Assessing Misspecification and Aggregation for Structured Preferences

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Question

General utility maximization subject to a budget constraint has been extensively studied

In applied work, more structure is often used

- Pro: tractability
- Con: (more) misspecification

How bad is the misspecification?

This Paper

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  - Individual maximization of general utility may yield little structure in aggregate (Sonnenschein-Mantel-Debreu)
  - Irrationalities may average away to yield structure in aggregate (Becker 1962)
  - What is an appropriate level of aggregation for quasilinear utility?

This Paper – Theory

- Measure misspecification for quasilinear utility
  - Units of dollars
- Aggregation result: if individuals approximately maximize quasilinear utility, a representative agent does
  - Generalizes aggregation result for exactly quasilinear utility (Gorman 1953)

This Paper – Empirics

With grocery store scanner data we find

- All individuals inconsistent with quasilinear utility
- Representative agent deterministically consistent
- Qualitative differences between quasilinear utility and general utility

Quasilinear

Suppose we have K goods ("x") plus a numeraire good ("y")

Quasilinear utility models choices as solving

$$\max_{x \in \mathbb{R}^K_+, y \in \mathbb{R}} \quad u(x) + y \quad \iff \quad \max_{x \in \mathbb{R}^K_+} \quad u(x) + I - p \cdot x$$

$$\text{s.t. } p \cdot x + y \leqslant I$$

Approximately Quasilinear

For observed x^* and all possible $x \in \mathbb{R}_+^K$,

$$u(x^*) - p \cdot x^* + \varepsilon \geqslant u(x) - p \cdot x$$

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- Could have $\varepsilon > 0$ because individual does not maximize quasilinear utility (or any utility)

Approximately Quasilinear

When is a finite dataset $\{(x^t, p^t)\}_{t=1}^T$ rationalized by ε-quasilinear utility?

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Interpretations of x^t:

- Quantity vector at time t for an individual
- Average of quantities at time t across individuals
- Population mean quantities at time t (prices are predetermined)

Approximate Law of Demand

How do we check if a dataset is ε-rationalized?

- Challenge: function u is unknown, infinite-dimensional
- Want something that just uses prices and quantities

Approximate Law of Demand

A first step is to consider a version of the law of demand.

- Exactly quasilinear:

$$(p^s - p^r) \cdot (x^s - x^r) \leqslant 0$$

- ε-quasilinear:

$$\frac{1}{2}(p^s - p^r) \cdot (x^s - x^r) \leqslant \varepsilon$$

Approximate Law of Demand

$$u(x^r) - p^r \cdot x^r + \varepsilon \geqslant u(x^s) - p^r \cdot x^s$$
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$$\iff$$

$$p^r \cdot (x^r - x^s) \leqslant u(x^r) - u(x^s) + \varepsilon$$
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$$\implies$$

$$\frac{1}{2}(p^s - p^r) \cdot (x^s - x^r) \leqslant \varepsilon$$

Pictures

A Revealed Preference Characterization

Theorem

For any dataset $\{(x^t, p^t)\}_{t=1}^T$ *and* $\varepsilon \geqslant 0$*, the following are equivalent:*

(i) $\{(x^t, p^t)\}_{t=1}^T$ *is* ε*-rationalized by quasilinear utility*

(ii) *There exist numbers* $\{u^t\}_{t=1}^T$ *that satisfy the following inequalities for all* r, s*:*

$$u^s \leqslant u^r + p^r \cdot (x^s - x^r) + \varepsilon$$

A Revealed Preference Characterization

Theorem (Continued)

(iii) *For all finite sequences* $\{t_m\}_{m=1}^{M}$ *with* $t_m \in \{1, \ldots, T\}$ *and* $M \geqslant 2$, *the inequality*

$$\frac{1}{M} \sum_{m=1}^{M} p^{t_m} \cdot (x^{t_m} - x^{t_{m+1}}) \leqslant \varepsilon$$

holds, where $(x^{t_{M+1}}, p^{t_{M+1}}) = (x^{t_1}, p^{t_1})$

(iv) $\{(x^t, p^t)\}_{t=1}^{T}$ *is* ε*-rationalized by a quasilinear utility function that is continuous, monotonic increasing, and concave*

Measuring Misspecification

We calculate the smallest ε^* such that a dataset is ε-quasilinear rationalized

- Measure of quasilinear misspecification
- ε^* is in units of dollars lost
- ε^*-quasilinear maximization is smallest model consistent with the data

Measuring Misspecification

Proposition

The measure of misspecification can be computed by:

$$\varepsilon^* = \min_{\substack{\varepsilon \in \mathbb{R}_+ \\ u^1,\ldots,u^T \in \mathbb{R}_+}} \varepsilon \quad s.t.$$

$$u^s \leqslant u^r + p^r \cdot (x^s - x^r) + \varepsilon \quad \textit{for all } r, s \in \{1, \ldots, T\}$$

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Equivalently:

$$\varepsilon^* = \min_{\varepsilon \in \mathbb{R}_+} \varepsilon \quad s.t. \quad \frac{1}{M} \sum_{m=1}^{M} p^{t_m} \cdot \left(x^{t_m} - x^{t_{m+1}}\right) \leqslant \varepsilon,$$

Aggregation

There is a theoretical link between the measure of misspecification and level of aggregation

Aggregation

- Individual datasets: $\{(x^{(i,t)}, p^t)\}_{t=1}^T$
- Aggregate dataset: $\{(\bar{x}^t, p^t)\}_{t=1}^T$
  - $\overline{x}^t = \frac{1}{n}\sum_{i=1}^n x^{(i,t)}$

Aggregation

Proposition

If each individual dataset is ε^i-rationalized by quasilinear utility, then the aggregate dataset is $\overline{\varepsilon}$-rationalized by quasilinear utility, where $\overline{\varepsilon} = \frac{1}{n}\sum_{i=1}^{n} \varepsilon^i$

- Gorman (1953): Exact quasilinear utility aggregates
- This paper: ε-quasilinear utility aggregates

Aggregation – Proof

For each cycle $\{t_m\}$ and individual i:

$$\frac{1}{M}\sum_{m=1}^{M} p^{t_m} \cdot \left(x^{i,t_m} - x^{i,t_{m+1}}\right) \leqslant \varepsilon^i$$

$$\Longrightarrow$$

$$\frac{1}{M}\sum_{m=1}^{M} p^{t_m} \cdot \left(\overline{x}^{t_m} - \overline{x}^{t_{m+1}}\right) \leqslant \overline{\varepsilon}$$

- Related to Shi, Shum, Song (2018) when $\varepsilon^i = 0$

Aggregation

Proposition (Convexity of ε^*)

Aggregation

Proposition (Convexity of ε^*)

The measures of misspecification satisfy

$$\overline{\varepsilon}^* \leqslant \frac{1}{n}\sum_{i=1}^{n} \varepsilon^{i*},$$

where $\overline{\varepsilon}^*$ is the measure of misspecification for the aggregate dataset and ε^{i*} is the measure of misspecification for the dataset for individual i

Empirical Question: how big is the gap in this inequality?

Statistical Inference

So far, deterministic results for datasets such as

- Indvidual prices and quantities: $\{(x^{(i,t)}, p^t)\}_{t=1}^T$
- Average prices and quantities: $\{(\bar{x}^t, p^t)\}_{t=1}^T$

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Statistical inference is possible with **population-level** dataset

- $\left\{(\mathbb{E}[X^{i,t}], p^t)\right\}_{t=1}^T$

Statistical Inference

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H_0 : For all finite sequences $\{t_m\}_{m=1}^{M}$,

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- Test of moment inequalities Details

Application

Our application uses Stanford Basket Data, previously used in Echenique, Lee, Shum (2011)

- From four grocery stores
- Panel dataset of $n = 494$ households
- $T = 26$ four-week time periods, June 1991 to June 1993
- $K = 375$ goods after aggregating to brand level

Application : Individual Data

All individuals are inconsistent with quasilinear utility

Table: Summary Statistics of ε^{i*}

Mean	31.96
Median	28.59
Min	5.73
Max	156.42

- Up to ε^{i*} lost each period
  - \$830.96 total with average ε^{i*}
- Average expenditure \$5,575.50 over all $T = 26$ time periods

Application : Aggregate Data

We fail to reject (exact) quasilinear utility with aggregated data

- The dataset $\{(\bar{x}^t, p^t)\}_{t=1}^T$ is consistent with quasilinear utility in a deterministic framework

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We fail to reject (exact) quasilinear utility with aggregated data

- The dataset $\{(\bar{x}^t, p^t)\}_{t=1}^T$ is consistent with quasilinear utility in a deterministic framework
- Two-sided 95% confidence interval for the representative agent's ε^* is

$$[0, 2.69]$$

  - The lowest household ε^{i*} (5.73) is higher than the upper bound on the representative agent's ε^* (2.69)

Application : Aggregate Data

Is this model "too big" – does aggregation somehow mechanically lead to rationalization?

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Not necessarily... if quantities have a density and are unrelated to prices,

$$P(\text{Aggregate Exactly Consistent}) = \frac{1}{T!}$$

Application Simulations

Quasilinear utility seems to be much better at describing aggregate scanner data than individual scanner data

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To understand role of aggregation, we do some simulations

- Each individual simulated dataset is of the form X^{*i}
- We simulate aggregate datasets $\overline{X}^*_{n_b} = \frac{1}{n_b}\sum_{i=1}^{n_b} X^{*i}$ constructed from different size samples n_b

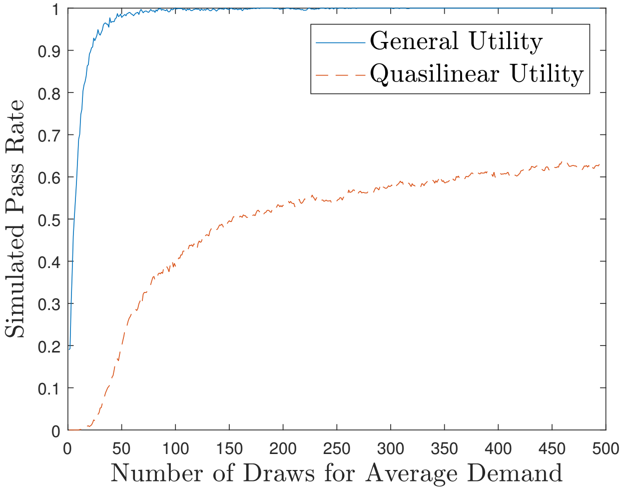
Role of Aggregation – Design 1

Design 1: $\overline{X}^*_{n_b}$ constructed from n_b individual datasets, each sampled with replacement from the original dataset

- First plot fraction of simulations with rationalization
- Then average $\overline{\varepsilon}^*$ across simulations

Design 1 – Pass Rate

Fraction of simulations with aggregate passing theory

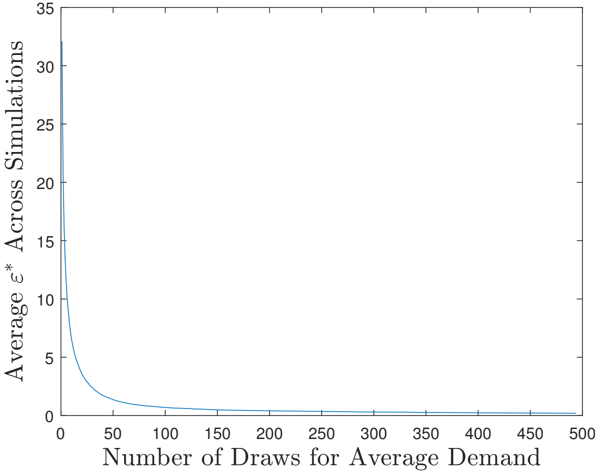


Design 1 – Pass Rate

63% of simulations with $n_b = 494$ individuals consistent with quasilinear utility, but 100% consistent with general utility

- Should be "more surprised" when aggregated dataset is consistent with quasilinear utility, than that it is consistent with general utility
- Maybe should also be surprised that general utility can explain the aggregate, since that model does not aggregate.

Role of Aggregation – Design 1



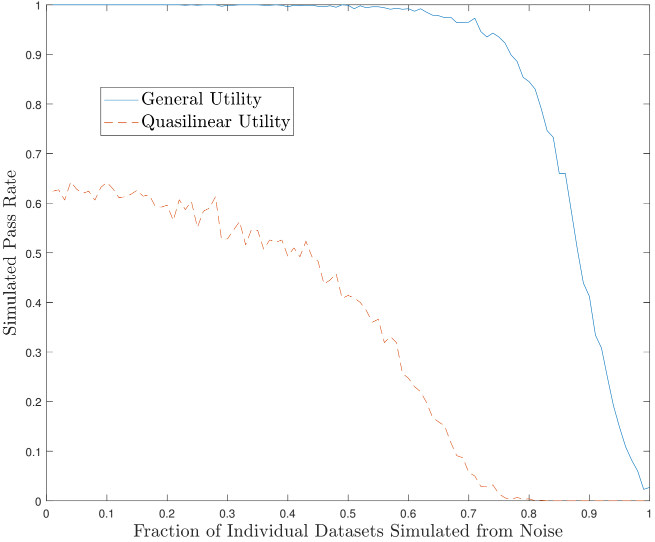
- Averaging over 50 people results in misspecification of $\approx$ \$2 on average

Role of Noise – Design 2

Each simulated individual dataset X^{*i} constructed as:

- Probability b of being drawn from the original collection of datasets
- Probability $1 - b$ of being generated from "noise"
  - X^{i*} constructed by filling in quantities each period, drawn i.i.d. from the collection of all quantities $\{\{X^{(i,t)}\}_{i=1}^{n}\}_{t=1}^{T}$

Role of Noise – Design 2



Related Literature

Measuring misspecification with revealed preference approach

- General consumer problem: Afriat (1973), Houtman and Maks (1985), Varian (1990), Echenique, Lee, Shum (2012), Halevy, Persitz, and Zrill (2018), Dziewulski (2018)
- Homothetic: Heufer and Hjerstrand (2017)
- Expected utility: Echenique, Imai, Saito (2018)

Misspecification, parameter recoverability, and aggregation

- Chetty (2012)

Structure arising from aggregation

- Birchenall (2016)
- More distantly, e.g. Hildenbrand (1983)

Conclusions

Theoretical:

- New measure of misspecification for quasilinear utility
- Aggregation theorem for approximately quasilinear utility
- Statistical inference with panel data or repeated cross sections
- Computable by linear programming

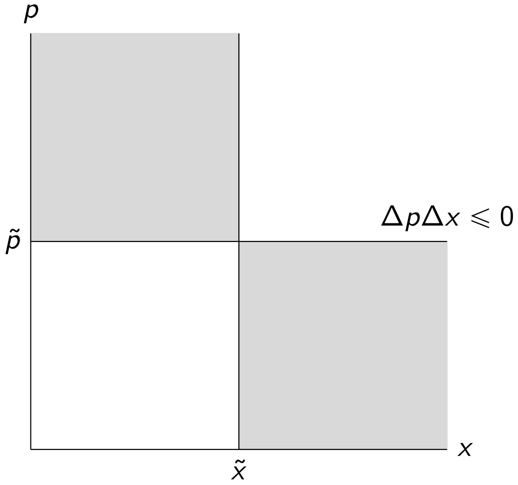
Conclusions

Empirical:

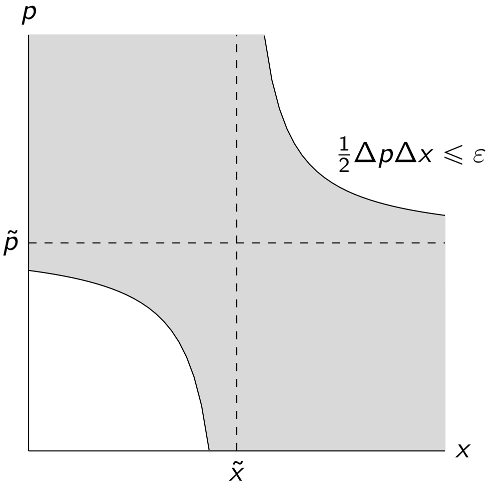
- All individuals are inconsistent with quasilinear utility but aggregate data is consistent
- General utility and quasilinear utility have different empirical aggregation properties
    - Aggregate over 10 individuals: general utility passes in 70% of simulations, quasilinear utility passes in 0%
    - Aggregate over 494 individuals: general utility always passes, quasilinear utility passes 63% of simulations

Graphical Law of Demand

Exact law of demand ($\varepsilon = 0$) tells us quantity up, price down



Graphical Approximate Law of Demand



Back

Testing Details

H_0 : For all finite sequences $\{t_m\}_{m=1}^{M}$,

$$\frac{1}{M}\sum_{m=1}^{M} p^{t_m} \cdot \left(\mathbb{E}\left[X^{(i,t_m)}\right] - \mathbb{E}\left[X^{(i,t_{m+1})}\right]\right) \leqslant \varepsilon$$

For each cycle, form a sample average,

$$\hat{\mu}_j := \frac{1}{M}\sum_{m=1}^{M} p^{t_m} \cdot \left(\overline{X}^{(t_m)} - \overline{X}^{(t_{m+1})}\right)$$

Reject when $\max_{j\in J} \hat{\mu}_j - \varepsilon > c_{1-\alpha}$ Back