Optimal Currency Areas with Labor Market Frictions

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Abstract

I study efficiency and optimal monetary policy in a two-country monetary union with frictional labor markets. With heterogeneity in labor market frictions, the constrained efficient allocation generically cannot be achieved even if productivity shocks affecting each country are the same. The second-best optimal policy targets smaller inflation and output gaps in the more sclerotic labor market. A quantitative calibration to the Eurozone implies welfare gains from redefining the union’s inflation target to put more weight on its sclerotic members.

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1 Introduction

Sustained imbalances and unemployment in the Eurozone in recent years have renewed attention on labor market frictions in the union, casting a fresh light on debates over the costs of the single currency and the appropriate conduct of stabilization. Are the conventional criteria for the optimality of the euro robust to the presence of these frictions? And should these frictions be accounted for in the optimal policy of the ECB?

Despite its rich history and recent resurgence, the theory of Optimal Currency Areas does not provide a framework to think through these questions. While much research has studied the consequences of sticky nominal wages\textsuperscript{1}, less has focused on the search and matching features of labor markets in a monetary union. This paper contributes to filling that gap.

In a two-country monetary union with frictional labor markets, I obtain two main results. First, even if countries are subject to the same shocks, heterogeneity in labor market frictions will generically induce inefficiency in a monetary union, modifying the Mundellian criterion for an optimal currency area.\textsuperscript{2} Heterogeneous frictions interact with the same shocks to require adjustment in the efficient terms of trade, inducing inefficient distortions in a monetary union with nominal rigidities. Second, given a necessary adjustment in the efficient terms of trade, the union-wide monetary policy should accommodate the more sclerotic member of the union at the second-best optimum.\textsuperscript{3} A more sclerotic labor market features a larger welfare loss and larger inflationary pressure from output fluctuations, motivating smaller price and output distortions there under the optimal policy. In a quantitative calibration matching the fluidity of labor markets across the Eurozone, this motivates welfare gains from redefining the union’s inflation target to put greater weight on its sclerotic members.

The model developed in this paper merges workhorse ingredients from the international macro and macro/labor traditions. From the first, production of distinct intermediate goods uses domestic labor and country-specific productivity; international trade is in differentiated final goods; and prices of varieties across the union are set à la Calvo (1983) in a common unit of account. From the second, workers and vacancies in each country match in the tradition of Diamond (1981), Mortensen (1982), and Pissarides (1984) (DMP); recruiting costs generate a surplus from each match; and wages split the bilateral surplus.

\textsuperscript{1}For instance, among recent analyses of monetary unions, Schmitt-Grohe and Uribe (2013) and Gali and Monacelli (2016) focus on the consequences of wage rigidity in particular, and Gourinchas, Philippou, and Vayanos (2017), Martin and Philippou (2017), and Chodorow-Reich, Karabarbounis, and Kekre (2019) study these consequences quantitatively for southern European economies during the recent recession.

\textsuperscript{2}Here I refer to the standard prescription since Mundell (1961) that countries form an optimal currency area if they are subject to the same shocks, obviating any need for adjustment in the terms of trade.

\textsuperscript{3}I define a more sclerotic labor market as one with higher hiring costs and/or a lower separation probability, following in the tradition of Blanchard and Gali (2010).
I begin by characterizing when such a monetary union will experience distortions, qualifying the conventional Mundellian criterion for optimal currency areas in the presence of heterogeneous labor market frictions across countries. I assume that the natural allocation — the equilibrium absent nominal rigidity — is constrained efficient: in each country, a production subsidy offsets the distortion from monopolistic competition and the Hosios (1990) condition is satisfied. Then variation in the natural terms of trade is a necessary and sufficient condition for inefficiency with nominal rigidity: the combination of a fixed exchange rate and sticky prices means that the union cannot replicate the efficient movement in relative prices without inefficient price dispersion, output distortions, or both. Away from a unitary intertemporal elasticity of substitution, the natural level of employment in each country will vary even if relative productivity is constant. The relative employment response in each country depends on labor market frictions, feeding back to affect relative production across countries and thus the natural terms of trade. Hence, the same shocks will generically interact with asymmetric labor market frictions to generate inefficiency in a monetary union.

In the presence of shocks necessitating distortions in the union, I then find that optimal policy is characterized by relative accommodation of the more sclerotic labor market. In particular, greater hiring costs or a lower separation probability in a particular union member tend to raise the deadweight costs of inefficient hiring in that country. This increases the welfare cost of output distortions and the slope of the Phillips curve in that country — which, in fact, are tightly linked when the natural allocation is constrained efficient, reflecting the connection between the curvature of social welfare and the sensitivity of firms’ real marginal costs to output. Since the more sclerotic labor market features greater welfare losses and greater price pressure from a given distortion in output, the optimal policy accommodates that market by targeting both a smaller output gap and smaller inflation/deflation at the second-best optimum. I demonstrate these results on optimal policy analytically in a limiting case which renders the planning problem a static one, and numerically in the more general case. I further demonstrate that these results are robust to (symmetric) real wage rigidity in each country, in which case inefficiency in real wages accompanies inefficiency in the terms of trade to necessitate distortions across the union.

In a quantitative calibration to the Eurozone, these insights imply welfare gains from redefining the union’s inflation target to place greater weight on its more sclerotic members. In keeping with the two-country environment studied in the paper, I calibrate the model to Germany and France, the largest members of the Eurozone. Applying the methodology of Elsby, Hobijn, and Sahin (2013) to characterize average inflow and outflow rates from

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I also provide an alternative calibration to Germany and Italy in the appendix. Italy, like France, features a labor market which is more sclerotic than that of Germany.
unemployment since the creation of the euro in 1999, I find that the French labor market is more sclerotic than that in Germany: it features smaller inflows and especially smaller outflows from unemployment. Such differences in the structure of labor markets are not accounted for in the harmonised index of consumer prices (HICP) used to define price stability in the ECB’s mandate. While an inflation targeting rule is not the optimal policy, raising the weight on French producer price inflation from 43% (under HICP-targeting) to 54% can achieve more than half of the welfare gains from the optimal policy.

The evidence on labor market flows underscores the sources and relevance of the heterogeneity emphasized by this paper. Across Eurozone economies, I demonstrate that outflow and inflow rates from unemployment correlate negatively with indexes of employment protection summarizing the information required of employers, severance requirements, and other costs and procedures regarding employee dismissal. While I abstract from endogenous separations and firing costs for parsimony, this suggests that the differences in flows which I model with differences in hiring costs and separation probabilities captures differences in such underlying institutions. Moreover, the evidence on flows validates the paper’s emphasis on heterogeneous labor market frictions. For instance, while (as my analytical results make clear) heterogeneity in Frisch elasticities of labor supply would qualitatively generate similar implications for efficiency and optimal policy in the union, such heterogeneity in preferences is not well suited to explain differences in the fluidity of labor markets across countries.

This paper sits at the intersection of literatures on monetary unions, the level and cyclical behavior of unemployment, and optimal monetary policy. The questions I ask are inspired by the seminal contributions of Mundell (1961), McKinnon (1963), and Kenen (1969) on Optimal Currency Areas, and Ljungqvist and Sargent (1998), Nickell and Layard (1999), and Blanchard and Wolfers (2000) on European institutions and unemployment. The model builds on Clarida, Gali, and Gertler (2002) and Gali and Monacelli (2008)’s more modern analyses of optimal policy in the open economy and monetary unions and Shimer (2010)’s analysis of labor market dynamics with DMP frictions. Like the closed economy analyses of Thomas (2008), Faia (2009), and especially Blanchard and Gali (2010), it is primarily concerned with optimal monetary policy in an environment with DMP frictions.

Indeed, this paper’s explicit normative focus distinguishes it from most other papers in the growing literature on DMP frictions in the open economy. Hairault (2002), Christiano, Trabandt, and Walentin (2011), Boz, Durdu, and Li (2015), and Bodenstein, Kamber, and Thoenissen (2018) explore the positive implications of labor market frictions for open economy business cycle dynamics. Campolmi and Faia (2011) and Abbritti and Mueller (2013) focus on a monetary union and are thus especially complementary with the present analysis. They study the consequences of restrictions on flows into and out of unemployment
for the volatility of macroeconomic aggregates such as inflation, inflation differentials, and unemployment differentials. I trace out the implications for optimal monetary policy.

A small set of papers in this literature also take a normative perspective; relative to these, my analysis focuses on a distinct set of questions and provides a number of closed form analytical results. Campolmi and Faia (2015) provide a welfare ranking of fixed versus floating exchange rate regimes and then evaluate how heterogeneity in unemployment insurance affects optimal policy. Instead, I take the monetary union as given and, building on Blanchard and Gali (2010), focus on how heterogeneity in labor market fluidity affects optimal policy. Eggertsson, Ferrero, and Raffo (2014) and Cacciatore, Ghironi, and Fiori (2016) evaluate the trade-offs in using structural reforms and monetary policy to undo steady-state distortions from inefficient labor or product markets in a monetary union. Instead, I assume labor market frictions are consistent with a constrained efficient steady-state so that, building on Gali and Monacelli (2008), I can study the interaction between heterogeneity in these frictions and macroeconomic shocks alone. Focusing on this narrower question in a parsimonious setting allows me to analytically characterize mechanisms that I expect are also relevant in the richer models studied in this literature numerically.

In revisiting the prescriptions for optimal monetary policy in a monetary union, my analysis complements recent work focusing on other policy tools in such an environment. Schmitt-Grohe and Uribe (2016) argue that macro-prudential policy can relax the second-best problem posed by fixed exchange rates and downward nominal wage rigidity. Farhi and Werning (2014, 2017) re-examine the desirability of labor mobility and fiscal transfers in a union with nominal rigidity. Farhi, Gopinath, and Itskhoki (2013) argue that distortionary taxes can in fact eliminate the relative price distortions in such an environment altogether. The focus in all of these analyses is primarily on how these policy tools can re-allocate demand to mitigate the inefficiency induced by the fixed exchange rate. In contrast, my analysis emphasizes how labor market frictions affect the welfare cost of supply-side adjustment to those demand imbalances, shaping the second-best response of monetary policy itself.\(^5\)

In this sense, my analysis is perhaps most complementary with Benigno (2004). Benigno (2004) characterizes optimal monetary policy in a union in which the degree of nominal rigidity differs across countries, whereas I characterize optimal policy in a union in which the fluidity of labor markets differs. I demonstrate that while these dimensions of heterogeneity affect welfare through different channels — the cost of inflation in the first case, versus the cost of output fluctuations in the second — they both imply it is optimal to accommodate

\(^5\)In this way, it relates to the analysis of Gali and Monacelli (2016), who argue that greater wage flexibility can be welfare reducing from the perspective of a single country in a monetary union because of its effect on inflation. Similarly, the effect of labor market frictions on the inflationary responses to output fluctuations is one key component of my result on second-best optimal policy in the union.
the more rigid member of the union. Importantly, however, they have opposite effects on the slope of the Phillips curve in each country. This means that to obtain accurate implications for optimal policy, quantitative analyses of monetary unions should match cross-country heterogeneity in the observed duration of nominal contracts and observed fluidity of labor markets, rather than simply being consistent with cross-country variation in the observed relationship between inflation and output overall.

In section 2 I outline the economic environment. In section 3 I study the natural allocation and then characterize when the union will experience distortions with nominal rigidities. In section 4 I characterize the second-best stabilization problem when the union must accept distortions. In section 5 I calibrate the model to the Eurozone to assess its quantitative predictions for optimal policy. Finally, in section 6 I conclude.

2 A monetary union with DMP frictions

In this section I characterize a global economy from period 0 onwards consisting of two countries in a monetary union subject to productivity shocks and featuring DMP labor markets. The environment merges standard elements of the international macro and macro/labor literatures. I detail the optimization problem facing Home agents and briefly describe the symmetric problem facing Foreign agents; the full description is provided in appendix A. Where appropriate, asterisks denote variables chosen by or endowed to Foreign agents. For expositional simplicity, time subscripts encode both the period and the state of the world.

**Households** Home is comprised of measure γ households and Foreign is comprised of measure 1 − γ households. Each household is comprised of measure one workers. Complete markets within and across households mean that we can focus on the problem faced by a representative household in each country. We exposit the model with trade in one-period state-contingent securities trading at prices $Q_{t,t+1}$ in period $t$, though no arbitrage in a full set of date- and state-contingent securities imply $Q_{t_1,t_2} = \prod_{s=t_1}^{t_2-1} Q_{s,s+1}$ for any $t_1 \leq t_2$ (where $Q_{t_1,t_1} = 1$). The representative household at Home then chooses a state-contingent sequence of consumption $c_t$, portfolio of one-period securities $B_{t+1}$, and labor force participation $u_t$ among its $1 - (1 - \delta)n_{t-1}$ unemployed members to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, n_t)$$  (1)
where flow utility is given by

\[ u(c_t, n_t) = \frac{c_t^{1-\sigma}}{1-\sigma} - \frac{\chi n_t^{1+\varphi}}{1+\varphi}, \]

(2)

\[ c_t = \left[ (\gamma)^c (c_{Ht})^{1-\sigma} + (1-\gamma)^c (c_{Ft})^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \]

(3)

\[ c_{Ht} = \left[ \int_0^1 c_{Ht}(j) \frac{\varepsilon}{\varepsilon-1} \frac{dj}{j} \right]^{\frac{1}{\varepsilon-1}}, \quad c_{Ft} = \left[ \int_0^1 c_{Ft}(j^*) \frac{\varepsilon}{\varepsilon-1} \frac{dj^*}{j^*} \right]^{\frac{1}{\varepsilon-1}}, \]

(4)

where \( c_{Ht}(j) \) denotes its consumption of Home-produced variety \( j \in [0,1] \) and \( c_{Ft}(j^*) \) denotes its consumption of Foreign-produced variety \( j^* \in [0,1] \); the evolution of employed workers in the household at each date and state

\[ n_t = (1-\delta)n_{t-1} + p(\theta_t)u_t, \]

(5)

where \( p(\theta_t) \) is the job-finding rate of unemployed workers described in further detail below; and the resource constraint at each date and state

\[ \int_0^1 P_{Ht}(j) c_{Ht}(j) dj + \int_0^1 P_{Ft}(j^*) c_{Ft}(j^*) dj^* + E_t Q_{t,t+1} B_{t+1} \leq W_t n_t + B_t - T_t, \]

(6)

where \( P_{Ht}(j) \) and \( P_{Ft}(j^*) \) denote the prices of varieties, \( W_t \) is the wage earned by each employed worker in the household,\(^6\) and \( T_t \) is a transfer from the government.\(^7,8\)

Analogously, the representative household in Foreign chooses a state-contingent sequence of \( c^*_t \) (and the nested components \( c^*_{Ht}, c^*_{Ft}, \{c^*_t(j^*)\}_{j^*=0}^1, \{c^*_t(j^*)\}_{j^*=0}^1 \), \( B^*_{t+1} \), and \( u^*_t \)) to maximize its utility. The separation probability \( \delta^* \) and job-finding probability \( p^*(\theta^*_t) \) may differ from those in Home. To compare the role of preferences with that of labor market frictions, the disutility of labor \( \chi^* \) and Frisch elasticity \( \varphi^* \) may also differ from those in Home. To focus on differences in labor markets across countries, I assume for simplicity that households have the same discount factor \( \beta \), coefficient of relative risk aversion \( \sigma \), inverse trade elasticity \( \varsigma \), and elasticity of substitution across varieties from each country \( \varepsilon \). Producer-currency pricing ensures that households face the same variety prices \( \{P_{Ht}(j)\} \) and \( \{P_{Ft}(j^*)\} \) expressed in the union’s common unit of account. Finally, complete markets imply that households face the same state prices \( Q_{t,t+1} \).

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\(^6\) This means in particular that incumbent and newly hired workers are paid the same wage. This is innocuous; given risk-sharing within the household, the wage paid to incumbent workers has no effect on the equilibrium allocation, as is well known in the literature.

\(^7\) Upper-case variables denote nominal variables denominated in the union’s unit of account. I follow Woodford (2003) and Gali (2008) in studying the “cashless limit” so that my results are unaffected by seignorage considerations across the union and utility fluctuations from real money balances.

\(^8\) Furthermore, we assume a standard no-Ponzi constraint \( \lim_{s \to \infty} E_t Q_{t,t+s} B_{t+s} \geq 0 \) at all \( t \).
Intermediate good producers  Perfectly competitive intermediate good producers recruit workers in their frictional labor market, produce, and sell to domestic retailers. With constant returns to scale in production and recruiting, there is a representative producer in each country. The representative producer at Home chooses a state-contingent sequence of vacancies $\nu_t$ and workers $n_t$ (expressed as a share of the measure $\gamma$ Home agents) to maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} Q_{0,t} \Pi_t$$

subject to the evolution of its workforce at each date and state

$$n_t = (1 - \delta)n_{t-1} + q(\theta_t)\nu_t,$$  \hspace{1cm} (8)

where $q(\theta_t)$ is the vacancy-filling rate described in further detail below, and the definition of flow profits at each date and state

$$\Pi_t = \gamma \left( P^I_t a_t [(1 - \delta)n_{t-1} + q(\theta_t)\nu_t - k\nu_t] - W_t [(1 - \delta)n_{t-1} + q(\theta_t)\nu_t] \right),$$  \hspace{1cm} (9)

where $P^I_t$ is the price of the intermediate good, $a_t$ is productivity, and — following Shimer (2010) — $k$ is the measure of incumbent workers required to manage a single vacancy.

Analogously, the representative producer in Foreign chooses the state-contingent sequence of $\nu^*_t$ and $n^*_t$ to maximize firm value. The vacancy-filling rate $q^*(\theta^*_t)$, price of intermediates $P^I_t$, productivity $a^*_t$, and recruiting cost $k^*$ can differ from those in Home.

Final good retailers  In each country, a unit measure of monopolistically competitive retailers produce differentiated varieties using their domestic intermediate good and sell them globally. If retailer $j$ at Home can update its price in period $t$, which occurs with probability $\iota$ following Calvo (1983), it chooses the price $P^{H_t}$ and state-contingent intermediate inputs $x_s(j)$ and varieties sold $y_s(j)$ (for $s \geq t$) to maximize

$$\mathbb{E}_t \sum_{s=t}^{\infty} Q_{t,s} \Pi^r_s(P^{H_t}; j)$$

subject to flow profits earned at each date and state given the price $P^{H_t}$

$$\Pi^r_s(P^{H_t}; j) = P^{H_t}y_s(j) - (1 + \tau^r)P^I_s x_s(j),$$  \hspace{1cm} (11)
where $\tau^r$ is a tax set by policy; the technology at each date and state

$$y_s(j) = x_s(j); \quad (12)$$

and the derived demand for its variety from households’ optimization at each date and state

$$y_s(j) = \left(\frac{P_{Ht}}{P_{Hs}}\right)^{-\varepsilon} (\gamma c_{Hs} + (1 - \gamma)c^*_H), \quad (13)$$

where the latter uses the assumption of producer-currency pricing and the definition of the CES price index

$$P_{Ht} = \left[\int_0^1 P_{Ht}(j)^{1-\varepsilon} dj\right]^\frac{1}{1-\varepsilon}. \quad (14)$$

If retailer $j$ cannot update its price in period $t$ it accommodates consumption demand at its preset price provided it earns non-negative profits.

Analogously, if variety $j^*$ in Foreign can update its price at $t$, which occurs with probability $\iota^*$, it chooses $P_{Ft}$ and the state-contingent sequence of $x_s^*(j^*)$, and $y_s^*(j^*)$ to maximize firm value. The variety takes as given the CES price index

$$P_{Ft} = \left[\int_0^1 P_{Ft}(j^*)^{1-\varepsilon} dj^*\right]^\frac{1}{1-\varepsilon}. \quad (15)$$

**Matching, search, and wage determination**

Given the assumption that labor is immobile across borders, with $u_t$ and $u_t^*$ unemployed workers searching for employment and $\nu_t$ and $\nu_t^*$ vacancies posted by producers, the Home and Foreign economies see

$$m(u_t, \nu_t) = \bar{m}(u_t)^{1-\eta}(\nu_t)^\eta, \quad m^*(u_t^*, \nu_t^*) = \bar{m}^*(u_t^*)^{1-\eta^*}(\nu_t^*)^{\eta^*}$$

aggregate matches, where $\bar{m}$ and $\bar{m}^*$ denote match efficiency and $\eta$ and $\eta^*$ denote the elasticity of matches with respect to vacancies in each country. Tightness in each country is

$$\theta_t = \frac{\nu_t}{u_t}, \quad \theta_t^* = \frac{\nu_t^*}{u_t^*}, \quad (16)$$

implying the vacancy-filling probabilities

$$q(\theta_t) = \frac{m(u_t, \nu_t)}{\nu_t}, \quad q^*(\theta_t^*) = \frac{m^*(u_t^*, \nu_t^*)}{\nu_t^*}.$$
and job-finding probabilities

\[ p(\theta_t) = \frac{m(u_t, \nu_t)}{u_t}, \quad p^*(\theta^*_t) = \frac{m^*(u^*_t, \nu^*_t)}{u^*_t}. \]

Search is random and wages are Nash bargained each period with worker bargaining shares \( \zeta \) and \( \zeta^* \) in Home and Foreign. In appendix E, I generalize this environment to accommodate real wage rigidity. The absence of real wage rigidity in the baseline analysis allows me to focus on the core source of inefficiency in a monetary union, one source of nominal rigidity coupled with a fixed exchange rate. Nonetheless, accounting for the possibility of real wage rigidity is useful when calibrating the model to the Eurozone at the end of the paper.

**Government budgets and policy** Budget balance in each country requires

\[
\int_0^1 \tau^P_t x_t(j) dj + T_t = 0, \tag{17}
\]

\[
\int_0^1 \tau^{*P}_t x^*_t(j^*) dj^* + T^*_t = 0. \tag{18}
\]

Furthermore, a union-wide policymaker sets a state-contingent path for the nominal interest rate \( i_t \) on one-period riskless nominal bonds, where no arbitrage implies

\[ i_t = \frac{1}{E_t Q_{t,t+1}}. \]

**Market clearing** For intermediate goods, it must be that

\[
\int_0^1 x_t(j) dj = \gamma a_t [(1 - \delta) n_{t-1} + q(\theta_t) \nu_t - k \nu_t], \tag{19}
\]

\[
\int_0^1 x^*_t(j^*) dj^* = (1 - \gamma) a^*_t [(1 - \delta^*) n^*_{t-1} + q^*(\theta^*_t) \nu^*_t - k^* \nu^*_t]. \tag{20}
\]

For final goods, it must be that

\[ \gamma c_{Ht}(j) + (1 - \gamma) c^*_{Ht}(j) = y_t(j), \tag{21} \]

\[ \gamma c_{Ft}(j^*) + (1 - \gamma) c^*_{Ft}(j^*) = y^*_t(j^*). \tag{22} \]

for each \( j \in [0,1] \) and \( j^* \in [0,1] \). Finally, in asset markets, the market for each available security must clear. Since Arrow securities are in zero net supply, Walras’ Law implies that

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9One source of real wage rigidity could be the simultaneous presence of nominal price and wage rigidity.
the sum of payoffs on portfolios at each date and state reflects the profits of global firms

$$\gamma B_t + (1 - \gamma)B^*_t = \Pi_t + \int_0^1 \Pi^*_t(j)\,dj + \Pi^*_t + \int_0^1 \Pi^{**}_t(j)\,dj. \quad (23)$$

**Equilibrium** The definition of an equilibrium is standard and provided in appendix B. In that appendix, I further outline the first order conditions of households and firms in this global economy. As derived therein, optimal risk-sharing in this environment requires

$$c_t = \Xi c^*_t$$

at all dates and states, where $\Xi$ is a constant reflecting the initial levels of wealth $B_0$ and $B^*_0$. Going forward, I assume:

**Assumption 1.** Initial endowments are such that $\Xi = 1$.

My results on constrained efficiency and the Ramsey problem studied in the subsequent sections assume a utilitarian objective. In the usual way, these results would generalize when $\Xi \neq 1$ provided that the Pareto weights used by the planner and in the Ramsey problem are $\Xi^\sigma$ times higher for Home than Foreign agents.

## 3 Inefficiency in a monetary union

I first characterize when the monetary union will experience distortions. I begin by characterizing the allocation absent nominal rigidity before turning to the one with sticky prices.

### 3.1 Natural allocation

Under standard conditions, the natural allocation will be constrained efficient — that is, consistent with the allocation chosen by a planner maximizing utilitarian social welfare subject to the economy’s technological constraints in production and recruiting:

**Proposition 1.** The natural allocation is constrained efficient if and only if in each country retailers are subsidized to offset the distortion from monopolistic competition ($\tau^r = \tau^{**} = -\frac{1}{\epsilon}$) and the Hosios (1990) condition holds ($\zeta = 1 - \eta$, $\zeta^* = 1 - \eta^*$).

Relative to the constrained efficient allocation, there are two sets of possible distortions in the competitive equilibrium: monopoly power among retailers in each country and search externalities imposed on others in a frictional labor market. A retailer subsidy can undo the first, a standard result in the monetary literature. The Hosios condition ensures that
the second is internalized by agents just as in the closed economy, since participants in the labor market only impose search externalities on others in the domestic market in view of the assumed lack of cross-country labor mobility.

3.2 Inefficiency with nominal rigidity

I now consider the environment with sticky prices. In the tradition of the literature on Optimal Currency Areas, I maintain the following assumption to focus on inefficiency induced by the interaction between nominal rigidity and the fixed exchange rate alone:

Assumption 2. The natural allocation is constrained efficient (implemented as described in Proposition 1).

Consistent with the literature (and evident in the linear-quadratic approximation to the optimal policy problem studied in the next section), variation in the natural terms of trade will then be a sufficient statistic for distortions in a monetary union with nominal rigidity. If the natural terms of trade adjust, the union must experience inefficient price dispersion, output distortions, or both. Hence, we can simplify the present question of interest by asking whether labor market frictions affect variation in the natural terms of trade.

Define Home’s terms of trade $s_t \equiv \frac{p_{Hu}}{p_{Ft}}$. Using the $n$ superscript to denote variables in the natural allocation, I show in appendix B that

$$s^n_t = \left( \frac{a^n_t \left[ n^n_t - k^* (1 - (1 - \delta^*) n^n_{t-1}) \theta^n_t \right]}{a_t \left[ n^n_t - k (1 - (1 - \delta) n^n_{t-1}) \theta^n_t \right]} \right)^{\varsigma}. \quad (24)$$

Intuitively, the greater the output of Foreign-produced goods relative to Home-produced goods, the more expensive must be Home-produced goods to clear markets. Variation in the natural terms of trade thus directly reflects variation in relative productivity as well as the variation in relative employment and hiring induced by productivity shocks. The latter generically depends on labor market features in each country, motivating the following useful benchmark case:

Definition 1. When countries are symmetric, they have the same preferences and technologies \( \{\chi a^{\varsigma-1}, \varphi, k, \delta, \bar{m}, \eta\} = \{\chi^*, a^{*\varsigma-1}, \varphi^*, k^*, \delta^*, \bar{m}^*, \eta^*\} \).\(^{10}\)

With log preferences (\(\sigma = 1\)) or symmetric countries, we can then prove:

Proposition 2. With \(\sigma = 1\) or symmetric countries, the natural terms of trade will be constant if and only if relative productivity is constant.

\(^{10}\)Here, \(a\) and \(a^*\) denote productivity in the deterministic steady-state in each country.
Log preferences ensure that employment in each country is simply a function of relative productivity, so the result then follows from (24). Intuitively, first consider the even more special Cole-Obstfeld parameterization, which further assumes \( \zeta = 1 \). In this case, employment in each country is invariant to productivity at home and abroad, generalizing results in Clarida et al. (2002), Blanchard and Gali (2010), and Shimer (2010) to the open economy with labor market frictions. In particular, these preferences ensure that the income, substitution, and terms-of-trade effects of domestic productivity shocks offset each other, and the risk-sharing and terms-of-trade effects from foreign productivity shocks offset each other, in the determination of domestic employment. Continuing to assume \( \sigma = 1 \) but now allowing \( \zeta \neq 1 \), I prove in the claim that while employment in each country now varies with productivity at home and abroad, it only does so if there is variation in relative productivity.

Away from log preferences, employment in each country will vary with productivity even if relative productivity is constant across countries. However, if countries are symmetric, employment will respond in the same way across countries. Hence, (24) again implies that the natural terms of trade are simply a function of relative productivity.

Away from these knife-edge cases, we instead obtain:

**Proposition 3.** With \( \sigma \neq 1 \) and asymmetric countries, the natural terms of trade will generically vary if relative productivity is constant.

In this case, employment in each country will vary even if relative productivity is constant. Since labor market frictions affect the employment responses to shocks, the same shocks will interact with heterogeneous frictions to lead to variation in the natural terms of trade.

Taken together, Propositions 2 and 3 allow us to characterize the circumstances under which a monetary union with nominal rigidity will face distortions:

**Proposition 4.** With \( \sigma = 1 \) or when countries are symmetric, the efficient allocation can be implemented if and only if relative productivity is constant. With \( \sigma \neq 1 \) and asymmetric countries, the efficient allocation generically cannot be implemented even if relative productivity is constant.

Hence, away from log preferences and with asymmetry across countries, Mundell (1961)’s business cycle synchronicity criterion for optimal currency areas breaks down: the efficient allocation cannot be implemented even though shocks are the same.\(^{11}\) Notably, this is the case even though the heterogeneity in labor market frictions is consistent with efficient fluctuations in the natural allocation.

\(^{11}\)Equivalently, to implement the efficient allocation, shocks in one country will have to systematically differ from those in the other country depending on their differences in labor markets.
4 Stabilization trade-offs and optimal monetary policy

I now characterize how differences in labor market frictions affect stabilization trade-offs and the second-best monetary policy when the efficient allocation is unattainable.

4.1 Sclerotic and fluid labor markets

Following Blanchard and Gali (2010), I will focus on differences in the fluidity of labor markets between countries:

**Definition 2.** If Home’s labor market is more sclerotic, it has higher hiring costs (higher $k$), lower match efficiency (lower $\overline{m}$), or a lower separation probability (lower $\delta$). If its labor market is more fluid, the opposite is true.

Since $k$ and $\overline{m}$ have isomorphic effects on employment and consumption (and analogously for $k^*$ and $\overline{m}^*$), I focus on differences in $\{k, \delta\}$ from $\{k^*, \delta^*\}$ in the results which follow.

4.2 Ramsey policy problem

Consider the problem faced by the union’s central bank seeking to choose monetary policy to maximize union-wide utilitarian social welfare in response to productivity shocks, chosen in period 0 with commitment. Then the Ramsey optimal monetary policy will implement the real allocation maximizing

$$U_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \gamma u(c_t, n_t) + (1 - \gamma) u^*(c_t^*, n_t^*) \right]$$

subject to the equilibrium conditions of the model. I introduce the following assumption to facilitate the analysis of this problem:

**Assumption 3.** The initial prices $\{P_{H-1}(j)\}$ and $\{P_{F-1}(j^*)\}$ are the same across retailers and, together with initial employment levels $n_{-1}$ and $n^*_{-1}$, are consistent with a zero-inflation, constrained efficient steady-state under the Ramsey optimal policy.

---

12 This follows from the fact that the state-contingent sequence of employment and consumption in the union given $\{k, \overline{m}\}$ is identical to that given $\{\overline{m} \delta k, 1\}$, and analogously for Foreign. This is easily proven using the equilibrium conditions outlined in appendix B, and in particular uses the fact that given the state-contingent sequence of labor market tightness $\theta_t$ in the first case, the second features tightness $\overline{m}^{\frac{1}{\delta}}\theta_t$.

13 In the context of the literature on optimal policy in open economies, I am studying the optimal policy problem chosen under cooperation across countries. This seems appropriate given my focus on the choice of a single policy instrument chosen by the union-wide central bank. An interesting question left for future work is how the Nash equilibrium of policies chosen non-cooperatively by multiple central banks would be affected by the presence of labor market frictions across countries, building on the analysis of Corsetti and Pesenti (2001), Clarida et al. (2002), Benigno and Benigno (2006), and others.
I follow Woodford (2003) in employing a primal approach and approximating the Ramsey optimal allocation up to first-order deviations from the deterministic steady-state. A linear approximation to the equilibrium conditions and quadratic approximation to social welfare suffices to characterize the optimum up to first order under Assumptions 2 and 3.

4.3 Additional notation

The analysis will be facilitated by some additional notation. Define \( \pi_{Ht} \) and \( \pi_{Ft} \) to be log producer-price inflation in each country, so

\[
\pi_{Ht} = \log P_{Ht} - \log P_{Ht-1}, \\
\pi_{Ft} = \log P_{Ft} - \log P_{Ft-1}.
\]

For any other endogenous variable \( z_t \) given \( z \), its value in the deterministic steady-state, and \( z_t^n \), its value in the natural allocation, define

\[
\hat{z}_t \equiv \log z_t - \log z, \\
\hat{z}_t^n \equiv \log z_t^n - \log z, \\
\tilde{z}_t \equiv \hat{z}_t - \hat{z}_t^n.
\]

I refer to the last variable as the “gap” in \( z_t \). Define the real marginal cost (in terms of domestic output) facing retailers in each country \( \mu_t \equiv \frac{P_I}{P_{Ht}} \) and \( \mu_t^* \equiv \frac{P_I^*}{P_{Ft}} \). Given Home’s terms of trade \( s_t \equiv \frac{P_H}{P_F} \), the symmetric CES demand system implies that

\[
\omega \equiv \frac{\gamma}{\gamma + (1 - \gamma)s_t^{1-\omega}}
\]

characterizes the steady-state expenditure share on Home-produced goods in each country (and thus \( 1 - \omega \) characterizes the expenditure share on Foreign-produced goods). Finally, define the effective units of labor engaged in production

\[
f(n_t, n_{t-1}) \equiv n_t - k(1 - (1 - \delta)n_{t-1})p^{-1} \left( \frac{n_t - (1 - \delta)n_{t-1}}{1 - (1 - \delta)n_{t-1}} \right),
\]

\[
f^*(n_t^*, n_{t-1}^*) \equiv n_t^* - k^*(1 - (1 - \delta^*)n_{t-1}^*)p^{*-1} \left( \frac{n_t^* - (1 - \delta^*)n_{t-1}^*}{1 - (1 - \delta^*)n_{t-1}^*} \right),
\]

given the evolution of employment in (5), the fact that the representative household will optimally choose all unemployed workers to participate in the labor market, and the consistency of tightness with aggregate vacancies and job search, with analogous conditions in
Foreign. We can thus define the elasticities evaluated at steady-state

\[
\epsilon^f_n \equiv f_n \frac{n}{f}, \quad \epsilon^f_n \equiv f_n \frac{n}{f}, \quad \epsilon^f_{n-1} \equiv f_{n-1} \frac{n}{f}, \quad \epsilon^f_{n-1} \equiv f_{n-1} \frac{n}{f}, \quad \epsilon^f_n \equiv f_n \frac{n}{f}, \quad \epsilon^f_n \equiv f_n \frac{n}{f},
\]

\[
\epsilon^*_{n} \equiv f_n^* \frac{n^*}{f^*}, \quad \epsilon^*_{n-1} \equiv f_{n-1}^* \frac{n^*}{f^*}, \quad \epsilon^*_{n-1} \equiv f_{n-1}^* \frac{n^*}{f^*}, \quad \epsilon^*_{n} \equiv f_n^* \frac{n^*}{f^*}, \quad \epsilon^*_{n} \equiv f_n^* \frac{n^*}{f^*},
\]

using the fact that \( n = n-1 \) and \( n^* = n^*_{-1} \) in steady-state. We can further write the output of intermediate goods produced in each country as

\[
x_t \equiv \gamma a_t f(n_t, n_{t-1}), \\
x^*_t \equiv (1 - \gamma) a_t f^*(n^*_t, n^*_{t-1}).
\]

### 4.4 Implementability constraints

I begin by characterizing a linear approximation to the equilibrium conditions constituting the implementability constraints facing the Ramsey policymaker.

We first obtain Phillips curves relating producer-price inflation to deviations in real marginal cost from their efficient level:

**Lemma 1.** Up to first order around the deterministic steady-state,

\[
\pi_{Ht} = \beta E_t \pi_{Ht+1} + \lambda \tilde{\mu}_t, \\
\pi_{Ft} = \beta E_t \pi_{Ft+1} + \lambda^* \tilde{\mu}_t^*,
\]

where \( \lambda \equiv \frac{\varphi(1-\beta(1-\delta))}{1-\delta} \) and \( \lambda^* \equiv \frac{\varphi^*(1-\beta(1-\delta^*))}{1-\delta^*} \).

These Phillips curves are standard in the literature. The sensitivity of inflation to real marginal cost in each country is rising in the degree of price flexibility \( \delta \) and \( \delta^* \).

The relationship between real marginal cost and employment in turn accounts for the frictional labor market in each country:

**Lemma 2.** Up to first order around the deterministic steady-state,

\[
\tilde{\mu}_t + \frac{\epsilon^f_{n-1}}{\epsilon^f_n} \beta E_t \tilde{\mu}_{t+1} = \phi \epsilon^f_n \tilde{n}_t + \phi_{-1} \epsilon^f_{n-1} \tilde{n}_{t-1} + \phi_{-1} \epsilon^f_{n-1} \beta E_t \tilde{n}_{t+1} \\
+ (\sigma - \varsigma) (1 - \omega) \left[ \phi^f_n \epsilon^*_{n} \tilde{n}^*_t + \phi^f_{n-1} \epsilon^*_{n-1} \tilde{n}^*_{t-1} + \frac{\epsilon^f_{n-1}}{\epsilon^f_n} \epsilon^*_{n} \beta E_t \tilde{n}^*_{t+1} \right],
\]

\[
\tilde{\mu}_t^* + \frac{\epsilon^f_{n-1}^*}{\epsilon^f_{n}^*} \beta E_t \tilde{\mu}_{t+1}^* = \phi^* \epsilon^f_{n} \tilde{n}^*_t + \phi^*_{-1} \epsilon^f_{n-1} \tilde{n}^*_{t-1} + \phi^*_{-1} \epsilon^f_{n-1} \beta E_t \tilde{n}^*_{t+1} \\
+ (\sigma - \varsigma) (1 - \omega) \left[ \phi^f_n \epsilon^*_{n} \tilde{n}^*_t + \phi^f_{n-1} \epsilon^*_{n-1} \tilde{n}^*_{t-1} + \frac{\epsilon^f_{n-1}^*}{\epsilon^f_{n}^*} \epsilon^*_{n} \beta E_t \tilde{n}^*_{t+1} \right],
\]

15
\[ + (\sigma - \varsigma) \omega \left[ \phi' \epsilon_n^f \tilde{n}_t + \epsilon_{n-1}^f \tilde{n}_{t-1} + \frac{\epsilon_{n-1}^f}{\epsilon_{n-1}^{f*}} \epsilon_{n}^{f*} \beta \mathbb{E}_t \tilde{n}_{t+1} \right], \quad (29) \]

where

\[ \phi \equiv \sigma + \phi - \frac{\epsilon_n^f}{\epsilon_n} - (\sigma - \varsigma) (1 - \omega) + \beta \left( \frac{\epsilon_{n-1}^f}{\epsilon_n} \right)^2 \left( \sigma + \frac{\phi - \epsilon_{n-1}^f}{\epsilon_{n-1}^f} - (\sigma - \varsigma) (1 - \omega) \right), \]

\[ \phi_{-1} \equiv \sigma - \frac{\epsilon_{n-1}^f}{\epsilon_{n-1}^{f*}} - (\sigma - \varsigma) (1 - \omega), \]

with analogous expressions for \( \phi^* \) and \( \phi_{-1}^* \) (in the proof for brevity), and \( \phi' \equiv 1 + \frac{\epsilon_{n-1}^f}{\epsilon_{n-1}^{f*}} \).

It is more notationally parsimonious to focus on the relationship between marginal cost and employment rather than marginal cost and output, though using

\[ \tilde{x}_t = \epsilon_n^f \tilde{n}_t + \epsilon_{n-1}^f \tilde{n}_{t-1} \quad (30) \]

and the analog in Foreign it is straightforward to go between the two.

To gain intuition behind Lemma 2, note that as hiring costs disappear \((k, k^* \rightarrow 0)\), \( \epsilon_n^f \rightarrow 1, \epsilon_{n-1}^f \rightarrow 0, \epsilon_n^{f*} \rightarrow 0 \), and analogously in Foreign, so that (28) and (30) simplify to

\[ \tilde{\mu}_t = (\sigma + \phi - (\sigma - \varsigma) (1 - \omega)) \tilde{x}_t + (\sigma - \varsigma) (1 - \omega) \tilde{x}^*_t, \]

and analogously for Foreign, as in the frictionless benchmark of Clarida et al. (2002) (who further assume \( \varsigma = 1 \)). When \( \sigma = \varsigma \), marginal cost only depends on output at home, and in particular is rising in it due to the income effect from higher consumption (controlled by \( \sigma \)) and the higher disutility from labor supply (controlled by \( \phi \)). When \( \sigma \neq \varsigma \), it also depends on output abroad: an increase in foreign output raises marginal cost through an income effect induced by international risk-sharing (controlled by \( \phi \)), but reduces marginal cost by causing an appreciation in the domestic terms of trade (controlled by \( \varsigma \)). An increase in domestic output has the opposite effects. These international considerations are stronger the larger are imports as a share of total expenditure \((1 - \omega)\).

Comparing this limiting case to (28), we see that labor market frictions have two consequences. First, they affect the sensitivity of marginal cost to contemporaneous output fluctuations at home \((\phi)\) and abroad \((\phi')\). For \( \phi \), labor market frictions affect the returns to scale in production \((\epsilon_n^f \text{ and } \epsilon_n^{f*})\), governing how many workers are needed to produce the incremental unit, and affect the elasticity of marginal production with respect to employment \((\epsilon_n^f \text{ and } \epsilon_n^{f*})\), governing how the productivity of the last worker falls with higher
employment. For both $\phi$ and $\phi'$, labor market frictions also mean that contemporaneous employment and thus output affects future recruiting costs (the terms multiplying $\beta$), which affects current match surplus and thus wages at $t$. Second and relatedly, labor market frictions imply that marginal cost at $t$ depends on domestic and foreign employment at $t-1$, domestic and foreign employment at $t+1$, and domestic marginal cost at $t+1$. Lagged employment affects the measure of unemployed workers and thus the recruiting component of marginal cost in each country at $t$, while future employment and marginal cost affects future recruiting costs, current match surplus, and thus current wages in each country at $t$.

The final implementability constraint facing the Ramsey policymaker concerns the constraint imposed by the fixed exchange rate in the monetary union. We obtain:

**Lemma 3.** Up to first order around the deterministic steady-state,

$$\hat{s}^n_t - \hat{s}^n_{t-1} = \left( \pi_{Ht} + \varsigma \left[ \epsilon^n_t \hat{n}_t + \epsilon_{n-1}^f \hat{n}_{t-1} - \epsilon^n_t \hat{n}_{t-1} - \epsilon^n_{n-1} \hat{n}_{t-2} \right] \right)$$

$$- \left( \pi_{Ft} + \varsigma \left[ \epsilon^n_{n^*} \hat{n}_t + \epsilon_{n^*}^{f^*} \hat{n}_{t-1}^* - \epsilon^n_{n^*} \hat{n}_{t-1} - \epsilon^n_{n^*} \hat{n}_{t-2} \right] \right).$$  \hspace{1cm} (31)

Consistent with the discussion in section 3.2, the combination of sticky prices and a fixed exchange rate hinders adjustment in the terms of trade. If the natural terms of trade $\hat{s}^n_t$ vary in response to macroeconomic shocks, this leads to costly inflation/deflation, employment distortions, or both. The higher is the trade elasticity ($\varsigma$), the larger the distortions in quantities (employment) relative to prices (inflation) given this constraint.

### 4.5 Social welfare

I now turn to a quadratic approximation to utilitarian social welfare. We obtain:

**Lemma 4.** Up to second order around the deterministic steady-state,

$$U_0 - U = -\frac{c^{1-\sigma}}{2} \sum_{t=0}^{\infty} \beta^t \left[ \omega \left( \frac{\epsilon}{\lambda} (\pi_{Ht})^2 + \phi \left( \epsilon^n_t \hat{n}_t \right)^2 + 2 \beta \phi_{-1} \epsilon^n_t \epsilon_{n-1}^f \hat{n}_t \hat{n}_{t+1} \right) \right. $$

$$ + (1 - \omega) \left( \frac{\epsilon}{\lambda} (\pi_{Ft})^2 + \phi^* \left( \epsilon_{n^*}^{f^*} \hat{n}_t^* \right)^2 + 2 \beta \phi^*_{-1} \epsilon_{n^*}^{f^*} \epsilon_{n^*}^{f^*} \hat{n}_t^* \hat{n}_{t+1} \right) $$

$$ + 2 \omega (1 - \omega) \left( \phi' \epsilon^n_t \epsilon_{n^*}^{f^*} \hat{n}_t \hat{n}_t^* + \phi^n_t \epsilon_{n^*}^{f^*} \hat{n}_t \hat{n}_{t-1} + \beta \epsilon_{n-1}^f \epsilon_{n^*}^{f^*} \hat{n}_t \hat{n}_{t+1} \right) \left. \right] + \text{tips} \hspace{1cm} (32)$$

where tips denotes terms independent of policy.

As is standard in the literature, the welfare loss from relative price dispersion in each country is rising in the stickiness of prices and the elasticity of substitution across varieties.
Importantly, the welfare loss from employment distortions in each country are controlled by the same terms which determine the increase in firms’ real marginal costs in response to an increase in employment, just as in the frictionless benchmark of Clarida et al. (2002). This follows from the constrained efficiency of the natural allocation in Assumption 2, which creates a tight link between the first derivatives of social welfare and the real marginal cost faced by firms with respect to employment in the competitive equilibrium.

4.6 Optimal policy in the $\beta \to 0$ limit

Given the implementability constraints and objective derived in Lemmas 1-4, I now characterize optimal policy. I begin with the $\beta \to 0$ limit, which renders the Ramsey problem a static one in period 0 admitting simple analytical results. In particular, the problem is to choose $\pi_{H0}$, $\pi_{F0}$, $\tilde{x}_0$, and $\tilde{x}_0^*$ to minimize

$$\frac{1}{2} \left[ \omega \left( \frac{\varepsilon}{\lambda} (\pi_{H0})^2 + \phi(\tilde{x}_0)^2 \right) + (1 - \omega) \left( \frac{\varepsilon}{\lambda^*} (\pi_{F0})^2 + \phi^*(\tilde{x}_0^*)^2 \right) + 2\omega(1 - \omega)(\sigma - \varsigma)\tilde{x}_0\tilde{x}_0^* \right]$$

subject to

$$\pi_{H0} = \lambda \phi \tilde{x}_0 + \lambda (\sigma - \varsigma) (1 - \omega) \tilde{x}_0^*; \quad \pi_{F0} = \lambda^* \phi^* \tilde{x}_0^* + \lambda^* (\sigma - \varsigma) \omega \tilde{x}_0,$$

$$\tilde{s}^n_0 = (\pi_{H0} + \varsigma \tilde{x}_0) - (\pi_{F0}^* + \varsigma \tilde{x}_0^*),$$

where

$$\phi = \sigma + \frac{\bar{\varphi} - \epsilon_l^n}{\epsilon_l^n} - (\sigma - \varsigma)(1 - \omega),$$

$$\phi^* = \sigma + \frac{\bar{\varphi}^* - \epsilon_l^{n*}}{\epsilon_l^{n*}} - (\sigma - \varsigma)\omega,$$

and I have used $\tilde{x}_0 = \epsilon_l^l \tilde{n}_0$ and $\tilde{x}_0^* = \epsilon_l^{n*} \tilde{n}_0^*$, since Assumption 2 implies $\tilde{n}_{-1} = \tilde{n}_0^*_{-1} = 0$.\footnote{To provide more economic intuition, I have taken limits of the planner’s objective and constraints as $\beta \to 0$ and then will solve for the optimum in period 0. It is straightforward to verify that the resulting allocation in period 0 is identical to that implied by the general Ramsey policy obtained in the next section (which in particular is optimal from a “timeless perspective”, following Woodford (2003)) as $\beta \to 0$.}

A key ingredient for the results which follow is that we can sign the effects of labor market frictions on $(\varphi - \epsilon_l^n)/\epsilon_l^n$ and the analog in Foreign, key components of the sensitivity

\footnote{It is again straightforward to establish that as hiring costs disappear $(k, k^* \to 0)$, (32) converges to the quadratic approximation to social welfare in Clarida et al. (2002) (who further assume $\varsigma = 1$). As in that paper, we note that distortions in the terms of trade from their efficient level do not enter separately in (32) because this has already been captured in deviations in employment from their efficient level.}
of marginal cost to output distortions as well as the welfare cost of output fluctuations in each country. Characterizing the elasticities in closed form,

\[
\frac{\varphi - \epsilon_n^f}{\epsilon_n^f} = \varphi + \frac{\frac{\delta}{\eta} \frac{1}{\frac{1}{\eta} - \frac{1}{\eta} \frac{1}{\frac{1}{\eta} - \frac{1}{\eta} \frac{1}{\frac{1}{\eta} - \delta} \frac{1}{\frac{1}{\eta} - \delta} (\theta)}}}{1 - \frac{1}{\eta} \frac{1}{\frac{1}{\eta} - \delta} \frac{1}{\frac{1}{\eta} - \delta}}.
\]

Recalling the earlier definition of sclerotic versus fluid labor markets, we can prove:

**Proposition 5.** Suppose \( \beta \to 0 \). Around the symmetric benchmark and at least for small \( \{k, k^*\} \) and unemployment rates \( \{1 - n, 1 - n^*\} \), the more sclerotic is Home’s labor market, the higher is \( (\varphi - \epsilon_n^f) / \epsilon_n^f \).

Intuitively, the more sclerotic is Home’s labor market, the lower are the returns to scale in production (lower \( \epsilon_n^f \)), and the more it is that an increase in production reduces the marginal productivity of labor (higher \( \epsilon_n^f \)). Both effects follow from the fact that an adjustment in employment leads to greater deadweight costs in a more sclerotic labor market, raising \( \phi \).

This result has implications not only for the optimal second-best policy, but also the natural terms of trade, building on Proposition 4:

**Proposition 6.** Suppose \( \beta \to 0 \) and consider the same positive productivity shock at Home and Foreign \( (\hat{a}_0 = \hat{a}_0^* > 0) \). Around the symmetric benchmark and at least for small \( \{k, k^*\} \) and unemployment rates \( \{1 - n, 1 - n^*\} \), if Home’s labor market is more sclerotic than Foreign’s then Home’s natural terms of trade appreciate if \( \sigma < 1 \), are unchanged if \( \sigma = 1 \), and depreciate if \( \sigma > 1 \).

Intuitively, assume an identical, positive productivity shock in each country. If \( \sigma > 1 \), employment in each country will fall. Cross-country heterogeneity in labor market frictions then interacts with the same shock to imply changes in the natural terms of trade. Consistent with Proposition 5, if Home is more sclerotic than Foreign, the same decrease in employment at Home reduces marginal cost by more than in Foreign. Hence, the relative price of Home-produced goods falls relative to Foreign-produced goods in the natural allocation. Consistent with Proposition 4, this is true irrespective of \( \varsigma \); the trade elasticity governs the magnitude of the change in the natural terms of trade, but not its sign.

Conditional on the natural terms of trade, we can then solve the Ramsey problem to characterize the second-best policy. A useful summary of the optimal policy is that it targets a zero weighted average of inflation rates across the union in period 0

\[
\xi \pi_{H0} + (1 - \xi) \pi_{F0} = 0.
\]
When countries are symmetric and the degree of price rigidity is also the same across countries, the weight on Home is particularly intuitive:

**Proposition 7.** Suppose $\beta \to 0$, countries are symmetric, and $\iota = \iota^*$. Then the optimal policy features $\xi = \omega$, the expenditure share on Home-produced goods.

Hence, in this case, the optimal policy is characterized by a consumer-price inflation targeting rule. This rule is of particular interest because it is the one suggested by the ECB’s mandate to maintain price stability in terms of the union-wide harmonized index of consumer prices (HICP). This generalizes a point made by Benigno (2004) to the presence of labor market frictions in each country.

Now allowing for differences in labor market fluidity, we obtain the paper’s second main result on relative accommodation in a monetary union:

**Proposition 8.** Suppose $\beta \to 0$ and $\iota = \iota^*$. Around the symmetric benchmark and at least for small $\{k, k^*\}$ and unemployment rates $\{1 - \eta, 1 - \eta^*\}$, the optimal weight on Home $\xi$ rises relative to $\omega$ as its labor market grows more sclerotic, unless $\varsigma$ is sufficiently below 1.

Intuitively, Proposition 5 demonstrated that the more sclerotic is Home’s labor market, the higher is $(\varphi - \epsilon f_n)/\epsilon f_n$. This raises $\phi$, governing both the welfare cost of output fluctuations and the slope of the Phillips curve at Home. Hence, the optimal policy should put more weight on minimizing producer-price inflation distortions at Home. At the Cole-Obstfeld parameterization, it is straightforward to in fact prove this result even for differences in labor markets far away from the symmetric benchmark.

Only if the trade elasticity is sufficiently above 1 (and thus $\varsigma$ is sufficiently below 1) will this result be reversed. Intuitively, at lower $\varsigma$, it becomes optimal to target larger inflation/deflation across the union to more closely replicate the adjustment in the terms of trade in the natural allocation, thus avoiding larger output distortions. Since a more sclerotic labor market generates greater inflationary/deflationary pressure from a given distortion in output, for sufficiently small $\varsigma$ the optimal policy in fact puts less weight on stabilizing producer-price inflation in that country so as to ease adjustment in the union overall. While this is true in terms of inflation, however, in terms of output the optimal policy still puts more weight on stabilizing output in the more sclerotic labor market.\(^{16}\)

### 4.7 Optimal policy in the general case

I now consider optimal policy in the more general case with positive $\beta$.

\(^{16}\)That is, the optimal policy can equivalently be summarized as setting a weighted average of output distortions to zero. Regardless of preference parameters, we can prove that around the symmetric benchmark the optimal weight on Home rises relative to $\omega$ as its labor market grows more sclerotic.
The first-order conditions of Ramsey problem are provided in appendix C.\footnote{Because of our maintained Assumptions 2 and 3, the resulting Ramsey policy is also optimal from a “timeless perspective” as defined by Woodford (2003).} It is straightforward to use these conditions to prove the analog of Proposition 7 in this setting:

**Proposition 9.** When countries are symmetric and \( \iota = \iota^* \), an inflation targeting rule

\[
\xi \pi_{Ht} + (1 - \xi)\pi_{Ft} = 0
\]  

(33)

with \( \xi = \omega \) implements the Ramsey optimal policy.

With heterogeneity across countries, an inflation targeting rule no longer implements the Ramsey optimal policy. While the dynamic interactions in price-setting and the labor market complicate a closed form characterization of the optimal policy, the first-order conditions constitute a system of linear stochastic difference equations easily characterized numerically using standard methods. As I demonstrate here using parameters in a range consistent with the literature and available evidence for the Eurozone, the insights from the prior section on relative accommodation of the more sclerotic union member remain robust.

I evaluate how the optimal policy varies as Home’s labor market grows more sclerotic. I calibrate the present framework to specific countries in the next section and discuss the relevant evidence for the Eurozone there. Based on that evidence, I first define the symmetric benchmark to be one with \( \delta = \delta^* = 0.030 \) and hiring costs \( k = k^* = 0.12 \), where the former means a quarterly separation probability of 3% and the latter implies a quarterly job-finding rate of 27% given the other parameters described below, at the more fluid end of countries in the Eurozone. I then vary \( \delta \) as low as 0.012 and \( k \) as high as 0.45, implying in the former case a quarterly separation probability of 1.2% and in the latter case a quarterly job-finding rate of 21%, consistent with the more sclerotic end of countries in the union.

For all other parameters, I maintain fixed, symmetric parameters to focus on heterogeneity in labor market fluidity alone. I set \( \gamma = 0.5 \) so that countries are the same size and \( \beta = 0.99 \) to match a 4% annual steady-state real interest rate. I set \( \sigma = 1 \) to be consistent with balanced growth and \( \varsigma^{-1} = 1.5 \) as in Backus, Kehoe, and Kydland (1994) and used extensively in the literature. I set \( \epsilon = 6 \) to imply a 20% steady-state mark-up, and \( \iota = \iota^* = 0.25 \) to match an average price duration of 4 quarters, consistent with evidence for the Eurozone from Alvarez, Dhyne, Hoeberichts, Kwapi, Bihan, Lunnenmann, Martins, Sabbatini, Stahl, Vermeulen, and Vilmunen (2006). I set \( \varphi = \varphi^* = 1 \) to deliver a unitary Frisch elasticity of labor supply. I set \( a = a^* = 1 \) and \( \bar{m} = \bar{m}^* = 0.5 \), free normalizations. I set \( \eta = \eta^* = 0.7 \) consistent with Petrongolo and Pissarides (2001), and I set \( \chi = \chi^* = 1.14 \) so that, together with \( k = k^* = 0.12 \), in the symmetric benchmark hiring costs per hire are 20%
of the quarterly wage. Finally, I assume independent AR(1) processes for log productivity in each country with persistence 0.9 and standard deviation of shocks 0.01.

The impulse responses to a depreciation in Home’s natural terms of trade in Figure 1 illustrate how the optimal policy features relative accommodation of the more sclerotic union member. Absent the price or exchange rate flexibility to costlessly replicate this movement in the natural terms of trade, Home (Foreign) experiences deflation (inflation) and inefficiently low (high) production for the first several quarters under the optimal policy. With symmetric labor markets these movements are mirror images across the union. But if hiring costs are higher or separation probabilities are lower at Home, the optimal policy features smaller deflation and smaller output gaps at Home, while it features larger inflation and output gaps in Foreign. My analytical results when \( \beta \to 0 \) trace these to greater welfare losses from and sensitivity of firms’ real marginal costs to output fluctuations in the more sclerotic market. Figure 2 demonstrates that these comparative statics of \( \phi \) are indeed borne out here.\(^{18}\)

These results are made especially sharp in the class of inflation targeting rules (33) in which the optimal weight on Home rises as its labor market grows more sclerotic. Even though such a rule cannot implement the optimal policy with heterogeneity across countries, it remains convenient to study because within this class we can summarize the optimal accommodation of Home with the choice of \( \xi \). Figure 3 demonstrates that as Home hiring costs rise or separation probabilities fall, the optimal rule puts greater weight on minimizing distortions at Home, especially pronounced in the first case.\(^{19,20}\) Figure 4 demonstrates that impulse responses under the optimal rule are quite similar to the optimal policy in Figure 1.

We can further compare policies in this class to the optimal policy by computing the lifetime change in consumption \( \psi \) which would render social welfare the same:

\[
\psi := \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \gamma u(c_t, n_t) + (1 - \gamma)u^*(c^*_t, n^*_t) \right] = \\
\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \gamma u(c_t, n_t) + (1 - \gamma)u^*(c^*_t, n^*_t) \right] \bigg|_{\text{optimal policy}}.
\]

\(^{18}\)The resulting effects on welfare and the Phillips Curve are not as extreme as these comparative statics of \( \phi \) suggest because \( \phi_{-1} \) is negative and becomes more negative as Home’s labor market grows more sclerotic. Provided employment is persistent, this mitigates the welfare losses and effects on real marginal cost from employment fluctuations. Appendix D depicts these comparative statics for \( \phi_{-1} \) as well as \( \{\phi^*, \phi^*_{-1}, \omega\} \).

\(^{19}\)As made clear in appendix D, the variation in \( \omega \) with parameters is tiny. The comparative statics of the optimal \( \xi \) thus reflect variation over and above any changes in \( \omega \), consistent with Proposition 8.

\(^{20}\)In appendix D I consider the sensitivity of this result to alternative values of \( \sigma \) and \( \varsigma \). I find that the result is robust to \( \sigma = 3 \), consistent with some of the lowest estimates of the intertemporal elasticity of substitution in the literature, as well as \( \varsigma = (4)^{-1} \), consistent with estimates of the trade elasticity in the international trade literature. However, I also find that the result can be reversed for even lower values of \( \varsigma \), such as \( \varsigma = (10)^{-1} \), consistent with Proposition 8.
Figure 1: response to depreciation in Home’s natural terms of trade (optimal policy)

Note: depreciation in Home’s natural terms of trade induced by 1% positive innovation to $a_t$ at $t = 0$. 
Figure 2: $\phi$ as Home’s labor market becomes more sclerotic

Note: $\phi$ controls the welfare costs from and sensitivity of real marginal costs to contemporaneous employment fluctuations. Its formula is given in Lemma 2. A more sclerotic labor market is one with higher $k$ or lower $\delta$, thus moving from the ends to the center of the figure.

With log preferences ($\sigma = 1$) and using the notation for social welfare in (25), it follows that

$$\log \psi = (1 - \beta) (U_0|_{\text{optimal policy}} - U_0),$$

$$= (1 - \beta) [(U_0|_{\text{optimal policy}} - U) - (U_0 - U)],$$

where we can approximate the terms in brackets up to second order using (32) provided that, as was assumed for the optimal policy in Assumption 3, the alternative policy is consistent with a zero-inflation, constrained efficient steady-state.

Using this approach, I find that an inflation targeting rule which recognizes union-wide differences in labor markets can achieve meaningful welfare gains relative to one that does not. Figure 5 displays the welfare losses from fluctuations under the optimal inflation targeting rule and the HICP-targeting rule with weight $\xi = \omega$ (which, as noted earlier, is indeed optimal absent any differences in labor markets across the union). In both cases, we use $\psi$ as defined in (34) to summarize the welfare loss versus the optimal policy. We see that as Home’s labor market gets more sclerotic, the welfare cost of suboptimal policies worsens. But simply optimizing the inflation targeting rule to account for differences in labor market frictions across the union eliminates more than half of these losses.

4.8 Robustness to real wage rigidity

The second-best problem facing policymakers in the present setting is purely due to the interaction between nominal rigidity and a fixed exchange rate, in keeping with the literature
Figure 3: optimal $\xi$ as Home’s labor market becomes more sclerotic

Note: the optimal $\xi$ is computed by minimizing the average welfare loss across many histories of shocks. Shaded markers depict the symmetric benchmark when $k = k^* = 0.12$ and $\delta = \delta^* = 0.030$. A more sclerotic labor market is one with higher $k$ or lower $\delta$, thus moving from the ends to the center of the figure.

on Optimal Currency Areas. In appendix E, I extend the model to feature real wage rigidity alongside these frictions, building on a large literature arguing that this friction generates empirically realistic unemployment fluctuations (e.g., Shimer (2005) and Hall (2005)).

In particular, letting $w_t \equiv \frac{W_t}{P_{Ht}}$ and $w^*_t \equiv \frac{W^*_t}{P_{Ft}}$ denote the product wages, I assume that

$$w_t = (1 - \alpha)w + \alpha \frac{W_{t}^{nb}}{P_{Ht}},$$

$$w^*_t = (1 - \alpha^*)w^* + \alpha^* \frac{W_{t}^{nb}}{P_{Ft}},$$

where $W_{t}^{nb}$ and $W_{t}^{nb*}$ are the wages which would be obtained under Nash bargaining and $w$ and $w^*$ are the product wages in the deterministic steady-state. This is consistent with the partially smoothed wage studied in Hall (2005) and Abbritti and Mueller (2013), and is identical (up to first order) to the specification in Blanchard and Gali (2010).

As I demonstrate in appendix E, the relative accommodation of the more sclerotic labor market remains robust to this setting. The natural allocation is no longer constrained efficient because real wages remain rigid with flexible prices. The optimal policy may no longer closely approximated by an inflation targeting rule, as the deviations in real wages from their efficient level in both countries may call for inflation/deflation across the union. But, with the same degree of real wage rigidity in each country, it remains the case that a more sclerotic labor market induces greater inflationary pressure and leads to greater welfare losses from output distortions, motivating relative accommodation of this member under the optimal policy.
Figure 4: response to depreciation in Home’s natural terms of trade (optimal $\xi$)

Note: depreciation in Home’s natural terms of trade induced by 1% positive innovation to $a_t$ at $t = 0$. 
Figure 5: welfare losses as Home’s labor market becomes more sclerotic

Note: average welfare loss from fluctuations under each policy is computed across many histories of shocks, and then compared to that under the optimal policy by computing ψ as in (34). Shaded markers depict the symmetric benchmark when $k = k^* = 0.12$ and $δ = δ^* = 0.030$. A more sclerotic labor market is one with higher $k$ or lower $δ$, thus moving from the ends to the center of the figure.

4.9 Relation to Benigno (2004)

The relative accommodation of the more sclerotic labor market under the optimal policy is especially complementary with the findings of Benigno (2004). Benigno (2004) focuses on cross-country heterogeneity in the degree of nominal rigidity ($\{1 - \iota, 1 - \iota^*\}$ in my notation), whereas I have focused on heterogeneity in the fluidity of labor markets. As is evident in the linear-quadratic approximation, these affect welfare through distinct theoretical channels: the cost of relative price dispersion in the first case, versus the cost of output fluctuations in the second. While both imply it is optimal to accommodate the more rigid member of the union,21 these dimensions of heterogeneity have opposite effects on the slope of the Phillips curve. This means that to reach accurate conclusions for optimal policy, quantitative models of monetary unions should separately match cross-country heterogeneity in the observed duration of nominal contracts and the observed fluidity of labor markets, rather than cross-country variation in the observed relationship between inflation and output overall. This informs my quantitative application to the Eurozone, to which I now turn.

21Indeed, the insights of Benigno (2004) hold within my model despite the presence of rich and diverse labor markets across countries. For instance, holding fixed Foreign flexibility ($\iota^*$), as Home prices approach perfect flexibility ($\iota \to 1$), optimal monetary policy fully seeks to stabilize inflation in Foreign ($\xi \to 0$).
Table 1: inflow and outflow rates from unemployment in the largest 10 Eurozone economies

<table>
<thead>
<tr>
<th>Country (code)</th>
<th>Unemployment rate</th>
<th>Mo. outflow rate</th>
<th>Mo. inflow rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Austria (AT)</td>
<td>4.8%</td>
<td>11.5%</td>
<td>0.6%</td>
</tr>
<tr>
<td>Belgium (BE)</td>
<td>7.7%</td>
<td>5.9%</td>
<td>0.5%</td>
</tr>
<tr>
<td>Finland (FI)</td>
<td>8.4%</td>
<td>13.6%</td>
<td>1.3%</td>
</tr>
<tr>
<td>France (FR)</td>
<td>8.8%</td>
<td>5.0%</td>
<td>0.5%</td>
</tr>
<tr>
<td>Germany (DE)</td>
<td>7.1%</td>
<td>8.8%</td>
<td>0.6%</td>
</tr>
<tr>
<td>Greece (GR)</td>
<td>15.6%</td>
<td>4.5%</td>
<td>0.8%</td>
</tr>
<tr>
<td>Italy (IT)</td>
<td>9.5%</td>
<td>6.6%</td>
<td>0.7%</td>
</tr>
<tr>
<td>Netherlands (NL)</td>
<td>5.0%</td>
<td>6.4%</td>
<td>0.4%</td>
</tr>
<tr>
<td>Portugal (PT)</td>
<td>9.0%</td>
<td>5.9%</td>
<td>0.6%</td>
</tr>
<tr>
<td>Spain (ES)</td>
<td>16.0%</td>
<td>9.8%</td>
<td>1.7%</td>
</tr>
</tbody>
</table>

Notes: monthly inflow and outflow rates are estimated over 1999-2019 using the methodology of Elsby et al. (2013), with further details provided in appendix G. Data for Greece only begins in 2000 when data becomes available from the OECD.

5 Calibration to the Eurozone

I now evaluate the quantitative importance of the insights in this paper for the Eurozone given the diversity in labor markets across members.

5.1 Labor market flows in the Eurozone

I first estimate labor market flows across Eurozone countries, key targets as I calibrate my model to the Eurozone in the next section. Shimer (2012) proposed a method to estimate the flow hazard rates at which a worker becomes unemployed (the inflow rate) or exits unemployment (the outflow rate) using U.S. data observed at discrete, monthly intervals. Elsby et al. (2013) extended it to produce comparable estimates across countries for use with OECD data through 2009. I employ the method of Elsby et al. (2013) to characterize inflow and outflow rates for Eurozone economies over 1999-2018 after the introduction of the single currency. Appendix G provides details on the methodology, data, and estimates.\(^{22}\)

Table 1 summarizes the average outflow and inflow rates over the sample period for the

\(^{22}\)In keeping with this empirical literature, I use the terms “outflow rate” and “inflow rate” because the estimates do not distinguish between flows across \{unemployment, employment\} and \{unemployment, non-participation\}. However, when I map these rates to my model (in which there is no equilibrium non-participation), the outflow rate will correspond to the model’s job-finding probability and the inflow rate will correspond to the model’s separation probability.
Figure 6: flow rates and employment protection in the Eurozone

Note: employment protection index is index for protection of permanent workers against individual and collective dismissal from the OECD, where 0 is the lowest and 6 the highest. The index is as of 2013 in each country (the latest available as of the time of writing).

10 largest Eurozone members by nominal GDP, along with the average unemployment rate over the same period. As is evident, there is considerable heterogeneity in the fluidity in labor markets across the union. Monthly outflow rates from unemployment range between 4.5-13.6%. Monthly inflow rates to unemployment range between 0.4-1.7%.

In Figure 6, I compare these rates to an index of employment protection constructed by the OECD for the protection of permanent workers against individual and collective dismissal. This aggregates information on the notification required of employers, severance requirements, and other costs and procedures regarding dismissal. As is evident, countries with smaller flow rates are ones with stricter employment protection.\textsuperscript{23} For parsimony I have abstracted from endogenous separations, firing costs, and other such features in the model. Nonetheless, the relationship between these flow rates and employment protection suggests that the differences in labor market fluidity captured by the present framework reflect these underlying differences in institutions.

5.2 Parameterization and model validation

I now evaluate the model’s implications for optimal policy in a calibration to the Eurozone. In keeping with model’s two-country structure, I calibrate the model to Germany and France.

\textsuperscript{23}Campolmi and Faia (2011) report a positive correlation between employment protection and inflation volatility across Eurozone members. Given my theoretical results that a more sclerotic labor market will feature a steeper Phillips curve, this is consistent with my findings here.
### Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Moment</th>
<th>Target</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Heterogeneity in labor markets</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.015</td>
<td>qtr. inflow prob., FR</td>
<td>1.5%</td>
<td>Table 1</td>
</tr>
<tr>
<td>$k$</td>
<td>0.56</td>
<td>qtr. outflow prob., FR</td>
<td>13.9%</td>
<td>Table 1</td>
</tr>
<tr>
<td>$\chi$</td>
<td>1.12</td>
<td>opp cost employment FR / DE</td>
<td>97%</td>
<td>repl. rates in Table 6</td>
</tr>
<tr>
<td>$\delta^*$</td>
<td>0.018</td>
<td>qtr. inflow prob., DE</td>
<td>1.8%</td>
<td>Table 1</td>
</tr>
<tr>
<td>$k^*$</td>
<td>0.16</td>
<td>recruiting costs / qtr. wage, DE</td>
<td>23%</td>
<td>MP (2016)</td>
</tr>
<tr>
<td>$\chi^*$</td>
<td>1.10</td>
<td>qtr. outflow prob., DE</td>
<td>23.2%</td>
<td>Table 1</td>
</tr>
<tr>
<td><strong>Other features of economy</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.43</td>
<td>nominal GDP FR/(FR+DE)</td>
<td>43%</td>
<td>OECD, 1999-2018</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.93</td>
<td>persistence log labor prod., FR</td>
<td>0.93</td>
<td>OECD, 1999-2018</td>
</tr>
<tr>
<td>$\rho^*$</td>
<td>0.87</td>
<td>persistence log labor prod., DE</td>
<td>0.87</td>
<td>OECD, 1999-2018</td>
</tr>
<tr>
<td>$\zeta_H$</td>
<td>0.0036</td>
<td>std. dev. log labor prod., FR</td>
<td>0.0036</td>
<td>OECD, 1999-2018</td>
</tr>
<tr>
<td>$\zeta_F$</td>
<td>0.0082</td>
<td>std. dev. log labor prod., DE</td>
<td>0.0082</td>
<td>OECD, 1999-2018</td>
</tr>
<tr>
<td>$\rho_{HF}$</td>
<td>0.61</td>
<td>corr log labor prod. FR, DE</td>
<td>0.61</td>
<td>OECD, 1999-2018</td>
</tr>
<tr>
<td>$\alpha = \alpha^*$</td>
<td>0.27</td>
<td>std. dev. product wages / unemp.</td>
<td>1.05</td>
<td>OECD, 1999-2018</td>
</tr>
</tbody>
</table>

Table 2: calibrated parameters for France (Home) and Germany (Foreign)

Note: all model-implied moments exactly hit the targets, so I do not separately report these. See text for values of other externally set parameters.

Table 1 indicates that the French labor market is more sclerotic than the German one: the inflow rate to unemployment is slightly lower and the outflow rate from unemployment is especially lower. To better capture the relative volatilities of employment and inflation in the data, I calibrate the version of the model allowing for (symmetric) real wage rigidity described in appendix E. I model France as Home and Germany as Foreign, and again define one period to be a quarter.

I first set parameters externally. I set $\beta = 0.99$, $\sigma = 1$, $\varsigma^{-1} = 1.5$, $\epsilon = 6$, $\iota = \iota^* = 0.25$, $\varphi = \varphi^* = 1$, $\eta = \eta^* = 0.7$, $\bar{m} = \bar{m}^* = 0.5$, and $\bar{a} = \bar{a}^* = 1$ as in the numerical exploration of the optimal policy in section 4.7. I refer the reader to the references discussed therein.

I calibrate the remaining parameters to match the heterogeneity in French and German labor markets as well as other features of these economies. The parameters and the targeted moment most closely associated with each parameter are summarized in Table 2.

I first discuss the parameters matching the heterogeneity in labor markets. I directly set

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24In appendix G, I provide an alternative calibration to Germany and Italy, noting that Italy is the third largest member of the union.
δ = 0.015 and δ∗ = 0.018 to match the quarterly separation probabilities implied by the monthly inflow rates to unemployment for France and Germany in Table 1.25 I jointly set k∗ and χ∗ to target recruiting costs which are 23% of the quarterly wage in Germany, following the estimates of Muehlemann and Pfeifer (2016), and a quarterly job-finding probability of 23.2% as implied by the monthly outflow rate from unemployment for Germany in Table 1 (which implies that I match the German unemployment rate having already matched the inflow rate, up to the error induced by time aggregation).26 I set χ such that the opportunity cost of employment in France is 97% of that in Germany, consistent with the fact that the net replacement rate during unemployment has been 97% of that in Germany over this period.27,28 I then set k to target a quarterly job-finding probability of 13.9% as implied by the monthly outflow rate from unemployment for France reported in Table 1 (which again implies that I match the unemployment rate). I note that in this calibration strategy, the direct evidence on recruiting costs in Germany provides a concrete target for the level of k∗, but the heterogeneity in outflow rates from unemployment disciplines the difference between k and k∗. The heterogeneity is what matters in the results which follow, and consistent with Figure 6, I interpret it as capturing differences in labor market institutions such as employment protection, not simply the technological costs of hiring workers.29

I turn now to the remaining parameters which match other features of the French and German economies. I use γ to target ω = 0.43, the average ratio of France’s nominal GDP relative to the sum of France and Germany over 1999-2018. I set the persistence, volatility, and correlation of productivity shocks to be consistent with the corresponding moments estimated on labor productivity for France and Germany over the 1999-2018 period.30 Finally, given these driving forces, I calibrate the degree of real wage rigidity across the union α = α∗ = 0.27 to match the weighted average of log product wage volatilities over 1999-2018.

25Given a monthly inflow hazard rate δmo, I assume the corresponding quarterly separation probability is δ = 1 − exp(−3δmo). I similarly relate monthly outflow hazard rates to the quarterly job-finding probability.

26In steady-state, n = (1 − δ)n + p(θ)(1 − (1 − δ)n) implies that n = \frac{p(θ)}{δ + p(θ)(1 − δ)}.

27In appendix G, I report these net replacement rates averaged across family situations for the ten largest members of the Eurozone. While I do not explicitly model unemployment insurance (UI), this calibration strategy accounts for the effects of cross-country differences in UI on workers’ opportunity cost of employment. 28By opportunity cost of employment here I refer to \chi γ−cσ−c¯Hnϕ at Home and χ∗(1 − γ)−cσ−c¯Fnϕ∗ at Foreign, where I have expressed both in terms of units of the domestically produced good. Recalling that in steady-state workers’ marginal product is one in each country (a = a∗ = 1), I thus calibrate χ so that χγ−cσ−c¯Hnϕ = 0.97 × χ∗(1 − γ)−cσ−c¯Fnϕ∗. I provide results isolating the effects of heterogeneity in \{χ, χ∗\} on the optimal monetary policy in appendix F.

29While the overall employment protection index reported in Figure 6 differs little between Germany and France, I note that this masks substantial variation within its subcomponents and in other indices. The subcomponents reveal, for instance, that France allows workers 5 years to make a claim of unfair dismissal, while Germany allows workers only 3 weeks. On regulation of temporary rather than permanent workers, the OECD ranks France the strictest of the countries in Figure 6, while Germany is the second most flexible.

30I estimate all second moments in the data after linearly detrending each series over the sample period.
Table 3: untargeted second moments

<table>
<thead>
<tr>
<th></th>
<th>$\hat{x}_t$</th>
<th>$\hat{x}_t^*$</th>
<th>$\hat{n}_t$</th>
<th>$\hat{n}_t^*$</th>
<th>$\hat{w}_t$</th>
<th>$\hat{w}_t^*$</th>
<th>$\pi_{Ht}$</th>
<th>$\pi_{Ft}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SD data</td>
<td>1.57%</td>
<td>1.75%</td>
<td>0.89%</td>
<td>1.23%</td>
<td>0.66%</td>
<td>1.50%</td>
<td>0.21%</td>
<td>0.34%</td>
</tr>
<tr>
<td>SD model</td>
<td>1.52%</td>
<td>2.55%</td>
<td>0.63%</td>
<td>1.07%</td>
<td>0.79%</td>
<td>1.03%</td>
<td>0.26%</td>
<td>0.19%</td>
</tr>
</tbody>
</table>

Notes: empirical second moments estimated on Q1/99-Q4/18 OECD data, where $\hat{x}_t$ and $\hat{x}_t^*$ are linearly detrended log GDP per capita, $\hat{n}_t$ and $\hat{n}_t^*$ are one minus linearly detrended harmonised unemployment rate, $\hat{w}_t$ and $\hat{w}_t^*$ are linearly detrended log labor compensation per employed person less log GDP deflator, and $\pi_{Ht}$ and $\pi_{Ft}$ are linearly detrended difference in log GDP deflator. Corresponding model moments are estimated after simulating many histories of shocks and assuming an HICP-targeting rule.

relative to the weighted average of unemployment volatilities over the same period (with weights $\omega = 0.43$ on France and $1 - \omega = 0.57$ on Germany). I generate the same moment in simulated data in the model assuming a HICP-targeting policy rule with $\xi = \omega = 0.43$.

Table 3 validates the quantitative fit of the model using a HICP-targeting rule versus untargeted second moments over the 1999-2018 period in the data. Second moments in output, employment, and product wages are quite consistent with the data, even though the only driving forces in the model are labor productivity in each country. In particular, the simulated output volatility in France is very close to that in the data, simulated employment volatilities in both countries are only slightly below those in the data, and the model replicates the lower volatility of product wages in France versus Germany. In terms of nominal prices, while the order of magnitude in producer-price inflation volatility is consistent with the data, the HICP-rule counterfactually implies that it is more volatile in France versus Germany, whereas the opposite is true in the data. This is perhaps not so surprising, as the actual monetary policy in the Eurozone likely differs from the strict HICP-targeting rule simulated here (that too only simulated for Germany and France).

5.3 Implications for optimal policy

I now assess the implications of the differences in labor market fluidity for optimal policy.

Relative to the HICP-targeting rule, the optimal inflation targeting rule features 11pp more weight on France. As summarized in the first two rows of Table 4, while the HICP-

---

31I note that direct evidence is consistent with my calibrated degree of real wage rigidity and assumption that real wage rigidity is the same in both countries: when I project the quarterly log difference in the product wage on the log difference in labor productivity, I obtain an estimated slope of 0.28 (standard error 0.08) for France and 0.27 (standard error 0.06) for Germany. These are remarkably consistent with the calibrated $\alpha = \alpha^* = 0.27$ and are statistically indistinguishable from one another.

32Absent real wage rigidity, the simulated volatility of unemployment in France would be 0.07% and in Germany would be 0.14%, both substantially lower than in the data. This underscores the importance of real wage rigidity in accounting for unemployment fluctuations as emphasized by the prior literature.
<table>
<thead>
<tr>
<th>Model</th>
<th>$\xi$</th>
<th>$\log \psi \times 10^{-6}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>HICP-targeting</td>
<td>0.43</td>
<td>15.8</td>
</tr>
<tr>
<td>Optimal $\xi$</td>
<td>0.54</td>
<td>7.0</td>
</tr>
<tr>
<td>HICP-targeting, $\alpha = \alpha^* = 1$</td>
<td>0.43</td>
<td>1.9</td>
</tr>
<tr>
<td>Optimal $\xi$, $\alpha = \alpha^* = 1$</td>
<td>0.48</td>
<td>0.2</td>
</tr>
<tr>
<td>Optimal $\xi$, $\alpha = \alpha^* = 1$ and $k = k^*$</td>
<td>0.43</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Table 4: HICP-targeting versus optimal inflation targeting rule

Notes: $\xi$ denotes weight on Home producer-price inflation in (33) and $\psi$ denotes the proportional change in consumption at all dates and states to render welfare the same as under the optimal policy, following (34).

targeting rule calls for $\xi = \omega = 0.43$, the optimal inflation targeting rule features $\xi = 0.54$. Consistent with the illustrative numerical experiments in Figure 5, re-weighting the inflation targeting rule in this way eliminates more half of the welfare losses from fluctuations relative to the optimal policy. The third and fourth rows of the table eliminate real wage rigidity in both France and Germany and keep all other parameters the same. It remains the case that the optimal inflation targeting rule features a higher weight on France ($\xi = 0.48$) but, evidently, real wage rigidity only amplifies this result in the baseline calibration. Continuing to assume no real wage rigidity, the fifth row of this table sets $k$ to the value of $k^*$ and leaves all other parameters unchanged. We now see that the optimal inflation targeting rule is consistent with HICP-targeting. In other words, the difference in hiring costs which rationalizes the difference in outflow rates from unemployment between these economies — and which may reflect differences in employment protection, as previously argued — quantitatively drives the departure from HICP-targeting in the optimal rule.

The model’s impulse responses shed further light on the important role of labor market frictions in determining the optimal policy. Figure 7 depicts impulse responses under the optimal inflation targeting rule to a one standard deviation negative productivity shock in France. Each panel compares the impulse responses under the benchmark labor market parameters to those in a counterfactual where $k$ is set equal to $k^*$. In the constrained efficient allocation, the negative productivity shock in France would lead to an appreciation in its terms of trade and a decrease in its real wage. A fixed exchange rate together with

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Footnotes:

33 In part this is because real wage rigidity amplifies the effects of labor market heterogeneity on the optimal policy, as characterized in appendix E. And in part it is because the volatility of shocks in Germany is calibrated to be higher than in France, so inflation targeting in the presence of real wage rigidity leads to larger welfare losses in Germany than France.

34 Now $\hat{x}$, $\hat{x}^*$, $\hat{n}$, and $\hat{n}^*$ refer to log deviations from the constrained efficient (rather than natural) allocation, since real wage rigidity in the natural allocation still renders it inefficient.
Figure 7: response to one standard deviation negative productivity shock in France (Home)
nominal rigidity renders France’s terms of trade inefficiently weak; real wage rigidity renders France’s real wage inefficiently high. On balance, the calibrated degree of real wage rigidity implies that the latter effect dominates and renders French output and employment inefficiently low, even though inflation is inefficiently positive. These patterns differ from the numerical experiments in section 4.7 in which inefficiency in the union arises solely from the fixed exchange rate and nominal rigidity. However, consistent with those earlier results, the optimal inflation targeting rule targets substantially smaller inefficiency in France than would be the case if its hiring costs and thus outflow rates from unemployment were comparable to Germany. In particular, whereas the French unemployment rate only rises by a maximum of 0.15pp in the former case, it would rise by 0.21pp in the latter.

6 Conclusion

This paper revisits the classic theory of Optimal Currency Areas in the presence of rich and diverse labor market frictions across the union. Shocks generically interact with heterogeneity in labor market frictions to induce inefficiency in a monetary union subject to the same shocks. Facing a second-best stabilization problem, the optimal monetary policy should relatively accommodate the more sclerotic labor market. A calibration matching the fluidity of labor markets underscores the quantitative importance of these insights for the Eurozone and, in particular, implies welfare gains from redefining the union’s inflation target to put greater weight on its more sclerotic members.

Future work can naturally build on the model of this paper to explore these and other features of labor markets in greater depth. Accounting for endogenous separations would permit a refined quantitative analysis of the effects of employment protection on optimal policy. And accounting for incomplete markets would permit a more complete analysis of unemployment insurance which, building on McKay and Reis (2016, 2019) and Kekre (2019), could partially relieve the constraints facing the central bank in a monetary union.

References


Proofs

Proposition 1

Proof. The constrained planner with a utilitarian objective faces

\[
\max_{c_{H_t}, c_{F_t}, c^*_{H_t}, c^*_{F_t}}, u_t \in [0, 1 - (1 - \delta)n_{t-1}], u^*_t \in [0, 1 - (1 - \delta^*)n^*_{t-1}], \theta_t, \theta^*_t, n_t, n^*_t} E_0 \sum_{t=0}^{\infty} \beta^t \left[ \gamma u(c_t, n_t) + (1 - \gamma)u^*(c^*_t, n^*_t) \right] \text{ s.t.}
\]

\[
c_t = \left[ (\gamma)^\varsigma (c_{H_t})^{1-\varsigma} + (1 - \gamma)^\varsigma (c_{F_t})^{1-\varsigma} \right]^{\frac{1}{1-\varsigma}},
\]

\[
c^*_t = \left[ (\gamma)^\varsigma (c^*_{H_t})^{1-\varsigma} + (1 - \gamma)^\varsigma (c^*_{F_t})^{1-\varsigma} \right]^{\frac{1}{1-\varsigma}},
\]

\[
n_t = (1 - \delta)n_{t-1} + p(\theta_t)u_t,
\]

\[
n^*_t = (1 - \delta^*)n^*_{t-1} + p^*(\theta^*_t)u^*_t,
\]

\[
\gamma c_{H_t} + (1 - \gamma)c^*_{H_t} = \gamma a_t[n_t - ku_t\theta_t],
\]

\[
\gamma c_{F_t} + (1 - \gamma)c^*_{F_t} = (1 - \gamma)a^*_t[n^*_t - k^*u^*_t\theta^*_t],
\]

where I already account for the fact that the planner will choose identical production levels across varieties from a particular country, and identical consumption of those varieties from residents of a particular country, given the symmetric tastes and technologies.

It is straightforward to use the first-order conditions for participation and tightness to show that

\[
u_t = 1 - (1 - \delta)n_{t-1}, \quad u^*_t = 1 - (1 - \delta^*)n^*_{t-1},
\]

since raising participation in the labor force will reduce the social cost of employing the same number of workers in each country.

It is also straightforward to use the first-order conditions for consumption to show that

\[c_{H_t} = c_{H_t}, \quad c_{F_t} = c^*_{F_t}, \quad c_t = c^*_t,
\]

given the symmetry of preferences.

Finally, we can use the first-order conditions for consumption, tightness, and employment to obtain

\[
a_t = \chi^\gamma \varsigma c_t^{\sigma - \varsigma} c_{H_t} n_t^\phi + \frac{1}{\eta} \frac{ka_t}{q(\theta_t)} \tag{38}
\]

\[
- (1 - \delta)E_t \beta \frac{c_t^{\sigma - \varsigma} c_{H_t}^\varsigma}{c_{H_t+1}^{\sigma - \varsigma} c_{H_t}^{\varsigma}} \left( 1 + \frac{1 - \eta}{\eta} (1 - p(\theta_{t+1})) \right) \frac{ka_{t+1}}{q(\theta_{t+1})}, \tag{39}
\]

\[
a^*_t = \chi^*(1 - \gamma)^{-\varsigma} c_t^{\sigma - \varsigma} c_{F_t} n_t^{\phi^*} + \frac{1}{\eta^*} \frac{k^*a^*_t}{q^*(\theta^*_t)} \tag{40}
\]
Comparing these to the conditions defining the natural allocation characterized in appendix B, it is clear that consumption and production will be efficient at all dates and states if and only if \(\tau^r = \tau^{r*} = -\frac{1}{\varepsilon}, \zeta = 1 - \eta, \text{ and } \zeta = 1 - \eta^*\).

\[ \boxed{(36)-(41), \text{ coupled with the constraints of (35), define the constrained efficient allocation.}} \]

**Propositions 2 and 3**

**Proof.** Using the equilibrium conditions defining the natural allocation in appendix B (and \(\tau^r = -\frac{1}{\varepsilon}, \text{ implied by Assumption 2}), \{c^n_{Ht}, n^n_t, \theta^n_t, c^n_{Ft}, n^n_t, \theta^n_t, s^n_t\} \text{ solve}

\[
\begin{align*}
a_t &= \chi \gamma^{-\sigma} (\gamma + (1 - \gamma)(s^n_t)^{\frac{1}{\varepsilon}} - 1) \frac{q_{\gamma}}{\gamma} (c^*_{Ht})^\sigma (n^n_t)^\phi + \frac{1}{1 - \zeta} \frac{k a_t}{q^n_t} \\
&\quad - (1 - \delta) \beta \frac{(\gamma + (1 - \gamma)(s^n_t)^{\frac{1}{\varepsilon}} - 1) \frac{q_{\gamma}}{\gamma} (c^*_{Ft})^\sigma (n^n_t)^\phi}{(\gamma + (1 - \gamma)(s^n_{t+1})^{\frac{1}{\varepsilon}} - 1) \frac{q_{\gamma}}{\gamma} (c^n_{Ht+1})^\sigma} \left(1 + \frac{\zeta^* (1 - p^*(\theta^n_{t+1}))}{1 - \zeta^* (1 - p^*(\theta^n_{t+1}))} \right) \frac{k a_{t+1}}{q^n_{t+1}},
\end{align*}
\]

\[

t^n_t = (1 - \delta) n^n_{t-1} + p(\theta^n_t)(1 - (1 - \delta)n^n_{t-1}),
\]

\[
c^n_{Ht} = \gamma a_t \left[n^n_t - k(1 - (1 - \delta)n^n_{t-1})\theta^n_t\right],
\]

\[
a^*_t = \chi^s (1 - \gamma)^{-\sigma} (\gamma (s^n_t)^{\frac{1}{\varepsilon}} - 1) \frac{q_{\gamma}^s}{\gamma} (c^n_{Ht})^\sigma (n^n_t)^\phi^* + \frac{1}{1 - \zeta^s} \frac{k a^*_t}{q^n_t} \\
&\quad - (1 - \delta^s) \beta \frac{(\gamma (s^n_t)^{\frac{1}{\varepsilon}} - 1) \frac{q_{\gamma}^s}{\gamma} (c^n_{Ft})^\sigma (n^n_t)^\phi^*}{(\gamma (s^n_{t+1})^{\frac{1}{\varepsilon}} - 1) \frac{q_{\gamma}^s}{\gamma} (c^n_{Ht+1})^\sigma} \left(1 + \frac{\zeta^* (1 - p^*(\theta^n_{t+1}))}{1 - \zeta^* (1 - p^*(\theta^n_{t+1}))} \right) \frac{k a^*_{t+1}}{q^n_{t+1}},
\]

\[
n^*_{t+1} = (1 - \delta^s) n^n_{t+1} + p^*(\theta^n_t)(1 - (1 - \delta^s)n^n_{t-1}),
\]

\[
c^n_{Ft} = (1 - \gamma) a^*_t \left[n^n_{t+1} - k^* (1 - (1 - \delta^s)n^n_{t-1})\theta^n_{t+1}\right],
\]

\[
s^n_t = \left(\frac{a^*_t \left[n^n_{t+1} - k^* (1 - (1 - \delta^s)n^n_{t-1})\theta^n_{t+1}\right]}{\gamma a_t \left[n^n_t - k(1 - (1 - \delta)n^n_{t-1})\theta^n_t\right]}\right)^\zeta.
\]

First suppose \(\sigma = 1\). If \(a^*_t/a_t\) is constant, it is clear that \(n^n_t = n, \theta^n_t = \theta, n^*_t = n^*, \theta^*_{t+1} = \theta^*\), and thus \(s^n_t = s\) are consistent with all of the equilibrium conditions. Conversely, if \(s^n_t\) is constant for arbitrary differences in labor markets across countries, reversing the argument makes clear that \(a^*_t/a_t\) must be constant (this is easier to see after taking a first-order approximation of the system around the deterministic steady-state and setting \(s^n_t = 0\) at all dates and states, which generically requires \(\hat{a}^*_t = \hat{a}_t\) at all dates and states).

Now suppose countries are symmetric, which means specifically that

\[
\{\chi a^{c-1}, \varphi, k, \delta, \bar{m}, \eta\} = \{\chi^* a^{c^* - 1}, \varphi^*, k^*, \delta^*, \bar{m}^*, \eta^*\}.
\]
If \( a^*_t / a_t \) is constant, it is straightforward to verify that \( n^n_t = n^n_{t-1} \), \( \theta^n_t = \theta^n_{t-1} \), and thus \( s^n_t = s \) are consistent with all of the equilibrium conditions. Conversely, if \( s^n_t \) is constant for arbitrary preferences, reversing the argument makes clear that \( a^*_t / a_t \) must be constant.

If \( \sigma \neq 1 \) and countries are asymmetric, even if \( a^*_t / a_t \) is constant, generically the equilibrium conditions imply \( n^n_t \neq n^n_{t-1} \) and \( \theta^n_t \neq \theta^n_{t-1} \). Since the response of employment and hiring to productivity depends on labor market frictions in each country, \( s^n_t \) will thus generically vary even though \( a^*_t / a_t \) is constant.

\[ \square \]

**Proposition 4**

*Proof.* As argued in the main text (and formalized in the linear-quadratic approximation to the Ramsey problem derived in Lemmas 1-4), the efficient allocation can be achieved if and only if there is no adjustment in the natural terms of trade. The claim thus follows immediately from Propositions 2 and 3.

\[ \square \]

**Lemma 1**

*Proof.* This proof is standard given Calvo (1983) price-setting. I refer to the equilibrium conditions with nominal rigidities in appendix B. Log-linearizing the optimal price-setting condition in (80) yields

\[
\frac{1}{1 - \beta(1 - \iota)} \hat{P}_{Ht} = \frac{\beta(1 - \iota)}{1 - \beta(1 - \iota)} \mathbb{E}_t \left( \pi_{Ht+1} + \frac{\hat{P}_{Ht+1}}{P_{Ht+1}} \right) + \hat{\mu}_t,
\]

given the definition of \( \mu_t \equiv \frac{\pi_t}{\hat{P}_{Ht}} \). Log-linearizing the evolution of \( P_{Ht} \) in (81) yields

\[
\hat{P}_{Ht} = \frac{1 - \iota}{\iota} \pi_{Ht}.
\]

Combining these and noting that \( \hat{\mu}^n_t = 0 \) in the natural allocation, we obtain the New Keynesian Phillips curve (26). With exactly analogous steps in Foreign, we obtain (27).

\[ \square \]

**Lemma 2**

*Proof.* I again refer to the equilibrium conditions in appendix B. First note that by the definition of \( f(n_t, n_{t-1}) \), we have

\[
f_n(n_t, n_{t-1}) = 1 - \frac{1}{\eta q(\theta_t)},
\]

42
\[
\begin{align*}
\mu_t f_{n}(n_t, n_{t-1}) + \\
\mathbb{E}_t \left( \left( \frac{D_{Ht+1}}{D_{Ht}} \right)^\sigma \left( \frac{a_t}{a_{t+1}} \right)^{\sigma-1} \frac{\gamma + (1 - \gamma)s_{t-1}^{\delta} \epsilon f_{n_{t}^{*}}}{\gamma + (1 - \gamma)s_{t+1}^{\delta} \epsilon f_{n_{t+1}^{*}}} \right)^{\frac{\sigma-1}{\sigma}} \frac{f(n_t, n_{t-1})}{f(n_{t+1}, n_t)} \mu_{t+1} f_{n-1}(n_{t+1}, n_t) = \\
\chi D_{Ht}^{-\sigma} a_t^{\sigma-1} \left( \gamma + (1 - \gamma)s_{t}^{\delta-1} \epsilon f_{n_{t}^{*}} \right)^{\frac{\sigma-1}{\sigma}} f(n_t, n_{t-1})^\sigma n_t^\varphi.
\end{align*}
\]

Furthermore, (71), (74), and (75) can be combined to give

\[
s_t = \left( \frac{D_{Ht}}{D_{Ft}} \right) \left( \frac{a_t^n}{a_{t+1}^n} \left[ n_{t}^{*} - k^{*}(1 - (1 - \delta)n_{t-1}^{*})\theta_{t}^{*} \right] \right)^{\varsigma}.
\]

Log-linearizing these conditions, making use of

\[
\dot{D}_{Ht} = 0
\]

around the zero inflation steady-state implied by (82), and taking differences from the same log-linearized conditions in the natural allocation (in which \( \mu_t = 1 \) per Assumption 2 and \( D_{Ht} = 1 \)), we obtain (28). With exactly analogous steps in Foreign we obtain (29), where

\[
\begin{align*}
\phi^* &\equiv \sigma + \frac{\varphi - \epsilon f_{n_{t}^{*}}}{\epsilon f_{n_{t}^{*}}} - (\sigma - \varsigma) \omega + \beta \left( \frac{\epsilon f_{n_{t}^{*}}}{\epsilon f_{n_{t+1}^{*}}} \right)^2 \left( \sigma + \frac{\varphi - \epsilon f_{n_{t+1}^{*}}}{\epsilon f_{n_{t+1}^{*}}} - (\sigma - \varsigma) \omega \right), \\
\phi_{t-1}^* &\equiv \sigma - \frac{\epsilon f_{n_{t-1}^{*}}}{\epsilon f_{n_{t-1}^{*}}} - (\sigma - \varsigma) \omega
\end{align*}
\]

were excluded from the claim for brevity.
Lemma 3

Proof. I again refer to the equilibrium conditions in appendix B. Log-linearizing (75) both in the sticky price equilibrium and in the natural allocation, in gap notation we have

$$\hat{s}_t^n = \hat{s}_t + \varsigma \left[ \bar{c}_{Ht} - \bar{c}_{Ft} \right].$$

(44)

Log-linearizing (86) yields

$$\hat{s}_t - \hat{s}_{t-1} = \pi_{Ht} - \pi_{Ft},$$

(45)

while log-linearizing (71) and (74) around the zero-inflation steady-state and using (43), the analog for Foreign, and gap notation yields

$$\bar{c}_{Ht} = \epsilon_n^f \tilde{n}_t + \epsilon_{n-1}^f \tilde{n}_{t-1}, \quad \bar{c}_{Ft} = \epsilon_n^f \tilde{n}_t + \epsilon_{n-1}^f \tilde{n}_{t-1},$$

(46)

Combining (44)-(46) yields (31), the constraint posed by membership in a monetary union.

Lemma 4

Proof. I again refer to the equilibrium conditions in appendix B. Given (71), (74), and (75),

$$s_t = \left( \frac{D_{Ht} a_t f^s(n_t^* n_{t-1}^*)}{D_{Ft} a_t f(n_t, n_{t-1})} \right)^\varsigma \equiv s_t(D_{Ht}, n_t^*, D_{Ft}, n_t^*, n_{t-1}^*),$$

defining the terms of trade as a function of price dispersion and employment in each country. Given (71), (75), (76), and (78),

$$c_t = \left( \gamma + (1 - \gamma) s_t^{\frac{1}{\varsigma} - 1} \right)^{\frac{1}{\varsigma}} \frac{1}{D_{Ht}} a_t f(n_t, n_{t-1}).$$

Analogously, (74), (75), (77), and (79) imply

$$c_t^* = \left( \gamma s_t^* \frac{1}{\varsigma} + (1 - \gamma) \right)^{\frac{1}{\varsigma}} \frac{1}{D_{Ft}} a_t^* f^s(n_t^*, n_{t-1}^*).$$

Plugging all of these into (25), it follows that utilitarian social welfare can be written

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \gamma \left( \frac{\left( \gamma + (1 - \gamma) s_t(\cdot) \right)^{\frac{1}{\varsigma} - 1}}{1 - \sigma} \frac{1}{D_{Ht}} a_t f(n_t, n_{t-1}) \right)^{1-\sigma} - \chi \frac{n_t^{1+\varphi}}{1+\varphi} \right] +$$
(1 − γ) \left( \frac{\left[ (γs_t(\cdot)^{1−\frac{1}{\sigma}} + (1−γ)\right)^{\frac{1}{\sigma}}}{1−σ} \frac{DF_t a_t^* f^*(n_t^*, n_{t−1}^*)}{\chi (n_t^*)^{1+ϕ^*}} \right)^{1−σ} \right)

where I have written \( s_t(\cdot) \) in place of \( s_t(D_{Ht}, n_t, n_{t−1}, D_{Ft}, n_t^*, n_{t−1}^*) \) for brevity.

Now consider a second order expansion of social welfare. I begin with terms involving price dispersion, which is itself second-order in inflation and hence we only need to consider a first-order approximation of social welfare in price dispersion. Straightforward algebra implies

\[ U_{D_{Ht}} D_{Ht}|_{ss} = -β' c^{1−σ} \omega \]

where recall \( \omega \equiv \frac{γ}{γ + (1−γ)s^{1−τ}} \). Then standard arguments under Calvo (1983) price-setting imply that a second-order approximation of (82) around the zero-inflation steady-state is

\[ \hat{D}_{Ht} = (1−ι) \hat{D}_{Ht−1} + \frac{1}{2} \epsilon 1−lip_{Ht}^2, \]

where we use Assumption 3 to justify the treatment of \( \hat{D}_{Ht−1} \) as itself a second-order term. Iterating backwards, it follows that

\[ \hat{D}_{Ht} = \frac{1}{2} \epsilon 1−lip_{Ht}^2 \sum_{τ=0}^{t} (1−ι)^{t−τ}p_{Hτ}^2, \]

so we have

\[ \sum_{t=0}^{∞} U_{D_{Ht}} D_{Ht}|_{ss} \hat{D}_{Ht} = -\frac{1}{2} c^{1−σ} \omega \epsilon \frac{1−ι}{ι(1−β(1−ι))} \sum_{t=0}^{∞} β^t p_{Ht}^2 \]

after collecting terms involving each \( (p_{Ht})^2 \). With analogous steps for price dispersion of Foreign-produced goods, we obtain

\[ \sum_{t=0}^{∞} U_{D_{Ft}} D_{Ft}|_{ss} \hat{D}_{Ft} = -\frac{1}{2} c^{1−σ} (1−ω) \epsilon \frac{1−ι^*}{ι^*(1−β(1−ι^*))} \sum_{t=0}^{∞} β^t (p_{Ft})^2. \]

Now consider the terms involving employment. The first derivative with respect to Home employment at any date evaluated at steady-state is

\[ U_{n_{1}|ss} = β't c^{1−σ} \omega \frac{n}{af(n, n)} \left[ f_{n} - \chi a^{σ−1} \left( γ + (1−γ)s^{1−1}\right)^{\frac{σ−ι}{ι−ι}} f(n, n)^{σ} n^{ϕ} + β f_{n−1} \right], \]

where the expression in brackets is equal to zero by the labor market equilibrium condition (42) evaluated at steady-state, given the constrained efficiency of the steady-state per
Assumption 2; an analogous result obtains for $U_{n_t^*}\mid_{ss}$. This has two implications. First, the first-order terms in $\tilde{n}_t$ and $\hat{n}_t^*$ drop out in the second-order expansion of social welfare, justifying linear approximations of the equilibrium constraints as sufficient to characterize the Ramsey optimal allocation up to first order, following Woodford (2003). Second, a first-order approximation of the Home labor market equilibrium condition (42) under the natural allocation (in which $\mu_t = 1$ and $D_{Ht} = 1$) can be written

$$
U_{n_t} n_t \mid_{ss} \hat{n}_t + U_{n_t n_{t-1}} n_{t-1} \mid_{ss} \hat{n}_t n_{t-1} + U_{n_t n_{t-1}} n_{t+1} \mid_{ss} \beta \mathcal{E}_t n_{t+1} +
$$

$$
U_{n_t n_t} n_t \mid_{ss} \hat{n}_t n_{t-1} + U_{n_t n_{t-1}} n_{t-1} \mid_{ss} \hat{n}_t n_{t-1} + U_{n_t n_{t-1}} n_{t+1} \mid_{ss} \beta \mathcal{E}_t n_{t+1} +
$$

$$
U_{n_t a_t} a_t \mid_{ss} \hat{a}_t + U_{n_t a_{t+1}} a_{t+1} \mid_{ss} \mathcal{E}_t \hat{a}_{t+1} + U_{n_t a_{t+1}} a_{t+1} \mid_{ss} \beta \mathcal{E}_t \hat{a}_{t+1} = 0,
$$

where we have used the fact that $\beta U_{n_t n_t} \mid_{ss} = U_{n_t n_{t-1}} \mid_{ss}$ and $\beta U_{n_t n_t} \mid_{ss} = U_{n_t n_{t+1}} \mid_{ss}$. An analogous result obtains using the Foreign labor market equilibrium condition. It follows that, by completing the square,

$$
\mathbb{E}_0 \sum_{t=0}^{\infty} \left[ \frac{1}{2} U_{n_t n_t} n_t^2 \mid_{ss} \hat{n}_t + U_{n_t n_{t-1}} n_{t-1} \mid_{ss} \hat{n}_t n_{t-1} + \frac{1}{2} U_{n_t n_t} n_{t+1} \mid_{ss} \hat{n}_t + U_{n_t n_{t-1}} n_{t+1} \mid_{ss} \hat{n}_t n_{t-1} + U_{n_t n_{t-1}} n_{t+1} \mid_{ss} \hat{n}_t n_{t-1} \right]
$$

$$
= \mathbb{E}_0 \sum_{t=0}^{\infty} \left[ \frac{1}{2} U_{n_t n_t} n_t^2 \mid_{ss} \hat{n}_t + U_{n_t n_{t-1}} n_{t-1} \mid_{ss} \hat{n}_t n_{t-1} + \frac{1}{2} U_{n_t n_t} n_{t+1} \mid_{ss} \hat{n}_t + U_{n_t n_{t-1}} n_{t+1} \mid_{ss} \hat{n}_t n_{t-1} + U_{n_t n_{t-1}} n_{t+1} \mid_{ss} \hat{n}_t n_{t-1} \right] + \text{tips},
$$

where \text{tips} is a function only of the natural allocation and is thus independent of policy.

Straightforward algebra implies

$$
U_{n_t n_t} n_t^2 \mid_{ss} = -\beta t c^{1-\sigma} \omega (\varepsilon_n) \hat{f}_n^2,
$$

$$
U_{n_t n_{t-1}} n_{t-1} \mid_{ss} = -\beta t c^{1-\sigma} \omega (\varepsilon_n) \hat{f}_n \hat{f}_{n_{t-1}},
$$

$$
U_{n_t n_{t-1}} n_{t-1} \mid_{ss} \hat{n}_t n_{t-1} = -\beta t c^{1-\sigma} (1 - \omega) \hat{f}_n \hat{f}_{n_{t-1}} \hat{f}_{n_{t-1}},
$$

$$
U_{n_t n_{t-1}} n_{t-1} \mid_{ss} \hat{n}_t n_{t-1} = -\beta t c^{1-\sigma} (1 - \omega) (\sigma - \varsigma) \hat{f}_n \hat{f}_{n_{t-1}} \hat{f}_{n_{t-1}},
$$

$$
U_{n_t n_{t-1}} n_{t-1} \mid_{ss} = -\beta t c^{1-\sigma} \omega (1 - \omega) (\sigma - \varsigma) \hat{f}_n \hat{f}_{n_{t-1}} \hat{f}_{n_{t-1}}.
$$

46
given the definitions of $\phi, \phi_{-1}, \phi^*, \phi^*_{-1},$ and $\phi'$ in Lemma 2.

Since second-order terms only involving productivity are independent of policy, this completes the claim. \hfill \Box

**Proposition 5**

*Proof.* Define

$$
\tilde{\phi} \equiv \frac{\varphi - \epsilon f_n}{\epsilon f_n} = \varphi + \frac{\frac{1}{2} \frac{k}{\eta q \sigma}}{1 - \frac{1}{2} \frac{k}{\eta q \sigma}} \left( \frac{1 - \eta}{\eta \delta} \right) \equiv \tilde{\phi}(\kappa, \delta),
$$

where $\kappa$ is the steady-state hiring costs per hired worker

$$
\kappa \equiv \frac{k}{q(\theta)}.
$$

In order to characterize the comparative statics of $\tilde{\phi}$ with respect to $k$ and $\delta$, I first characterize the comparative statics of $\kappa$ with respect to $k$ and $\delta$.

Given the equilibrium conditions in appendix B evaluated at steady-state and $\beta \to 0$, we have that $\kappa, \kappa^*, \omega$ are jointly determined by

$$
0 = v(\kappa, \omega; k, \delta) \equiv 1 - \chi \gamma \frac{\sigma}{\xi} \omega \frac{\sigma}{\xi} \tilde{a}^{-1}(1 - \delta \kappa) \sigma n(\kappa; k, \delta)^{\varphi + \sigma} - \frac{1}{\eta} \kappa,
$$

$$
0 = v^*(\kappa^*, \omega; k^*, \delta^*) \equiv 1 - \chi^* (1 - \gamma) \frac{\sigma}{\xi} (1 - \omega) \frac{\sigma}{\xi} \tilde{a}^{-1}(1 - \delta^* \kappa^*) \sigma n^*(\kappa^*; k^*, \delta^*)^{\varphi^* + \sigma} - \frac{1}{\eta^*} \kappa^*;
$$

$$
0 = \Omega(\kappa, \kappa^*, \omega; k, \delta, k^*, \delta^*) \equiv \omega - \frac{\gamma}{\gamma + (1 - \gamma) \left( \frac{\tilde{a}^{n^*(\kappa^*; k^*, \delta^*)}(1 - \delta^* \kappa^*)}{an(\kappa; k, \delta)(1 - \delta \kappa)} \right)^{1 - \xi}},
$$

where

$$
n(\kappa; k, \delta) = \frac{p(q^{-1}(k/\kappa))}{\delta + (1 - \delta)p(q^{-1}(k/\kappa))},
$$

$$
n^*(\kappa^*; k^*, \delta^*) = \frac{p^*(q^{-1}(k^*/\kappa^*))}{\delta^* + (1 - \delta^*)p^*(q^{-1}(k^*/\kappa^*))}
$$

denote the steady-state employment rates. Differentiating with respect to $k$ yields

$$
\begin{align*}
\nu_k d\kappa + \nu_\omega d\omega + \nu_k = 0, \\
\nu_{k^*} d\kappa^* + \nu_{\omega^*} d\omega^* = 0,
\end{align*}
$$

47
\[ \Omega_k \frac{dk}{dk} + \Omega_κ^* \frac{dκ^*}{dk} + dω \frac{dk}{dk} + Ω_k = 0. \]

At the symmetric benchmark,

\[ v_κ|_{sym} = v_κ^*|_{sym}, \]
\[ v_ω|_{sym} = -\frac{1 - ω}{ω} v_ω^*|_{sym}, \]
\[ Ω_κ|_{sym} = -Ω_κ^*|_{sym}. \]

Hence, we can write the system at the symmetric benchmark as

\[ v_κ|_{sym} \frac{d(κ - κ^*)}{dk}|_{sym} + \frac{1}{1 - ω} v_ω|_{sym} \frac{dω}{dk}|_{sym} + v_k = 0, \]
\[ Ω_κ|_{sym} \frac{d(κ - κ^*)}{dk}|_{sym} + \frac{dω}{dk}|_{sym} + Ω_k|_{sym} = 0, \]

which imply

\[ \frac{d(κ - κ^*)}{dk}|_{sym} = -v_κ|_{sym} + \frac{1}{1 - ω} v_ω|_{sym} Ω_κ|_{sym}, \]
\[ \frac{dω}{dk}|_{sym} = Ω_κ|_{sym} v_k|_{sym} - Ω_κ^*|_{sym} v_κ|_{sym}. \]

Analogous steps when differentiating the original system by \( δ \) yields

\[ \frac{d(κ - κ^*)}{dδ}|_{sym} = -v_δ|_{sym} + \frac{1}{1 - ω} v_ω|_{sym} Ω_δ|_{sym}, \]
\[ \frac{dω}{dδ}|_{sym} = Ω_κ|_{sym} v_δ|_{sym} - Ω_δ|_{sym} v_κ|_{sym}. \]

With these in hand, we can use (47) to solve for \( \frac{dk}{dk} \) and analogously for \( \frac{dk}{dδ} \). Doing this yields

\[ \frac{dk}{dk}|_{sym} = \frac{-v_κ|_{sym} \left( 1 - \frac{ω}{1 - ω} v_κ|_{sym} Ω_κ|_{sym} \right) + v_ω|_{sym} Ω_k|_{sym}}{v_κ|_{sym} - \frac{1}{1 - ω} v_ω|_{sym} Ω_κ|_{sym}}, \]

\[ \frac{dk}{dδ}|_{sym} = \frac{-v_δ|_{sym} \left( 1 - \frac{ω}{1 - ω} v_κ|_{sym} Ω_κ|_{sym} \right) + v_ω|_{sym} Ω_δ|_{sym}}{v_κ|_{sym} - \frac{1}{1 - ω} v_ω|_{sym} Ω_κ|_{sym}}. \]
Now, straightforward differentiation yields

\[
v_\kappa = \frac{1}{\kappa} \chi^\frac{\alpha-\kappa}{1-\omega} \omega^\frac{\xi-\kappa}{1-\omega} a^{\sigma-1} (1 - \delta \kappa)^\sigma n^{\phi+\sigma} \left[ \sigma \frac{\delta \kappa}{1 - \delta \kappa} - (\varphi + \sigma) \frac{\kappa}{n} \frac{\partial n}{\partial \kappa} - \frac{\delta \kappa}{1 - \eta \kappa} \right],
\]

\[
v_\kappa = \frac{1}{k} \chi^\frac{\alpha-\kappa}{1-\omega} \omega^\frac{\xi-\kappa}{1-\omega} a^{\sigma-1} (1 - \delta \kappa)^\sigma n^{\phi+\sigma} \left[ -(\varphi + \sigma) \frac{k}{n} \frac{\partial n}{\partial \kappa} \right],
\]

\[
v_\delta = \frac{1}{\delta} \chi^\frac{\alpha-\kappa}{1-\omega} \omega^\frac{\xi-\kappa}{1-\omega} a^{\sigma-1} (1 - \delta \kappa)^\sigma n^{\phi+\sigma} \left[ \sigma \frac{\delta \kappa}{1 - \delta \kappa} - (\varphi + \sigma) \frac{\delta}{n} \frac{\partial n}{\partial \delta} \right],
\]

\[
v_\omega = \frac{1}{\omega} \chi^\frac{\alpha-\kappa}{1-\omega} \omega^\frac{\xi-\kappa}{1-\omega} a^{\sigma-1} (1 - \delta \kappa)^\sigma n^{\phi+\sigma} \left[ - \left( \frac{\zeta - \sigma}{1 - \varsigma} \right) \right],
\]

\[
\Omega_\kappa = \frac{1}{\kappa} \omega (1 - \omega) (1 - \varsigma) \left[ - \frac{\kappa}{n} \frac{\partial n}{\partial \kappa} + \frac{\delta \kappa}{1 - \delta \kappa} \right],
\]

\[
\Omega_k = \frac{1}{k} \omega (1 - \omega) (1 - \varsigma) \left[ - \frac{k}{n} \frac{\partial n}{\partial k} \right].
\]

Substituting above, we obtain

\[
\frac{d \kappa}{d k} \bigg|_{\text{sym}} = \frac{k}{k} A_1,
\]

\[
\frac{d \kappa}{d \delta} \bigg|_{\text{sym}} = \frac{\kappa}{\delta} A_2,
\]

where

\[
A_1 = \frac{(\varphi + \sigma) \frac{k}{n} \frac{\partial n}{\partial k} \left( 1 + \omega \frac{(s - \sigma)}{\sigma \frac{\delta \kappa}{1 - \delta \kappa} - (\varphi + \sigma) \frac{\delta}{n} \frac{\partial n}{\partial \kappa} - \frac{\delta \kappa}{1 - \eta \kappa} \right) + (1 - \omega) (s - \sigma) \frac{k}{n} \frac{\partial n}{\partial k} - \frac{\delta \kappa}{1 - \delta \kappa} \right)}{\sigma \frac{\delta \kappa}{1 - \delta \kappa} - (\varphi + \sigma) \frac{\delta}{n} \frac{\partial n}{\partial \kappa} - \frac{\delta \kappa}{1 - \eta \kappa}},
\]

\[
A_2 = \frac{(\varphi + \sigma) \frac{k}{n} \frac{\partial n}{\partial k} \left( 1 + \omega \frac{(s - \sigma)}{\sigma \frac{\delta \kappa}{1 - \delta \kappa} - (\varphi + \sigma) \frac{\delta}{n} \frac{\partial n}{\partial \kappa} - \frac{\delta \kappa}{1 - \eta \kappa} \right) - (1 - \omega) (s - \sigma) \left[ - \frac{\delta}{n} \frac{\partial n}{\partial \delta} + \frac{\delta \kappa}{1 - \delta \kappa} \right] - \frac{\delta \kappa}{1 - \delta \kappa} \right)}{\sigma \frac{\delta \kappa}{1 - \delta \kappa} - (\varphi + \sigma) \frac{\delta}{n} \frac{\partial n}{\partial \kappa} - \frac{\delta \kappa}{1 - \eta \kappa}},
\]

Now note that

\[
\frac{\partial n}{\partial \kappa} = \frac{n}{\kappa} \left( \frac{\delta}{\delta + (1 - \delta) p(\theta)} \right) \frac{\eta}{1 - \eta} > 0,
\]

\[
\frac{\partial n}{\partial k} = \frac{n}{k} \left( \frac{\delta}{\delta + (1 - \delta) p(\theta)} \right) \frac{\eta}{1 - \eta} < 0,
\]

\[
\frac{\partial n}{\partial \delta} = \frac{n}{\delta} \left( \frac{\delta}{\delta + (1 - \delta) p(\theta)} \right) < 0.
\]

49
Since $k \to 0$ implies $\kappa \to 0$, we have that $k \to 0$ implies
\[
A_1 \to 1 > 0,
\]
\[
A_2 \to (1 - p(\theta)) \frac{1 - \eta}{\eta} > 0,
\]
and thus $\frac{d\kappa}{dk}|_{sym} > 0$ and $\frac{d\kappa}{d\delta}|_{sym} > 0$ at least for $k$ sufficiently small.

Now return to the comparative statics of $\tilde{\phi} = \tilde{\phi}(\kappa, \delta)$. First considering the comparative static with respect to hiring costs $k$, we have that
\[
\frac{d\tilde{\phi}}{dk} = \frac{\partial \tilde{\phi}}{\partial \kappa} \frac{d\kappa}{dk}.
\]
Since
\[
\frac{\partial \tilde{\phi}}{\partial \kappa} \propto -\epsilon_n \left( \frac{d\epsilon_n^f}{d\kappa} \right) - (\varphi - \epsilon_n^f) \left( \frac{d\epsilon_n^f}{d\kappa} \right),
\]
\[
= \epsilon_n^f \left( \frac{1}{(1 - \frac{1}{\eta} \kappa)^2} \left( \frac{1 - \eta}{\delta \eta^2} \right) \right) + (\varphi - \epsilon_n^f) \left( \frac{1}{(1 - \delta \kappa)^2} \left( \frac{1 - \eta}{\eta} - \delta \right) \right),
\]
\[
> 0,
\]
and we know from the above results that $\frac{d\kappa}{dk}|_{sym} > 0$, we can conclude
\[
\frac{d\tilde{\phi}}{dk}|_{sym} > 0
\]
as claimed.

Finally consider the comparative static with respect to the separation probability $\delta$, where
\[
\frac{d\tilde{\phi}}{d\delta} = \frac{\partial \tilde{\phi}}{\partial \delta} + \frac{\partial \tilde{\phi}}{\partial \kappa} \frac{d\kappa}{d\delta}.
\]
The challenge here is that while
\[
\frac{\partial \tilde{\phi}}{\partial \delta} \propto -\epsilon_n \left( \frac{\partial \epsilon_n^f}{\partial \delta} \right) - (\varphi - \epsilon_n^f) \left( \frac{\partial \epsilon_n^f}{\partial \delta} \right),
\]
\[
= -\epsilon_n \left( \frac{\frac{1}{\eta} \kappa}{1 - \frac{1}{\eta} \kappa} \left( \frac{1 - \eta}{\delta^2} \right) \right) - (\varphi - \epsilon_n^f) \left( \frac{1 - \frac{1}{\eta} \kappa}{(1 - \delta \kappa)^2} \right),
\]
\[
< 0,
\]
and
\[
50
\]
we know from (50) and the above results that
\[
\frac{\partial \tilde{\phi}}{\partial \kappa} \bigg|_{\text{sym}} \frac{d\kappa}{d\delta} \bigg|_{\text{sym}} > 0.
\]

In economic terms, while a lower separation probability directly raises the welfare cost of output fluctuations (\(\frac{\partial \tilde{\phi}}{\partial \delta} < 0\)), in equilibrium it also reduces hiring costs per hire \(\kappa\) in the present framework (\(\frac{d\kappa}{d\delta} \bigg|_{\text{sym}} > 0\)), which has an offsetting effect on the welfare cost of output fluctuations (since \(\frac{\partial \tilde{\phi}}{\partial \kappa} > 0\)). The fact that a lower separation probability reduces hiring costs per hire is mitigated in the more general \(\beta\) case where, by raising the continuation value of a match, a lower separation probability incentivizes vacancy posting and thus raise hiring costs per hire. Nonetheless, this \(\beta \to 0\) limit serves to clarify a more important difference from classic DMP models which typically assume risk neutrality. In the present environment, a lower separation probability tends to raise the marginal rate of substitution between labor and consumption, and thus dis-incentivize vacancy posting.

If the unemployment rate \(1 - n\) is small (in which case \(1 - p(\theta)\) is also small),
\[
\frac{\partial \tilde{\phi}}{\partial \kappa} \bigg|_{\text{sym}} \frac{d\kappa}{d\delta} \bigg|_{\text{sym}}
\]
is an order of magnitude smaller than
\[
\frac{\partial \tilde{\phi}}{\partial \delta} \bigg|_{\text{sym}}
\]
and thus
\[
\frac{d\tilde{\phi}}{d\delta} \bigg|_{\text{sym}} < 0
\]
as claimed. \(\square\)

**Proposition 6**

*Proof.* I refer to the conditions defining the natural allocation in appendix B. At the \(\beta \to 0\) limit, (58)-(60), (64), (65), and (67) imply
\[
f_n(n_0^n, n_{-1}^n) = \chi a_0^{\sigma - 1} \left( \gamma + (1 - \gamma) (s_0^n)^{\frac{1}{\gamma - 1}} \right)^{\frac{\sigma - \varsigma}{\gamma - \varsigma}} f(n_0^n, n_{-1}^n)^{\sigma} (n_0^n)^{\tilde{\phi}}.
\]

Analogously, (61)-(63), (64), (66), and (68) imply
\[
f_n^*(n_0^{*n}, n_{-1}^{*n}) = \chi^* a^{*\sigma - 1} \left( \gamma (s_0^{*n})^{\frac{1}{\gamma - 1}} + (1 - \gamma) \right)^{\frac{\sigma - \varsigma}{\gamma - \varsigma}} f^*(n_0^{*n}, n_{-1}^{*n})^{\sigma} (n_0^{*n})^{\tilde{\phi}^*}.
\]
Finally, (60), (63), and (64) imply

\[ s^n_0 = \left( \frac{a^*_0 f(n^n_0, n^{n-1}_0)}{a_0 f(n^n_0, n^{n-1}_0)} \right)^c. \]

Log-linearizing these, combining, and assuming \( \hat{a}_0 = \hat{a}^*_0 \), straightforward algebra implies

\[ \hat{s}^n_0 = \zeta \frac{(1 - \sigma) (\tilde{\phi} - \tilde{\phi}^*)}{\phi^* \phi - (\sigma - \zeta)^2 \omega (1 - \omega)} \hat{a}_0, \]

where \( \tilde{\phi} \equiv \frac{\varphi - L^n}{\epsilon^n} \) and \( \tilde{\phi}^* \equiv \frac{\varphi^* - L^{n*}}{\epsilon^{n*}} \) as defined in the proof of Proposition 5. Since

\[ \phi = \sigma + \tilde{\phi} - (\sigma - \zeta)(1 - \omega), \]
\[ \phi^* = \sigma + \tilde{\phi}^* - (\sigma - \zeta)\omega, \]

at the symmetric benchmark

\[ (\phi\phi^* - (\sigma - \zeta)^2 \omega (1 - \omega)) |_{sym} = \left( \sigma + \tilde{\phi} \right) \left( \zeta + \tilde{\phi} \right) > 0. \]

It follows that around the symmetric benchmark,

\[ \hat{s}^n_0 \propto (1 - \sigma) \left( \tilde{\phi} - \tilde{\phi}^* \right) \hat{a}_0. \]

Given the comparative statics of \( \tilde{\phi} - \tilde{\phi}^* \) around the symmetric benchmark provided in the proof of Proposition 8, the result follows. \( \square \)

**Proposition 7**

**Proof.** The first-order conditions of the Ramsey problem can be combined to yield

\[ \xi = \frac{\Theta}{\Theta + \Theta^*} \]

where

\[ \Theta \equiv \omega \frac{\varepsilon \lambda \phi + 1}{\lambda (\lambda \phi - \lambda^* (\sigma - \zeta) \omega + \zeta)} + \omega (1 - \omega) (\sigma - \zeta) \frac{\varepsilon}{\lambda^* \phi^* - \lambda (\sigma - \zeta) (1 - \omega) + \zeta}, \]
\[ \Theta^* \equiv (1 - \omega) \frac{\varepsilon \lambda^* \phi^* + 1}{\lambda^* (\lambda^* \phi^* - \lambda (\sigma - \zeta) (1 - \omega) + \zeta)} + \omega (1 - \omega) (\sigma - \zeta) \frac{\varepsilon}{\lambda \phi - \lambda^* (\sigma - \zeta) \omega + \zeta}. \]
When $\iota = \iota^*$, we have that $\lambda = \lambda^*$. When countries are symmetric in terms of labor markets, we have that $\phi = \phi^*$, $\epsilon_{fn} = \epsilon_{fn}^*$, and $\epsilon_{fn}^* = \epsilon_{fn}^*$. Taken together, these imply that

\[
\lambda \phi - \lambda^* (\sigma - \varsigma) \omega + \varsigma
\]

\[
= \lambda \left( \varsigma + \frac{\phi - \epsilon_{fn}^*}{\epsilon_{fn}} \right) + \varsigma,
\]

\[
= \lambda^* \left( \varsigma + \frac{\phi^* - \epsilon_{fn}^*}{\epsilon_{fn}^*} \right) + \varsigma,
\]

\[
= \lambda^* \phi^* - \lambda (\sigma - \varsigma)(1 - \omega) + \varsigma.
\]

Hence, $\xi$ simplifies to

\[
\xi = \frac{\omega (\varepsilon \lambda \phi + 1) + \omega (1 - \omega)(\sigma - \varsigma) \varepsilon \lambda}{\omega (\varepsilon \lambda \phi + 1) + \omega (1 - \omega)(\sigma - \varsigma) \varepsilon \lambda + (1 - \omega) (\varepsilon \lambda \phi^* + 1) + \omega (1 - \omega)(\sigma - \varsigma) \varepsilon \lambda^*}.
\]

Then, since

\[
\omega (\varepsilon \lambda \phi + 1) + \omega (1 - \omega)(\sigma - \varsigma) \varepsilon \lambda
\]

\[
= \omega \left( \varepsilon \lambda \left( \sigma + \frac{\phi - \epsilon_{fn}^*}{\epsilon_{fn}} \right) + 1 \right),
\]

\[
= \frac{\omega}{1 - \omega} (1 - \omega) \left( \varepsilon \lambda^* \left( \sigma + \frac{\phi^* - \epsilon_{fn}^*}{\epsilon_{fn}^*} \right) + 1 \right),
\]

\[
= \frac{\omega}{1 - \omega} \left[ (1 - \omega) (\varepsilon \lambda^* \phi^* + 1) + \omega (1 - \omega)(\sigma - \varsigma) \varepsilon \lambda^* \right],
\]

we obtain

\[
\xi = \omega
\]

as claimed. \(\square\)

**Proposition 8**

**Proof.** Define $\bar{\phi} \equiv \frac{\phi - \epsilon_{fn}}{\epsilon_{fn}}$ and $\bar{\phi}^* \equiv \frac{\phi^* - \epsilon_{fn}^*}{\epsilon_{fn}^*}$. Since $\iota = \iota^*$ and thus $\lambda = \lambda^*$, we can write

\[
\Theta = \frac{\varepsilon \lambda \left( \sigma + \bar{\phi} - (\sigma - \varsigma)(1 - \omega) \right) + 1}{\lambda \left( \varsigma + \bar{\phi} \right) + \varsigma} + \omega (1 - \omega)(\sigma - \varsigma) \frac{\varepsilon}{\lambda \left( \varsigma + \bar{\phi}^* \right) + \varsigma},
\]
\[ \Theta^* = (1 - \omega) \frac{\varepsilon \lambda (\sigma + \tilde{\phi} - (\sigma - \varsigma)\omega)}{\lambda \left( \varsigma + \tilde{\phi} \right) + \varsigma} + 1 + \omega \left( \lambda \left( \varsigma + \tilde{\phi} \right) + \varsigma \right) \varepsilon, \]

where the optimal \( \xi = \frac{\Theta}{\Theta^* + \Theta} \) as demonstrated in the proof of Proposition 7. Defining \( \xi \) as a function of \( \tilde{\phi}, \tilde{\phi}^*, \) and \( \omega \), we can differentiate it with respect to each argument holding all others fixed. Doing so and evaluating at the symmetric benchmark yields

\[
\begin{align*}
\frac{\partial (\xi/\omega)}{\partial \tilde{\phi}} \bigg|_{sym} &\propto \varepsilon \varsigma - 1, \\
\frac{\partial (\xi/\omega)}{\partial \tilde{\phi}^*} \bigg|_{sym} &= -\frac{\partial (\xi/\omega)}{\partial \tilde{\phi}} \bigg|_{sym}, \\
\frac{\partial (\xi/\omega)}{\partial \omega} \bigg|_{sym} &= 0.
\end{align*}
\]

It follows that

\[
\begin{align*}
\frac{d(\xi/\omega)}{dk} \bigg|_{sym} &\propto (\varepsilon \varsigma - 1) \frac{d(\tilde{\phi} - \tilde{\phi}^*)}{dk} \bigg|_{sym}, \\
\frac{d(\xi/\omega)}{d\delta} \bigg|_{sym} &\propto (\varepsilon \varsigma - 1) \frac{d(\tilde{\phi} - \tilde{\phi}^*)}{d\delta} \bigg|_{sym}.
\end{align*}
\]

so we now focus on evaluating the derivatives on the right-hand side. Writing \( \tilde{\phi} \) and \( \tilde{\phi}^* \) as functions of \( \{\kappa \equiv \frac{k}{q(\theta)}, \delta\} \) and \( \{\kappa^* \equiv \frac{k^*}{q^*(\theta^*)}, \delta^*\} \) (as in the proof of Proposition 5), at the symmetric benchmark

\[
\begin{align*}
\frac{d(\tilde{\phi} - \tilde{\phi}^*)}{dk} \bigg|_{sym} &= \frac{\partial \tilde{\phi}}{\partial \kappa} \bigg|_{sym} \frac{d(\kappa - \kappa^*)}{dk} \bigg|_{sym}, \\
\frac{d(\tilde{\phi} - \tilde{\phi}^*)}{d\delta} \bigg|_{sym} &= \frac{\partial \tilde{\phi}}{\partial \delta} \bigg|_{sym} + \frac{\partial \tilde{\phi}}{\partial \kappa} \bigg|_{sym} \frac{d(\kappa - \kappa^*)}{d\delta} \bigg|_{sym}.
\end{align*}
\]

Using (48) and (49) derived in the proof of Proposition 5,

\[
\begin{align*}
\frac{d(\kappa - \kappa^*)}{dk} \bigg|_{sym} &= \frac{\kappa}{k} \left( \varphi + \varsigma \right) \frac{k}{n} \frac{\partial n}{\partial k} - \frac{1}{1 - \delta \kappa} \left( \varphi + \varsigma \right) \frac{k}{\delta \kappa} \frac{\partial n}{\partial k} \frac{\partial \kappa}{\partial n} - \frac{1 - \frac{1}{\delta \kappa}}{1 - \frac{1}{\delta \kappa}}, \\
\frac{d(\kappa - \kappa^*)}{d\delta} \bigg|_{sym} &= \frac{\kappa}{\delta} \left( \varphi + \varsigma \right) \frac{\delta}{n} \frac{\partial n}{\partial \delta} - \frac{1}{1 - \delta \kappa} \left( \varphi + \varsigma \right) \frac{\kappa}{\delta \kappa} \frac{\delta}{n} \frac{\partial \kappa}{\partial n} - \frac{1 - \frac{1}{\delta \kappa}}{1 - \frac{1}{\delta \kappa}}.
\end{align*}
\]

For \( k = k^* \) and thus \( \kappa = \kappa^* \) sufficiently small at the symmetric benchmark, we have \( \frac{d(\kappa - \kappa^*)}{dk} \bigg|_{sym} > 0 \) and \( \frac{d(\kappa - \kappa^*)}{d\delta} \bigg|_{sym} > 0 \). Since \( \frac{\partial \delta}{\partial \kappa} \bigg|_{sym} > 0 \) as demonstrated in the proof of
Proposition 5, we can conclude from (53) that
\[ \frac{d(\tilde{\phi} - \tilde{\phi}^*)}{dk}\big|_{\text{sym}} > 0. \]

And since \( \frac{\partial(\tilde{\phi} - \tilde{\phi}^*)}{\partial \phi}\big|_{\text{sym}} < 0 \) is again an order of magnitude larger than \( \frac{\partial \tilde{\phi}}{\partial \delta}\big|_{\text{sym}} \frac{d(\kappa - \kappa^*)}{d\delta}\big|_{\text{sym}} > 0 \) when \( 1 - n = 1 - n^* \) and thus \( 1 - p(\theta) = 1 - p^*(\theta^*) \) are sufficiently small, we can conclude from (54) that
\[ \frac{d(\tilde{\phi} - \tilde{\phi}^*)}{d\delta}\big|_{\text{sym}} < 0. \]

Thus, (51) and (52) imply
\[ \frac{d(\xi/\omega)}{dk}\big|_{\text{sym}} \propto (\epsilon \varsigma - 1), \]
\[ \frac{d(\xi/\omega)}{d\delta}\big|_{\text{sym}} \propto - (\epsilon \varsigma - 1). \]

Provided \( \varsigma \geq \frac{1}{\epsilon} \), we have that \( \frac{d(\xi/\omega)}{dk}\big|_{\text{sym}} > 0 \) and \( \frac{d(\xi/\omega)}{d\delta}\big|_{\text{sym}} < 0 \). If \( \varsigma < \frac{1}{\epsilon} \), we instead have that \( \frac{d(\xi/\omega)}{dk}\big|_{\text{sym}} < 0 \) and \( \frac{d(\xi/\omega)}{d\delta}\big|_{\text{sym}} > 0 \). Since \( \frac{1}{\epsilon} < 1 \), the stated claim follows.

Proposition 9

Proof. I refer to the first-order conditions of the Ramsey problem in appendix C. Given a state-contingent sequence of \( \pi_{Ht}, \mu_t, \tilde{n}_t, \psi_{\pi_{Ht}}, \psi_{\mu t}, \) and \( \pi_{st} \) solving (26), (28), (31), (91), (92), and (93) where \( \pi_{Ft} = -\frac{\omega}{1 - \omega} \pi_{Ht}, \bar{\mu}_t^* = -\frac{\omega}{1 - \omega} \bar{\mu}_t, \bar{n}_t^* = -\frac{\omega}{1 - \omega} \bar{n}_t, \psi_{\pi_{Ft}} = -\psi_{\pi_{Ht}}, \psi_{\mu^* t} = -\psi_{\mu t}, \) I will prove that (27), (29), (94), (95), and (96) are also satisfied. It will thus follow that the optimal policy can be implemented using an inflation targeting rule with weight \( \omega \) on producer-price inflation at Home.

Since \( \nu = \nu^* \) and thus \( \lambda = \lambda^* \), (27) is implied by (26), \( \pi_{Ft} = -\frac{\omega}{1 - \omega} \pi_{Ht}, \) and \( \bar{\mu}_t^* = -\frac{\omega}{1 - \omega} \bar{\mu}_t. \)

Since labor markets are symmetric, (29) is implied by (28), \( \bar{\mu}_t^* = -\frac{\omega}{1 - \omega} \bar{\mu}_t \) and \( \bar{n}_t^* = -\frac{\omega}{1 - \omega} \bar{n}_t. \)

Since \( \lambda = \lambda^* \), (94) is implied by (91), \( \pi_{Ft} = -\frac{\omega}{1 - \omega} \pi_{Ht}, \) and \( \psi_{\pi_{Ft}} = -\psi_{\pi_{Ht}}. \)

Since \( \lambda = \lambda^* \) and labor markets are symmetric, (95) is implied by (92), \( \psi_{\pi_{Ft}} = -\psi_{\pi_{Ht}}, \) and \( \psi_{\mu^* t} = -\psi_{\mu t}. \)

Finally, since labor markets are symmetric, (96) is implied by (93), \( \bar{n}_t^* = -\frac{\omega}{1 - \omega} \bar{n}_t \) and \( \psi_{\mu^* t} = -\psi_{\mu t}. \)
Appendix

A Optimization problems facing Foreign agents

In this appendix I detail the optimization problems facing Foreign agents which were excluded from the main text for brevity.

The representative household chooses a state-contingent sequence of \( c_t^*, B_{t+1}^*, u_t^* \) to maximize

\[
\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u^*(c_t^*, n_t^*)
\]

subject to

\[
u^*(c_t^*, n_t^*) = \frac{c_t^{1-\sigma} - \lambda_{-1} n_t^{1+\varphi}}{1+\varphi}, \]
\[
c_t^* = \left[ \left( \gamma \right)^{c_{Ht}^*} + (1-\gamma)^{c_{Ft}^*} \right]^{\frac{1}{1-\gamma}},
\]
\[
c_{Ht}^* = \left[ \int_0^1 c_{Ht}^*(j)^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}},
\]
\[
c_{Ft}^* = \left[ \int_0^1 c_{Ft}^*(j)^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}},
\]
\[
n_t^* = (1-\delta^*) n_{t-1}^* + p^*(\theta_t^*) u_t^*,
\]
\[
\int_0^1 P_{Ht}(j) c_{Ht}^*(j) dj + \int_0^1 P_{Ft}(j^*) c_{Ft}^*(j^*) dj^* + \mathbb{E}_t Q_{t,t+1} B_{t+1}^* \leq W_t u_t^* + B_t^* - T_t^*;
\]

as well as the no-Ponzi constraint \( \lim_{s \to \infty} \mathbb{E}_t Q_{t,t+s} B_{t+s}^* \geq 0 \) at all \( t \).

The representative intermediate good producer chooses a state-contingent sequence of \( \nu_t^* \) and \( n_t^* \) to maximize

\[
\mathbb{E}_0 \sum_{t=0}^{\infty} Q_{0,t} \Pi_t^*
\]

subject to

\[
n_t^* = (1-\delta^*) n_{t-1}^* + q^*(\theta_t^*) \nu_t^*,
\]
\[
\Pi_t^* = (1-\gamma) \left( P_{t+1}^* a_t^* \left[ (1-\delta^*) n_{t+1}^* + q^*(\theta_t^*) \nu_t^* - k^* \nu_t^* \right] - W_t^* \left[ (1-\delta^*) n_{t+1}^* + q^*(\theta_t^*) \nu_t^* \right] \right).
\]

If retailer \( j^* \in [0,1] \) can update its price in period \( t \), it chooses \( P_{Ft} \) and a state-contingent sequence of \( x_s^*(j^*) \) and \( y_s^*(j^*) \) (for \( s \geq t \)) to maximize

\[
\mathbb{E}_t \sum_{s=t}^{\infty} Q_{t,s} \Pi_s^* (P_{Ft}; j^*)
\]
subject to

$$\Pi_s^r(\mathcal{P}_{Ft}: j^*) = \mathcal{P}_{Ft} y_s^r(j^*) - (1 + \tau^r) P_{s}^l x_s^r(j^*),$$
$$y_s^r(j^*) = x_s^r(j^*),$$
$$y_s^r(j^*) = \left( \frac{P_{Ft}}{P_{Fs}} \right)^{-\varepsilon} (\gamma c_{Fs} + (1 - \gamma) c_{Fs}).$$

If retailer $j^*$ cannot update its price in period $t$ it accommodates consumption demand at its preset price provided it earns non-negative profits.

### B Equilibrium

In this appendix I formally define the equilibrium and derive the equilibrium conditions for the environment described in the main text.

#### B.1 Definition of equilibrium

The definition of equilibrium is standard:

**Definition B.1.** An equilibrium is a state-contingent sequence of consumption, labor force participation, vacancies, tightness, employment, intermediate goods, and final goods, as well as nominal wages, prices, and profits, such that given a set of initial prices and portfolios and a state-contingent path for nominal interest rates and lump-sum taxes:

1. households solve (1) subject to (3)-(6);
2. producers solve (7) subject to (8)-(9);
3. retailers solve (10) subject to (11)-(13) if they can update prices and accommodate demand if not;
4. analogous conditions to those above are all satisfied in Foreign;
5. tightness is consistent with aggregate vacancies and job-seekers according to (16);
6. wages are Nash bargained each period (as detailed in appendix B.5);
7. government budgets are balanced according to (17) and (18);
8. the intermediate good markets clear according to (19) and (20);
9. the final good markets clear according to (21) and (22);
10. global asset markets in state-contingent securities and firm shares clear according to (23).

B.2 Equilibrium conditional on arbitrary final goods prices

I now derive equilibrium conditions conditional on arbitrary final goods prices consistent with producer-currency pricing. In the subsequent sections I complete the description of the natural allocation and then the equilibrium with nominal rigidity.

First, by households’ optimal intratemporal allocation of consumption

\[ \frac{c_{Ht}}{c_{Ft}} = \frac{\gamma}{1 - \gamma} \left( \frac{P_{Ht}}{P_{Ft}} \right)^{-\frac{1}{\gamma}}, \]

\[ \frac{c^*_{Ht}}{c^*_{Ft}} = \frac{\gamma}{1 - \gamma} \left( \frac{P_{Ht}}{P_{Ft}} \right)^{-\frac{1}{\gamma}}. \]

Given the definition of the terms of trade

\[ s_t \equiv \frac{P_{Ht}}{P_{Ft}}, \]

it follows that

\[ s_t = \left( \frac{\gamma}{1 - \gamma} \frac{c_{Ft}}{c_{Ht}} \right)^{\frac{1}{\gamma}} = \left( \frac{\gamma}{1 - \gamma} \frac{c^*_{Ft}}{c^*_{Ht}} \right)^{\frac{1}{\gamma}}. \]

Now consider households’ optimal choice of portfolios, which imply

\[ \beta \frac{P_t}{P_{t+1}} \frac{c^-_{t+1} \sigma}{c^-_{t+1}} = Q_{t,t+1} = \beta \frac{P_t}{P_{t+1}} \frac{c^-_{t+1} \sigma}{c^-_{t+1}} \]

at all dates and states. We thus obtain the standard international risk-sharing condition

\[ c_t = \Xi c^*_t, \]

where \( \Xi \) reflects endowments in period 0. Following Assumption 1, endowments are such that \( \Xi = 1 \). Since households have the same homothetic preferences and face the same prices, it follows that

\[ c_{Ht} = c^*_{Ht}, \]

\[ c_{Ft} = c^*_{Ft}, \]
so that the terms of trade further satisfy

$$s_t = \left( \frac{\gamma}{1 - \gamma} c_{Ft}^* \right) \zeta.$$  

Turn now to equilibrium in the labor market. Producers’ first-order condition for vacancy posting requires

$$P_t^I a_t - W_t - P_t^I \frac{k a_t}{q(\theta_t)} + (1 - \delta) E_t Q_{t, t+1} P_t^I \frac{k a_{t+1}}{q(\theta_{t+1})} = 0, \quad P_t^I a_t^* - W_t^* - P_t^I^* \frac{k^* a_t^*}{q^*(\theta_t^*)} + (1 - \delta^*) E_t Q_{t, t+1} P_t^I^* \frac{k^* a_{t+1}^*}{q^*(\theta_{t+1}^*)} = 0$$

for an interior optimum in vacancy posting to exist. As shown in appendix B.5, the Nash bargained wage in each country is

$$W_t = -P_t \frac{u_n(c_t, n_t)}{u_c(c_t, n_t)} + \frac{\zeta}{1 - \zeta} \left[ P_t^I \frac{k a_t}{q(\theta_t)} - (1 - \delta) E_t Q_{t, t+1} (1 - p(\theta_{t+1})) P_t^I \frac{k a_{t+1}}{q(\theta_{t+1})} \right],$$

$$W_t^* = -P_t^I \frac{u_n^*(c_t^*, n_t^*)}{u_c^*(c_t^*, n_t^*)} + \frac{\zeta^*}{1 - \zeta^*} \left[ P_t^I^* \frac{k^* a_t^*}{q^*(\theta_t^*)} - (1 - \delta^*) E_t Q_{t, t+1} (1 - p^*(\theta_{t+1}^*)) P_t^I^* \frac{k^* a_{t+1}^*}{q^*(\theta_{t+1}^*)} \right].$$

Given the assumed preferences, the marginal rate of substitution between labor and consumption in each country is:

$$-P_t \frac{u_n(c_t, n_t)}{u_c(c_t, n_t)} = P_{Ht} \chi^{\gamma} c_t^{\sigma - \gamma} c_{Ht}^{\varphi} n_t^{\varphi},$$

$$-P_t^I \frac{u_n^*(c_t^*, n_t^*)}{u_c^*(c_t^*, n_t^*)} = P_{Ft} \chi^* (1 - \gamma)^{-\gamma} c_t^{\sigma - \gamma} c_{Ft}^{\varphi} n_t^{\varphi}.$$  

Combining the Nash bargained wages with optimal vacancy posting, labor market equilibrium can be summarized by

$$P_t^I a_t = P_{Ht} \chi^{\gamma} c_t^{\sigma - \gamma} c_{Ht}^{\varphi} n_t^{\varphi} + \frac{1}{1 - \zeta} P_t^I \frac{k a_t}{q(\theta_t)} - (1 - \delta) E_t Q_{t, t+1} \left( 1 + \frac{\zeta}{1 - \zeta} (1 - p(\theta_{t+1})) \right) P_t^I \frac{k a_{t+1}}{q(\theta_{t+1})},$$

$$P_t^I a_t^* = P_{Ft} \chi^* (1 - \gamma)^{-\gamma} c_t^{\sigma - \gamma} c_{Ft}^{\varphi} n_t^{\varphi}^* + \frac{1}{1 - \zeta^*} P_t^I^* \frac{k^* a_t^*}{q^*(\theta_t^*)} - (1 - \delta^*) E_t Q_{t, t+1} \left( 1 + \frac{\zeta^*}{1 - \zeta^*} (1 - p^*(\theta_{t+1}^*)) \right) P_t^I^* \frac{k^* a_{t+1}^*}{q^*(\theta_{t+1}^*)}.$$

In the usual way, labor market frictions generate a wedge between the marginal rate of
transformation and marginal rate of substitution between consumption and labor in each country. Moreover, provided workers have some bargaining power \((\zeta, \zeta^* > 0)\), the equilibrium wage will be high enough that households will optimally ensure all initially unemployed members participate in the labor market:

\[
\begin{align*}
u_t &= 1 - (1 - \delta)n_{t-1}, \\
u_t^* &= 1 - (1 - \delta^*)n_{t-1}^*.
\end{align*}
\]

The employment rate in each market is thus

\[
\begin{align*}n_t &= (1 - \delta)n_{t-1} + p(\theta_t)(1 - (1 - \delta)n_{t-1}), \\
n_t^* &= (1 - \delta^*)n_{t-1}^* + p^*(\theta_t^*)(1 - (1 - \delta^*)n_{t-1}^*).\end{align*}
\]

Lastly, consider final good market clearing for each variety produced by each country. Integrating over varieties, we obtain

\[
\begin{align*}\int_0^1 \gamma c_{Ht}(j) dj + \int_0^1 (1 - \gamma)c_{Ht}^*(j) dj &= \int_0^1 y_t(j) dj, \\
\int_0^1 \gamma c_{Ft}(j^*) dj + \int_0^1 (1 - \gamma)c_{Ft}^*(j^*) dj &= \int_0^1 y_t^*(j^*) dj^*.
\end{align*}
\]

Substituting in households’ optimal consumption of each variety

\[
\begin{align*}c_{Ht}(j) &= \left(\frac{P_{Ht}(j)}{P_{Ht}}\right)^{-\varepsilon} c_{Ht}, \\
c_{Ft}(j^*) &= \left(\frac{P_{Ft}(j^*)}{P_{Ft}}\right)^{-\varepsilon} c_{Ft}, \\
c_{Ht}^*(j) &= \left(\frac{P_{Ht}(j)}{P_{Ht}}\right)^{-\varepsilon} c_{Ht}^*, \\
c_{Ft}^*(j^*) &= \left(\frac{P_{Ft}(j^*)}{P_{Ft}}\right)^{-\varepsilon} c_{Ft}^*.
\end{align*}
\]

and making use of retailers’ production technologies \(y_t(j) = x_t(j)\) and \(y_t^*(j^*) = x_t^*(j^*)\) and intermediate good market clearing, we obtain

\[
\begin{align*}
\left(\int_0^1 \left(\frac{P_{Ht}(j)}{P_{Ht}}\right)^{-\varepsilon} dj\right) \left[\gamma c_{Ht} + (1 - \gamma)c_{Ht}^*\right] &= \gamma a_t \left(n_t - k\nu_t\right), \\
\left(\int_0^1 \left(\frac{P_{Ft}(j^*)}{P_{Ft}}\right)^{-\varepsilon} dj^*\right) \left[\gamma c_{Ft} + (1 - \gamma)c_{Ft}^*\right] &= (1 - \gamma)a_t^* \left(n_t^* - k^*\nu_t^*\right).
\end{align*}
\]

Defining indices of price dispersion

\[
D_{Ht} \equiv \int_0^1 \left(\frac{P_{Ht}(j)}{P_{Ht}}\right)^{-\varepsilon} dj, \tag{55}
\]
Together with results of the previous subsection, we obtain the following system of equilibria:

\[ D_{Ft} \equiv \int_0^1 \left( \frac{P_{Fl}(j^*)}{P_{Fl}} \right)^{-\varepsilon} dj^*, \]  

we have

\[ \gamma c_{Ht} + (1 - \gamma)c_{Ht}^* = \frac{1}{D_{Ht}} \gamma a_t (n_t - k\nu_t), \]
\[ \gamma c_{Ft} + (1 - \gamma)c_{Ft}^* = \frac{1}{D_{Ft}} (1 - \gamma)a_t^* (n_t^* - k^*\nu_t^*). \]

Since \( c_{Ht}^* = c_{Ht} \) and \( c_{Ft}^* = c_{Ft}^* \), and making use of the definition of market tightness in each country, it follows that

\[ c_{Ht} = \frac{1}{D_{Ht}} \gamma a_t (n_t - k(1 - (1 - \delta)n_{t-1})\theta_t), \]
\[ c_{Ft}^* = \frac{1}{D_{Ft}} (1 - \gamma)a_t^* (n_t^* - k^*(1 - (1 - \delta^*)n_{t-1}^*)\theta_t^*). \]

### B.3 Natural allocation

When all retailers can set prices after the realization of productivity shocks (\( \nu = \nu^* = 1 \)), their optimal price-setting conditions imply

\[ P_{Ht}(j) = \frac{\varepsilon}{\varepsilon - 1} (1 + \tau^r) P_{l.t}^I, \]
\[ P_{Ft}(j^*) = \frac{\varepsilon}{\varepsilon - 1} (1 + \tau^{rs}) P_{l.t}^{I*}. \]

Together with results of the previous subsection, we obtain the following system of equilibrium conditions defining the natural allocation (denoted with \( n \) superscripts):

\[ \frac{1}{\varepsilon - 1}(1 + \tau^r) a_t = \chi(\nu_t) \gamma^c (c_{Ht}^{n})^\alpha (c_{Ht}^{n})^\xi (n_t^{n})^{\phi} + \frac{1}{1 - \delta} \frac{1}{\varepsilon - 1} \frac{1}{(1 + \tau^r)} q(\theta_t^{n}) \left( 1 + \frac{\zeta^2}{1 - \zeta} (1 - p(\theta_t^{n})) \right) \frac{ka_t}{\varepsilon - 1}(1 + \tau^r) q(\theta_t^{n}), \]
\[ n_t^{n} = (1 - \delta)n_{t-1}^{n} + p(\theta_t^{n})(1 - (1 - \delta)n_{t-1}^{n}), \]
\[ c_{Ht}^{n} = \gamma a_t \left[ n_t^{n} - k(1 - (1 - \delta)n_{t-1}^{n})\theta_t^{n} \right], \]
\[ \frac{1}{\varepsilon - 1}(1 + \tau^{rs}) a_t^* = \chi^* (\nu_t^*) \gamma^c (c_{Ft}^{n})^\alpha (c_{Ft}^{n})^\xi (n_t^{n})^{\phi^*} + \frac{1}{1 - \delta} \frac{1}{\varepsilon - 1} \frac{1}{(1 + \tau^{rs})} q^*(\theta_t^{jn}) \left( 1 + \frac{\zeta^*}{1 - \zeta^*} (1 - p^*(\theta_t^{jn})) \right) \frac{k^* a_t^*}{\varepsilon - 1}(1 + \tau^{rs}) q^*(\theta_t^{jn}), \]
\[ n_t^{jn} = (1 - \delta^*)n_{t-1}^{jn} + p^*(\theta_t^{jn})(1 - (1 - \delta^*)n_{t-1}^{jn}), \]
\[ c_{Ft}^{n} = (1 - \gamma)a_t^* \left[ n_t^{jn} - k^*(1 - (1 - \delta^*)n_{t-1}^{jn})\theta_t^{jn} \right], \]
\[ s_t^n = \left( \frac{\gamma}{1 - \gamma} \frac{c_{Fi}^n}{c_{Hi}^n} \right)^{\varsigma}, \quad (64) \]

\[ c_{Fi}^n = c_{Fi}^*, \quad (65) \]

\[ c_{Hi}^n = c_{Hi}^*, \quad (66) \]

\[ c_t^n = \left( (\gamma)^{\varsigma} (c_{Hi}^n)^{1-\varsigma} + (1 - \gamma)^{\varsigma} (c_{Fi}^n)^{1-\varsigma} \right)^{\frac{1}{1-\varsigma}}, \quad (67) \]

\[ c_t^* = \left( (\gamma)^{\varsigma} (c_{Hi}^*)^{1-\varsigma} + (1 - \gamma)^{\varsigma} (c_{Fi}^*)^{1-\varsigma} \right)^{\frac{1}{1-\varsigma}}. \quad (68) \]

As is evident, the natural allocation is fully determined without reference to nominal prices and wages in the global economy, reflecting a standard real/nominal dichotomy.

**B.4 Sticky price equilibrium**

We now turn to the case with more general \( \iota \) and \( \iota^* \). Consider the problem facing a retailer which can update its price \( P_{Hi} \) in period \( t \). Its optimal price satisfies

\[
\mathbb{E}_t \sum_{s=t}^{\infty} (1 - \iota)^{s-t} Q_{t,s} y_s \left( P_{Hi} - \frac{\epsilon}{\epsilon - 1} (1 + \tau^r) P_s^t \right) = 0
\]

where

\[ y_s = \left( \frac{P_{Hi}}{P_{Hi}^s} \right)^{-\epsilon} (\gamma c_{Hi}^s + (1 - \gamma) c_{Hi}^s). \]

Since only a randomly drawn fraction \( \iota \) of retailers can update prices each period, the producer-price index in (14) evolves according to

\[(P_{Hi})^{1-\epsilon} = (1 - \iota)(P_{Ht-1})^{1-\epsilon} + \iota(P_{Hi})^{1-\epsilon}, \]

while the index of price dispersion in (55) evolves according to

\[ D_{Hi} = (1 - \iota)D_{Ht-1} \left( \frac{P_{Ht-1}}{P_{Ht}} \right)^{-\epsilon} + \iota \left( \frac{P_{Hi}}{P_{Ht}} \right)^{-\epsilon}. \]

Analogous conditions obtain in Foreign. Given the other conditions derived in appendix B.2, it is straightforward to summarize the equilibrium with a system in which the real and nominal sides of the global economy are now jointly determined, given a particular choice of union-wide monetary policy summarized by a state-contingent path of the riskless nominal interest rate

\[ 1 + i_t = \frac{1}{\mathbb{E}_t Q_{t,t+1}}. \]

Here, I proceed directly to characterize the set of implementable real allocations given
possible paths of \( \{i_t\} \), serving as constraints in the optimal policy problem studied in the main text:

\[
\frac{P_{t}^{I}}{P_{Ht}} a_{t} = \chi \gamma^{-c}(c_{t})^{\sigma-c}(c_{Ht})^{c}(n_{t})^{\varphi} + \frac{1}{1-\zeta} \frac{P_{t}^{I}}{\varrho q(\theta_{t})} k a_{t} - (1-\delta)E_{t} \beta \frac{(c_{t})^{\sigma-c}(c_{Ht})^{c}}{(c_{t+1})^{\sigma-c}(c_{Ht+1})^{c}} \left( 1 + \frac{\zeta}{1-\zeta} (1-p(\theta_{t+1})) \right) \frac{P_{t+1}^{I}}{P_{Ht+1}^{I}} k a_{t+1},
\]

\( n = (1-\delta)n_{t-1} + p(\theta_{t})(1-(1-\delta)n_{t-1}) \),

\( c_{Ht} = \frac{1}{D_{Ht}} \gamma a_{t} \left[ n_{t} - k(1-(1-\delta)n_{t-1}) \theta_{t} \right] \),

\[
\frac{P_{t}^{I}}{P_{Ft}} a_{t}^{*} = \chi^{*}(1-\gamma)^{-c}(c_{t}^{*})^{\sigma-c}(c_{Ft}^{*})^{c}(n_{t}^{*})^{\varphi^{*}} + \frac{1}{1-\zeta^{*}} \frac{P_{t}^{I*}}{\varrho^{*} q^{*}(\theta_{t}^{*})} k^{*} a_{t}^{*} - (1-\delta^{*})E_{t} \beta \frac{(c_{t}^{*})^{\sigma-c}(c_{Ft}^{*})^{c}}{(c_{t+1})^{\sigma-c}(c_{Ft+1})^{c}} \left( 1 + \frac{\zeta^{*}}{1-\zeta^{*}} (1-p^{*}(\theta_{t+1}^{*})) \right) \frac{P_{t+1}^{I*}}{P_{Ft+1}^{I*}} k^{*} a_{t+1}^{*},
\]

\( n^{*} = (1-\delta^{*})n_{t-1}^{*} + p^{*}(\theta_{t}^{*})(1-(1-\delta^{*})n_{t-1}^{*}) \),

\( c_{Ft}^{*} = \frac{1}{D_{Ft}} (1-\gamma) a_{t}^{*} \left[ n_{t}^{*} - k^{*}(1-(1-\delta^{*})n_{t-1}^{*}) \theta_{t}^{*} \right] \),

\( s_{t} = \left( \frac{\gamma}{1-\gamma} \frac{c_{Ft}^{*}}{c_{Ht}} \right)^{\varsigma} \),

\( c_{Ft} = c_{Ft}^{*} \),

\( c_{Ht} = c_{Ht}^{*} \),

\( c_{t} = ((\gamma)^{c}(c_{Ht})^{1-\varsigma} + (1-\gamma)^{c}(c_{Ft})^{1-\varsigma}) \frac{1}{1-\varsigma} \),

\( c_{t}^{*} = ((\gamma)^{c}(c_{Ht}^{*})^{1-\varsigma} + (1-\gamma)^{c}(c_{Ft}^{*})^{1-\varsigma}) \frac{1}{1-\varsigma} \),

\[
\mathbb{E}_{t} \sum_{s=t}^{\infty} (1-\ell)^{s-t} \beta^{s-t} \frac{(c_{t})^{\sigma-c}(c_{Ht})^{c}}{(c_{s})^{\sigma-c}(c_{Hs})^{c}} \left[ \left( \frac{P_{Ht}}{P_{Hs}} \right)^{-\varphi} (\gamma c_{Hs} + (1-\gamma) c_{Hs}) \right] \times \left( \frac{P_{Ht}}{P_{Hs}} - \frac{\ell}{\epsilon - 1} \right)^{\varphi} = 0,
\]

\( (P_{Ht})^{1-\varphi} = (1-\ell) (P_{Ht-1})^{1-\varphi} + \ell (P_{Ht})^{1-\varphi} \),

\( D_{Ht} = (1-\ell) D_{Ht-1} \left( \frac{P_{Ht-1}}{P_{Ht}} \right)^{-\varphi} + \ell \left( \frac{P_{Ht}}{P_{Ht}} \right)^{-\varphi} \),

\[
\mathbb{E}_{t} \sum_{s=t}^{\infty} (1-\ell)^{s-t} \beta^{s-t} \frac{(c_{t}^{*})^{\sigma-c}(c_{Ft}^{*})^{c}}{(c_{s}^{*})^{\sigma-c}(c_{Ft}^{*})^{c}} \left[ \left( \frac{P_{Ft}}{P_{Fs}} \right)^{-\varphi^{*}} (\gamma c_{Fs} + (1-\gamma) c_{Fs}^{*}) \right] \times \left( \frac{P_{Ft}}{P_{Fs}} - \frac{\ell}{\epsilon - 1} \right)^{\varphi^{*}} = 0,
\]

\( (P_{Ft})^{1-\varphi^{*}} = (1-\ell^{*}) (P_{Ft-1})^{1-\varphi^{*}} + \ell^{*} (P_{Ft})^{1-\varphi^{*}} \),

63
\[ D_{Ft} = (1 - \iota^*) D_{Ft-1} \left( \frac{P_{Ft-1}}{P_{Ft}} \right)^{-\varepsilon} + \iota^* \left( \frac{P_{Ft}}{P_{Ft}} \right)^{-\varepsilon}, \]  

\( s_t = \frac{P_{Ht}}{P_{Ft}}. \)  

The monetary policy which implements such an allocation is simply given by

\[ 1 + \iota_t = \frac{1}{\mathbb{E}_t \beta \frac{P_t}{P_{t+1}} \frac{c_{t+1}}{c_t}} = \frac{1}{\mathbb{E}_t \beta \frac{P_t}{P_{t+1}} \frac{c_{t+1}}{c_t}}, \]

where it is straightforward to verify that second equality is satisfied at the given allocation using the equilibrium conditions above.

### B.5 Wage determination

Finally, I derive the Nash bargained wages used in the derivation of equilibrium. I focus on the problem at Home; the wage at Foreign can be derived analogously.

Following Shimer (2010), at Home let \( \tilde{v}_{nt}(\hat{W}) \) denote the marginal value to the representative household of employing an additional worker at wage \( \hat{W} \) and the equilibrium wage \( \{W_s\}_{s=t+1}^{\infty} \) thereafter, and let \( \tilde{J}_{nt}(\hat{W}) \) denote the marginal value to the representative producer of employing an additional worker at wage \( \hat{W} \) and the equilibrium wage \( \{W_s\}_{s=t+1}^{\infty} \) thereafter. Then Nash bargained wages solve

\[ W_t = \underset{\hat{W}}{\text{arg max}} \left[ \tilde{v}_{nt}(\hat{W}) \right]^{\xi} \left[ \tilde{J}_{nt}(\hat{W}) \right]^{1-\xi}. \]  

(87)

To compute \( \tilde{v}_{nt}(\hat{W}) \), let \( \hat{v}_t(B, n, W; \epsilon, \hat{W}) \) be the value to the representative household of \( n \) workers employed at wage \( W \) and \( \epsilon \) workers employed at wage \( \hat{W} \) in period \( t \). Then

\[ \hat{v}_t(B, n, W; \epsilon, \hat{W}) = \max_{\{c_H(j)\},\{c_F(j^*)\},B_{t+1}} u(c, n + \epsilon) + \beta \mathbb{E}_t v_{t+1}(B_{t+1}, n + \epsilon) \text{ s.t.} \]

\[ \int_0^1 P_{Ht}(j) c_{H}(j) dj + \int_0^1 P_{Ft}(j^*) c_{F}(j^*) dj^* + \mathbb{E}_t Q_{t,t+1} B_{t+1} \leq Wn + \hat{W} \epsilon + B - T_t, \]

so we have

\[ \tilde{v}_{nt}(\hat{W}) \equiv \frac{\partial}{\partial \epsilon} \hat{v}_t(B_t, n_t, W_t; \epsilon, \hat{W})|_{\epsilon=0}. \]  

(88)

To compute \( \tilde{J}_{nt}(\hat{W}) \) at Home, let \( \hat{J}_t(n, W; \epsilon, \hat{W}) \) be the profit for the representative producer of \( n \) workers employed at \( W \) and \( \epsilon \) workers employed at \( \hat{W} \) after vacancy posting
costs have been sunk. Then

\[ \hat{J}_t(n, W; \epsilon, \hat{W}) = (P_t^I a_t - W)n + (P_t^I a_t - \hat{W})\epsilon + \mathbb{E}_t Q_{t,t+1} J_{t+1}(n + \epsilon), \]

so we have

\[ \tilde{J}_{nt}(\hat{W}) \equiv \frac{\partial}{\partial \epsilon} \hat{J}_t(n_t, W_t; \epsilon, \hat{W})|_{\epsilon=0}. \]  

(89)

Now, evaluating the right-hand side of (88) using the Envelope Theorem, the marginal value to the representative household of an additional worker employed at \( \hat{W} \) is

\[ \tilde{v}_{nt}(\hat{W}) = \frac{\hat{W}}{P_t^I} u_{ct} + u_{nt} + \mathbb{E}_t v_{nt+1}(B_{t+1}, n_t). \]

Evaluating the right-hand side of (89), the marginal value to the representative producer of an additional worker employed at \( \hat{W} \) is

\[ \tilde{J}_{nt}(\hat{W}) = P_t^I a_t - \hat{W} + \mathbb{E}_t Q_{t,t+1} J_{nt+1}(n_t). \]

It follows that the maximization problem in (87) yields

\[ \hat{W} + P_t^I \frac{u_{nt}}{u_{ct}} + P_t^I \frac{1}{u_{ct}} \beta \mathbb{E}_t v_{nt+1}(B_{t+1}, n_t) = \frac{\zeta}{1 - \zeta} \left( P_t^I a_t - \hat{W} + \mathbb{E}_t Q_{t,t+1} J_{nt+1}(n_t) \right). \]

Evaluating this at the equilibrium wage \( \hat{W} = W_t \) yields

\[ W_t + P_t^I \frac{u_{nt}}{u_{ct}} + P_t^I \frac{1}{u_{ct}} \beta \mathbb{E}_t v_{nt+1}(B_{t+1}, n_t) = \frac{\zeta}{1 - \zeta} P_t^I \frac{ka_t}{q(\theta_t)}. \]  

(90)

where I have used the representative producer’s first-order condition with respect to vacancies \( \nu_t \) on the right hand side. Then, since

\[ v_{nt+1}(B_{t+1}, n_t) = (1 - \delta)(1 - p(\theta_{t+1})) \frac{1}{P_{t+1}} u_{ct+1} \times \]

\[ \left[ W_{t+1} + P_{t+1} \frac{u_{nt+1}}{u_{ct+1}} + P_{t+1} \frac{1}{u_{ct+1}} \beta \mathbb{E}_{t+1} v_{nt+2}(B_{t+2}, n_{t+1}) \right], \]

\[ = (1 - \delta)(1 - p(\theta_{t+1})) \frac{1}{P_{t+1}} u_{ct+1} \frac{\zeta}{1 - \zeta} P_{t+1}^I \frac{ka_{t+1}}{q(\theta_{t+1})} \]

where I have used the Envelope Theorem in the first equality and (90) at \( t + 1 \) in the second, we can write (90) as

\[ W_t + P_t^I \frac{u_{nt}}{u_{ct}} + \mathbb{E}_t Q_{t,t+1} (1 - \delta)(1 - p(\theta_{t+1})) \frac{\zeta}{1 - \zeta} P_{t+1}^I \frac{ka_{t+1}}{q(\theta_{t+1})} = \frac{\zeta}{1 - \zeta} P_t^I \frac{ka_t}{q(\theta_t)}. \]
where I have used $Q_{t,t+1} = \frac{P_{t+1} - P_t}{P_t} \pi_{t+1}$ by households’ intertemporal optimization. Re-arranging yields the Nash bargained wage described in appendix B.2.

C First-order conditions characterizing optimal policy

In this appendix I characterize the first-order conditions of maximizing the quadratic objective in Lemma 4 subject to the linear implementability constraints in Lemmas 1-3, implicitly defining the Ramsey optimal allocation. Letting $\psi_{\pi t}$, $\psi_{\mu}$, $\psi_{\pi t}$, $\psi_{\mu t}$, and $\psi_{st}$ denote the multipliers on the constraints, we obtain:

\[
\omega \frac{\epsilon - \pi_{\pi t}}{\lambda} + \psi_{\pi t} - \psi_{\pi t-1} - \psi_{st} = 0, \tag{91}
\]

\[
- \lambda \psi_{\pi t} + \psi_{\mu} + \frac{\epsilon_{n-1}^f}{\epsilon_n^f} \psi_{\mu t-1} = 0, \tag{92}
\]

\[
\omega \left[ \phi \left( \epsilon_n^f \right)^2 \tilde{n}_t + \beta \phi_{-1} \epsilon_n^f \epsilon_{n-1}^f \mathbb{E}_t \tilde{n}_{t+1} + \phi_{-1} \epsilon_n^f \epsilon_{n-1}^f \tilde{n}_{t-1} \right] \\
+ \omega (1 - \omega) \left[ (\sigma - \varsigma) \left[ \phi \epsilon_n^f \epsilon_n^* \tilde{n}_t^* + \epsilon_n^f \epsilon_n^* \tilde{n}_{t-1}^* + \beta \epsilon_n^f \epsilon_n^* \mathbb{E}_t \tilde{n}_t^* \right] \\
- \phi \epsilon_n^f \psi_{\mu} - \beta \phi_{-1} \epsilon_n^f \mathbb{E}_t \psi_{\mu t+1} - \phi_{-1} \epsilon_n^f \psi_{\mu t-1} \\
- \omega (\sigma - \varsigma) \left[ \phi \epsilon_n^f \psi_{\mu^* t} + \epsilon_{n-1}^f \epsilon_n^* \psi_{\mu^* t-1} + \beta \epsilon_n^f \epsilon_n^* \mathbb{E}_t \psi_{\mu^* t+1} \right] \\
- \varsigma \left[ \epsilon_n^f \psi_{st} + \beta \left( -\epsilon_n^f + \epsilon_n^f \right) \mathbb{E}_t \psi_{st+1} - \beta^2 \epsilon_n^f \mathbb{E}_t \psi_{st+2} \right] = 0, \tag{93}
\]

\[
(1 - \omega) \frac{\epsilon}{\lambda} \pi_{\pi t} + \psi_{\pi t} - \psi_{\pi t-1} + \psi_{st} = 0, \tag{94}
\]

\[
- \lambda \psi_{\mu} + \psi_{\mu^*} + \frac{\epsilon_{n-1}^f}{\epsilon_n^f} \psi_{\mu^* t-1} = 0, \tag{95}
\]

\[
(1 - \omega) \left[ \phi^* \left( \epsilon_n^* \right)^2 \tilde{n}_t^* + \beta \phi_{-1} \epsilon_n^* \epsilon_{n-1}^* \mathbb{E}_t \tilde{n}_{t+1}^* + \phi_{-1} \epsilon_n^* \epsilon_{n-1}^* \tilde{n}_{t-1}^* \right] \\
+ \omega (1 - \omega) \left[ (\sigma - \varsigma) \left[ \phi^* \epsilon_n^* \epsilon_n^* \tilde{n}_t^* + \epsilon_n^* \epsilon_n^* \tilde{n}_{t-1}^* + \beta \epsilon_n^* \epsilon_n^* \mathbb{E}_t \tilde{n}_t^* \right] \\
- \phi^* \epsilon_n^* \psi_{\mu^*} - \beta \phi_{-1} \epsilon_n^* \mathbb{E}_t \psi_{\mu^* t+1} - \phi_{-1} \epsilon_n^* \psi_{\mu^* t-1} \\
- (1 - \omega) \left[ (\sigma - \varsigma) \left[ \phi^* \epsilon_n^* \psi_{\mu} + \epsilon_{n-1}^f \epsilon_n^* \psi_{\mu t+1} + \beta \epsilon_n^f \epsilon_n^* \mathbb{E}_t \psi_{\mu t+1} \right] \\
+ \varsigma \left[ \epsilon_n^* \psi_{st} + \beta \left( -\epsilon_n^* + \epsilon_n^* \right) \mathbb{E}_t \psi_{st+1} - \beta^2 \epsilon_n^* \mathbb{E}_t \psi_{st+2} \right] \right] = 0, \tag{96}
\]

where $\psi_{\pi t-1} = \psi_{\mu-1} = \psi_{\pi p-1} = \psi_{\mu^*-1} = 0$. 
D Supplemental numerical results

In this appendix I provide supplemental numerical results for the exploration of the optimal monetary policy in section 4.3 of the main text.

D.1 Additional comparative statics

I first provide additional comparative statics of interest as Home’s labor market grows more sclerotic, accompanying Figure 2 in the main text.

Figure 8 provides the comparative statics for $\phi_{-1}$. We see that a more sclerotic labor market features more negative $\phi_{-1}$. Provided that employment is persistent, this offsets the amplified $\phi$ in Figure 2 in determining the welfare losses and effects on real marginal cost from employment fluctuations.

Figure 9 provides the comparative statics for the employment rates in each country $n$ and $n^*$, as well as the global expenditure share on Home-produced goods $\omega$. As is standard, higher $k$ and lower $\delta$ feature offsetting effects on $n$, so that a more sclerotic labor market need not have higher steady-state unemployment.\(^{35}\) Because $\sigma > \varsigma$ in this parameterization, the risk-sharing effects of changes in foreign employment dominate their terms of trade effects, so that greater employment at Home raises marginal cost and thus lowers employment in Foreign. It follows that greater employment at Home is associated with greater relative

\(^{35}\)Quantitatively, I also note that the variation in employment is well within the variation observed in Eurozone economies summarized in Table 1.
Figure 9: $\bar{n}$, $\bar{n}^*$, and $\omega$ as Home’s labor market becomes more sclerotic

Note: $\omega$ is the global expenditure share on Home-produced goods. A more sclerotic labor market is one with higher $k$ or lower $\delta$, and thus involves moving from the ends to the center of the figure.
Figure 10: $\phi^*$ and $\phi^*_{-1}$ as Home’s labor market becomes more sclerotic

Note: $\phi^*$ and $\phi^*_{-1}$ control the welfare costs from and sensitivity of real marginal cost to employment fluctuations in Foreign. Their formulas are given in the proof of Lemma 2. A more sclerotic labor market is one with higher $k$ or lower $\delta$, and thus involves moving from the ends to the center of the figure.

production of Home-produced goods. Since $\varsigma < 1$ in this parameterization, this raises the global expenditure share on Home-produced goods $\omega$.

Figure 10 then provides the comparative statics for $\phi^*$ and $\phi^*_{-1}$. Higher employment in Home is associated with lower $\phi^*$ and $\phi^*_{-1}$ because $\sigma > \varsigma$ and $\omega$ is rising in Home employment. However, we see that these spillovers are modest in size.

**D.2 Alternative values of $\sigma$ or $\varsigma$**

I now characterize how the relationship between labor market frictions and the optimal policy varies with the coefficient of relative risk aversion $\sigma$ and inverse trade elasticity $\varsigma$. I continue to focus on the optimal weight on Home producer-price inflation $\xi$ in the rule (33).

Recall that in the parameterization in the main text, I set $\sigma = 1$ and $\varsigma = (1.5)^{-1}$. The
Figure 11: optimal $\xi$ as Home’s labor market becomes more sclerotic

Note: the optimal $\xi$ is computed by minimizing the average welfare loss across many histories of shocks. Shaded markers depict the symmetric benchmark in which $k = k^* = 0.12$ and $\delta = \delta^* = 0.030$. A more sclerotic labor market is one with higher $k$ or lower $\delta$, thus moving from the ends to the center of the figure.

former is consistent with balanced growth. The latter is consistent with Backus et al. (1994) and used extensively in the international macro literature.

For alternative parameterizations still within the typical range of the literature, I continue to find that the optimal $\xi$ rises as Home’s labor market grows more sclerotic. Figure 11 reproduces this result for the baseline parameterization and demonstrates that the qualitative patterns remain the same for $\sigma = 3$ or $\varsigma = (4)^{-1}$. The former is consistent with some of the lowest estimates of the intertemporal elasticity of substitution in the literature (see, for instance, the survey in Hall (2009)). The latter is consistent with trade elasticity estimates in the international trade literature (see, for instance, the survey in Costinot and Rodriguez-Clare (2014)), which typically estimate a trade elasticity higher than that estimated in the international macro literature.

However, for sufficiently low $\varsigma$ I find that the optimal $\xi$ falls as Home’s labor market grows more sclerotic, consistent with Proposition 8. For instance, for $\varsigma = (10)^{-1}$, Figure 11 demonstrates that the optimal $\xi$ falls as Home’s labor market grows more sclerotic. Figure 12 depicts the impulse responses under this optimal rule when $\varsigma = (10)^{-1}$. As is evident, when Home’s labor market is more sclerotic the optimal policy now accommodates larger relative inflation/deflation at Home. Nonetheless, even in this case in terms of output and employment the optimal policy continues to target smaller distortions at Home than in Foreign, consistent with the discussion of this result in the main text.
Figure 12: response to depreciation in Home’s natural terms of trade (optimal $\xi$, $\varsigma = (10)^{-1}$)

Note: depreciation in Home’s natural terms of trade induced by 1% positive innovation to $a_t$ at $t = 0$. 

71
E Real wage rigidity

In this appendix I demonstrate that the paper’s result on relative accommodation of the more sclerotic union member is robust to real wage rigidity.

E.1 Environment and equilibrium revisited

The environment is exactly as in section 2, except product wages are given by

\[
\frac{W_t}{P_{Ht}} = (1 - \alpha)w + \alpha \frac{W_{nb}^t}{P_{Ht}},
\]

\[
\frac{W^*_t}{P_{Ft}} = (1 - \alpha^*)w^* + \alpha^* \frac{W_{nb}^{*t}}{P_{Ft}},
\]

where \(W_{nb}^t\) and \(W_{nb}^{*t}\) are the wages which would be obtained under Nash bargaining and \(w\) and \(w^*\) are the product wages in the deterministic steady-state.

The Nash bargained nominal wage \(W_{nb}^t\) derived in appendix B.5 is

\[
W_{nb}^t = P_{Ht} \chi^{-\gamma} c_t^{\sigma - \gamma} c_{Ht} n_{it}^\phi + \frac{\zeta}{1 - \zeta} \left[ P_{t}^I \frac{ka_t}{q(\theta_t)} - E_t Q_{t,t+1} (1 - \delta)(1 - p(\theta_{t+1})) P_{t+1}^I \frac{ka_{t+1}}{q(\theta_{t+1})} \right]
\]

Combined with firms’ optimal vacancy posting, labor market equilibrium is thus characterized by

\[
P_I^t a_t = P_{Ht} \chi^{-\gamma} c_t^{\sigma - \gamma} c_{Ht} n_{it}^\phi + \frac{1}{1 - \zeta} P_t^I \frac{ka_t}{q(\theta_t)} - (1 - \delta) E_t Q_{t,t+1} \left( 1 + \frac{\zeta}{1 - \zeta} (1 - p(\theta_{t+1})) \right) P_{t+1}^I \frac{ka_{t+1}}{q(\theta_{t+1})} + (W_t - W_{nb}^t).
\]

Given the form of the equilibrium wage \(W_t\), it follows that

\[
P_I^t a_t = P_{Ht} \chi^*(1 - \gamma)^{-\gamma} c_t^{\sigma - \gamma} c_{Ht}^{\phi} n_{it}^{\phi^*} + \frac{1}{1 - \zeta^*} P_t^I \frac{k a_t^*}{q^*(\theta_t^*)} - (1 - \delta^*) E_t^* Q_{t,t+1} \left( 1 + \frac{\zeta^*}{1 - \zeta^*} (1 - p^*(\theta_{t+1})) \right) P_{t+1}^I \frac{k a_{t+1}^*}{q^*(\theta_{t+1})} + (W_t - W_{nb}^t).
\]

Analogously, in Foreign, we have

\[
W_{nb}^{*t} = P_{Ft}^* \chi^*(1 - \gamma)^{-\gamma} c_t^{\sigma^* - \gamma} c_{Ft}^{\phi^*} n_{it}^{\phi^*} + \frac{\zeta^*}{1 - \zeta^*} \left[ P_t^{I*} \frac{k a_t^{*t}}{q^*(\theta_t^*)} - E_t^* Q_{t,t+1} (1 - \delta^*)(1 - p^*(\theta_{t+1})) P_{t+1}^{I*} \frac{k a_{t+1}^{*t}}{q^*(\theta_{t+1})} \right].
\]
The remainder of the equilibrium is as characterized in appendix B.

### E.2 Stabilization trade-offs and optimal policy

We study the stabilization problem in this environment again employing a linear-quadratic approximation.

We replace Assumption 2 with:

**Assumption E.1.** A retailer subsidy offsets the distortions from monopolistic competition ($\tau^{*} = \tau^{*} = -\frac{1}{2}$) and the Hosios condition is satisfied in each country ($\zeta = 1 - \eta$ and $\zeta^{*} = 1 - \eta^{*}$).

This ensures that in steady-state, the natural allocation (which has flexible prices but rigid real wages) remains constrained efficient as in the main text. However, this allocation will now be generically inefficient in response to shocks because of the rigidity in real wages. For that reason, deviations from the natural allocation will no longer be the appropriate measure of inefficiency in this economy. For any variable $z_t$, we thus redefine the tilde notation to be

\[
\tilde{z}_t \equiv \log z_t - \log z_t^{ce}
\]

and further define

\[
\tilde{z}_t^{ce} \equiv \log z_t^{ce} - \log z,
\]

where the $ce$ superscript denotes the value of that variable in the allocation with flexible prices, no rigidity in real wages, and assumption E.1, which is constrained efficient.

We continue to make Assumption 3 as in the main text. In the results which follow, it will be useful to refer to the product wages $w_t \equiv \frac{W}{P_Ht}$ and $w^*_t \equiv \frac{W^*_t}{P^{*}_{Ft}}$ and Nash bargained product wages $w^{nb}_t \equiv \frac{W^{nb}_t}{P_Ht}$ and $w^{*nb}_t \equiv \frac{W^{*nb}_t}{P^{*}_{Ft}}$.

#### E.2.1 Implementability constraints

Lemma 1 characterizing the New Keynesian Phillips curves is unchanged.

Lemma 2 must now be generalized to account for real wage rigidity in the labor market:
Lemma E.1. Up to first order around the deterministic steady-state,

\[
\ddot{\mu}_t + \frac{\epsilon_n^{l_{-1}}}{\epsilon_n} \beta \bar{E}_t \dot{\mu}_{t+1} = \phi \epsilon_n^f \ddot{n}_t + \phi_{-1} \epsilon_{n-1}^f \ddot{n}_{t-1} + \phi_{-1} \epsilon_{n-1}^f \beta \bar{E}_t \ddot{n}_{t+1}
\]

\[
+ (\sigma - \varsigma) (1 - \omega) \left[ \phi' \epsilon_n^f \ddot{n}_t + \epsilon_{n-1}^f \ddot{n}_{t-1} + \frac{\epsilon_n^f}{\epsilon_n} \beta \bar{E}_t \ddot{n}_{t+1} \right]
\]

\[- (1 - \alpha) \frac{w_n}{af(n,n)} \frac{1}{\epsilon_n^f} \ddot{w}^{nb}_t,
\]

\[
\ddot{\mu}_t + \frac{\epsilon_n^{l_{-1}}}{\epsilon_n} \beta \bar{E}_t \dot{\mu}_{t+1} = \phi^* \epsilon_n^f \ddot{n}_t + \phi_{-1} \epsilon_{n-1}^f \ddot{n}_{t-1} + \phi_{-1} \epsilon_{n-1}^f \beta \bar{E}_t \ddot{n}_{t+1}
\]

\[
+ (\sigma - \varsigma) \omega \left[ \phi' \epsilon_n^f \ddot{n}_t + \epsilon_{n-1}^f \ddot{n}_{t-1} + \frac{\epsilon_n^f}{\epsilon_n^*} \epsilon_n^f \beta \bar{E}_t \ddot{n}_{t+1} \right]
\]

\[- (1 - \alpha^*) \frac{w^* n^*}{a^* f^*(n^*,n^*)} \frac{1}{\epsilon_n^*} \ddot{w}^{nb}_t,
\]

where \( \phi, \phi_{-1}, \phi^*, \phi^*_{-1}, \) and \( \phi' \) are as defined in Lemma 2.

There are two differences relative to Lemma 2. First, we express these dynamics in terms of employment distortions relative to the constrained efficient allocation, rather than the natural allocation. Second, real wage rigidity together with adjustment in the Nash bargained wages mean that generically there must be inefficiency in employment and/or distortions in real marginal cost, which in turn drives inflation according to Lemma 1.

We now characterize the adjustment in Nash bargained product wages:

Lemma E.2. Up to first order around the deterministic steady-state,

\[
\ddot{w}^{nb}_t = \ddot{w}^{nce} + \epsilon_n^w \ddot{\mu}_t + \epsilon_{n+1}^w \beta \bar{E}_t \dot{\mu}_{t+1} + \epsilon_n^w \epsilon_n^f \ddot{n}_t + \epsilon_{n-1}^w \epsilon_n^f \ddot{n}_{t-1} + \epsilon_n^w \epsilon_n^f \beta \bar{E}_t \ddot{n}_{t+1}
\]

\[
+ \epsilon_n^w \epsilon_n^f \ddot{n}^* + \epsilon_{n-1}^w \epsilon_n^f \ddot{n}^*_{t-1} + \epsilon_n^w \epsilon_n^f \beta \bar{E}_t \ddot{n}^*_{t+1},
\]

\[
\ddot{w}^{nb}_t = \ddot{w}^{nce} + \epsilon_n^w \ddot{\mu}_t + \epsilon_{n+1}^w \beta \bar{E}_t \dot{\mu}_{t+1} + \epsilon_n^w \epsilon_n^f \ddot{n}_t + \epsilon_{n-1}^w \epsilon_n^f \ddot{n}_{t-1} + \epsilon_n^w \epsilon_n^f \beta \bar{E}_t \ddot{n}_{t+1}
\]

\[
+ \epsilon_n^w \epsilon_n^f \ddot{n}^* + \epsilon_{n-1}^w \epsilon_n^f \ddot{n}^*_{t-1} + \epsilon_n^w \epsilon_n^f \beta \bar{E}_t \ddot{n}^*_{t+1},
\]

where

\[
\epsilon_n^w = (1 - \Phi) \frac{1}{1 - (1 - \delta) \beta (1 - p(\theta))},
\]

\[
\epsilon_{n+1}^w = -(1 - \Phi) \frac{(1 - \delta) (1 - p(\theta))}{1 - (1 - \delta) \beta (1 - p(\theta))},
\]

\[
\epsilon_n^w = \Phi \left( \sigma + \frac{\varphi}{\epsilon_n^f} - (\sigma - \varsigma) (1 - \omega) \right)
\]

74
Lemma E.3. Up to first order around the deterministic steady-state,

\[
\delta_t^{ce} - \delta_{t-1}^{ce} = \left( \pi_{Ht} + \varsigma \left[ \epsilon_n^t \tilde{n}_t + \epsilon_{n-1}^t \tilde{n}_{t-1} - \epsilon_{n-1}^t \tilde{n}_{t-2} \right] \right)
- \left( \pi_{Ft} + \varsigma \left[ \epsilon_n^t \tilde{n}_t^* + \epsilon_{n-1}^t \tilde{n}_{t-1}^* - \epsilon_{n-1}^t \tilde{n}_{t-2}^* \right] \right).
\]

where \( \Phi \equiv \frac{\chi_n \varsigma - \varsigma \epsilon_n^t}{\epsilon_n} \) is the steady-state ratio of the marginal rate of substitution between consumption and labor and the product wage, with analogous expressions for \( \epsilon_n^w \).

Absent inefficiency in real marginal cost and employment, the Nash bargained product wage is given by the constrained efficient product wage. Inefficiency in real marginal cost and employment change the Nash bargained product wage based on their impact on the marginal rate of substitution between consumption and labor and on present and future hiring costs.

Finally, Lemma 3 is replaced by

**Lemma E.3.** Up to first order around the deterministic steady-state,
We now have on the left-hand side the constrained efficient terms of trade and on the right-hand side employment distortions relative to the constrained efficient allocation.

### E.2.2 Social welfare

Lemma 4 is replaced by

**Lemma E.4.** Up to second order around the deterministic steady-state,

\[
U_0 - U = -\frac{c^{1-\sigma}}{2} \sum_{t=0}^{\infty} E_t \beta^t \left[ \omega \left( \frac{\varepsilon}{\lambda} (\pi_{Ht})^2 + \phi \left( \epsilon_n^f \right)^2 \left( \bar{n}_t \right)^2 + 2\beta \phi_{-1} \epsilon_n^f \epsilon_n^{f*} \bar{n}_t \bar{n}_{t+1} \right) 
+ (1 - \omega) \left( \frac{\varepsilon}{\lambda^*} (\pi_{Ft})^2 + \phi^* \left( \epsilon_n^{f*} \right)^2 \left( \bar{n}_t^* \right)^2 + 2\beta \phi^*_{-1} \epsilon_n^{f*} \epsilon_n^{f*} \bar{n}_t \bar{n}_{t+1} \right) 
+ 2\omega (1 - \omega) (\sigma - \varsigma) \left( \phi' \epsilon_n^{f*} \bar{n}_t \bar{n}_{t+1} + \epsilon_n^{f*} \epsilon_n^{f*} \bar{n}_t \bar{n}_{t+1} + \beta \epsilon_n^{f*} \epsilon_n^{f*} \bar{n}_t \bar{n}_{t+1} \right) \right] + \text{tips}
\]

where \text{tips} denotes terms independent of policy.

This is the same as Lemma 4 except again inefficiency in employment is captured by its deviation from the constrained efficient rather than natural allocation.

### E.2.3 Optimal policy

We now consider the problem of maximizing the welfare objective in Lemma E.4 subject to the implementability constraints in Lemmas 1 and E.1-E.3. As is evident, even if the constrained efficient terms of trade are constant, there will be distortions in this environment if the constrained efficient product wages adjust. And because of the monetary union, it will generally be the case that the optimal policy will call for distortions in both countries even if only the constrained efficient product wage in one country needs to adjust.

The first-order conditions characterizing the optimal policy generalize (91)-(96) in appendix C. Here we numerically explore the optimal policy by introducing real wage rigidity into the analysis of section 4.7. We assume the same parameters as in the numerical analysis of that section except for \(\alpha = \alpha^* < 1\). We obtain two main insights.

First, the result on relative accommodation of the more sclerotic labor market remains robust to symmetric real wage rigidity across the union. We continue to focus on the optimal inflation targeting rule of the form (33). In Figure 13, we compare the optimal weight on Home producer-price inflation \(\xi\) absent real wage rigidity \((\alpha = \alpha^* = 1)\), reproduced from Figure 3, with that under higher degrees of real wage rigidity \((\alpha = \alpha^* = 0.75\) and \(\alpha = \alpha^* = 0.5\)). It remains the case that more weight should be placed on Home as its labor
market grows more sclerotic. In the case of a higher \( k \), this result is in fact amplified with more real wage rigidity.

Second, while the optimal policy departs from an inflation targeting rule in the presence of real wage rigidity (even with symmetric countries), it remains the case that putting more weight on the more sclerotic labor market eliminates incremental welfare losses from labor market heterogeneity. I again summarize the consumption-equivalent of welfare losses under an alternative policy to that under the optimal policy with \( \psi \), given by (34). In Figure 14 I characterize \( \psi \) under the HICP-targeting rule \( \xi = \omega \) and the optimal inflation targeting rule when \( \alpha = \alpha^* = 0.5 \), to be compared to the case without real wage rigidity in Figure 5 in the main text. First, we see that in the symmetric benchmark, a positive value of \( \psi \) indicates that the inflation targeting rules are suboptimal, consistent with the suboptimality of inflation targeting found by Blanchard and Gali (2010) in a closed economy with real wage rigidity (though I note that HICP-targeting remains optimal within the class of inflation targeting rules). Second, as \( k \) rises, the welfare losses from the HICP-targeting rule grow, but optimizing over the weight on Home \( \xi \) can eliminate nearly all of these incremental losses. Interestingly, as \( \delta \) falls, the welfare losses from HICP-targeting in fact fall, though again by optimizing over \( \xi \) we can further reduce welfare losses relative to the optimal policy.

While not the focus of the paper, we can also ask how the optimal policy would respond to asymmetric degrees of real wage rigidity across countries. Figure 15 depicts the optimal \( \xi \) as Home real wage rigidity varies (with \( \alpha \) between 0.1 and 1), assuming that Foreign has no real wage rigidity (\( \alpha^* = 1 \)) and all labor market parameters are at their symmetric benchmark.

Figure 13: optimal \( \xi \) as Home’s labor market becomes more sclerotic

Note: the optimal \( \xi \) is computed by minimizing the average welfare loss across many histories of shocks. Shaded markers depict the symmetric benchmark in which \( k = k^* = 0.12 \) and \( \delta = \delta^* = 0.030 \). A more sclerotic labor market is one with higher \( k \) or lower \( \delta \), thus moving from the ends to the center of the figure.
values. As is evident, ξ should fall as Home real wage rigidity strengthens. This is consistent with the closed economy results of Blanchard and Gali (2010): inflation targeting generates welfare losses in the presence of real wage rigidity, so the optimal inflation targeting rule should place less weight on stabilizing inflation in the country with more real rigidity.

F Heterogeneity in opportunity costs of employment

In this appendix I provide analytical and numerical results regarding the effects of heterogeneity in the disutilities of labor \( \{\chi, \chi^*\} \) on optimal monetary policy in the union. As noted in the main text, these parameters can capture the effects of heterogeneity in the opportunity cost of employment across countries. I indeed use them in this way in the calibration to the Eurozone in section 5.

F.1 Optimal policy in the \( \beta \to 0 \) limit

I begin with analytical results in the \( \beta \to 0 \) limit, as in section 4.6.

We can first characterize the effect of lower \( \chi \) on \( (\varphi - \epsilon_{n}^{f})/\epsilon_{n}^{f} \):

**Proposition F.1.** Suppose \( \beta \to 0 \). Around the symmetric benchmark and at least for small \( \{k, k^*\} \), the lower is \( \chi \), the higher is \( (\varphi - \epsilon_{n}^{f})/\epsilon_{n}^{f} \).

Intuitively, a lower opportunity cost of employment encourages vacancy creation, leading to
higher hiring costs per hire. This is as in a more sclerotic labor market characterized in Proposition 5, implying that \( \phi \) rises for the same reasons as described in that result.

Since a lower \( \chi \) raises \( \phi \) through this channel, it means that Home inflation is more sensitive to output fluctuations and Home output fluctuations themselves are more costly in welfare terms. For both reasons, we obtain an analogous result to Proposition 8:

**Proposition F.2.** Suppose \( \beta \to 0 \) and \( \iota = \iota^* \). Around the symmetric benchmark and at least for small \( \{k, k^*\} \), the optimal weight on Home \( \xi \) rises relative to \( \omega \) as \( \chi \) decreases, unless \( \zeta \) is sufficiently below 1.

### F.2 Optimal policy in the general case

I now numerically demonstrate that these results hold in the more general case. Starting with the same symmetric parameterization in section 4.7, I lower \( \chi \) and keep all other parameters unchanged. The first panel of Figure 16 demonstrates that \( \phi \) rises as \( \chi \) falls, consistent with Proposition F.1. The second panel demonstrates that the optimal inflation targeting rule then places more weight on Home, consistent with Proposition F.2.

### G Supplemental results for calibration to Eurozone

In this appendix I provide supplemental results accompanying the quantitative calibration to the Eurozone in section 5.
Figure 16: $\phi$ and optimal $\xi$ as Home’s opportunity cost of employment falls

Note: $\phi$ controls the welfare costs from and sensitivity of real marginal costs to contemporaneous employment fluctuations. Its formula is given in Lemma 2. The optimal $\xi$ is computed by minimizing the average welfare loss across many histories of shocks. Shaded markers depict the symmetric benchmark in which $\chi = \chi^* = 1.14$. A lower opportunity cost of employment is one with lower $\chi$, moving from the right to the left of the figure.

G.1 Measurement of labor market flows

I first provide additional details on the measurement of labor market flows for the Eurozone countries summarized in the main text.

G.1.1 Methodology

I exactly follow the approach in Elsby et al. (2013) who in turn build on Shimer (2012), and so I only briefly the summarize the steps here.

The outflow hazard rate from unemployment $f$ is estimated using data on unemployment by duration as well as overall unemployment. For instance, if monthly data were available, the outflow probability during month $t$ could be recovered from the stock of unemployed workers at the end of month $t$, less those who report being unemployed for less than one month, divided by the stock of unemployed workers at the end of month $t-1$. Elsby et al. (2013) demonstrate how to apply this approach using data sampled less frequently, and using data on duration spells longer than one month to improve the precision of the estimates. The latter is particularly relevant for European economies for which short-term unemployment is quite noisly measured, but will not bias the estimated outflow rate only if there is no duration dependence in outflow rates. I test the hypothesis of no duration dependence for each country using the approach further described in their paper.

Given the outflow rate $f$, the inflow rate $s$ over period $t$ is implied by how the unemploy-
<table>
<thead>
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<th>Country (code)</th>
<th>Sample size (persons)</th>
<th>p-value</th>
<th>Null rejected?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Austria (AT)</td>
<td>57,500</td>
<td>0%</td>
<td>yes</td>
</tr>
<tr>
<td>Belgium (BE)</td>
<td>37,500</td>
<td>2%</td>
<td>no</td>
</tr>
<tr>
<td>Finland (FI)</td>
<td>36,000</td>
<td>0%</td>
<td>yes</td>
</tr>
<tr>
<td>France (FR)</td>
<td>182,500</td>
<td>0%</td>
<td>yes</td>
</tr>
<tr>
<td>Germany (DE)</td>
<td>170,750</td>
<td>0%</td>
<td>yes</td>
</tr>
<tr>
<td>Greece (GR)</td>
<td>67,500</td>
<td>3%</td>
<td>no</td>
</tr>
<tr>
<td>Italy (IT)</td>
<td>150,000</td>
<td>0%</td>
<td>yes</td>
</tr>
<tr>
<td>Netherlands (NL)</td>
<td>109,375</td>
<td>0%</td>
<td>yes</td>
</tr>
<tr>
<td>Portugal (PT)</td>
<td>56,430</td>
<td>7%</td>
<td>no</td>
</tr>
<tr>
<td>Spain (ES)</td>
<td>160,000</td>
<td>1%</td>
<td>yes</td>
</tr>
</tbody>
</table>

Table 5: hypothesis test for duration dependence in outflow rate from unemployment

Notes: as in Elsby et al. (2013), the null hypothesis is that the probabilities an unemployed worker completes their spell within $d$ months, for $d \in \{3, 6, 12\}$, are equal.

I use three sets of data on unemployment for each country from the OECD: annual data on unemployment, annual data on unemployment by duration, and quarterly data on unemployment rates. I use data over 1999-2018 period after the introduction of the euro.

To test the hypothesis of no duration dependence in outflow rates from unemployment, I also require information on the sample size used in each country’s labor force surveys to estimate unemployment. I use the latest available survey size reported on the website of each country’s appropriate statistical agency.

G.1.3 Additional details on estimated flows

In Table 5, I provide information regarding the hypothesis test that there is no duration dependence in outflow rates from unemployment. The first column provides the sample size assumed for that country’s labor force surveys necessary to perform the test. The second column provides the $p$-value associated with the null that the probabilities an unemployed worker completes their spell within $d$ months, for $d \in \{3, 6, 12\}$, are equal. The third column summarizes whether or not we reject the null at a 1% significance level. Comparing this table to Table 2 in Elsby et al. (2013), it is notable that we can reject the null hypothesis.
for France, Germany, Italy, and Spain over the more recent sample period. For countries in which I reject the null hypothesis, I use only data on unemployment duration less than one month to estimate the outflow rate; for those in which I cannot, I use data for longer durations to estimate the outflow rate with greater precision, optimally weighting the various estimates as explained in Elsby et al. (2013).

In Figure 17, I depict the estimated outflow and inflow rates by year. For the countries and years which overlap with the analysis of Elsby et al. (2013), the estimates are broadly consistent with Figures 2 and 3 in that paper.

G.2 Net replacement rates

I now provide evidence on net replacement rates in unemployment used to discipline the heterogeneity in opportunity costs of employment in the main text.

I use the annual estimates provided by the OECD over 2001-2018. Averaging over this period, Table 6 reports the net replacement rate in unemployment for a worker at two months of unemployment duration, earning the average wage prior to job loss, and including housing-related benefits. The OECD reports these replacement rates for workers in various family situations. The last column is a simple arithmetic average across these. It is based on the last column that, in the calibration provided in the main text, I calibrate an opportunity cost of employment in France which is 75%/77% = 97% of that in Germany.

G.3 Alternative calibration: Germany and Italy

I now provide an alternative calibration to Germany and Italy to complement that for Germany and France in the main text. By Table 1, the Italian labor market features a lower outflow rate from unemployment but slightly higher inflow rate to unemployment than the German labor market. I model Italy as Home and Germany as Foreign.

I parameterize the model as follows. The externally set parameters are identical to the calibration in the main text. The calibrated parameters are summarized in Table 7. I directly set \( \delta = 0.021 \) and \( \delta^* = 0.018 \) to match the quarterly separation probabilities implied by the monthly inflow rates to unemployment for Italy and Germany in Table 1. I again set \( k^* \) and \( \chi^* \) to target recruiting costs which are 23% of the quarterly wage in Germany and a quarterly job-finding probability of 23.2% as implied by the monthly outflow rate from unemployment for Germany in Table 1. I set \( \chi \) such that the opportunity cost of employment in Italy is 94% of that in Germany, consistent with the fact that the net replacement rate during unemployment has been 94% of that in Germany over this period, per Table 6. I then set

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36 These data only become available starting in 2001.
Figure 17: estimates of monthly outflow and inflow rates by year
Table 6: net replacement rates in unemployment in the largest 10 Eurozone economies

| Children: | Single | Couple | | | Avg |
|-----------|--------|--------|------------------|---|---|---|
| Partner wage / avg wage: | n/a | n/a | 0% | 67% | 100% | 0% | 67% | 100% |
| Size: | | | | | | | | |
| Single | Couple | 
| 0 2 | 0 0 2 2 2 2 | | | | | | |
| Austria (AT) | 58% 70% | 63% 75% 78% 75% 79% 82% | 72% |
| Belgium (BE) | 62% 67% | 56% 71% 78% 59% 73% 80% | 68% |
| Finland (FI) | 57% 78% | 66% 75% 78% 78% 79% 82% | 74% |
| France (FR) | 67% 71% | 66% 80% 84% 68% 82% 84% | 75% |
| Germany (DE) | 60% 73% | 60% 85% 87% 74% 89% 91% | 77% |
| Greece (GR) | 32% 44% | 34% 57% 63% 44% 63% 67% | 51% |
| Italy (IT) | 61% 70% | 64% 76% 79% 70% 79% 80% | 72% |
| Netherlands (NL) | 72% 71% | 73% 82% 85% 80% 79% 82% | 78% |
| Portugal (PT) | 90% 90% | 88% 97% 99% 87% 99% 100% | 94% |
| Spain (ES) | 59% 74% | 59% 76% 80% 73% 84% 86% | 74% |

Table 6: net replacement rates in unemployment in the largest 10 Eurozone economies

Notes: each column reports for a given family situation, the average net replacement rate over 2001-2018 for an unemployed worker with duration 2 months, earning 100% of the average wage prior to job loss, and including housing-related benefits.

\( k \) to target a quarterly job-finding probability of 18.0% as implied by the monthly outflow rate from unemployment for Italy reported in Table 1. I use \( \gamma \) to target \( \omega = 0.37 \), the average ratio of Italy’s nominal GDP relative to the sum of Italy and Germany over 1999-2018. I set the persistence, volatility, and correlation of productivity shocks to be consistent with the corresponding moments estimated on labor productivity for Italy and Germany over the 1999-2018 period. Finally, given these driving forces, I calibrate the degree of real wage rigidity across the union \( \alpha = \alpha^* \) to match the weighted average of log product wage volatilities over 1999-2018 relative to the weighted average of unemployment volatilities over the same period (with weights \( \omega = 0.37 \) on Italy and \( 1 - \omega = 0.63 \) on Germany). I generate the same moment in simulated data in the model assuming a HICP-targeting policy rule with \( \xi = \omega = 0.37 \).

Table 8 compares the quantitative fit of the model using a HICP-targeting rule to untargeted second moments over the 1999-2019 period in the data. In general we see that the model underpredicts volatilities of quantities for Italy. This suggests that labor productivity shocks alone are incapable of generating the observed fluctuations in Italian aggregates over this period, consistent with the focus of the recent literature on financial conditions, fiscal policy, and other driving forces in Southern Europe (as emphasized by Gourinchas et al.

84
Table 7: calibrated parameters for Italy (Home) and Germany (Foreign)

Note: all model-implied moments exactly hit the targets, so I do not separately report these. See text for values of other externally set parameters.

(2017), Martin and Philippon (2017), and Chodorow-Reich et al. (2019), among others). As in the data, however, the HICP-targeting rule does capture the fact that inflation volatility is higher in Italy than Germany.

Relative to the HICP-targeting rule, the optimal inflation targeting rule again features 11pp more weight on Italy. As summarized in the first two rows of Table 9, while the HICP-targeting rule calls for $\xi = \omega = 0.37$, the optimal inflation targeting rule features $\xi = 0.48$. Re-weighting the inflation targeting rule in this way again eliminates nearly half of the welfare losses from fluctuations relative to the optimal policy. The third and fourth rows of the table eliminate real wage rigidity in both Italy and Germany and keep all other parameters the same. It remains the case that the optimal inflation targeting rule features a higher weight on Italy. Finally, the fifth row of this table sets $k$ to the value of $k^*$ and leaves all other parameters unchanged. We again see that the optimal inflation targeting rule is consistent with HICP-targeting, so that the difference in hiring costs which rationalize the difference in outflow rates from unemployment between these economies quantitatively drives the departure from HICP-targeting in the optimal inflation targeting rule.
Table 8: untargeted second moments

Notes: empirical second moments estimated on Q1/99-Q4/18 OECD data, where $\hat{x}_t$ and $\hat{x}_t^*$ are linearly detrended log GDP per capita, $\hat{n}_t$ and $\hat{n}_t^*$ are one minus linearly detrended harmonised unemployment rate, $\hat{\omega}_t$ and $\hat{\omega}_t^*$ are linearly detrended log labor compensation per employed person less log GDP deflator, and $\pi_{Ht}$ and $\pi_{Ft}$ are linearly detrended difference in log GDP deflator. Corresponding model moments are estimated after simulating many histories of shocks and assuming an HICP-targeting rule.

<table>
<thead>
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<th></th>
<th>SD data</th>
<th>3.05%</th>
<th>1.75%</th>
<th>1.73%</th>
<th>1.23%</th>
<th>1.27%</th>
<th>1.50%</th>
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<th>0.34%</th>
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<td>0.55%</td>
<td>1.17%</td>
<td>0.77%</td>
<td>1.04%</td>
<td>0.30%</td>
<td>0.18%</td>
<td></td>
</tr>
</tbody>
</table>

Table 9: HICP-targeting versus optimal inflation targeting rule

Notes: $\xi$ denotes weight on Home producer-price inflation in (33) and $\psi$ denotes the proportional change in consumption at all dates and states to render welfare the same as under the optimal policy, following (34).

<table>
<thead>
<tr>
<th>Model</th>
<th>$\xi$</th>
<th>$\psi \times 10^{-6}$</th>
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</thead>
<tbody>
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<td>HICP</td>
<td>0.37</td>
<td>21.5</td>
</tr>
<tr>
<td>Optimal $\xi$</td>
<td>0.48</td>
<td>10.9</td>
</tr>
<tr>
<td>HICP, $\alpha = \alpha^* = 1$</td>
<td>0.37</td>
<td>2.1</td>
</tr>
<tr>
<td>Optimal $\xi$, $\alpha = \alpha^* = 1$</td>
<td>0.42</td>
<td>0.4</td>
</tr>
<tr>
<td>Optimal $\xi$, $\alpha = \alpha^* = 1$, $k = k^*$</td>
<td>0.37</td>
<td>0.0</td>
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</table>