Static Information Design

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Mechanism Design and Information Design

- **Basic Mechanism Design**:
  - Fix an economic environment and information structure
  - Design the rules of the game to get a desirable outcome
- **Information Design**
  - Fix an economic environment and rules of the game
  - Design an information structure to get a desirable outcome

Mechanism Design and Information Design

- **Basic Mechanism Design**:
  - Can compare particular mechanisms..
    - e.g., first price auctions versus second price auctions
  - Can work with space of all mechanisms...
    - without loss of generality, let each agent's action space be his set of types...revelation principle
    - e.g., Myerson's optimal mechanism
- **Information Design**
  - Can compare particular information structures
    - Linkage Principle: Milgrom-Weber 82
    - Information Sharing in Oligopoly: Novshek and Sonnenschein 82
  - Can work with space of all information structures
    - without loss of generality, let each agent's type space be his set of actions......revelation principle

Information Design

Leading Cases:

1. Many (perhaps informed) players and an uninformed information designer (or "mediator"):
   - Incomplete Information Correlated Equilibrium literature of the 1980s and 1990s (Forges 93)
2. One uninformed player (a "receiver") and an informed information designer (or "sender")
   - "Bayesian Persuasion": Kamenica-Genzkow 11
3. Many (perhaps informed) players *and* an informed information designer
   - "Bayes correlated equilibrium": Bergemann-Morris 13 & 16 and various applications....

Roger defining Mechanism Design

- Myerson Mechanism Design:
  - Bayesian games with communication (game is fixed)
  - Bayesian collective choice problems (mechanism is chosen by designer)
  - both combined in Myerson (1982, 1987)
- Both have truth-telling (honesty) and obedience constraints
- "information design" = "Bayesian games with communication" − truth-telling + informed information designer/mediator
- Informed principal literature

This Talk

1. General Framework with informed players and *omniscient* Information Designer
2. Analyze leading example
   - one uninformed player (Kamenica-Genzkow leading example)
   - one informed player
   - many (two) uninformed players
3. Information Designer who does not know players' information (putting back truth-telling constraints) = "private persuasion"
4. Applications and Elaborations

Setup

- Maintained Environment: Fix players $1,...,I$; payoff states Θ; prior on states $\psi \in \Delta(\Theta)$
- Basic Game $G : (A_i, u_i)_{i=1..,I}$ where $u_i : A \times \Theta \to \mathbb{R}$
- Information Structure S: $(T_i)_{i=1..,I}$ and $\pi : \Theta \to \Delta(T)$

Information Designer's Problem

- *Decision rule* $\sigma : T \times \Theta \to A$ is *obedient* for (G, S) if, for all i, t_i and t_i',

$$\sum_{a_{-i}, t_{-i}, \theta} u_i\left((a_i, a_{-i}), \theta\right) \sigma\left(a \middle| t, \theta\right) \pi\left(t \middle| \theta\right) \psi(\theta)$$
$$\geq \sum_{a_{-i}, t_{-i}, \theta} u_i\left(\left(a_i', a_{-i}\right), \theta\right) \sigma\left(a \middle| t, \theta\right) \pi\left(t \middle| \theta\right) \psi(\theta);$$

  Obedient decision rule σ is a *Bayes correlated equilibrium*. Characterizes implementability.

- Information designer with payoff $v : A \times \Theta \to \mathbb{R}$ picks a Bayes correlated equilibrium $\sigma \in BCE(G, S)$ to maximize

$$V_S(\sigma) \equiv \sum_{a, t, \theta} \psi(\theta) \pi\left(t \middle| \theta\right) \sigma\left(a \middle| t, \theta\right) v(a, \theta).$$

Benchmark Investment Example

- a firm is deciding whether to invest or not:
- binary state: $\theta \in \{B, G\}$, bad or good
- binary action: $a \in \{\text{Invest, Not Invest}\}$
- payoffs

	bad state B	good state G
Invest	-1	x
Not Invest	0	0

  with $0 < x < 1$
- prior probability of each state is $\frac{1}{2}$
- firm is uninformed
- information designer (government) seeks to maximize probability of investment (independent of state)
- leading example of Kamenica-Gentzkow 11

Decision Rule

- p_θ is probability of investment, conditional on being in state θ

	bad state B	good state G
Invest	p_B	p_G
Not Invest	$1 - p_B$	$1 - p_G$

- interpretation: firm observes only "action recommendation," drawn according to (p_B, p_G)

Decision Rule without Government

- With no information, decision rule is

	bad state B	good state G
Invest	0	0
Not Invest	1	1

- probability of investment is 0

Obedience Constraints

- if "advised" to invest, invest has to be a best response:

$$\begin{aligned} -\frac{1}{2}p_B + \frac{1}{2}p_G x &\geq 0 \Leftrightarrow \\ p_G &\geq \frac{p_B}{x} \end{aligned}$$

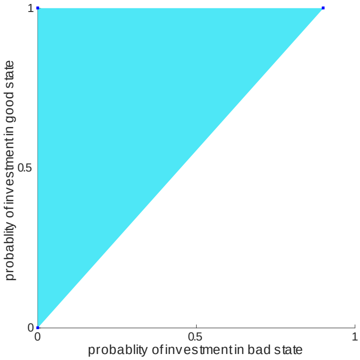
- if "advised" to not invest, not invest has to be a best response

$$\begin{aligned} -\frac{1}{2}\left(1 - p_B\right) y - \left(1 - p_G\right) x &\geq 0 \Leftrightarrow \\ p_G &\geq \frac{p_B}{x} + 1 - \frac{1}{x} \end{aligned}$$

- because $x < 1$, investment constraint is binding
- always invest ($p_B = 1$ and $p_G = 1$) cannot happen in equilibrium
- the full information equilibrium has invest only in good state ($p_B = 0$ and $p_G = 1$)

Bayes Correlated Equilibria

equilibrium outcomes (p_B, p_G) for $x = 0.9$



- always invest ($p_B = 1$ and $p_G = 1$) cannot happen in equilibrium
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Information Design

- recommendation maximizing the probability of investment:

$$p_B = x,\ p_G = 1$$

- best BCE

	B	G
Invest	x	1
Not Invest	$1-x$	0

One Informed Player

- Firm receives a signal which is "correct" with probability $q > 1/2$.
- Formally, the firm observes a signal g or b, with signals g and b being observed with conditionally independent probability q when the true state is G or B respectively:

	bad state B	good state G
bad signal b	q	$1-q$
good signal g	$1-q$	q

- Write p_θ^t for the probability of investing in state $\theta \in \{B, G\}$ if the signal is $t \in \{b, g\}$; a decision rule is then a quadruple $\left(p_B^b, p_G^b, p_B^g, p_G^g\right)$.
- Same constraints signal by signal essentially as before...
  - A firm observing a good signal will then have an incentive to invest (when told to invest) if

$$p_G^g \geq \frac{1-q}{q}\frac{p_B^g}{x}.$$

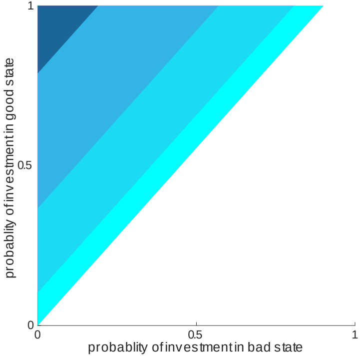
    and not invest (when told to not invest) if

$$p_G^g \geq \frac{1-q}{q}\frac{p_B^g}{x} + \frac{1-q}{q} - \frac{1}{x}.$$

- If private information of the firm is sufficiently noisy, or $q \leq \frac{1}{1+x}$, the binding constraint is still the investment constraint.

One Informed Player: Bayes Correlated Equilibrium

projecting $\left(p_B^g, p_G^g, p_B^b, p_G^b\right)$ on $\left(p_B, p_G\right)$, we get equilibrium set (for $x = 0.9$ and $q = 0.5$, 0.575, 0.7 and 0.875)



- As we will discuss later in more detail, more information of the firm implies more constraints on the information designer and so shrinking of the attainable outcomes

Two Firms

- payoffs almost as before....

$\theta = B$	I	N
I	$-1+\varepsilon$	-1
N	0	0

$\theta = G$	I	N
I	$x+\varepsilon$	x
N	0	0

- ...up to ε term
- maintain $x > \frac{1}{2}$ to reduce cases
- first assume that information designer (government) wants to maximize the sum of probabilities that firms invest....
- if $\varepsilon = 0$, problem is exactly as before firm by firm; doesn't matter if and how signals are correlated
- we will consider what happens when $|\varepsilon| \approx 0$ (so the analysis cannot change very much)
- will now have profile of action recommendations depending on the state

Two Firms: Strategic Complementarities

- If $\varepsilon > 0$, optimal rule is

$\theta = B$	I	N
I	$\frac{x+\varepsilon}{1-\varepsilon}$	0
N	0	$\frac{1-x-2\varepsilon}{1-\varepsilon}$

$\theta = G$	I	N
I	1	0
N	0	0

- the probability of any one firm investing is still about x..
- ....and signals are public

Two Firms: Strategic Substitutes

- If $\varepsilon < 0$, optimal rule is

$\theta = B$	I	N
I	0	$x + \varepsilon$
N	$x + \varepsilon$	$1 - 2x - 2\varepsilon$

$\theta = G$	I	N
I	1	0
N	0	0

- the probability of any one firm investing if the state is bad is still about x....
- ....and signals are private

Other Objectives

- Suppose the government was interested in maximizing the probability of at least one firm investing
- This can always be achieved with probability 1....

$\theta = B$	I	N
I	0	$\frac{1}{2}$
N	$\frac{1}{2}$	0

$\theta = G$	I	N
I	1	0
N	0	0

- This is true for $\varepsilon = 0$ and by continuity for $|\varepsilon|$ independent of the sign...
- Compare Ely (2015), Arieli (2015)

Benevolent Information Designer

- In one firm case, if government had the same objective as the firm, he would always give them full information...
- But in the two firm case, a benevolent government maximizing the (joint) profits of the two firms might still manipulate information in order to correct for externalities and coordinate behavior
- In game

$\theta = B$	I	N
I	$-1+\varepsilon+z$	-1
N	z	0

$\theta = G$	I	N
I	$x+\varepsilon+z$	x
N	z	0

benevolent government will behave as an investment maximizing government if z is large enough

Access to Players' Information

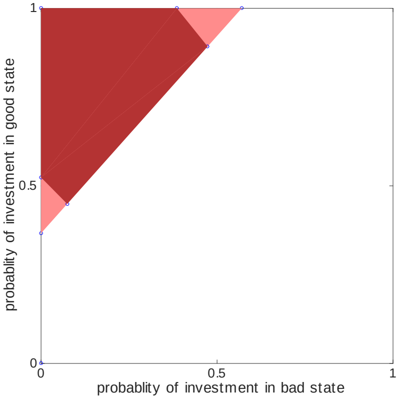
- We want to assume that information designer knows the state θ...
- ...but what should we assume about what information designer knows about players' information? Consider three scenarios:
  1. Omniscient Designer: the designer knows all players' information too...[**maintained assumption so far**]
  2. Communicating Designer: the designer can condition his announcements about the state only on players' reports of their types
  3. Non-Communicating Designer: the designer can tell players about the state but without conditioning on players' information

Back to One Informed Player: Communicating Designer

- Kotolinin et al. 15
- as before, firm observes a signal $t \in T$ and government makes a recommendation to invest p_θ^t as a function of reported signal t and state θ
- incentive constraint: add truth-telling to obedience
- to insure truth-telling, differences in recommendations must be bounded across states

Communicating Designer

- adding truth-telling constraints...($x = 0.9$, $q = 0.7$)



- communicating (red), omniscient (pink)
- truthtelling imposes constraints at low and high ends

Non-communicating designer

- firm observes his signal
- government offers a recommendation, independent of the signal, depending on the true state
- KG 11 and Kotolinin et al. 15
- in the example (and Kotolin 15), communicating and non-communicating designer can attain the same set of outcomes

Taxonomy

	Single Agent	Many Agent Uninformed Designer	Many Agent Informed Designer
Omniscient	.	Bayesian Solution	Bayes Correlated Equilibrium
Communicating	Kolotilin et al	Communication Equilibrium	.
Non Communicating	KG informed receiver	Strategic Form Correlated Equilibrium	.

Applications and Elaborations

1. Information Sharing and Strategic Substitutes
2. Metaphorical Information Designers and Robust Predictions
3. Comparing Information
4. Concavification and its many player generalizations
5. Information designer with adversarial equilibrium selection
6. Incomplete information correlated equilibrium literature

1. Information Sharing

- Classic Question: are oligopolists better off if they share their information?
- Consider oligopolists uncertain about level of demand (intercept of linear demand curve); two effects in conflict:
  1. Firms would like their output to be as uncorrelated with others' output as possible (strategic substitutes, c.f., $\varepsilon < 0$ case above)
  2. But would like to be as informed as possible about the state of demand
- Resolution:
  - If inverse demand curve is flat enough... i.e., small strategic substitutability...effect 2 wins and full sharing is optimal
  - If inverse demand curve is very steep...i.e., large strategic substitutability...effect 1 wins and no sharing is optimal
  - In intermediate cases, optimal to have firms observe imperfect information about demand, with conditionally independent signals, and thus signals which are as uncorrelated as possible conditional on their accuracy
- Bergemann Morris 13 Ecta

2. Metaphorical Information Designers

- Literal Interpretation of Information Design Problem: informed player can commit ex ante to information structure
  - In many applications, this commitment is an iffy assumption
- Literal Interpretation of Mechanism Design Problem: informed player can commit ex ante to mechanism
  - In many applications, this commitment is an iffy assumption
- But in mechanism design, "mechanism designer" is often a metaphor; how about in information design....?
- For example...
  - Adversarial information designer can tell us the worst case outcome....
  - Information designer may identify range of possible values for some variable
  - More generally, can identify "robust predictions" of a model

Metaphorical Information Designer: Applications

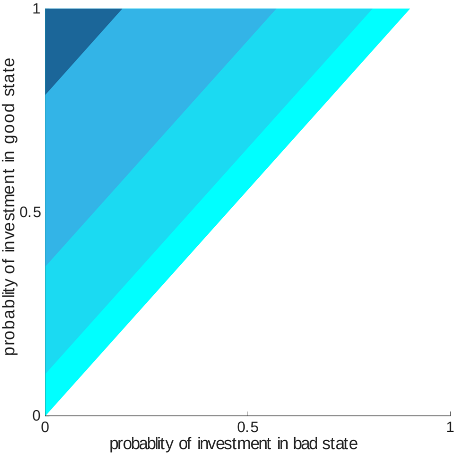
1. First Price Auction
   - QUESTION: For an arbitrary fixed symmetric distribution of values (common, private or interdependent), what is the lowest possible expected revenue across information structures?
   - ANSWER: positive but low
   - METHOD: consider information designer minimizing revenue
2. Linear Best Response Games with common and idiosyncratic shocks
   - QUESTION: What is the maximum variance of aggregate actions, consistent with a given variance of common shocks?
   - ANSWER: Unbounded
   - METHOD: consider information designer maximizing aggregate variance

Robust Predictions

1. First Price Auction: robust low bound on revenue
   - Bergemann-Brooks-Morris 16
2. Aggregate Variance: none
   - Bergemann-Heumann-Morris 15

3. Ordering Information

in one informed player example, more information shrunk attainable outcomes on primitives... for $x = 0.9$ and $q = 0.5$, 0.575, 0.7 and 0.875:



Ordering Information

- Intuition: more information for the player imposes more constraints on the information designer and reduces the set of outcomes she can induce
- Say that information structure S "is more incentive constrained than" ($=$ more informed than) S' if it gives rise to a smaller set of BCE outcomes than S' in all games
  - in one player case, this ordering corresponds to Blackwell's sufficiency ordering
  - in many player case, corresponds to "individual sufficiency" ordering
- Bergemann-Morris 16, see also Lehrer et al 10 and 11

Nice Properties of Individual Sufficiency Ordering

- Reduces to Blackwell in one player case
- Transitive
- Two information structures are each individually sufficient for each other if and only if they share the same higher order beliefs about Θ
- S is individually sufficient for S' if and only if giving extra signals to S' equals S plus an appropriate correlation device

4. Concavification

- We described two step procedure for solving information design problem (with one or many players):
  1. Characterize all implementable decision rules
  2. Pick the designer's favorite
- Concavification procedure (with one player) [Aumann-Maschler 95 and Kamenica-Gentzkow 11]
  - Identify information designer's utility for every belief of the single player
  - Identify utility from optimal design by concavification, identifying information design only implicitly
- Many player generalization: Taneva et al 16
- Always nice interpretation, sometimes (but not always) useful in solving information design problem

5. Adversarial Equilibrium Selection

- Suppose that an information designer gets to make a communication $\Phi : T \times \Theta \to \Delta(M)$; new game of incomplete information (G, S, Φ)
- Write $E(G, S, \Phi)$ for the set of Bayes Nash equilibria of (G, S, Φ) and Write $V_S^*(C, \beta)$ for the information designer's utility
- We have been studying the maxmax problem

$$\max_C \max_\beta V_S^*(C, \beta)$$

  using a revelation principle argument to show that this equals

$$\max_{\sigma \in BCE(G,S)} V_S(\sigma)$$

- The maxmin problem

$$\max_C \min_\beta V^*(S, C, \beta)$$

  does not have a revelation principle characterization
- maxmin problem relates to Carroll 15, Taneva et al 16, Kajiii Morris 97

6. Incomplete Information Correlated Equilibrium

- Decision rule $\sigma : T \times \Theta \rightarrow A$ is *incentive compatible* for (G, S) if, for each i, t_i and a_i, we have

$$\sum_{a_{-i}, t_{-i}, \theta} u_i \left(\left(a_i, a_{-i} \right), \theta \right) \sigma \left(a | t, \theta \right) \pi \left(t | \theta \right) \psi \left(\theta \right) \tag{1}$$

$$\geq \sum_{a_{-i}, t_{-i}, \theta} u_i \left(\left(\delta \left(a_i \right), a_{-i} \right), \theta \right) \sigma \left(a | \left(t_i', t_{-i} \right), \theta \right) \pi \left(t | \theta \right) \psi \left(\theta \right);$$

for all t_i' and $\delta_i : A_i \rightarrow A_i$.

- Decision rule $\sigma : T \times \Theta \rightarrow A$ is *join feasible* for (G, S) if $\sigma \left(a | t, \theta \right)$ is independent of θ, i.e., $\sigma \left(a | t, \theta \right) = \sigma \left(a | t, \theta' \right)$ for each $t \in T$, $a \in A$,and $\theta, \theta' \in \Theta$.
- Solution Concepts:
  - Bayes correlated equilibrium = obedience
  - Communication equilibrium = incentive compatibility (and thus obedience) and join feasibility
  - etc...

Our Methodology Papers

- "Robust Predictions in Incomplete Information Games," Ecta 13
- "Bayes Correlated Equilibrium and The Comparison of Information Structures," TE 16
- This material...
  - summary and some one player examples in "Information Design, Bayesian Persuasion and Bayes Correlated Equilibrium," AER P&P forthcoming
  - full treatment in preparation

Summary

- Much recent work fits under the label "information design"
- A subset studies optimal information design appealing to a revelation principle argument
- Talk described an integrated perspective, some applications and further theoretical issues