\xi_\alpha}\left[(\upsilon - n) + \alpha\mu_x + \frac{1-\zeta}{\alpha - 1 + \zeta}\lambda_x - \delta_x - \hat{r} - (\theta - 1)\,\Omega n\right], \\
g &= \frac{1}{\xi_\gamma}\left[\qquad\qquad \gamma\mu_h + \frac{1-\varepsilon}{\gamma - 1 + \varepsilon}\lambda_h - \delta_h - \hat{r} - (\theta - 1)\,\Omega n\right],
\end{aligned}
$$

where

$$\xi_\alpha \equiv \alpha - 1 + \theta > 0, \qquad \xi_\gamma \equiv \gamma - 1 + \theta > 0.$$

For log utility, $\theta = 1$, or if variety is not valued, if $\Omega = 0$,

the second equation alone determines $g$, and

$g$ does not depend on the technology parameters $\mu_x, \delta_x, \lambda_x$.

# 5. Balanced growth paths

PROPOSITION 6: On a BGP,

a. an increase in $\mu_h, \lambda_h$ or a decrease in $\delta_h$ raises $g$ and reduces $n$;

b. a decrease in $\hat{r}$ raises $g$, and

—reduces $n$ if $(\theta - 1)\,\Omega \geq 0$, and

—has an ambiguous effect on $n$ if $(\theta - 1)\,\Omega < 0$;

c. an increase in $\mu_x, \lambda_x$ or a decrease in $\delta_x$ raises $n$, and

—raises $g$ if $(\theta - 1)\,\Omega < 0$,

—has no effect on $g$ if $(\theta - 1)\,\Omega = 0$, and

—reduces $g$ if $(\theta - 1)\,\Omega > 0$.

# 5. Balanced growth paths: log utility

For $\theta = 1$,

$$g = \mu_h + \frac{1}{\gamma}\left[\frac{1-\varepsilon}{\gamma-(1-\varepsilon)}\lambda_h - \delta_h - \hat{r}\right].$$

Then

an increase $1/\rho$ (monopoly power) reduces $g$ (through $\varepsilon$);

an increase in $\alpha$ (thinner tail for technology) reduces $g$ (through $\varepsilon$);

an increase in $\gamma$ (thinner tail for skill) reduces $g$ (direct effect),

although the effect through $\varepsilon$ works the other way.

If $\alpha = \gamma$, then for any $\theta$, $\Omega$,

$$n - \upsilon = \gamma\left(\mu_x - \mu_h\right) + \frac{1-\varepsilon}{\gamma-(1-\varepsilon)}\left(\lambda_x - \lambda_h\right) - \left(\delta_x - \delta_h\right).$$

If $\alpha \neq \gamma$, then the hazard rates get different weights.

# 6. Conclusions

In the model here, per capita income growth comes from growth in technology and growth in skill.

They work through TFP growth and growth in variety.

On a BGP, skill and technology (TFP) grow at a common rate, $g$.

But the skill parameters $(\mu_h, \lambda_h, \delta_h)$ are more evidently more important for determining $g$.

The parameters governing skill and technology enter more symmetrically—but with opposite signs—in determining growth in variety.

# 6. Conclusions

The novel feature of the model here is that decisions to invest in technology and skill display a sort of ‘strategic complementarity.’

Questions for further work:

1. What types of empirical evidence could be used to assess the model?
2. Do transition paths for the model look like those for rapidly growing countries, where technology gets “ahead” because of inflows from abroad?
3. Investment in the competitive equilibrium is inefficient. What policies would bring it closer to the efficient level?