Motivational Ratings

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Focus: Ratings that incentivize effort (moral hazard).

Hospitals, physicians; schools, teachers; companies, executives, etc.

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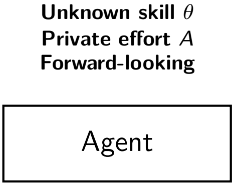
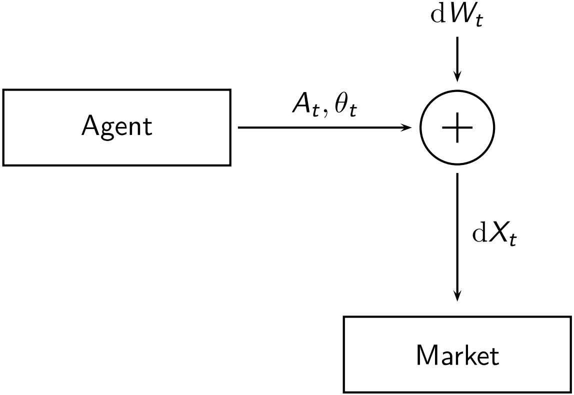
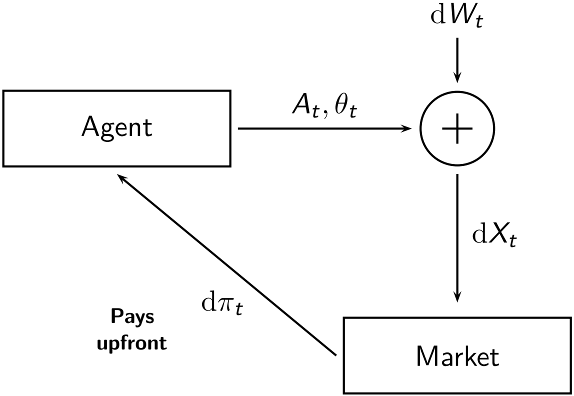
Goal: What is the best rating system?

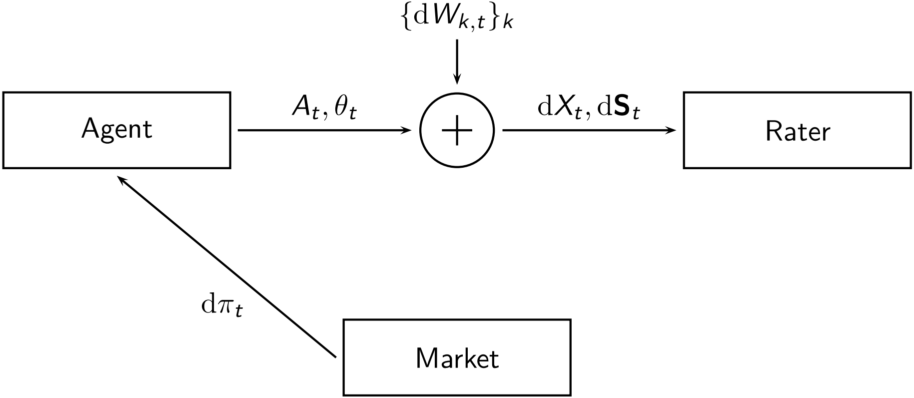

Unknown skill θ
Private effort A
Forward-looking
Agent

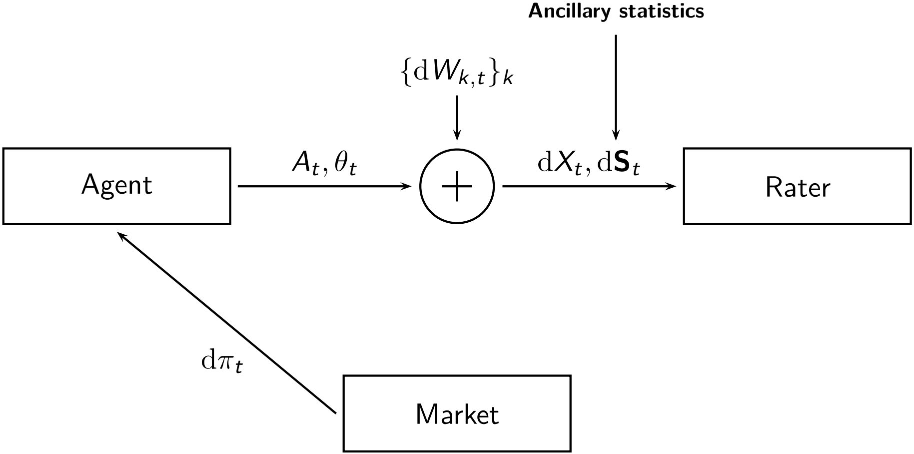


Competitive
Rational expectations

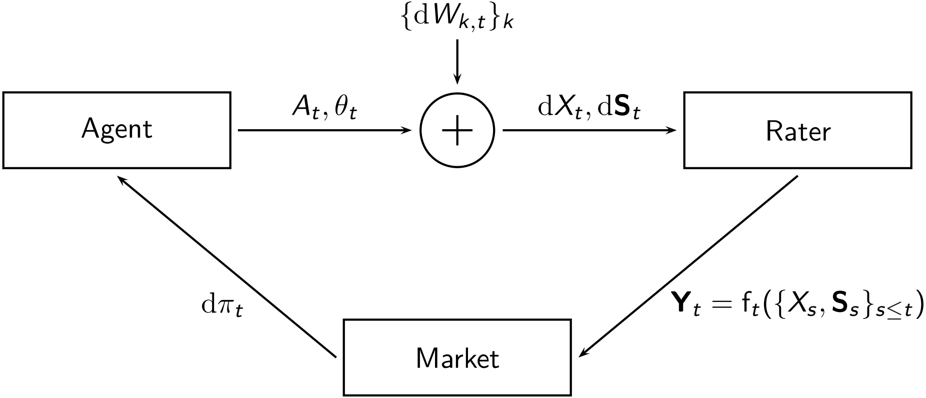


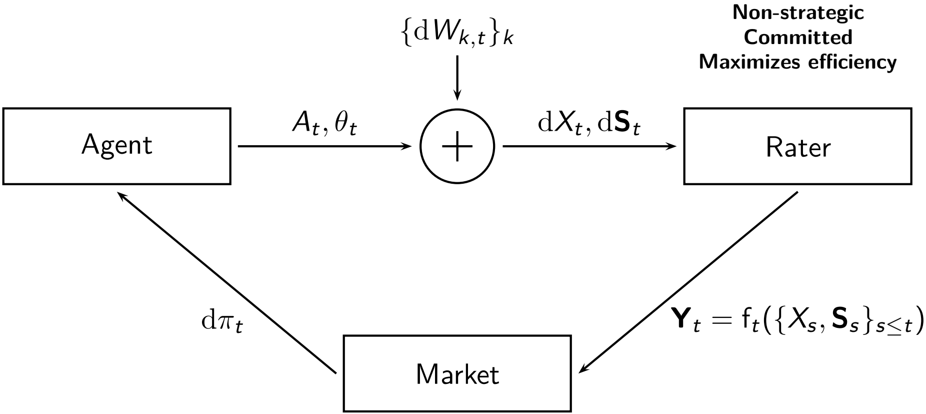
$\mathrm{d}W_t$
Agent
A_t, θ_t
$+$
$\mathrm{d}X_t$
$\mathrm{d}\pi_t$
Pays
upfront
Market


$\{\mathrm{d}W_{k,t}\}_k$
Agent
A_t, θ_t
+
$\mathrm{d}X_t, \mathrm{d}\mathbf{S}_t$
Rater
$\mathrm{d}\pi_t$
Market

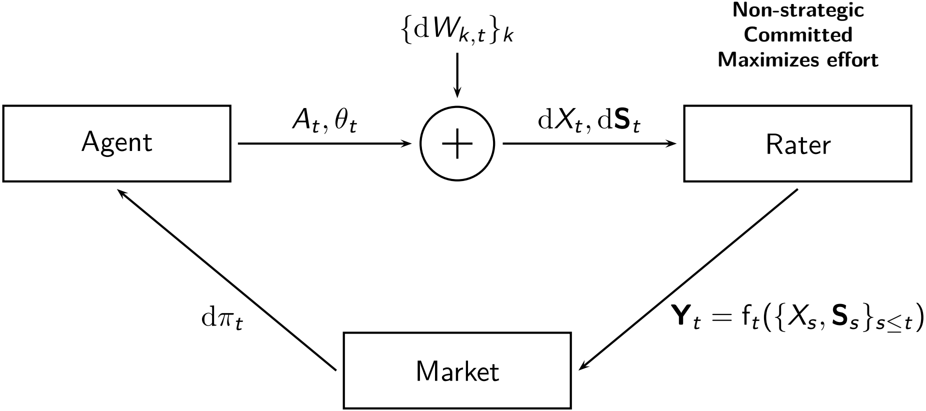


Ancillary statistics
$\{\mathrm{d}W_{k,t}\}_k$
Agent
A_t, θ_t
$+$
$\mathrm{d}X_t, \mathrm{d}\mathbf{S}_t$
Rater
$\mathrm{d}\pi_t$
Market

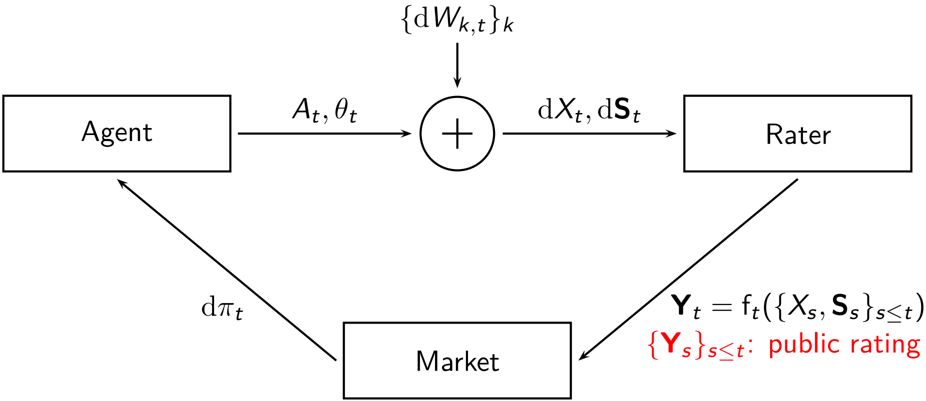

$\{\mathrm{d}W_{k,t}\}_k$
Agent
A_t, θ_t
$\mathrm{d}X_t, \mathrm{d}\mathbf{S}_t$
Rater
$\mathrm{d}\pi_t$
$\mathbf{Y}_t = \mathrm{f}_t(\{X_s, \mathbf{S}_s\}_{s \leq t})$
Market

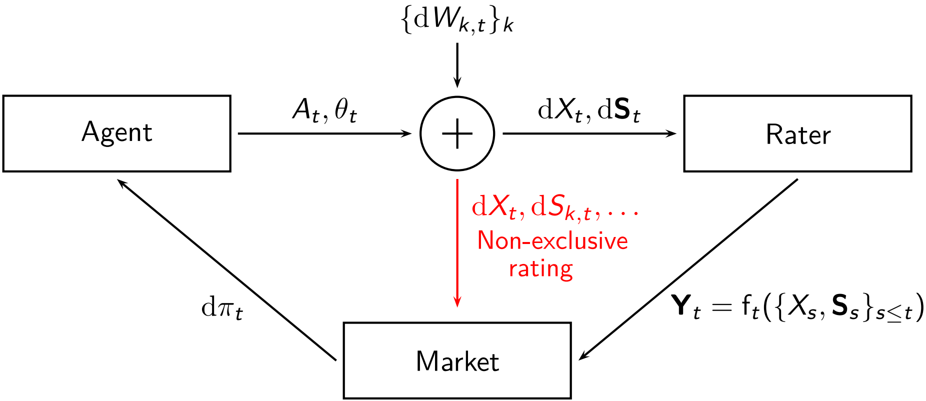


Non-strategic
Committed
Maximizes efficiency
$\{\mathrm{d}W_{k,t}\}_k$
Agent
A_t, θ_t
$\mathrm{d}X_t, \mathrm{d}\mathbf{S}_t$
Rater
$\mathrm{d}\pi_t$
$\mathbf{Y}_t = \mathrm{f}_t(\{X_s, \mathbf{S}_s\}_{s \le t})$
Market



Non-strategic
Committed
Maximizes effort
$\{\mathrm{d}W_{k,t}\}_k$
Agent
A_t, θ_t
$+$
$\mathrm{d}X_t, \mathrm{d}\mathbf{S}_t$
Rater
$\mathrm{d}\pi_t$
$\mathbf{Y}_t = \mathrm{f}_t(\{X_s, \mathbf{S}_s\}_{s \leq t})$
Market


$\{\mathrm{d}W_{k,t}\}_k$
Agent
A_t, θ_t
$\mathrm{d}X_t, \mathrm{d}\mathbf{S}_t$
Rater
$\mathrm{d}\pi_t$
Market
$\mathbf{Y}_t = \mathsf{f}_t(\{X_s, \mathbf{S}_s\}_{s\le t})$
$\{\mathbf{Y}_s\}_{s\le t}$: public rating



$\{\mathrm{d}W_{k,t}\}_k$
Agent
A_t, θ_t
$\mathrm{d}X_t, \mathrm{d}\mathbf{S}_t$
Rater
$\mathrm{d}X_t, \mathrm{d}S_{k,t}, \ldots$
Non-exclusive rating
$\mathrm{d}\pi_t$
$\mathbf{Y}_t = \mathsf{f}_t(\{X_s, \mathbf{S}_s\}_{s \leq t})$
Market

	Confidential	Public
Exclusive	$\mathbf{Y}_t$	$\{\mathbf{Y}_s\}_{s \leq t}$
Non-exclusive	$\mathbf{Y}_t, \{X_s, S_{k,s}\}_{s \leq t, k \leq K_0}$	$\{\mathbf{Y}_s, X_s, S_{k,s}\}_{s \leq t, k \leq K_0}$

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Model

Continuous time $t \geq 0$, infinite horizon.

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The relevant processes are:

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- Effort: $A_t \in \mathbf{R}_+$; **privately** known by the agent.
- Ability: $\theta_t \in \mathbf{R}$; **unknown** to all.
- Output: $X_t \in \mathbf{R}$.
- Ancillary Statistics: $\mathbf{S}_t \in \mathbf{R}^{K-1}$.

Ability Process:

$$\mathrm{d}\theta_t = -\theta_t \, \mathrm{d}t + \gamma \, \mathrm{d}W_t^\theta,$$

with $\theta_0 \sim \mathcal{N}(0, \gamma^2/2)$, $\gamma > 0$, and W^θ a standard B.M.

Rate of mean-reversion: 1.

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Output Process:

$$\mathrm{d}X_t = (A_t + \theta_t)\mathrm{d}t + \sigma_1 \mathrm{d}W_{1,t},$$

with $X_0 = 0$, $\sigma_1 > 0$, and W_1 a standard B.M. ($W_1 \perp W^\theta$).

Signal Processes, $k = 2, \ldots, K$:

$$\mathrm{d}S_{k,t} = (\alpha_k A_t + \beta_k \theta_t)\,\mathrm{d}t + \sigma_k\,\mathrm{d}W_{k,t},$$

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We also write $S_1 := X$ $(\alpha_1 := 1, \beta_1 := 1)$.

Payoffs

Given a (cumulative) transfer process π, realized payoffs are:

$$\begin{aligned} \text{Market:} \quad & \int_0^\infty e^{-rt}(\ \mathrm{d}X_t \quad - \quad \mathrm{d}\pi_t\ ), \\ \text{Agent:} \quad & \int_0^\infty e^{-rt}(\ \mathrm{d}\pi_t \quad - \quad c(A_t)\,\mathrm{d}t\ ), \end{aligned}$$

where the discount rate is $r > 0$, and $c(0) = c'(0) = 0$ and $c'' > 0$.

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$$\text{Market:} \quad \int_0^\infty e^{-rt}\big(\overbrace{\mathrm{d}X_t}^{\substack{\text{Revenue} \\ \text{on } [t,\, t+\mathrm{d}t)}} - \mathrm{d}\pi_t\big),$$

$$\text{Agent:} \quad \int_0^\infty e^{-rt}\big(\mathrm{d}\pi_t - c(A_t)\,\mathrm{d}t\big),$$

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where the discount rate is $r > 0$, and $c(0) = c'(0) = 0$ and $c'' > 0$.

Recall that $\mathbf{E}[\mathrm{d}X_t] = A_t\,\mathrm{d}t$.

Hence, **efficiency** requires $c'(A_t) = 1\ \forall t$.

The Optimization Program

The market has some expectation A^* about the agent's effort.

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The rater's goal: to maximize the argmax (A_t) over $\mathcal{M}$.

Why is Transparency Suboptimal?

Consider:

$$\mathrm{d}X_t = A_t \,\mathrm{d}t + \mathrm{d}W_{1,t}, \quad \mathrm{d}S_{2,t} = \theta_t \,\mathrm{d}t + \mathrm{d}W_{2,t}.$$

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If instead the market only observes:

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One can do better than disclosing the sum of the signals.

Rating Processes

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Throughout, we impose:

1. For all Δ, $(\mathbf{Y}_t, \mathbf{S}_t - \mathbf{S}_{t-\Delta})$ is **normal and stationary**.
2. The map $\Delta \mapsto \mathbf{Cov}[\mathbf{Y}_t, \mathbf{S}_{t-\Delta}]$ is absolutely cts., with integrable and square integrable Radon-Nikodym derivative.
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We usually work with scalar $Y_t = \mathbf{E}^*[\theta_t \mid \mathcal{M}_t]$ ("Direct ratings").

Deterministic Information Quality Implies Normality

Lemma (Normal Representation)
Let $\mathbf{Y}$ *be a progressively measurable process on* $\mathcal{I}$ *such that:*

1. $\forall T > t + \tau > t$, $\mathbf{Cov}[\mathbf{Y}_T, \mathbf{S}_{t+\tau} \mid \mathcal{I}_t]$ *is a function of* (t, T, τ), *differentiable in* τ, *with uniformly Lipschitz cts. derivative in* t.
2. $\forall T > t$, $\mathbf{Cov}[\mathbf{Y}_T, \theta_t \mid \mathcal{I}_t]$ *is a function of* (t, T).
3. $\forall t$, $\mathbf{E}[\mathbf{Y}_t^2] < \infty$ *and* $E[\mathbf{Y}_t] = 0$.

Then, for all $\Delta \geq 0$, $(\mathbf{Y}_t, \mathbf{S}_t - \mathbf{S}_{t-\Delta})$ *is normally distributed.*

Methods that Qualify:

Exponential smoothing. (Business Week's b-school ranking.)

$$Y_t = \int_{s \le t} e^{-a(t-s)} \, \mathrm{d}X_s.$$

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Methods that Don't: Coarse ratings.

What can the rater Do? More Generally:

$$\begin{array}{ccccc}
\cdots & \mathrm{d}X_s & \cdots & \mathrm{d}X_{t-\mathrm{d}t} & \mathrm{d}X_t \\
 & \vdots & & \vdots & \vdots \\
\cdots & \mathrm{d}S_{k,s} & \cdots & \mathrm{d}S_{k,t-\mathrm{d}t} & \mathrm{d}S_{k,t} \\
 & \vdots & & \vdots & \vdots \\
\cdots & \mathrm{d}S_{K,s} & \cdots & \mathrm{d}S_{K,t-\mathrm{d}t} & \mathrm{d}S_{K,t}
\end{array}$$

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$$\begin{array}{ccccc} \cdots & \mathrm{d}X_s & \cdots & \mathrm{d}X_{t-\mathrm{d}t} & \mathrm{d}X_t \\ & \vdots & & \vdots & \vdots \\ \cdots & \mathrm{d}S_{k,s} & \cdots & \mathrm{d}S_{k,t-\mathrm{d}t} & \mathrm{d}S_{k,t} \\ & \vdots & & \vdots & \vdots \\ \cdots & \mathrm{d}S_{K,s} & \cdots & \mathrm{d}S_{K,t-\mathrm{d}t} & \mathrm{d}S_{K,t} \end{array}$$

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\begin{array}{cccccc}
\cdots & \mathrm{d}X_s & \cdots & \mathrm{d}X_{t-\mathrm{d}t} & \mathrm{d}X_t \\
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\end{array}
$$

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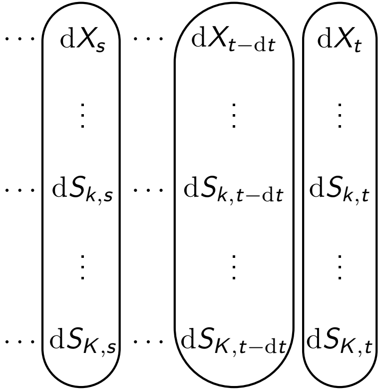

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$$e^{-\delta(t-s)} \quad + \quad e^{-\delta \mathrm{d}t} \quad + \quad 1$$

$$\begin{array}{ccccccc}
\cdots & \mathrm{d}X_s & \cdots & \mathrm{d}X_{t-\mathrm{d}t} & \mathrm{d}X_t \\
 & \vdots & & \vdots & \vdots \\
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\end{array}$$

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$$
\begin{array}{cccccccc}
\cdots & \mathrm{d}X_s & \cdots & \mathrm{d}X_{t-\mathrm{d}t} & \mathrm{d}X_t & & u_1(\cdot) \\
 & \vdots & & \vdots & \vdots & & \\
\cdots & \mathrm{d}S_{k,s} & \cdots & \mathrm{d}S_{k,t-\mathrm{d}t} & \mathrm{d}S_{k,t} & & \\
 & \vdots & & \vdots & \vdots & & \\
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\end{array}
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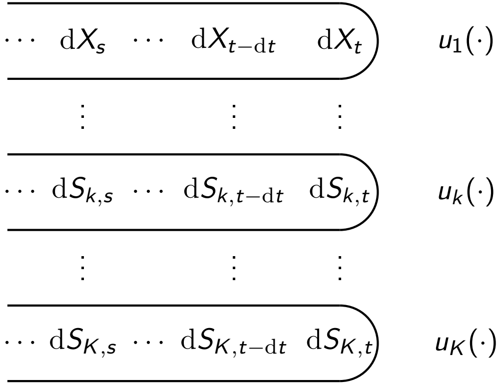

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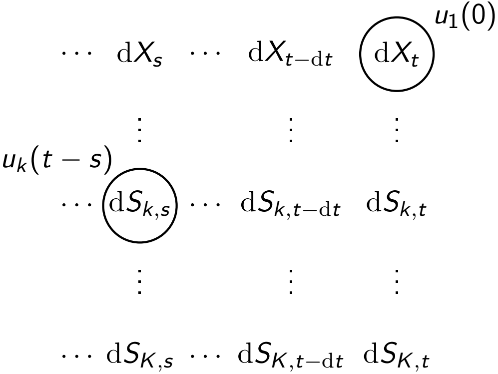

Lemma (Analytic Representation)

Fix a rating process Y. Given a conjectured A^*, there exist unique vector-valued functions u_k, $k = 1, \ldots, K$, such that, for all t,

$$Y_t = \sum\nolimits_k \int_{s \le t} u_k(t-s)(\mathrm{d}S_{k,s} - \alpha_k A^* \, \mathrm{d}s).$$

Main Results for the Confidential/Exclusive Case

The unique optimal **confidential** rating system is

$$u_k(t) = d_k \frac{\sqrt{r}}{\lambda} e^{-rt} + \frac{\beta_k}{\sigma_k^2} e^{-\kappa t}.$$

$$d_k := (\kappa^2 - r^2) m_\beta \frac{\alpha_k}{\sigma_k^2} - (\kappa^2 - 1) m_{\alpha\beta} \frac{\beta_k}{\sigma_k^2},$$

with

$$\begin{aligned}
\lambda &:= (\kappa - 1)\sqrt{r}(1 + r) m_{\alpha\beta} + (\kappa - r)\sqrt{\Delta}, \\
m_\beta &:= \sum_k \frac{\beta_k^2}{\sigma_k^2}, \quad m_{\alpha\beta} := \sum_k \frac{\alpha_k \beta_k}{\sigma_k^2}, \quad m_\alpha := \sum_k \frac{\alpha_k^2}{\sigma_k^2}, \\
\Delta &:= (\kappa + r)^2 (m_\alpha m_\beta - m_{\alpha\beta}^2) + (1 + r)^2 m_{\alpha\beta}^2, \quad \kappa := \sqrt{1 + \gamma^2 \sum_k \frac{\beta_k^2}{\sigma_k^2}}.
\end{aligned}$$

That is,

$$Y_t = \int_{s \le t} \sum_k \left(d_k \frac{\sqrt{r}}{\lambda} e^{-r(t-s)} + \frac{\beta_k}{\sigma_k^2} e^{-\kappa(t-s)} \right) (\mathrm{d}S_{k,s} - \alpha_k A^* \, \mathrm{d}s).$$

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One state is the rater's belief $\nu_t := \mathbf{E}^*[\theta_t \mid \mathcal{I}_t]$.

$$\mathrm{d}\nu_t \quad = \quad - \kappa \nu_t \,\mathrm{d}t + \frac{\gamma^2}{\kappa + 1} \sum_k \frac{\beta_k}{\sigma_k^2} (\mathrm{d}S_{k,t} - \alpha_k A^* \,\mathrm{d}t)$$

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The other is some incentive state I_t.

$$\mathrm{d}I_t = - rI_t \,\mathrm{d}t + \frac{\sqrt{r}}{\lambda} \sum_k d_k \left(\mathrm{d}S_{k,t} - \alpha_k A^* \,\mathrm{d}t\right).$$

Two states are needed: keeping track of ν_t isn't enough.

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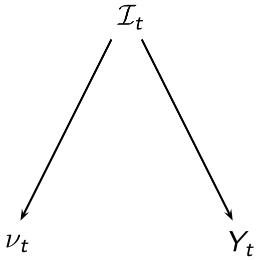

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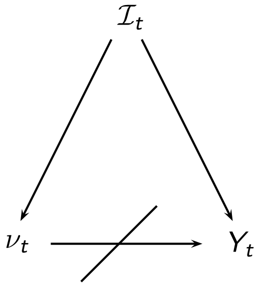

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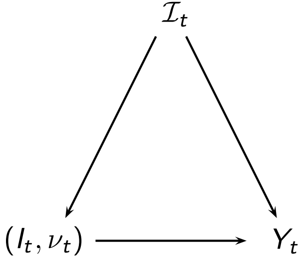

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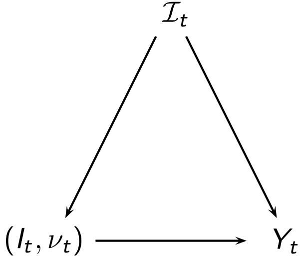

The rating process $Y = I + \nu$ isn't Markov.

Reality Check

Ratings are not Markov: widely documented for credit rating.

Altman and Kao (1992), Carty and Fons (1993), Altman (1998), Nickell et al. (2000), Bangia et al. (2002), Lando and Skødeberg (2002), Hamilton and Cantor (2004), etc.

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Mixture rating models: shown to explain economic differences.

Two-state: Frydman and Schuerman (2008);
HMM: Giampieri et al. (2005);
Rating momentum: Stefanescu et al. (2006).

Implication: Benchmarking

As an example, suppose there is one signal (output):

$$\alpha_k = \alpha, \beta_k = \beta, \sigma_k = \sigma.$$

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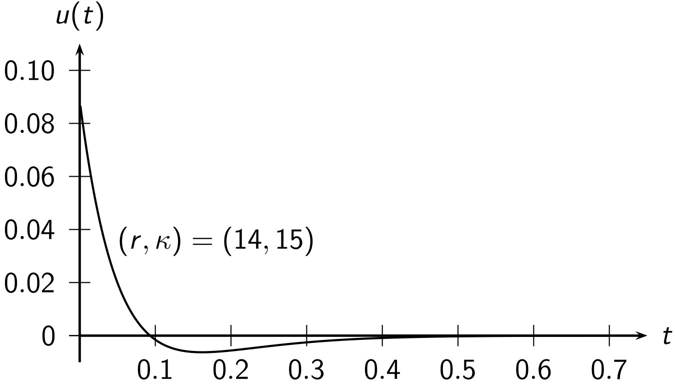
So the incentive state isn't always "added." It may be subtracted.



u(t)
0.10
0.08
0.06
0.04
0.02
0
(r, κ) = (14, 15)
0.1
0.2
0.3
0.4
0.5
0.6
0.7
t

Reality Check

Benchmarking: Prior-year performance widely used for incentives.

> *When standards are based on prior-year performance, managers might avoid unusually positive performance outcomes, since good current performance is penalized in the next period through an increased standard.* —Murphy, 2001.

Implementation

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Can induce any $A \in [0, \overline{A}]$ by $\lambda Y + (1 - \lambda)W$, some $\lambda \in [0, 1]$.

Two-state mixture Markov ratings (plus noise) are wlog.

In Conclusion. . .

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Our analysis shows why:

Insisting on transparency or even publicness isn't optimal.

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Insisting on transparency or even publicness isn't optimal.

And, more surprisingly:

Two-state mixture Markov models are "robust."

Ratings aren't Markovian.

Benchmarking can be optimal.

Technical Aspects

Focus on scalar ratings (wlog).

Lemma.

The effort A^* induced by a confidential process Y solves

$$c'(A^*) \propto \mathbf{Corr}[Y,\theta] \cdot \frac{\sum_k \int_{t\geq 0} \alpha_k u_k(t) e^{-rt}\,\mathrm{d}t}{\sqrt{\mathbf{Var}[Y]}}$$

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We first guess what optimal ratings look like.

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Main difficulties:

- Non-standard calculus of variation.
  (Multidimensional objective with single-dimensional input, time-delayed controls.)
- Set of FOC is a continuum.

We then verify that the guess is correct.

To do so, we define an auxiliary principal-agent problem.

The agent is as in the main model.

The principal pays the agent, as the market does in the main model, but her payoffs includes the objective of the intermediary in the main model.

As in the main model, the agent maximizes

$$\mathbf{E}\left[\int_{s\geq t} e^{-r(s-t)}(\mu_s - c(A_s))\,\mathrm{d}s \mid \mathcal{M}_t\right],$$

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where $\rho, \phi > 0$ and ν_t is the belief under "full transparency".

Note: if $\mathbf{E}[\mu] = 0$,

$$\mathbf{E}\left[\mu_t(\mu_t - \nu_t)\right] = 0 \Leftrightarrow \mathbf{Var}[\mu_t] = \mathbf{Cov}[\mu_t, \nu_t].$$

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We want μ to be a belief process.

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For any Gaussian (μ, ν):

$$\mathbf{E}[\nu_t \mid \mu_t] = \mathbf{E}[\nu_t] + \frac{\mathbf{Cov}[\mu_t, \nu_t]}{\mathbf{Var}[\mu_t]} \left(\mu_t - \mathbf{E}[\mu_t]\right).$$

$\implies$ A (mean-normalized) process is a market belief iff $\mathbf{Var}[\mu_t] = \mathbf{Cov}[\mu_t, \nu_t]$.

With a carefully chosen ϕ, as the principal becomes increasingly patient,

$$\mathbf{E}\left[\mu_t(\mu_t - \nu_t)\right] \to 0 \text{ and } c'(A) \to c'(A^*)$$

where A^* is the conjectured optimal effort level derived from the FOC, and μ converges to the conjectured optimal rating.

Overview of the Other Cases

Public Ratings

The unique optimal **public** rating system is

$$u_k(t) = \widetilde{d}_k \frac{\sqrt{r}}{\lambda} e^{-\sqrt{r}t} + \frac{\beta_k}{\sigma_k^2} e^{-\kappa t}.$$

Here,

$$\widetilde{d}_k := \frac{\kappa - \sqrt{r}}{\kappa - r} d_k + \lambda \frac{\sqrt{r} - 1}{\kappa - r} \frac{\beta_k}{\sigma_k^2}.$$

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In common:

A two-state rating system.

One state is the belief.

No signal gets discarded.

Benchmarking can arise.

Differences:

Impulse response is the harmonic mean between the discount rate and the rate of mean-reversion.

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Impulse response is the harmonic mean between the discount rate and the rate of mean-reversion.

With homogeneous signals, $\tilde{d}_k = 0$: transparency is best.

Exclusive vs. Non-Exclusive Information

Suppose some (not all) signals are openly available to the market.

In common:

New features:
(With homogeneous signals)

A two-state rating system.

Private:

$$u_k = \hat{d}_k e^{-rt} + \frac{\beta_k}{\sigma_k^2} e^{-\kappa t}$$

Better informed market.

Public:

$$u_k = \check{d}_k e^{-\delta t} + \frac{\beta_k}{\sigma_k^2} e^{-\kappa t}$$

Public information and ratings can be substitutes.

Multi-Dimensional Actions

The analysis extends to multi-dimensional actions (separable cost).

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with cost $c(a_1, a_2) = c \cdot (a_1^2 + a_2^2)$. The best confidential system is

$$u_1(t) = \frac{\sqrt{r}}{\sigma_1} e^{-rt}, \quad u_2(t) = \frac{e^{-\kappa t}}{\sigma_2^2},$$

and effort

$$c'(a_1) = \frac{\kappa - 1}{4\sqrt{r}\sigma_1}, \quad c'(a_2) = \frac{\kappa - 1}{2(r+\kappa)\sigma_2^2}.$$

How do Different Signals get Weighted?

The confidential process can be rewritten as

$$u_k(t) = \frac{\beta_k}{\sigma_k^2}\left[\left((\kappa^2 - r^2)\frac{\alpha_k}{\beta_k} - (\kappa^2 - 1)\frac{m_{\alpha\beta}}{m_\beta}\right)\frac{\sqrt{r}m_\beta}{\lambda}e^{-rt} + e^{-\kappa t}\right].$$

Fixing the SNR $\frac{\beta_k}{\sigma_k^2}$, signals are ordered according to the ratio $\frac{\alpha_k}{\beta_k}$: the higher the ratio, the larger the weight (whether positive or not).

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Variance non-monotone in r.

Limits in the Confidential Case

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When mean-reversion tends to 0:

Effort converges to a finite limit; no transparency.