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Exchange Rates and Asset Prices in a Global Demand System

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Abstract

Using cross-country holdings, we estimate a demand system for financial assets across 36 countries. Based on the estimated demand system and market clearing, we decompose exchange rates, long-term yields, and equity prices into three sources of variation: macro variables, policy variables (i.e., short-term rates, debt quantities, and foreign exchange reserves), and latent demand. The former two account for 58 percent of the variation in exchange rates, and the remaining variation due to latent demand is geographically concentrated. Policy variables account for 66 percent of the variation in long-term yields. Macro variables account for 63 percent of the variation in equity prices.

JEL: E52, F31, G12

Keywords: Asset demand system, Asset pricing, Foreign exchange reserves, Monetary policy, Portfolio choice

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I. Introduction

Global investors hold financial assets across many countries and have exchange rate exposure not only through short-term debt but also long-term debt and equity. The portfolio decisions of these investors across countries and asset classes are important for exchange rates, long-term yields, and equity prices. Government policies are also important. Conventional monetary policy determines the short-term rate. Fiscal policy and unconventional monetary policy affect the supply of domestic debt. Central banks hold foreign debt through foreign exchange reserves, affecting the residual supply of foreign debt. All these forces determine exchange rates and asset prices through market clearing of global financial markets.

We develop an asset pricing model to study sources of variation in exchange rates, long-term yields, and equity prices across 36 countries from 2002 to 2017. We model investors at the country level to match cross-country holdings of short-term debt, long-term debt, and equity in the International Monetary Fund's (IMF) Coordinated Portfolio Investment Survey. We start with market clearing identities for these three asset classes across all countries. The supply of each asset must equal the demand across all investor countries and foreign exchange reserves. Every asset pricing model is ultimately a model of asset demand that arises from portfolio choice, combined with market clearing. However, no previous paper has taken the portfolio-choice implications seriously enough to match cross-country holdings together with asset prices. This paper is the first attempt to do so.

Starting with a portfolio-choice model, Koijen and Yogo (2019) show that an investor's optimal portfolio weights can be expressed as a logit function of observed asset characteristics and unobserved (to the econometrician) latent demand. Following their approach, we specify the asset demand of investor countries and foreign exchange reserves to depend on the market-to-book ratio (equivalently, yield in the case of debt), the real exchange rate, and macro variables such as gross domestic product (GDP), real GDP per capita, and inflation. Asset demand also depends on risk measures including equity volatility and sovereign debt ratings. Bilateral variables including exports, imports, and the distance between investor and issuer countries are important for explaining cross-country financial investment (Portes, Rey, and Oh 2001; Portes and Rey 2005). We estimate the demand system by instrumental variables because asset prices and latent demand are jointly endogenous. The identifying assumption is that monetary policy shocks affect asset demand only through asset prices, conditional on observed characteristics in the demand specification. The estimated demand elasticities are 26 for short-term debt, 1.9 for long-term debt, and 1.5 for equity. That is, equity demand decreases by 1.5 percent per 1 percent price increase.

Based on the estimated demand system and market clearing, we decompose the variance of annual changes in exchange rates and asset prices. Macro variables account for 37 percent, short-term rates account for 7 percent, debt quantities account for 3 percent, and foreign exchange reserves account for 11 percent of the variation in exchange rates. These fundamental sources jointly account for 58 percent of the variation in exchange rates. The importance of latent demand that accounts for the remaining 42 percent is geographically concentrated in large investor countries. In particular, latent demand of US and European countries substituting across asset classes accounts for 22 percent of the variation in exchange rates.

Policy variables are the primary determinants of long-term yields. Short-term rates account for 7 percent, debt quantities account for 49 percent, and foreign exchange reserves account for 10 percent of the variation in long-term yields. These policy variables jointly account for 66 percent of the variation in long-term yields. In contrast, macro variables and short-term rates are the primary determinants of equity prices. Macro variables account for 63 percent, and short-term rates account for 6 percent of the variation in equity prices.

We provide additional context to the variance decomposition through two case studies that cover important events in our sample: global monetary easing and the European sovereign debt crisis. First, we examine how exchange rates and long-term yields relate to short-term rates and debt quantities in the euro area, Japan, Switzerland, and the UK. Our sample contains significant variation in short-term rates and debt quantities across the four regions because of changes in monetary policy that include quantitative easing. Averaged across the four regions, the exchange rate depreciates by 2.6 percent per 1 percentage point decrease in the short-term rate. A change in debt quantities that decreases the long-term yield by 10 basis points also depreciates the exchange rate by 0.8 percent.

Second, we decompose long-term yield spreads between Germany and the US as well as the southern euro countries and Germany. On the one hand, the relative timing of monetary easing accounts for almost all of the variation in the long-term yield spread between Germany and the US. Short-term rates account for 32 percent, and debt quantities account for 54 percent of the variation in the long-term yield spread. On the other hand, the relative macro and fiscal experiences account for most of the variation in long-term yield spreads between the southern euro countries and Germany. Macro variables account for 43 percent, and debt quantities account for 28 percent of the variation in long-term yield spreads. Latent demand, primarily of euro area investors, accounts for 28 percent of the variation in long-term yield spreads.

A. *Related Literature*

We relate this paper to the previous literature in the context of a two-country model of financial markets. Each country has a short-term debt and an equity market, so there four market clearing equations in total. There are five asset prices that enter the market clearing equations: short-term rates in each country, equity prices in each country, and the exchange rate. In traditional models of international finance, the consumption goods market determines the exchange rate, so that financial markets do not directly determine the exchange rate. Motivated by the empirical failure of traditional models, an alternative approach known as "portfolio balance models" relies entirely on financial markets for exchange rate determination.

In a model with only short-term debt, Kouri (1976) uses market clearing in the foreign short-term debt market for exchange rate determination, assuming that the foreign short-term rate is fixed.[1] Kouri and De Macedo (1978) resolve the problem of five asset prices in four market clearing equations by introducing a fifth equation for relative cash demand with the exchange rate as the relative price. Hau and Rey (2006) use market clearing in equity markets to jointly determine equity prices and the exchange rate. They introduce a third equation for the spot exchange rate, which depends on the imbalance between domestic demand for foreign equity and vice versa. Gabaix and Maggiori (2015) assume segmentation in short-term debt markets. Only speculators can hold both countries' debt, whose limited arbitrage capacity determines the exchange rate.

We take this literature that focuses on bilateral models in a more empirical direction that easily accommodates market clearing across multiple asset classes and many countries. We add long-term debt markets to study long-term yields in conjunction with short-term rates and equity prices. As in Hau and Rey (2006), we allow for substitution across asset classes so that the demand for long-term debt and equity could feed back into exchange rates. We use market clearing across all three asset classes for exchange rate determination, conditional on central bank policy that determines short-term rates.

Previous papers estimate demand elasticities of institutions and households for long-term debt and equity. They find less elastic demand than what asset pricing models would imply if assets within the same asset class were close substitutes. Krishnamurthy and Vissing-Jørgensen (2007) estimate demand elasticities for US Treasury debt. Koijen et al. (2018) estimate demand elasticities for sovereign debt in the euro area. Koijen and Yogo (2019) estimate demand elasticities for the cross section of US equity, and Koijen, Richmond, and

[1] Alternatively, Blanchard, Giavazzi, and Sa (2005) use market clearing in the domestic equity market for exchange rate determination, assuming that the domestic equity price is fixed. This assumption is unappealing in our context because equity prices are direct objects of interest.

Yogo (2019) estimate demand elasticities for the cross-section of US and UK equity. These papers use institutional and household holdings within a country, while we use aggregate holdings at the country level. An advantage of our approach is that we estimate demand elasticities for all countries and asset classes, based on a demand system that allows for all substitution effects. The previous papers implicitly rule out substitution effects outside the countries and asset classes that their data cover.

Motivated by arbitrage pricing theory or the intertemporal capital asset pricing model, an empirical literature tests for a low-dimensional factor structure in global stock (Fama and French 2012), bond (Dahlquist and Hasseltoft 2013; Jotikasthira, Le, and Lundblad 2015), and currency returns (Lustig, Roussanov, and Verdelhan 2011). These papers find both common and local factors across countries within each asset class. Asness, Moskowitz, and Pedersen (2013) find common factors in value and momentum returns across countries and asset classes. Like this literature, we develop an asset pricing model that covers the three asset classes across all developed and many emerging markets. An important difference is that our model matches cross-country holdings together with asset prices. The literature on factor models uses only data on asset prices and factors, ignoring the portfolio-choice implications and portfolio holdings data. In ongoing work, we use the demand system approach to decompose the factor returns in global stock, bond, and currency markets.

The remainder of the paper is organized as follows. In Section II, we describe the data on cross-country holdings and asset prices. We also present some reduced-form facts that support the formal analysis. In Section III, we present an asset demand system that matches observed cross-country holdings and implies flexible substitution within and across asset classes. We also discuss how the asset demand system and market clearing determine exchange rates and asset prices. In Section IV, we discuss the estimation methodology and present the estimated demand system. In Section V, we decompose exchange rates and asset prices, based on the estimated demand system and market clearing. Section VI concludes.

II. Data on Cross-Country Holdings and Asset Prices

We briefly describe the data on cross-country holdings and asset prices. We refer the reader to Appendix A for further details. We then present some reduced-form facts that support the formal analysis in the subsequent sections.

A. Data Construction

The Coordinated Portfolio Investment Survey contains each investor country's year-end holdings of foreign financial assets in US dollars by asset class and issuer country. The three asset classes are short-term debt (i.e., one year or less in maturity), long-term debt (i.e., greater than one year in maturity), and equity. The data also contain foreign exchange reserves, which are central bank holdings of foreign financial assets. IMF aggregates foreign exchange reserves across all central banks for confidentiality. Therefore, we treat the aggregate portfolio of foreign exchange reserves as an investor unit.

We measure the supply of each asset by aggregating holdings across all investor countries and foreign exchange reserves. For short- and long-term debt, our measure corresponds to the total amount held by foreign investors. At the country level, domestically held debt is both an asset and a liability, so it does not count towards supply or wealth according to the national accounting identities. In contrast, domestically held equity is an asset that is part of wealth. Therefore, we measure the supply of equity as total stock market capitalization by country, published by the World Bank. We construct the amount of domestically held equity by subtracting the aggregate foreign holdings from total stock market capitalization.

Short-term rates are 3-month interbank rates from Thomson Reuters Datastream. We use the 10-year benchmark government bond yields from Datastream and (in a few cases of missing data) OECD's Monetary and Financial Statistics. We construct the zero-coupon yield curve by country through Nelson and Siegel (1987), assuming that the 10-year benchmark yield is the par yield. Throughout the paper, the long-term yield refers to the 10-year zero-coupon yield, which we assume is the representative price of long-term debt in the cross-country holdings data. Equity returns and book-to-market equity for countries in the MSCI ACWI Index are from Datastream and MSCI, respectively. Exchange rates are from IMF's International Financial Statistics. Throughout the paper, exchange rates are in US dollars per local currency unit. We use year-end values of all exchange rates and asset prices to align with the year-end values of cross-country holdings.

The merged sample of cross-country holdings and asset prices covers 2002 to 2017. The financial assets are held by 88 investor countries plus foreign exchange reserves. We refer to Table A1 in Appendix A for a complete list of investor countries, which we group by MSCI classification for presentation purposes. There are 36 issuer countries with complete data on asset prices and characteristics. This covers all 22 countries in the MSCI World Index and 14 of 21 countries in the MSCI Emerging Markets Index. Ten of the 36 countries are in the euro area, and the Hong Kong dollar is pegged to the US dollar. Therefore, our sample contains 25 independent exchange rates relative to the US dollar. We aggregate all issuer countries outside the 36 countries as an "outside asset" for each asset class.

B. Summary of Global Financial Markets

Table 1 summaries financial markets across 36 countries in 2017. The size of the US short-term debt market was \$822.7 billion, of which foreign exchange reserves held 34 percent. The US long-term debt market was \$7,402 billion, of which foreign exchange reserves held 28 percent. The US equity market was \$32,121 billion, of which foreign exchange reserves held 1 percent. Thus, foreign exchange reserves hold a significant share of US debt but not equity.

Foreign exchange reserves also hold a significant share of debt in the euro area and Japan, which are important reserve currencies. Foreign exchange reserves held 50 and 31 percent of German short- and long-term debt, respectively. Foreign exchange reserves held 36 and 22 percent of Japanese short- and long-term debt, respectively. Among the emerging markets, foreign exchange reserves are important in China. Foreign exchange reserves held 18 and 25 percent of Chinese short- and long-term debt, respectively. The large size of foreign exchange reserves suggests that central banks play an important role in managing exchange rates and the term structure of interest rates globally.

Table 2 reports the top ten investors by asset class in 2017. Foreign exchange reserves are the largest investor in both short- and long-term debt markets. They held \$917 billion of short-term debt and \$4,398 billion of long-term debt across all countries. Unsurprisingly, large developed countries such as Germany, Japan, the UK, and the US appear in the top ten list. Ireland, Luxembourg, and the Cayman Islands also appear in the top ten list for both short- and long-term debt. These countries are offshore financial centers through which other countries invest because of favorable regulation and taxes. We refer to Appendix A for a further discussion of offshore financial centers and how we construct the cross-country holdings data to avoid double counting.

C. Relative Asset Quantities and Prices

Figure 1 is a scatter plot of relative long-term debt quantity versus price for the euro area, Japan, Switzerland, and the UK. The horizontal axis measures the supply of each country's long-term debt relative to the US. It is each country's log long-term debt quantity in local currency units minus the US log long-term debt quantity (in US dollars). The vertical axis measures the price of each country's long-term debt relative to the US. It is each country's log long-term debt price plus the log exchange rate (in US dollars per local currency unit) minus the US log long-term debt price. The scatter plot reveals a negative relation that is consistent with a downward-sloping demand curve. When the supply of Japanese long-term debt is high relative to the US, its relative price inclusive of the exchange rate is low.

Our finding extends the evidence on downward-sloping demand for long-term debt to an international context (Krishnamurthy and Vissing-Jørgensen 2012; Greenwood and Vayanos 2014).

Figure 2 repeats the same exercise for equity markets. The horizontal axis measures the supply of each country's equity relative to the US. The vertical axis measures the price of each country's equity relative to the US. Again, the scatter plot reveals a negative relation that is consistent with a downward-sloping demand curve. The slope of the demand curve reveals the degree to which investors view the equity of different countries to be close substitutes. The slope would be virtually flat if investors viewed the equity of different countries to be near-perfect substitutes. Figures 1 and 2 foreshadow estimates of low demand elasticities for long-term debt and equity in Section IV.

D. *Predictability of Asset Returns*

Let $e_t(n)$ be the log exchange rate in US dollars per country n's currency unit. Let $\Delta e_{t+1}(n)$ be the change in the log exchange rate from year t to $t+1$. Let $y_t(n)$ be country n's continuously compounded 3-month rate in year t. We define a panel regression model for exchange rates:

$$\Delta e_{t+1}(n) = \kappa_n + \psi(y_t(n) - y_t(\text{US})) + \nu_{t+1}(n), \tag{1}$$

where the regressors are the interest rate differential and country fixed effects. Under the uncovered interest rate parity, $\psi = -1$. That is, high-interest currencies must depreciate on average for expected returns to be constant across countries.

We estimate the panel regression on an annual sample of 25 exchange rates from 2002 to 2017. Table 3 reports an estimated coefficient of -0.03 with a standard error of 0.32. That is, we reject the hypothesis of uncovered interest rate parity.

Exchange rates are notoriously difficult to predict. However, a high real exchange rate predicts depreciation of the nominal exchange rate under the purchasing power parity. We define the log real exchange rate in year t as $e_t(n) - z_t(n)$, where $z_t(n)$ is the US log consumer price index (CPI) minus country n's log CPI. Table 3 reports that the real exchange rate predicts changes in nominal exchange rates with a statistically significant coefficient of -0.36.

Let $r_{t+1}(n, l)$ be the continuously compounded return in US dollars for asset class l in country n from year t to $t+1$. The return in US dollars is the sum of the return in local currency units plus the change in the log exchange rate. Therefore, the real exchange rate should predict foreign asset returns in US dollars. Let $y_t(\text{US})$ be the US continuously compounded 1-year yield in year t. We define a panel regression model for excess returns:

$$r_{t+1}(n,l) - y_t(\text{US}) = \kappa_{n,l} + \psi_l(p_t(n,l) + \theta_l(e_t(n) - z_t(n))) + \nu_{t+1}(n,l), \tag{2}$$

where $p_t(n,l)$ is log market-to-book for asset class l in country n. For debt, log market-to-book is equal to minus maturity times the continuously compounded yield. For equity, log market-to-book is a valuation measure, for which high valuation predicts subsequently low returns.

Table 3 reports estimates of panel regression (2) for short-term debt, long-term debt, and equity. The log real exchange rate and log market-to-book are reliable predictors of excess returns for all three asset classes. For short-term debt, the estimated coefficient is −0.31 on the log real exchange rate and −7.52 on log market-to-book. For long-term debt, the estimated coefficient is −0.37 on the log real exchange rate and −0.45 on log market-to-book. For equity, the estimated coefficient is −0.88 on the log real exchange rate and −0.29 on log market-to-book. We will refer to these estimates of expected returns for the demand estimation in Section IV.

III. Demand System Asset Pricing

We present an asset demand system that matches observed cross-country holdings and implies flexible substitution within and across asset classes. We also discuss how the asset demand system and market clearing determine exchange rates and asset prices. In Appendix B, we show that the demand system approach collapses to the capital asset pricing model (CAPM) when investors are homogeneous and have identical portfolios.

A. Financial Markets

There are N issuer countries that we index as $n = 1, \ldots, N$. Within each country, there are three asset classes: 1) short-term debt, 2) long-term debt, and 3) equity. We index the three asset classes as $l = 1, 2, 3$. $P_t(n,l)$ denotes the market-to-book ratio for asset class l in country n at time t. In the case of debt, the market-to-book ratio is simply the price per unit of face value. $Q_t(n,l)$ denotes the total book value (in country n's currency units) for asset class l in country n at time t. We refer to $P_t(n,l)$ and $Q_t(n,l)$ as the price and quantity of asset class l in country n, respectively. $E_t(n)$ denotes the nominal exchange rate in US dollars per country n's currency unit at time t. $Z_t(n)$ denotes the US CPI in dollars relative to country n's CPI in its currency units. Thus, $E_t(n)/Z_t(n)$ is the real exchange rate.

To clarify the notation, consider an example of UK long-term debt. $P_t(n,2)$ is the market price (in pounds) per pound of face value. $Q_t(n,2)$ is the total face value of debt in pounds. $E_t(n)$ is the exchange rate in US dollars per pound. Thus, $P_t(n,2)Q_t(n,2)$ is the total market value of debt in pounds, and $P_t(n,2)E_t(n)Q_t(n,2)$ is the total market value of debt in US dollars.

We use lowercase letters to denote the logarithm of the corresponding uppercase variables. For example, $p_t(n,l) = \log(P_t(n,l))$, $q_t(n,l) = \log(Q_t(n,l))$, and $e_t(n) = \log(E_t(n))$. We use bold letters to denote column vectors or matrices, whose elements are the corresponding variables. For example, $\mathbf{P}_t$ is a matrix whose (n,l)th element is $P_t(n,l)$. $\mathbf{E}_t$ is a column vector whose nth element is $E_t(n)$.

B. *Market Clearing*

There are I investor countries that we index as $i = 1, \dots, I$. Each investor i allocates wealth $A_{i,t}$ at time t across three asset classes in N issuer countries. The investor could also allocate wealth to countries outside the N issuer countries, which we represent as an outside asset (indexed as $n = 0$) within each asset class. Without loss of generality, we write investor i's portfolio weight in country n and asset class l at time t as

$$w_{i,t}(n,l) = w_{i,t}(n|l)w_{i,t}(l). \tag{3}$$

The first term on the right side represents the portfolio weight in country n within asset class l. The second term represents the aggregate portfolio weight in asset class l.

The portfolio weights must sum to one within each asset class: $\sum_{n=0}^{N} w_{i,t}(n|l) = 1$. The aggregate portfolio weights must also sum to one across all asset classes: $\sum_{l=1}^{3} w_{i,t}(l) = 1$. Let $O_{i,t} = \sum_{l=1}^{3} A_{i,t}w_{i,t}(0,l)$ be the total investment in outside assets across all asset classes. We write the investor's wealth as

$$A_{i,t} = \frac{O_{i,t}}{1 - \sum_{l=1}^{3}\sum_{n=1}^{N} w_{i,t}(n,l)}. \tag{4}$$

Market clearing for each country n and asset class l at time t is

$$\begin{aligned} P_t(n,l)E_t(n)Q_t(n,l) &= \sum_{i=1}^{I} A_{i,t}w_{i,t}(n,l;\mathbf{P}_t,\mathbf{E}_t) \\ &= \sum_{i=1}^{I} \frac{O_{i,t}w_{i,t}(n,l;\mathbf{P}_t,\mathbf{E}_t)}{1 - \sum_{l=1}^{3}\sum_{n=1}^{N} w_{i,t}(n,l;\mathbf{P}_t,\mathbf{E}_t)}. \end{aligned} \tag{5}$$

The left side is asset supply. The right side is asset demand, which is wealth times the portfolio weight aggregated across all investors. The notation in equation (5) emphasizes that both wealth and the portfolio weights depend on the entire vector of asset prices and exchange rates.

Market clearing is a system of $3N$ equations in $3N$ asset prices and $N-1$ exchange rates. By defining all exchange rates to be US dollars per local currency unit, we normalize the exchange rate for the US to be one. Conditional on realized short-term debt prices (equivalently, short-term rates), we have a system of $3N$ equations in N long-term debt prices, N equity prices, and $N-1$ exchange rates. We obtain an exactly determined system by assuming that the Federal Reserve adjusts the supply so that the US short-term debt market clears at a given short-term rate. Therefore, we could use the demand system (5) as an international asset pricing model once we specify a model of portfolio weights.

C. Characteristics-Based Demand

Koijen and Yogo (2019) develop a characteristics-based model of asset demand, in which the portfolio weights are a logit function of asset prices, observed characteristics, and unobserved latent demand. The logit function implies portfolio weights that are strictly positive and sum to one. The exponential-linear specification is ideal for fitting observed holdings that are log-normally distributed. The presence of latent demand makes the model sufficiently flexible to match actual holdings.

Koijen and Yogo (2019) derive characteristics-based demand from the mean-variance portfolio, which is the solution of a portfolio-choice problem. The key assumptions are that returns have a factor structure and that expected returns and factor loadings depend on the assets' own characteristics. These assumptions essentially make an asset's own characteristics sufficient for its contribution to the expected return and risk of the overall portfolio. We refer to Appendix B for further details about the relation between characteristics-based demand and the mean-variance portfolio.

In the present context of cross-country holdings across three asset classes, a nested logit specification is a natural extension of the logit specification in Koijen and Yogo (2019). The inner nest $w_{i,t}(n|l)$ in equation (3) describes how an investor substitutes across countries within an asset class. The outer nest $w_{i,t}(l)$ describes how an investor substitutes across asset classes. As we describe below, a nested logit specification allows for imperfect substitution across asset classes.

1. Allocation within Asset Class

We model the portfolio weight in country n within asset class l at time t as

$$w_{i,t}(n|l) = \frac{\delta_{i,t}(n,l)}{1 + \sum_{m=0}^{N} \delta_{i,t}(m,l)}, \tag{6}$$

where

$$\delta_{i,t}(n,l) = \exp\{\beta_l(p_t(n,l) + \theta_l(e_t(n) - z_t(n))) + \gamma_l' \mathbf{x}_{i,t}(n,l) + \epsilon_{i,t}(n,l)\}. \tag{7}$$

Asset demand depends on log market-to-book, the log real exchange rate, a vector of observed characteristics $\mathbf{x}_{i,t}(n,l)$, and latent demand $\epsilon_{i,t}(n,l)$. We index the coefficients β_l, θ_l, and γ_l by l to allow for heterogeneous demand elasticities across asset classes.[2] By the budget constraint, the portfolio weight in the outside asset within asset class l at time t is

$$w_{i,t}(0|l) = \frac{1}{1 + \sum_{m=0}^{N} \delta_{i,t}(m,l)}. \tag{8}$$

Log market-to-book and the log real exchange rate enter asset demand because they predict excess returns, as reported in Table 3. In fact, the coefficient θ_l in asset demand (7) is the same as the coefficient θ_l in the predictive regression (2). That is, we impose a single index restriction on log market-to-book and the log real exchange rate, which is the combination of the two variables that best predicts excess returns. Because excess returns are in US dollars, we implicitly assume that investors care about currency-hedged expected returns for portfolio choice.

We index the observed characteristics not only by issuer n but also by investor i to allow for bilateral variables such as export shares, import shares, and distance. Thus, expected returns and perceived risk for the same asset could vary across investors. For example, investors could perceive farther countries as having lower expected returns or higher risk because of informational frictions that increase with distance. Similarly, latent demand represents unobserved (to the econometrician) differences in expected returns and perceived risk across investors and assets.

[2] The coefficient restrictions $\beta_l < 1$ and $\theta_l \in (0,1)$ are sufficient for the existence and uniqueness of equilibrium (Koijen and Yogo 2019, Proposition 2).

2. Allocation across Asset Classes

We model the aggregate portfolio weight in asset class l at time t as

$$w_{i,t}(l) = \frac{\left(1 + \sum_{m=0}^{N} \delta_{i,t}(m,l)\right)^{\lambda_l} \exp\{\alpha_l + \xi_{i,t}(l)\}}{\sum_{k=1}^{3} \left(1 + \sum_{m=0}^{N} \delta_{i,t}(m,k)\right)^{\lambda_k} \exp\{\alpha_k + \xi_{i,t}(k)\}}, \tag{9}$$

where $\lambda_l \in [0,1]$. The first term inside the parentheses in the numerator, which is also the denominator in the inner nest (6), is called the "inclusive value" in a nested logit model. To understand the role of inclusive value, suppose that the coefficient on short-term debt price is negative. A decrease in short-term debt prices across several countries makes short-term debt more attractive as an asset class, reflected by an increase in the inclusive value of short-term debt. The outer nest (9) then implies an increase in the aggregate portfolio weight in short-term debt. Thus, the inclusive value connects changing asset prices and characteristics in the inner nest to respective changes in portfolio weights in the outer nest.

In addition to the inclusive value, equation (9) depends on asset-class fixed effects α_l and asset-class latent demand $\xi_{i,t}(l)$. Asset-class latent demand represents unobserved (to the econometrician) differences in expected returns and perceived risk across investors and asset classes. Because the budget constraint implies that there are only two degrees of freedom, we normalize $\alpha_3 + \xi_{i,t}(3) = 0$ for equity.

By market clearing (5), a demand shock could affect prices in other asset classes through two channels. The first channel is a substitution effect. A demand shock could change the inclusive value of an asset class and thereby change the aggregate portfolio weights (9) across asset classes. The second channel is a wealth effect. A demand shock could change the investors' wealth (4), which affects the demand for other asset classes. As we discussed above, market clearing across all asset classes determines exchange rates, conditional on short-term rates. Thus, a demand shock in long-term debt or equity markets could affect exchange rates through both substitution and wealth effects.

3. Special Cases

When $\lambda_l = 1$ for all asset classes in equation (9), the portfolio weight simplifies to the logit specification (Koijen and Yogo 2019):

$$w_{i,t}(n,l) = \frac{\delta_{i,t}(n,l)}{3 + \sum_{k=1}^{3} \sum_{m=0}^{N} \delta_{i,t}(m,k)}. \tag{10}$$

In this expression, we have normalized $\alpha_l + \xi_{i,t}(l) = 0$ because asset-class latent demand is not separately identified from latent demand within asset classes.

When $\lambda_l = 0$ for all asset classes in equation (9), the portfolio weight simplifies to

$$w_{i,t}(n,l) = \frac{\delta_{i,t}(n,l)}{1 + \sum_{m=0}^{N} \delta_{i,t}(m,l)} \frac{\exp\{\alpha_l + \xi_{i,t}(l)\}}{\sum_{k=1}^{3} \exp\{\alpha_k + \xi_{i,t}(k)\}}. \tag{11}$$

That is, the allocation across asset classes does not depend on inclusive value. As we discussed above, this means that only wealth effects are present and not substitution effects. Blanchard, Giavazzi, and Sa (2005) emphasize this special case that even when the overall allocation to equity is constant, the allocation across countries within equity affects exchange rates through the wealth effect.

These two cases clarify that λ_l that is an important parameter that determines the strength of substitution across asset classes. Higher values of λ_l imply stronger substitution across asset classes. That is, a demand shock has stronger effects on prices in other asset classes.

IV. Demand Estimation

Demand estimation is separable into two parts, within asset class and across asset classes. We describe the estimation methodology and present the estimated demand system.

A. Observed Characteristics

To operationalize asset demand (6), we must specify observed characteristics that explain portfolio choice across countries. The macro variables are log nominal GDP, log GDP per capita, and inflation. The risk measures are equity volatility and sovereign debt ratings. We convert the rating to a continuous measure equal to minus one times the 5-year default probability, so that a higher measure implies a higher rating. Bilateral variables are the export share, the import share, and distance. We refer to Appendix A for further details about how we construct these variables.

For the equity market, we capture home bias through a dummy for own country ownership. We include fixed effects for the issuer country's MSCI classification to allow for time-invariant unobserved characteristics. We include investor fixed effects to allow for cross-sectional variation in the outside asset weight. We include year fixed effects to allow for time-series variation in the outside asset weight. The combination of investor and year fixed effects means that the variation within issuer countries, rather than variation across issuer countries and the outside asset, identifies the demand elasticities.

We treat the observed characteristics as exogenous for the purposes of demand estimation. Our exercise is in the same spirit as asset pricing models that assume an exogenous endowment process (Lucas 1978) or term structure models that assume an exogenous short-term rate (Cox, Ingersoll, and Ross 1985). The ultimate goal, beyond the scope of this paper, is to endogenize the macro variables together with exchange rates and asset prices. Doing so for all 36 countries is a formidable task that is beyond the reach of current international macro models.

B. Estimation Methodology

1. Demand within Asset Class

Dividing equation (6) by equation (8) and taking the logarithm, we have

$$\log\left(\frac{w_{i,t}(n|l)}{w_{i,t}(0|l)}\right) = \beta_l \underbrace{(p_t(n,l) + \theta_l(e_t(n) - z_t(n)))}_{\text{log price}} + \gamma_l' \mathbf{x}_{i,t}(n) + \epsilon_{i,t}(n,l). \tag{12}$$

This is a separate panel regression model for each asset class l, whose observations are investor i's holding of issuer n in year t. Equation (12) says that the demand for US equity relative to UK equity depends on their relative characteristics. Thus, the investor substitutes from US equity to UK equity if the characteristics of UK equity become relatively more attractive (e.g., lower prices).

Log price is a single index of log market-to-book and the log real exchange rate, based on the estimated coefficient θ_l from the predictive regression (2). The single index restriction respects the economic reason that these two variables enter asset demand, which is their relation to expected returns. In addition, the single index restriction helps with identification because only one instrument is necessary for log price.

The identifying assumption is that the short-term rate is exogenous to latent demand conditional on observed characteristics. Formally, the conditional moment restriction is

$$\mathbb{E}[\epsilon_{i,t}(n,l)|y_t(n), \mathbf{x}_{i,t}(n)] = 0. \tag{13}$$

Monetary policy is endogenous to economic conditions captured by observed characteristics $\mathbf{x}_{i,t}(n)$. For example, the central bank follows a Taylor (1993) rule that responds to output and inflation. Monetary policy shocks are correlated with asset prices and exchange rates, making it a relevant instrument. The validity of the instrument relies on the assumption that latent demand does not directly respond to monetary policy shocks. That is, monetary policy shocks affect asset demand only through asset prices.

The primary threat to identification is omitted variables in the demand specification. Suppose that monetary policy depends on an omitted variable that is not part of observed characteristics. If this omitted variable is also information that investors use for portfolio choice, it invalidates the identifying assumption. That is, the short-term rate is not exogenous to asset demand conditional on observed characteristics.

The literature proposes various ways to identify exogenous variation in monetary policy. They include high-frequency identification around key policy announcements (e.g., target rate change or asset purchase program), identification through heteroscedasticity (Rigobon 2003), and the narrative approach (Romer and Romer 1989). In principle, any of these identified shocks to monetary policy could complement or replace moment restriction (13). Moments describing how asset prices respond to monetary policy shocks would be informative about demand elasticities because lower elasticities imply larger asset price reactions.

2. Demand across Asset Classes

Dividing equation (9) for short- or long-term debt (i.e., $l = 1, 2$) by the same equation for equity (i.e., $l = 3$) and taking the logarithm, we have

$$\begin{aligned}\log\left(\frac{w_{i,t}(l)}{w_{i,t}(3)}\right) =& \lambda_l \log\left(1 + \sum_{m=0}^{N} \delta_{i,t}(m,l)\right) - \lambda_3 \log\left(1 + \sum_{m=0}^{N} \delta_{i,t}(m,3)\right) + \alpha_l + \xi_{i,t}(l) \\ =& -\lambda_l \log(w_{i,t}(0|l)) + \lambda_3 \log(w_{i,t}(0|3)) + \alpha_l + \xi_{i,t}(l). \end{aligned} \tag{14}$$

The second line follows from equation (8), which relates the inclusive value to the portfolio weight in the outside asset. This is a panel regression model, whose observations are investor i's aggregate holding of asset class l relative to equity in year t. Equation (14) says that the demand for short-term debt relative to equity depends on their relative inclusive value. Thus, the investor substitutes from short-term debt to equity if the characteristics of equity become relatively more attractive (e.g., lower prices).

The panel regression (14) has two endogenous regressors, the logarithms of $w_{i,t}(0|l)$ and $w_{i,t}(0|3)$. We maintain the identifying assumption that the short-term rate is exogenous to asset-class latent demand conditional on observed characteristics. Formally, the conditional moment restriction is

$$\mathbb{E}[\xi_{i,t}(l)|\mathbf{y}_t, \mathbf{x}_{i,t}] = 0. \tag{15}$$

To construct an instrument for $w_{i,t}(0|l)$, we first estimate the reduced-form regression corresponding to equation (12),

$$\log\left(\frac{w_{i,t}(n|l)}{w_{i,t}(0|l)}\right) = \pi_l y_t(n) + \phi_l' \mathbf{x}_{i,t}(n) + \eta_{i,t}(n,l). \tag{16}$$

Let $\widehat{\delta}_{i,t}(n,l)$ be the predicted value of this regression. We construct an instrument as

$$\widehat{w}_{i,t}(0|l) = \frac{1}{1+\sum_{m=0}^{N}\widehat{\delta}_{i,t}(m,l)}. \tag{17}$$

We then estimate the panel regression (14) with two instruments, which are the logarithms of $\widehat{w}_{i,t}(0|l)$ and $\widehat{w}_{i,t}(0|3)$.

C. Estimated Demand System

1. Demand within Asset Class

Table 4 reports estimated coefficients for demand within asset class. For short-term debt, the coefficient on log price is −70.43. Although there is no simple analytic expression for the demand elasticity, we compute it numerically to be 26. That is, short-term debt demand decreases by 26 percent per 1 percent price increase. With a maturity of 0.25 years, the demand elasticity with respect to yield is 6.5. That is, short-term debt demand decreases by 6.5 percent per 1 percentage point yield decrease. For long-term debt, the coefficient on log price is −1.24, which implies a demand elasticity of 1.9. With a maturity of 10 years, the demand elasticity with respect to yield is 19. For equity, the coefficient on log price is −0.71, which implies a demand elasticity of 1.5.

The signs of the coefficients on the macro variables are similar across asset classes. The coefficients on log nominal GDP and log GDP per capita are positive and less than one, which means that asset demand increases less than one-to-one with the issuer country's size and wealth. Demand for short- and long-term debt decreases in inflation, but it increases in inflation for equity. Long-term debt demand decreases by 3 percent per 1 percentage point increase in inflation.

Asset demand decreases in the two risk measures across all asset classes. For equity, the coefficient on equity volatility is −3.68, which means that demand decreases by 3.68 percent per 1 percentage point increase in equity volatility. For long-term debt, the coefficient on sovereign debt rating is 0.06, which means that demand decreases by 6 percent per 1 percentage point increase in the 5-year default probability.

The bilateral variables are highly significant determinants of asset demand across all asset classes. The coefficients on export and import shares are positive, which means that investors hold assets of countries with which they trade more (Lane and Milesi-Ferretti 2008). For equity, the coefficient on the export share is 0.30, which means that demand increases by 30 percent per 1 percentage point increase in exports as a share of the geometric average of GDP. Even after controlling for trade activity, distance is a highly significant determinant of asset demand (Portes, Rey, and Oh 2001; Portes and Rey 2005). For equity, the coefficient on distance is -0.12, which means that demand decreases by 12 percent per 1,000 km increase in distance. Although there are several potential explanations, the previous literature favors the hypothesis that informational frictions increase with distance.

In addition to local bias, there is a strong home bias in the equity market. The coefficient on the dummy for own country ownership is 7.12. That is, demand for own equity is seven times higher than that for foreign equity, controlling for observed characteristics.

2. Demand across Asset Classes

Table 5 reports estimated coefficients for demand across asset classes. The coefficient on log outside asset weight is $\lambda_1 = 0.24$ for short-term debt, $\lambda_2 = 0.18$ for long-term debt, and $\lambda_3 = 0.45$ for equity. For all three asset classes, we reject the hypothesis that the coefficient is equal to zero. This means that substitution across asset classes are important for exchange rates and asset prices.

V. Decomposition of Exchange Rates and Asset Prices

Based on the estimated demand system and market clearing, we decompose exchange rates and asset prices into five sources of variation. They are changes in macro variables, short-term rates, debt quantities, foreign exchange reserves, and latent demand. We also provide additional context to the variance decomposition through two case studies that cover important events in our sample: global monetary easing and the European sovereign debt crisis.

A. Market Clearing

In our empirical implementation, we have market clearing (5) for long-term debt and equity across 36 countries. For the short-term debt market, we only have 27 market clearing equations because 10 of the 36 countries have a common short-term rate in the euro area. Because all exchange rates are defined in US dollars per local currency unit, the exchange rate for the US is always one. In the counterfactual experiments, we assume that the Federal Reserve adjusts the supply so that the US short-term debt market clears at a given short-term rate (and an exchange rate of one). Similarly, the Hong Kong dollar is pegged to the US dollar, and the Danish krone is pegged to the euro. We assume that the respective central banks maintain their currency pegs in counterfactual experiments. That is, the central bank adjusts the supply so that its short-term debt market clears at a given short-term rate and currency peg.

Market clearing defines an implicit function for exchange rates and asset prices, which we express as

$$\begin{bmatrix} \mathbf{e}_t \\ \mathbf{p}_t(2) \\ \mathbf{p}_t(3) \end{bmatrix} = \mathbf{g}(\mathbf{x}_t, \mathbf{z}_t, \mathbf{O}_t, \mathbf{p}_t(1), \mathbf{Q}_t, \epsilon_t, \xi_t). \tag{18}$$

The left side is a vector of exchange rates, long-term debt prices, and book-to-market equity. The right side is a function of observed characteristics, relative CPI, outside assets, short-term debt prices, asset quantities, latent demand, and asset-class latent demand.

We order and group the variables in the right side of equation (18) as follows.

1. Macro variables: $\mathbf{x}_t$, $\mathbf{z}_t$, $\mathbf{O}_t$, and $\mathbf{Q}_t(3)$.
2. Short-term rates: $\mathbf{p}_t(1)$.
3. Debt quantities: $\mathbf{Q}_t(1)$ and $\mathbf{Q}_t(2)$.
4. Foreign exchange reserves: Submatrix of ϵ_t and ξ_t for foreign exchange reserves only.
5. Latent demand: Submatrix of ϵ_t and ξ_t for investor countries.

Macro variables are a set of variables that government policy does not determine directly and would evolve exogenously in an endowment economy. Central bank policy determines short-term rates. Fiscal policy and unconventional monetary policy determine quantities of short- and long-term debt. We separate latent demand into that of foreign exchange reserves, determined by central bank policy, and investor countries.

We change the macro variables in equation (18) from their values in year t to $t+1$, then compute the counterfactual vector of exchange rates and asset prices that would clear all markets. We sequentially repeat the same procedure for short-term rates, debt quantities, foreign exchange reserves, and latent demand. The actual realized change in exchange rates and asset prices from year t to $t+1$ is equal to the sum of the changes across these counterfactual experiments. Thus, we have a variance decomposition of annual changes in exchange rates and asset prices.

B. Variance Decomposition of Exchange Rates and Asset Prices

Table 6 reports a variance decomposition of exchange rates, weighted by the size of the corresponding short-term debt market. The weighting is equivalent to constructing a value-weighted portfolio of exchange rates relative to the US dollar. Macro variables account for 37 percent, short-term rates account for 7 percent, debt quantities account for 3 percent, and foreign exchange reserves account for 11 percent of the variation in exchange rates. These fundamental sources jointly account for 58 percent of the variation in exchange rates. Latent demand, which is uncorrelated with observed characteristics in the demand specification, accounts for the remaining 42 percent.

In Table 7, we further decompose the contribution of latent demand to the variance of exchange rates by geographic group and asset class. Recall from Table 2 that a significant amount of short-term debt investment passes through offshore financial centers (i.e., Ireland, Luxembourg, and the Cayman Islands). Consistent with this fact, latent demand of offshore financial centers substituting across short-term debt markets accounts for 25 percent of the variation in exchange rates. Developed countries in North America and Europe are large investors across all asset classes. Consequently, asset-class latent demand of North American and European investors, which capture substitution across asset classes, account for 8 and 14 percent of the variation in exchange rates, respectively. Hau and Rey (2004) also find that substitution across asset classes is important for exchange rates. An important takeaway of Table 7 is that the importance of latent demand is geographically concentrated in large investor countries.

Table 6 reports a variance decomposition of long-term yields, weighted by the size of the corresponding long-term debt market. Policy variables are the primary determinants of long-term yields. Short-term rates account for 7 percent, and debt quantities account for 49 percent of the variation in long-term yields. Foreign exchange reserves account for 10 percent of the variation in long-term yields, confirming the importance of central bank interventions in foreign debt markets. Latent demand accounts for 36 percent of the variation in long-term yields, of which developed European countries account for 23 percent.

Table 6 also reports a variance decomposition of book-to-market equity, weighted by the size of the corresponding equity market. Macro variables and short-term rates are the primary determinants of book-to-market equity. Macro variables account for 63 percent, and short-term rates account for 6 percent of the variation in book-to-market equity. Latent demand accounts for 30 percent of the variation in book-to-market equity. Developed European and Pacific countries each account for 13 and 12 percent of the variation in book-to-market equity, respectively. Emerging markets are also important, accounting for 8 percent of the variation in book-to-market equity.

As a point of reference, Table C1 in Appendix C reports reduced-form regressions of annual changes in exchange rates and asset prices onto contemporaneous changes in the macro variables. We obtain an R^2 of 35 percent for exchange rates, 24 percent for long-term debt prices, and 53 percent for book-to-market equity. Therefore, it should not be surprising that fundamentals have high explanatory power in Table 6. The problem with the reduced-form regressions is that the signs of the estimated coefficients are inconsistent across asset classes and difficult to interpret. In contrast, the estimated coefficients for asset demand in Table 4 have a clear economic meaning, and their signs are consistent across asset classes.

C. A Case Study of Global Monetary Easing

As additional context to the variance decomposition in Table 6, we closely examine the output for four regions: the euro area, Japan, Switzerland, and the UK. According to the second step of the variance decomposition, short-term rates account for 7 percent of the variation in exchange rates. Figure 3 illustrates the second step of the variance decomposition for the four regions. The vertical axis is annual changes in exchange rates that relate to contemporaneous changes in short-term rates on the horizontal axis. In all four regions, monetary easing is associated with a depreciation of the exchange rate. Averaged across the four regions, the exchange rate depreciates by 2.6 percent per 1 percentage point decrease in the short-term rate.

According to the third step of the variance decomposition, debt quantities account for 3 percent of the variation in exchange rates and 49 percent of the variation in long-term yields. Figure 4 illustrates the third step of the variance decomposition for the four regions. The horizontal axis is annual changes in long-term yields that relate to contemporaneous changes in debt quantities. The vertical axis is annual changes in exchange rates that relate to contemporaneous changes in debt quantities. In all four regions, a decrease in debt quantities is associated with a lower long-term yield and a depreciation of the exchange rate. Averaged across the four regions, a change in debt quantities that decreases the long-term yield by 10 basis points also depreciates the exchange rate by 0.8 percent.

Based on a vector autoregression methodology, Clarida and Gali (1994), Eichenbaum and Evans (1995), and Inoue and Rossi (2018) find that both conventional and unconventional monetary policy affect exchange rates. Based on an event study methodology, Gagnon et al. (2011) and Krishnamurthy and Vissing-Jørgensen (2011) find that unconventional monetary policy affects long-term yields. Fundamentally, unconventional monetary policy concerns changes in the supply of long-term debt and their impact on exchange rates and asset prices through substitution effects. The demand system approach models this mechanism directly, based on market clearing of global financial markets. Our approach is suited for studying the simultaneous and cumulative impact of conventional and unconventional monetary policy across many countries.

D. A Case Study of the European Sovereign Debt Crisis

As additional context to the variance decomposition in Table 6, Table 8 reports a variance decomposition of the long-term yield spread between Germany and the US. Remarkably, short-term rates and debt quantities account for almost all of the variation in the long-term yield spread. Short-term rates account for 32 percent, and debt quantities account for 54 percent of the variation in the long-term yield spread.

Figure 5 shows the time series of the annual change in the long-term yield spread between Germany and the US and its decomposition into changes due to short-term rates and debt quantities. This figure shows high correlation between the overall change in the long-term yield spread and the change due to short-term rates. The covariance between the two time series divided by the variance of the overall change in the long-term yield spread is 32 percent, which is the share of variance due to short-term rates reported in Table 8. The figure also shows high correlation between the overall change in the long-term yield spread and the change due to debt quantities. The covariance between the two time series divided by the variance of the overall change in the long-term yield spread is 54 percent, which is the share of variance due to debt quantities reported in Table 8.

Figure C1 in Appendix C is a simple scatter plot that explains this finding. The upper panel shows that the long-term yield spread between Germany and the US is positively related to the interest rate differential between the two countries. That is, the relative level factor drives the long-term yield spread. The lower panel shows that the long-term yield spread is positively related to the relative long-term debt quantity. In both scatter plots, the data points in the upper right are 2002, 2003, and 2008 when the US had relatively expansionary monetary policy and a lower long-term yield. The data points in the lower left are 2016 and 2017 when Germany had relatively expansionary monetary policy and a lower long-term yield.

Table 8 also reports a variance decomposition of long-term yield spreads between southern euro countries (i.e., Greece, Italy, and Portugal) and Germany. Because countries in the euro area share a common short-term rate, the variance decomposition gives a sensible answer that short-term rates account for none of the variation in long-term yield spreads. Especially after 2008, southern euro countries had very different macro and fiscal experiences relative to Germany. Consistent with this fact, macro variables account for 43 percent, and debt quantities account for 28 percent of the variation in long-term yield spreads. Latent demand of European countries accounts for 20 percent, and latent demand of offshore financial centers accounts for 6 percent of the variation in long-term yield spreads. This finding that only the latent demand of euro area investors matters is consistent with currency bias in sovereign debt portfolios (Maggiori, Neiman, and Schreger 2018).

Figure 6 shows the time series of the annual change in the long-term yield spread between southern euro countries and Germany and their decomposition into changes due to macro variables, debt quantities, and latent demand. The most influential observations are the sharp increase in the long-term yield spread with the onset of the European sovereign debt crisis in 2011, followed by a sharp decrease when the European Central Bank intervened in 2012. Consistent with the variance decomposition in Table 8, the figure shows high correlation between the overall change in the long-term yield spread and the changes due to macro variables, debt quantities, and latent demand. A close examination reveals an interesting difference between Greece versus Italy and Portugal. For Greece, macro variables, which include equity volatility and sovereign debt ratings, account for the sharp increase in the credit spread. For Italy and Portugal, latent demand, which capture perceived rather than realized risk, account for the sharp increase in the credit spread. This finding is consistent with the narrative that Greece had a realized solvency problem, while investors perceived Italy and Portugal to be still solvent but vulnerable.

VI. Conclusion

Based on a demand system approach, we have developed an international asset pricing model to study sources of variation in exchange rates and asset prices. We conclude with two broad lessons. First, the significance of substitution effects across asset classes highlights the need to study exchange rates, long-term yields, and equity prices jointly. Second, government policy is important for exchange rates and asset prices. Short-term rates account for 7 percent of the variation in exchange rates and long-term yields and 6 percent of the variation in book-to-market equity. Debt quantities account for 49 percent of the variation in long-term yields. Foreign exchange reserves account for 11 and 10 percent of the variation in exchange rates and long-term yields, respectively. Through these policy tools, central banks play an important role in managing exchange rates and the term structure of interest rates globally.

In the spirit of portfolio balance models in international finance, we have focused entirely on financial markets in this paper. Future work could extend our framework to the real side of the economy. Recent work on international macro models emphasize the need for latent demand (i.e., asset demand shocks unrelated to fundamentals) to resolve longstanding puzzles in international finance (Blanchard, Giavazzi, and Sa 2005; Gabaix and Maggiori 2015; Itskhoki and Mukhin 2017). A critique of this literature is that latent demand is an unmeasurable "wedge" without a structural interpretation. Our approach shows otherwise, that latent demand can be estimated from cross-country holdings. Therefore, the variance decompositions in this paper are moments to discipline a new generation of international macro models.

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Table 1
Market values of Financial Assets

Country	Short-term debt Billion US$	Short-term debt Share in reserves	Long-term debt Billion US$	Long-term debt Share in reserves	Equity Billion US$	Equity Share in reserves
Developed markets: North America						
Canada	168.7	0.09	859	0.13	2,367	0.00
United States	822.7	0.34	7,402	0.28	32,121	0.01
Developed markets: Europe						
Austria	8.3	0.64	309	0.15	151	0.00
Belgium	82.1	0.17	364	0.12	438	0.00
Denmark	19.0	0.35	215	0.07	592	0.00
Finland	21.7	0.39	183	0.13	204	0.01
France	313.5	0.18	1,931	0.17	2,749	0.01
Germany	203.2	0.50	1,747	0.31	2,262	0.01
Israel	0.6	0.00	29	0.01	231	0.00
Italy	66.4	0.11	1,042	0.07	735	0.00
Netherlands	130.1	0.24	1,398	0.08	1,100	0.01
Norway	18.5	0.11	221	0.08	287	0.00
Portugal	9.6	0.00	119	0.15	76	0.00
Spain	53.0	0.09	763	0.08	889	0.00
Sweden	99.3	0.12	386	0.07	699	0.00
Switzerland	33.6	0.19	112	0.03	1,686	0.00
United Kingdom	352.9	0.06	1,948	0.08	3,246	0.01
Developed markets: Pacific						
Australia	95.9	0.09	638	0.14	1,508	0.00
Hong Kong	34.6	0.15	100	0.01	4,351	0.00
Japan	441.4	0.36	473	0.22	6,223	0.00
New Zealand	2.6	0.14	51	0.08	95	0.00
Singapore	63.7	0.24	88	0.06	787	0.00
Emerging markets						
China	104.9	0.20	237	0.25	8,711	0.00
Colombia	0.2	0.00	60	0.01	121	0.00
Czech Republic	15.5	0.00	34	0.02	25	0.00
Greece	4.3	0.00	32	0.16	51	0.00
Hungary	0.3	0.00	36	0.04	32	0.00
India	10.6	0.00	78	0.01	2,332	0.00
Malaysia	7.0	0.28	63	0.03	456	0.00
Mexico	10.7	0.08	258	0.02	417	0.00
Philippines	1.4	0.00	28	0.10	290	0.00
Poland	0.1	0.00	111	0.07	201	0.00
Russia	0.5	0.00	61	0.00	623	0.00
South Africa	0.2	0.00	63	0.01	1,231	0.00
South Korea	21.5	0.42	137	0.18	1,772	0.00
Thailand	3.4	0.00	28	0.00	549	0.00

The market value of short- or long-term debt is the total amount held by foreign investors. The market value of equity is total stock market capitalization. All market values are in billion US dollars at year-end 2017.

TABLE 2
TOP TEN INVESTORS BY ASSET CLASS

Short-term debt		Long-term debt		Equity	
Investor	Billion US$	Investor	Billion US$	Investor	Billion US$
Reserves	917	Reserves	4,398	United States	32,799
Ireland	527	Japan	2,176	China	8,192
United States	488	United States	2,165	Japan	5,343
Luxembourg	361	Germany	2,002	Hong Kong	4,198
France	215	Luxembourg	1,995	United Kingdom	2,867
Cayman Islands	188	France	1,489	Canada	2,846
United Kingdom	126	Ireland	1,317	France	1,965
Hong Kong	111	United Kingdom	1,038	Luxembourg	1,952
Singapore	84	Netherlands	909	India	1,826
Switzerland	55	Cayman Islands	834	Australia	1,641

IMF aggregates foreign exchange reserves across all central banks for confidentiality. All market values are in billion US dollars at year-end 2017.

Table 3
Predictability of Asset Returns

Variable	Exchange rate		Short-term debt	Long-term debt	Equity
Interest rate differential	-0.03				
	(0.79)				
Log real exchange rate		-0.36	-0.31	-0.37	-0.88
		(0.08)	(0.08)	(0.11)	(0.30)
Log market-to-book			-7.78	-0.45	-0.29
			(3.49)	(0.13)	(0.22)
Observations	375	375	375	540	540

Interest rate differential is the foreign 3-month rate minus the US 3-month rate. For debt, log market-to-book is equal to minus maturity times the continuously compounded yield. All specifications include country fixed effects. Robust standard errors clustered by year are reported in parentheses. The annual sample period is 2002 to 2017.

Table 4
Estimated Demand within Asset Class

Variable	Short-term debt	Long-term debt	Equity
Log price	-70.43	-1.24	-0.71
	(8.60)	(0.20)	(0.39)
Log GDP	0.88	0.80	0.85
	(0.02)	(0.01)	(0.01)
Log GDP per capita	0.89	0.77	0.48
	(0.08)	(0.05)	(0.04)
Inflation	-0.06	-0.03	0.04
	(0.02)	(0.01)	(0.01)
Volatility	-1.84	-0.51	-3.68
	(0.37)	(0.20)	(0.50)
Rating	0.03	0.06	0.03
	(0.02)	(0.01)	(0.01)
Export share	0.25	0.28	0.30
	(0.03)	(0.02)	(0.03)
Import share	0.10	0.13	0.10
	(0.03)	(0.02)	(0.03)
Distance	-0.14	-0.13	-0.12
	(0.01)	(0.00)	(0.00)
Dummy: Own country			7.12
			(0.14)
Observations	17,293	31,252	30,202
R^2	0.43	0.49	0.67

Equation (12) is the estimation equation. Sovereign debt rating is a continuous measure equal to minus one times the 5-year default probability. All specifications include fixed effects for the issuer country's MSCI classification, the investor country, and year. Heteroskedasticity-robust standard errors are reported in parentheses. The annual sample period is 2002 to 2017.

Table 5
Estimated Demand across Asset Classes

Variable	Symbol	Estimate
Log outside asset weight:		
Short-term debt	λ_1	0.24
		(0.06)
Long-term debt	λ_2	0.18
		(0.08)
Equity	λ_3	0.45
		(0.04)
Dummy:		
Short-term debt	α_1	-2.46
		(0.25)
Long-term debt	α_2	0.44
		(0.25)
Observations		2,339

Equation (14) is the estimation equation. The annual sample period is 2002 to 2017.

Table 6
Variance Decomposition of Exchange Rates and Asset Prices

Share of variance	Exchange rate	Long-term debt	Equity
Macro variables	0.37	-0.02	0.63
	(0.06)	(0.09)	(0.12)
Short-term rates	0.07	0.07	0.06
	(0.04)	(0.02)	(0.03)
Debt quantities	0.03	0.49	-0.03
	(0.05)	(0.05)	(0.04)
Reserves	0.11	0.10	0.05
	(0.04)	(0.03)	(0.02)
Latent demand	0.42	0.36	0.30
	(0.07)	(0.05)	(0.12)
North America	0.05	0.04	-0.01
	(0.03)	(0.01)	(0.07)
Europe	0.12	0.23	0.13
	(0.04)	(0.04)	(0.04)
Pacific	0.03	0.05	0.12
	(0.01)	(0.01)	(0.03)
Offshore financial centers	0.21	0.03	-0.02
	(0.06)	(0.02)	(0.02)
Emerging markets	0.01	0.01	0.08
	(0.01)	(0.00)	(0.04)
Other countries	0.01	0.00	-0.01
	(0.00)	(0.00)	(0.01)
Observations	375	540	540

Variance is value-weighted by the size of the corresponding asset market. Heteroskedasticity-robust standard errors are reported in parentheses. The annual sample period is 2002 to 2017.

Table 7
Variance Decomposition of Exchange Rates by Latent Demand

	Within asset class			Across
Share of variance	Short-term debt	Long-term debt	Equity	asset classes
Latent demand	0.21	0.01	-0.01	0.21
	(0.08)	(0.03)	(0.03)	(0.06)
North America	-0.04	0.00	0.01	0.08
	(0.06)	(0.00)	(0.01)	(0.05)
Europe	-0.03	0.01	0.00	0.14
	(0.07)	(0.01)	(0.01)	(0.06)
Pacific	0.01	-0.01	0.01	0.01
	(0.04)	(0.01)	(0.00)	(0.04)
Offshore financial centers	0.25	-0.01	0.00	-0.03
	(0.09)	(0.01)	(0.00)	(0.05)
Emerging markets	0.02	0.02	-0.02	0.00
	(0.01)	(0.02)	(0.02)	(0.00)
Other countries	0.01	0.00	0.00	0.00
	(0.00)	(0.00)	(0.00)	(0.00)

Variance is value-weighted by the size of the corresponding asset market. Heteroskedasticity-robust standard errors are reported in parentheses. The annual sample period is 2002 to 2017.

Table 8
Variance Decomposition of Long-Term Yield Spreads

Share of variance	Germany −US	Southern euro − Germany
Macro variables	-0.14	0.43
	(0.18)	(0.09)
Short-term rates	0.32	0.00
	(0.11)	(0.00)
Debt quantities	0.54	0.28
	(0.17)	(0.06)
Reserves	0.13	0.02
	(0.33)	(0.01)
Latent demand	0.14	0.28
	(0.14)	(0.09)
North America	-0.05	0.01
	(0.06)	(0.00)
Europe	0.14	0.20
	(0.14)	(0.07)
Pacific	0.05	0.00
	(0.04)	(0.00)
Offshore financial centers	-0.01	0.06
	(0.11)	(0.02)
Emerging markets	-0.02	0.00
	(0.02)	(0.00)
Other countries	0.02	0.00
	(0.02)	(0.01)
Observations	15	45

The southern euro countries are Greece, Italy, and Portugal. Heteroskedasticity-robust standard errors are reported in parentheses. The annual sample period is 2002 to 2017.

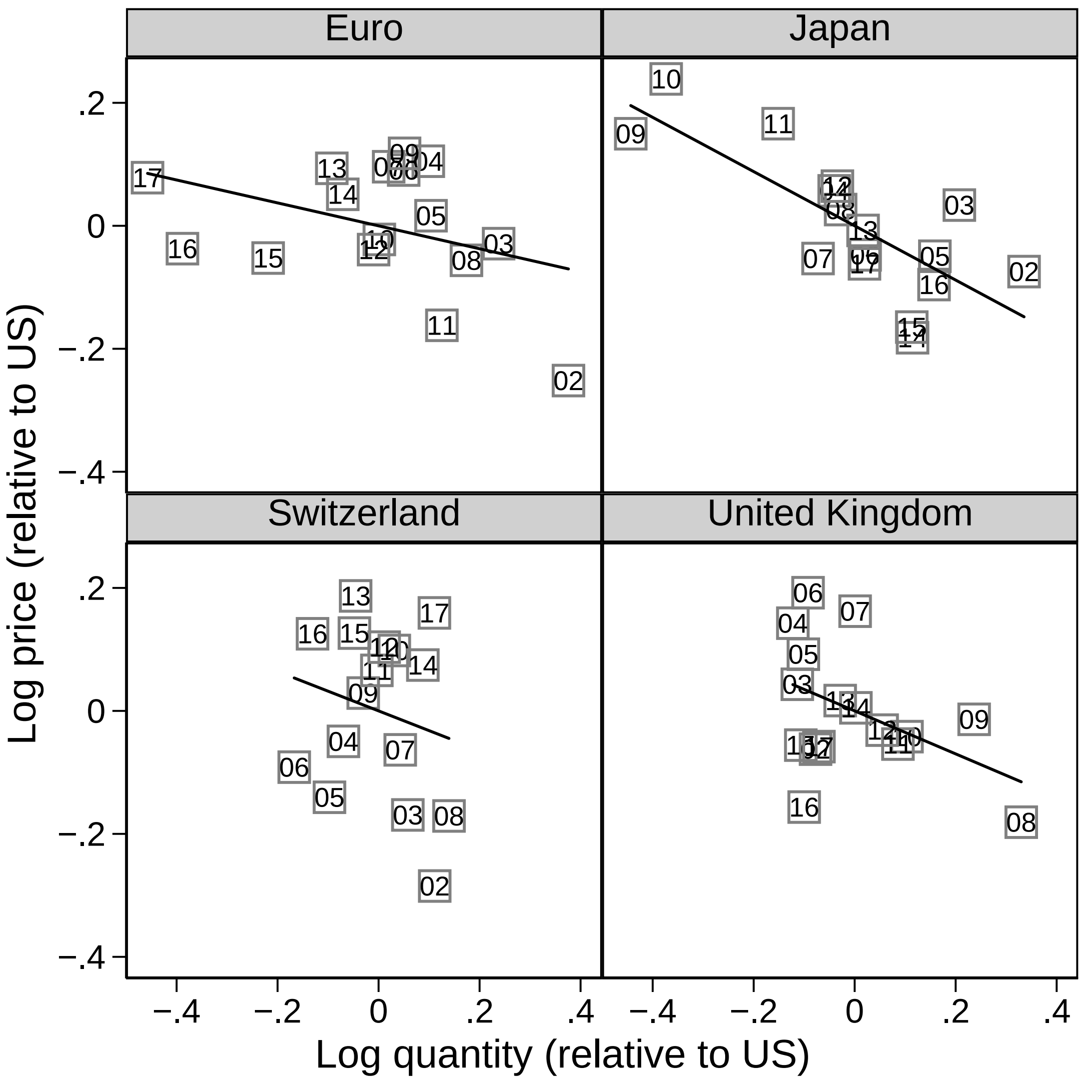

Figure 1. Relative Long-Term Debt Quantity and Price

The horizontal axis is each country's log long-term debt quantity in local currency units minus the US log long-term debt quantity (in US dollars). The vertical axis is each country's log long-term debt price plus the log exchange rate (in US dollars per local currency unit) minus the US log long-term debt price. For the euro area, log price is a weighted average across countries, based on the size of the long-term debt market. The two digit number indicates year (e.g., 02 is 2002). Each panel reports the linear regression line.

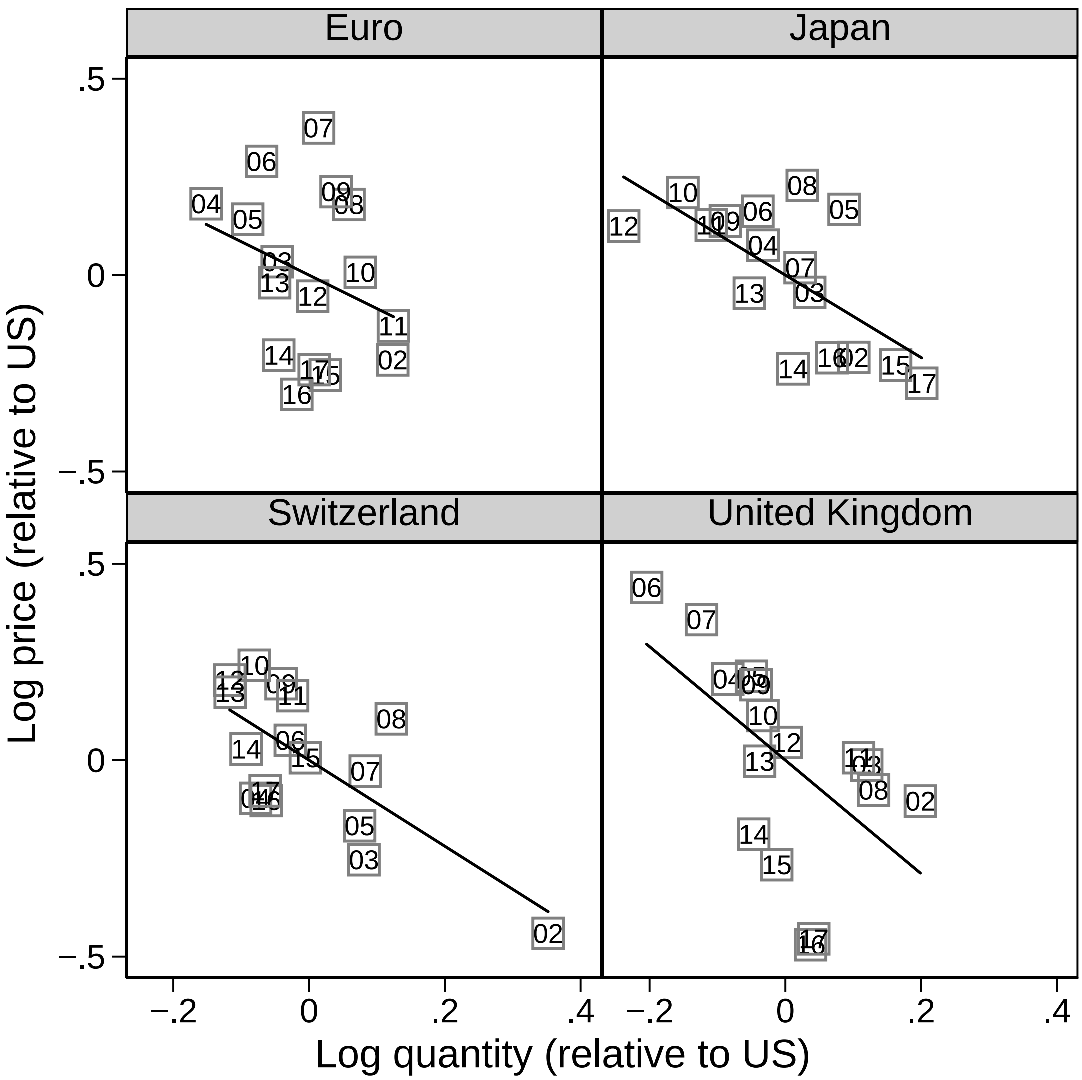

Figure 2. Relative Equity Quantity and Price

The horizontal axis is each country's log equity quantity in local currency units minus the US log equity quantity (in US dollars). The vertical axis is each country's log book-to-market equity plus the log exchange rate (in US dollars per local currency unit) minus the US log book-to-market equity. For the euro area, log price is a weighted average across countries, based on the size of the equity market. The two digit number indicates year (e.g., 02 is 2002). Each panel reports the linear regression line.

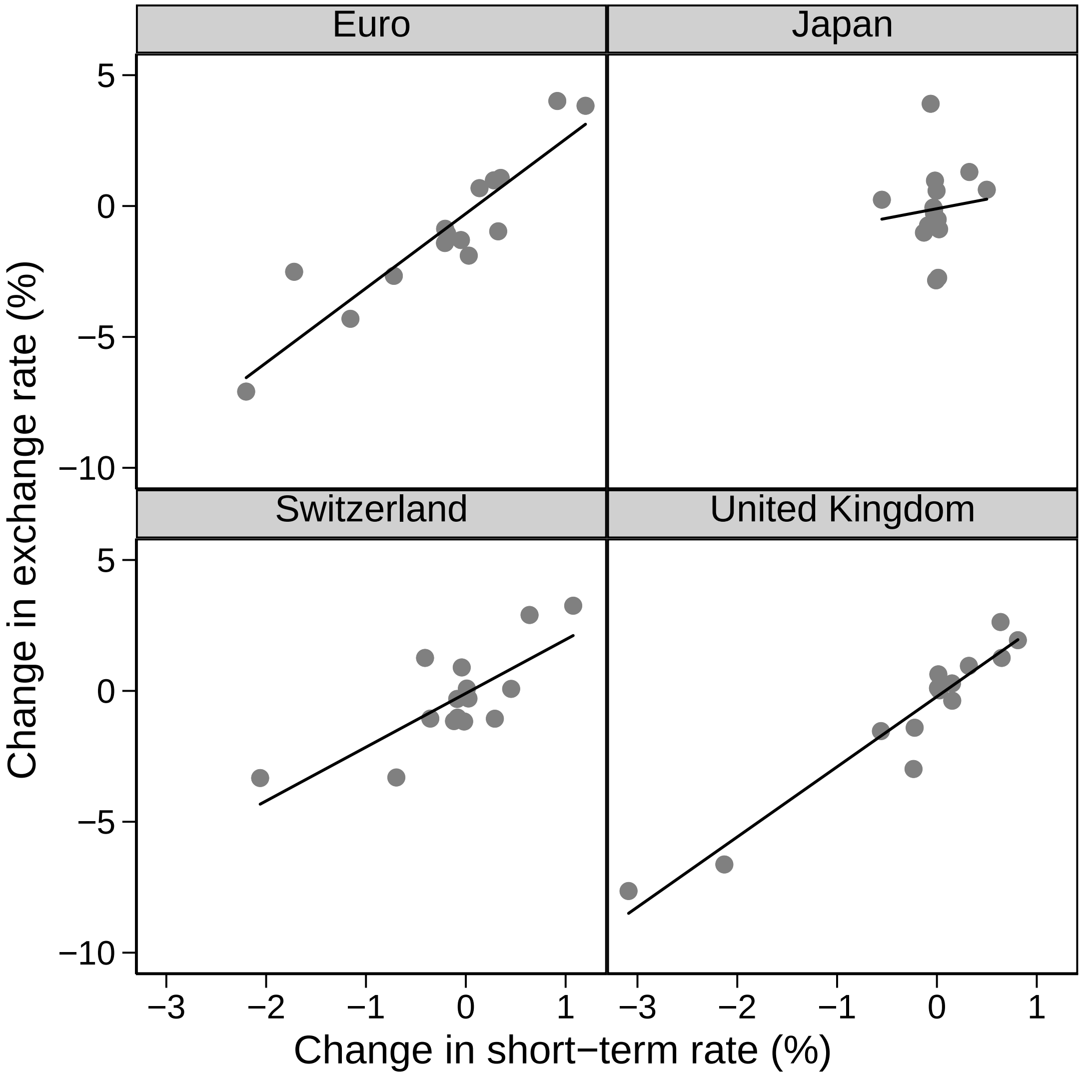

Figure 3. How Exchange Rates Relate to Short-Term Rates
Each panel reports the linear regression line. The annual sample period is 2002 to 2017.

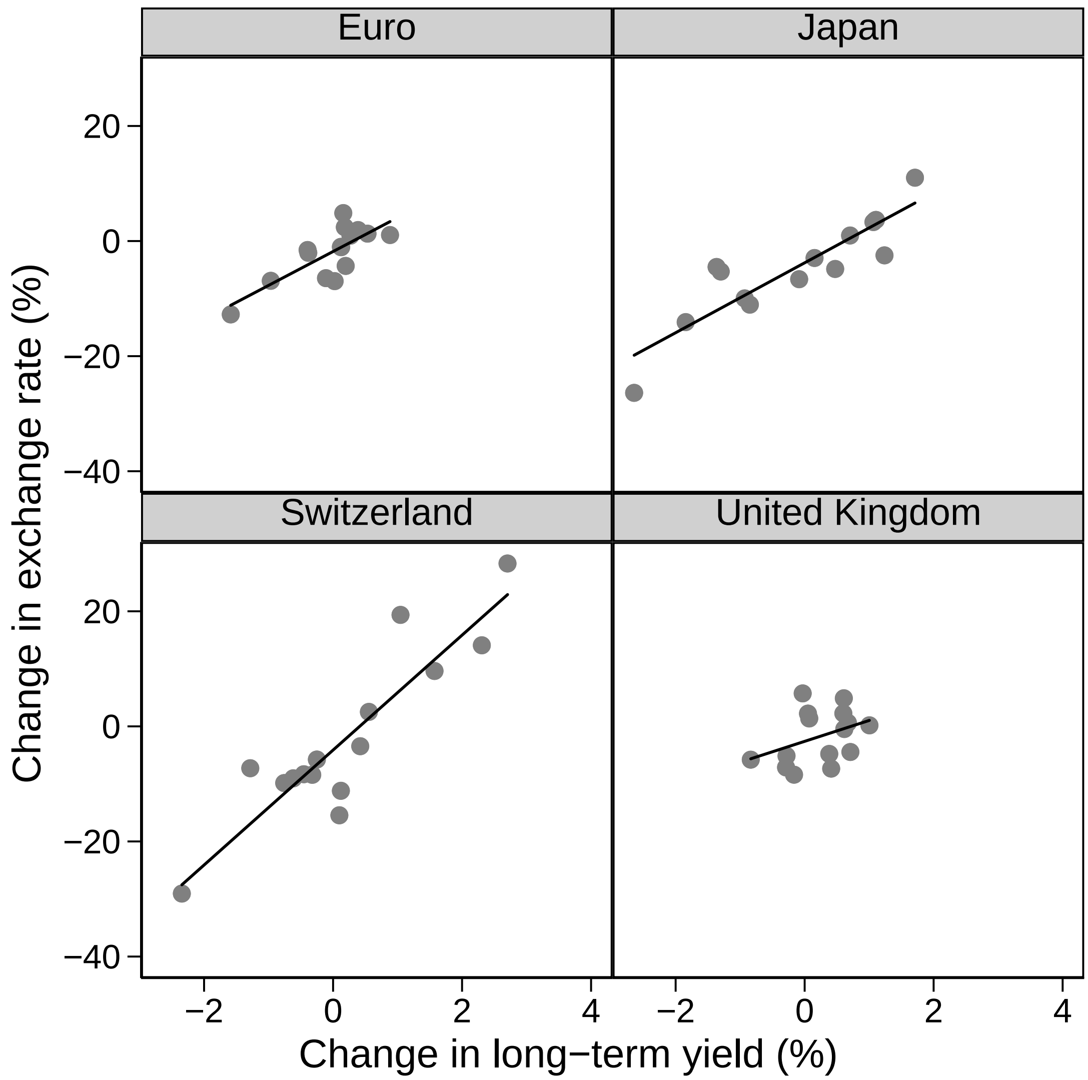

Figure 4. How Long-Term Yields and Exchange Rates Relate to Debt Quantities

For the euro area, yield is a weighted average across countries, based on the size of the long-term debt market. Each panel reports the linear regression line. The annual sample period is 2002 to 2017.

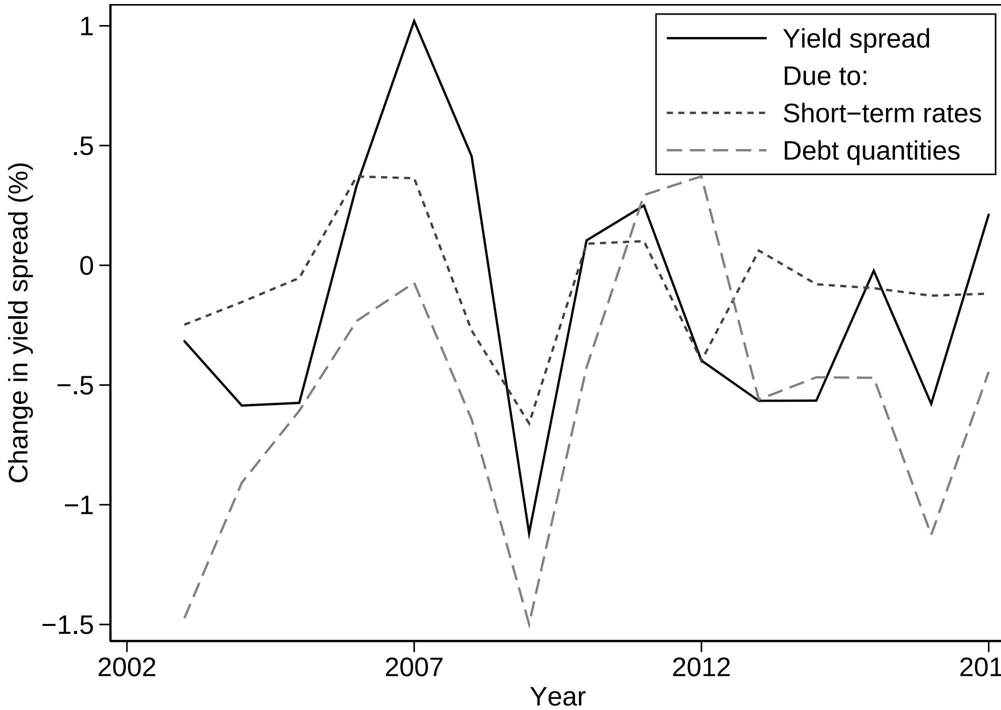

Figure 5. Change in the Long-Term Yield Spread between Germany and the US

The annual change in the long-term yield spread is decomposed into macro variables, short-term rates, debt quantities, foreign exchange reserves, and latent demand. This figure reports the changes due to short-term rates and debt quantities only.

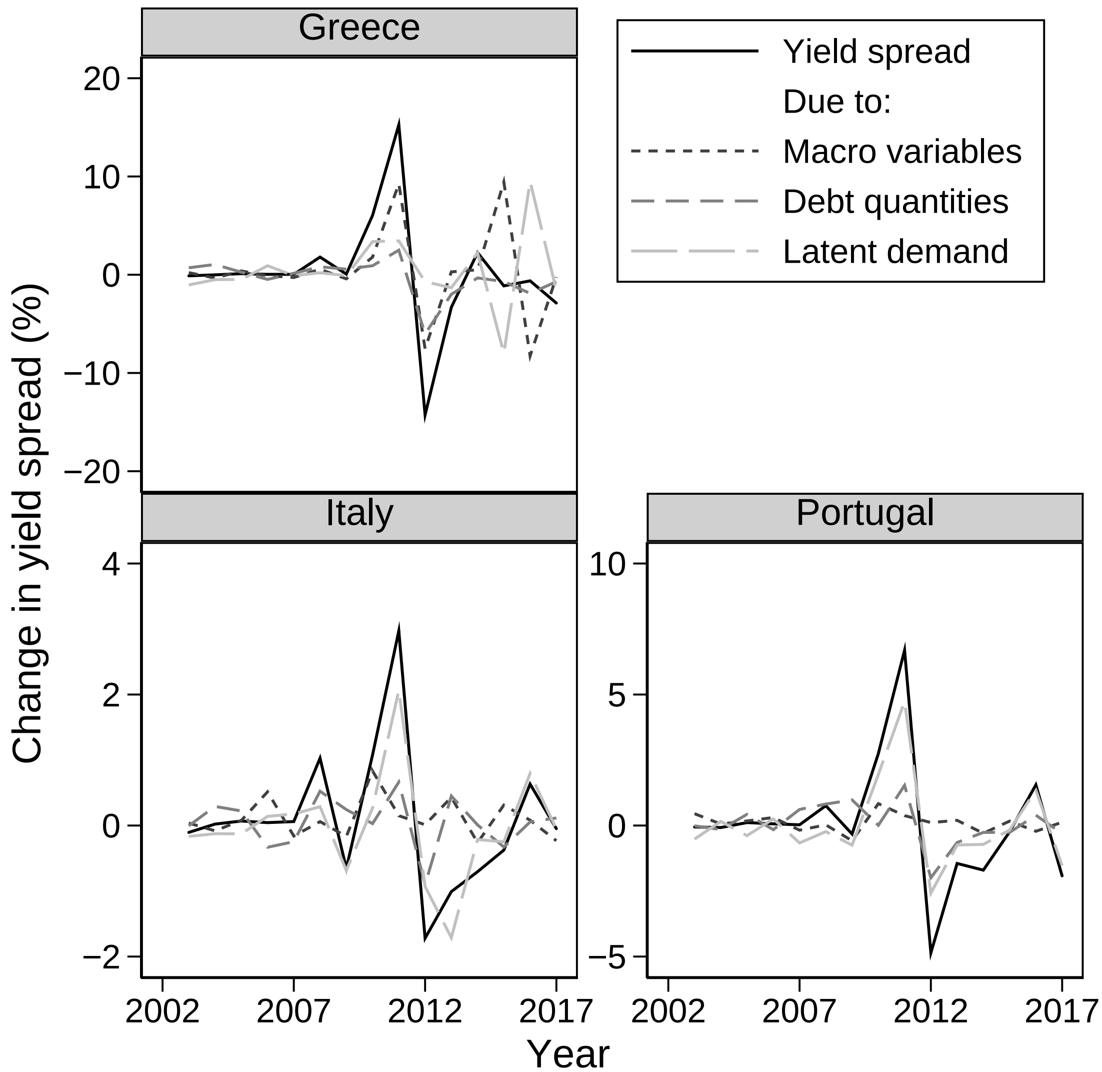

Figure 6. Change in the Long-Term Yield Spread between Southern Euro Countries and Germany

The annual change in the long-term yield spread is decomposed into macro variables, short-term rates, debt quantities, foreign exchange reserves, and latent demand. This figure reports the changes due to macro variables, debt quantities, and latent demand only.

Appendix A. Data Construction

A. Cross-Country Holdings

Table A1 contains the complete list of 88 investor countries. The 36 countries in bold are also issuer countries, for which we have complete data on asset prices and characteristics. Based on their MSCI classification, we group the countries into developed markets in three regions (North America, Europe, and Pacific) and emerging markets. We define offshore financial centers as countries whose ratio of portfolio assets to GDP is above five (Zoromé 2007, Table 8). They are Bermuda, the Cayman Islands, Guernsey, Ireland, the Isle of Man, Jersey, Luxembourg, and the Netherlands Antilles. The "other countries" in Table A1 are neither part of the MSCI ACWI Index nor an offshore financial center.

Mutual funds and investment companies domicile in offshore financial centers because of favorable regulation and taxes. Consequently, offshore financial centers have large amounts of investment from investor countries that pass through to issuer countries. IMF's Coordinated Portfolio Investment Survey counts these investments twice, once as investment from an investor country and again as investment to an issuer country (International Monetary Fund 2002, p. 72). To eliminate double counting, we do not count investment from an investor country to an offshore financial center. This is equivalent to treating the aggregate holdings of an offshore financial center as an investor unit without breaking them apart by the ultimate investor, which would require some assumptions and imputations.

IMF does not report confidential holdings. For each investor-issuer pair, we impute the missing observation based on the last (if available) or next observed holding. The assumption is that the quantity held in local currency units does not change from one year to the next, and the investment amount in US dollars changes only because of asset returns inclusive of exchange rate appreciation. If the data are always confidential for an investor-issuer pair, we assume that the investment amount is zero.

IMF reports small holdings less than \$0.5 million as zero. For these cases, we first distinguish an actual zero from a censored zero based on the panel dimension. If the reported amount has always been zero for an investor-issuer pair, we assume that the investment amount is an actual zero. If the reported amount has been positive in the past, we assume that the investment amount is a censored zero. We estimate a censored regression model of log investment amount onto investor and issuer fixed effects by year and asset class. We impute the censored observation as the predicted value from the regression, conditional on censoring at \$0.5 million.

Long-term debt includes both government and corporate debt. Unfortunately, there is no tractable way to separate the two for all countries. Moreover, corporate bond yields are

Table A1
List of Investor and Issuer Countries

Country	Country
Developed markets: North America	*Offshore financial centers*
Canada	Bermuda
United States	Cayman Islands
Developed markets: Europe	Guernsey
Austria	Ireland
Belgium	Isle of Man
Denmark	Jersey
Finland	Luxembourg
France	Netherlands Antilles
Germany	*Other countries*
Israel	Albania
Italy	Argentina
Netherlands	Aruba
Norway	Bahamas
Portugal	Bahrain
Spain	Barbados
Sweden	Belarus
Switzerland	Bolivia
United Kingdom	Bulgaria
Developed markets: Pacific	Costa Rica
Australia	Curacao
Hong Kong	Cyprus
Japan	Estonia
New Zealand	Gibraltar
Singapore	Honduras
Emerging markets	Iceland
Brazil	Kazakhstan
Chile	Kosovo
China	Kuwait
Colombia	Latvia
Czech Republic	Lebanon
Egypt	Lithuania
Greece	Macao
Hungary	Macedonia
India	Malta
Indonesia	Mauritius
Malaysia	Mongolia
Mexico	Panama
Pakistan	Romania
Peru	Saudi Arabia
Philippines	Slovakia
Poland	Slovenia
Russia	Ukraine
South Africa	Uruguay
South Korea	Vanuatu
Thailand	Venezuela
Turkey	West Bank and Gaza

The countries in bold are issuer countries with complete data on asset prices and characteristics.

unavailable for many of the 36 countries in our sample. Therefore, we simply assume that the government bond yield is the representative price of long-term debt in the cross-country holdings data. Another measurement issue is that a small share of debt may be denominated in a foreign currency. Because there is no tractable way to separate the foreign-currency debt, we simply assume that all debt is denominated in local currency.

B. Observed Characteristics

Nominal GDP in US dollars and real GDP per capita, based on purchasing power parity in constant international dollars, are from the World Bank. CPI inflation rates are from IMF's International Financial Statistics. Exports and imports are from the UN Comtrade Database. We compute the export share for each pair of countries as exports divided by the geometric average of their nominal GDP. We compute the import share for each pair of countries analogously. The physical distance between each pair of countries is from the GeoDist Database (Mayer and Zignago 2011).

For each country and at each year-end, we estimate the standard deviation of monthly returns in US dollars over the past 12 months. We annualize equity volatility by multiplying the monthly standard deviation by $\sqrt{12}$. Sovereign debt ratings are from S&P Capital IQ Entity Ratings. We use the long-term debt rating in local currency (if available) or foreign currency. We convert the rating to a continuous measure based on the 5-year default probability: 0 percent for AAA to AA−, 1.48 percent for A(+/−), 2.02 percent for BBB(+/−), 2.55 percent for BB(+/−), 5.32 percent for B(+/−), and 33.53 percent for CCC+ and below. Our measure is minus one times the 5-year default probability, so that a higher measure implies a higher rating.

Appendix B. Relation between Demand System Asset Pricing and the CAPM

We relate the demand system approach to a traditional model of international finance, based on the CAPM. Investors have heterogeneous expectations and disagree about expected returns. Let

$$\mu_i(n,l) = \mathbb{E}_i[\Delta p_{t+1}(n,l) + \Delta e_{t+1}(n,l) - y_t(\text{US})] \tag{B1}$$

be investor i's expectation of excess returns on asset class l in country n over the US interest rate. To simplify exposition, we assume that investors have homogeneous expectations about risk. Excess returns have a one-factor structure, where Ω is the vector of factor loadings and

ω is the vector of idiosyncratic variances. The covariance matrix of excess returns is

$$\Sigma = \Omega\Omega' + \text{diag}(\omega), \tag{B2}$$

where the second term is a diagonal matrix of idiosyncratic variances.

Investors have a mean-variance objective function, so their optimal portfolio is the mean-variance portfolio (Markowitz 1952):

$$\mathbf{w}_i = \Sigma^{-1}\mu_i. \tag{B3}$$

The Woodbury matrix identity implies that the inverse of the covariance matrix (B2) is

$$\Sigma^{-1} = \text{diag}(\omega)^{-1}\left(\mathbf{I} - \frac{\Omega\Omega'\text{diag}(\omega)^{-1}}{1 + \Omega'\text{diag}(\omega)^{-1}\Omega}\right), \tag{B4}$$

Therefore, the optimal portfolio weight in country n and asset class l is

$$w_i(n,l) = \frac{\mu_i(n,l)}{\omega(n,l)} - \frac{\Omega(n,l)}{\omega(n,l)} \underbrace{\frac{\Omega'\text{diag}(\omega)^{-1}\mu_i}{1 + \Omega'\text{diag}(\omega)^{-1}\Omega}}_{\text{scalar}}. \tag{B5}$$

Equation (B5) implies higher portfolio weights on assets with higher expected returns (the first term) and lower risk (the second term). Investors have different portfolios because they have heterogeneous expectations about expected returns. Demand system asset pricing amounts to matching equation (B5) to portfolio holdings data, by modeling expected returns and factor loadings as a function of observed characteristics and latent demand. We refer to Koijen and Yogo (2019) for technical details about how equation (B5) maps to the logit functional form.

If investors have homogeneous and rational expectations about expected returns, they have identical portfolios. Then equation (B3) and market clearing (5) imply the CAPM:

$$\mu(n,l) = \sigma^2\Omega(n,l), \tag{B6}$$

where σ^2 is the variance of the market portfolio return. That is, expected excess returns are linear in factor loadings. In a traditional model of international finance, the consumption goods market determines the exchange rate, and the CAPM determines all asset prices.

Appendix C. Supplemental Tables and Figures

Table C1
Regressions of Changes in Exchange Rates and Asset Prices

Variable	Exchange rate	Long-term debt	Equity
Log GDP	0.54	-0.02	-0.11
	(0.07)	(0.05)	(0.05)
Log GDP per capita	-0.22	0.01	0.00
	(0.08)	(0.06)	(0.00)
Inflation	-0.02	-0.30	0.13
	(0.06)	(0.09)	(0.07)
Volatility	-0.35	0.13	-0.67
	(0.08)	(0.05)	(0.05)
Rating	-0.02	0.33	0.02
	(0.03)	(0.10)	(0.03)
Export share	0.08	-0.05	0.04
	(0.12)	(0.12)	(0.10)
Import share	-0.10	0.21	-0.24
	(0.13)	(0.14)	(0.12)
Observations	509	539	539
R^2	0.35	0.24	0.53

Observations are value-weighted by the size of the corresponding asset market. All regressors are in first differences. Sovereign debt rating is a continuous measure equal to minus one times the 5-year default probability. All coefficients are standardized. Heteroskedasticity-robust standard errors are reported in parentheses. The annual sample period is 2002 to 2017.

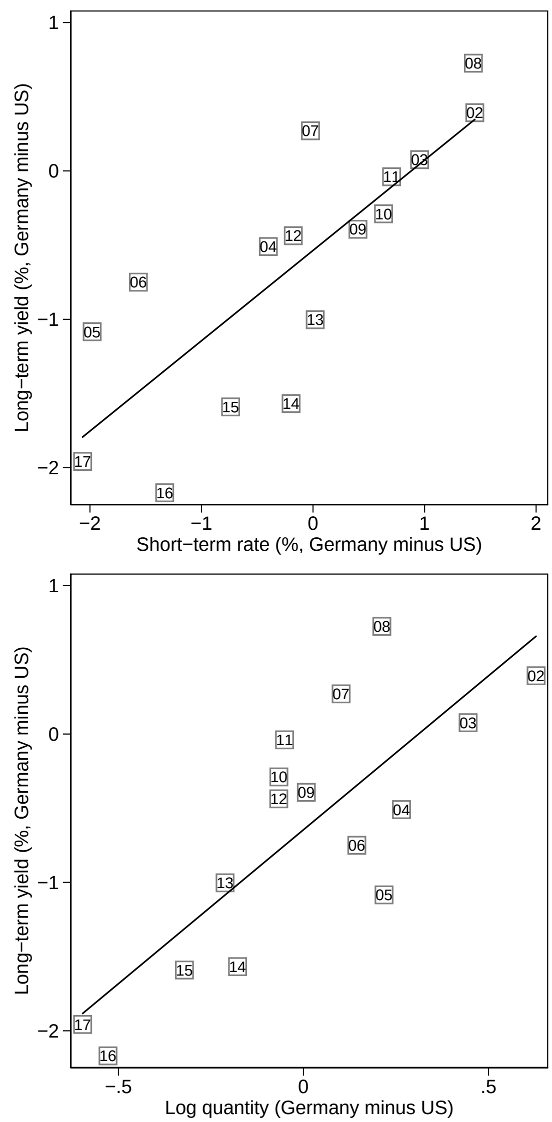

Figure C1. Long-Term Yield Spread between Germany and the US

In the lower panel, the horizontal axis is the German log long-term debt quantity in euros minus the US log long-term debt quantity (in US dollars). The two digit number indicates year (e.g., 02 is 2002). Each panel reports the linear regression line.