Monetary Policy, Redistribution, and Risk Premia

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Abstract

We study the transmission of monetary policy through risk premia in a heterogeneous agent New Keynesian environment. Heterogeneity in households’ marginal propensity to take risk (MPR) summarizes differences in portfolio choice on the margin. An unexpected reduction in the nominal interest rate redistributes to households with high MPRs, lowering risk premia and amplifying the stimulus to the real economy. Quantitatively, this mechanism rationalizes the role of news about future excess returns in driving the stock market response to monetary policy shocks and amplifies their real effects by 1.3-1.4 times.

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1 Introduction

A growing literature finds that expansionary monetary policy lowers risk premia. This has been established for the equity premium in stock markets, the term premium in nominal bonds, and the external finance premium on risky corporate debt. The basic New Keynesian framework as in Woodford (2003) and Gali (2008) does not capture this aspect of monetary policy transmission. As noted by Kaplan and Violante (2018), this is equally true for emerging heterogeneous agent New Keynesian models in which heterogeneity in the marginal propensity to consume enriches the transmission mechanism but still cannot explain the associated movements in risk premia.

This paper demonstrates that a New Keynesian model with heterogeneous households differing instead in risk-bearing capacity can quantitatively rationalize the observed effects of policy on risk premia, amplifying the transmission to the real economy. An expansionary monetary policy shock lowers the risk premium on capital if it redistributes to households with a high marginal propensity to take risk (MPR), defined as the marginal propensity to save in capital relative to save overall. With heterogeneity in risk aversion, portfolio constraints, rules of thumb, background risk, or beliefs, high MPR households borrow in the bond market from low MPR households to hold leveraged positions in capital. By generating unexpected inflation, raising profit income relative to labor income, and raising the price of capital, an expansionary monetary policy shock redistributes to high MPR households and thus lowers the market price of risk. In a calibration matching portfolio heterogeneity in the U.S. economy, this rationalizes the observed role of news about lower future excess returns in driving the increase in the stock market. The real stimulus is amplified by 1.3-1.4 times relative to an economy without heterogeneity in portfolios and MPRs.

Our baseline environment enriches a standard New Keynesian model with Epstein and Zin (1991) preferences and heterogeneity in risk aversion. Households consume, supply labor subject to adjustment costs in nominal wages, and choose a portfolio of nominal bonds and capital. Production is subject to aggregate TFP shocks. Monetary policy follows a Taylor (1993) rule. Heterogeneity in risk aversion generates heterogeneity in MPRs and exposures to a monetary policy shock. Epstein-Zin preferences imply that this heterogeneity is distinct from households’ intertemporal elasticities of substitution. We first analytically characterize the effects of a monetary policy

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1See Bernanke and Kuttner (2005), Hanson and Stein (2015), and Gertler and Karadi (2015).
shock in a simple two-period version of this environment, providing an organizing framework for the quantitative analysis of the infinite horizon which follows.

An expansionary monetary policy shock lowers the risk premium by redistributing wealth to households with a high marginal propensity to save in capital relative to save overall — that is, a high MPR. Redistribution to high MPR households lowers the risk premium because of asset market clearing: if households on aggregate wish to increase their portfolio share in capital, its expected return must fall relative to that on bonds. An expansionary monetary policy shock redistributes across households by revaluing their initial balance sheets: it deflates nominal debt, raises the profits earned using capital, and raises the price of capital. More risk tolerant households hold leveraged positions in capital and have a higher MPR. Hence, an expansionary monetary policy shock will redistribute to these households and lower the risk premium.

The reduction in the risk premium amplifies the transmission of monetary policy to the real economy. Conditional on the real interest rate — which reflects the degree of nominal rigidity and the monetary policy rule — a decline in the required excess return on capital is associated with an increase in investment. The increase in investment crowds in consumption by raising household wealth. The stimulus to consumption and investment implies an increase in output overall.

These results are robust to heterogeneity beyond risk aversion. We consider a richer environment in which households may also face portfolio constraints or follow rules-of-thumb; households may be subject to idiosyncratic background risk; and households may have subjective beliefs regarding the value of capital. Because each of these still imply that households holding more levered positions in capital will be those with high MPRs, they continue to imply that expansionary monetary policy will lower the risk premium through redistribution, amplifying real transmission.

Accounting for the risk premium effects of monetary policy is important given empirical evidence implying that it may be a key component of the transmission mechanism. We refresh this point from Bernanke and Kuttner (2005) using the structural vector autoregression instrumental variables (SVAR-IV) approach in Gertler and Karadi (2015). We find that a monetary policy shock resulting in a roughly 0.2pp reduction in the 1-year Treasury yield leads to a 1.9pp increase in the real S&P 500 return. Using a Campbell and Shiller (1988) decomposition and accounting for estimation uncertainty, 20% – 100% of this increase is driven by lower future excess returns, challenging existing New Keynesian frameworks where essentially all of the
effect on the stock market operates through higher dividends or lower risk-free rates.

Extending the model to the infinite horizon, we investigate whether a calibration to the U.S. economy is capable of rationalizing these facts. We match the heterogeneity in wealth, labor income, and financial portfolios in the Survey of Consumer Finances, together disciplining the exposures to a monetary policy shock and MPRs. We use global solution methods to solve the model. To make the computational burden tractable, we model three groups of households: two groups corresponding to the small fraction with high wealth relative to labor income, but differing in their risk tolerance and thus portfolio share in capital, and one group corresponding to the large fraction holding little wealth relative to labor income. In the data, the high-wealth, high-leverage households are disproportionately those with private business wealth, while the high-wealth, low-leverage households are disproportionately retirees.

We find that the redistribution across households with heterogeneous MPRs can quantitatively explain the risk premium effects of an expansionary monetary policy shock. Notably, the redistribution relevant for this result is between wealthy households holding heterogeneous portfolios, rather than between the asset-poor and asset-rich. Using the same Campbell-Shiller decomposition as was used on the data, over 30% of the return on equity in our baseline parameterization arises from news about lower future excess returns, compared to 0% in a representative agent counterfactual. Consistent with the analytical results, the redistribution to high-MPR households is amplified with a more persistent shock and thus larger debt deflation; higher stickiness and thus a larger increase in profit income relative to labor income; or higher investment adjustment costs and thus a larger increase in the price of capital.

Further consistent with the analytical results, the reduction in the risk premium through redistribution in turn amplifies the effect of policy on the real economy. In both our baseline and counterfactual representative agent economies, we study monetary policy shocks which deliver a 0.2pp decline in the 1-year nominal yield on impact. Our model amplifies the response of quantities by 1.3-1.4 times: the peak investment, consumption, and output responses are 2.0pp, 0.4pp, and 0.7pp, while the counterparts in the representative agent economy are 1.6pp, 0.3pp, and 0.5pp.

Related literature Our paper contributes to the rapidly growing literature on heterogeneous agent New Keynesian (HANK) models by studying the transmission of monetary policy through risk premia. We build on Doepke and Schneider (2006) in our
measurement of household portfolios, informing the heterogeneity in exposures to a monetary policy shock. The redistributive effects of monetary policy in our framework follow Auclert (2019). We demonstrate that it is the covariance of these exposures with MPRs rather than MPCs which matters for policy transmission through risk premia. Like Kaplan, Moll, and Violante (2018) and Luetticke (2021), we study an environment with bonds and capital. And like Alves, Kaplan, Moll, and Violante (2020), Auclert, Rognlie, and Straub (2020), and Melcangi and Sterk (2020) we study the effects of monetary policy shocks on asset prices. Unlike these models, in our framework assets differ in exposure to aggregate risk rather than in liquidity, allowing us to account for the important role of risk premia in driving the change in asset prices.

In doing so, we bring to the HANK literature many established insights from heterogeneous agent and intermediary-based asset pricing. The wealth distribution is a crucial determinant of the market price of risk as in other models with heterogeneous risk aversion (e.g., Garleanu and Panageas (2015)), segmented markets (e.g., He and Krishnamurthy (2013)), rules-of-thumb (e.g., Chien, Cole, and Lustig (2012)), background risk (e.g., Constantinides and Duffie (1996)), or heterogeneous beliefs (e.g., Geanakoplos (2009)).\textsuperscript{2} We build on this literature by focusing on the changes in wealth induced by a monetary policy shock in a production economy with nominal rigidities. In studying this question we follow Alvarez, Atkeson, and Kehoe (2009) and Drechsler, Savov, and Schnabl (2018), who study the effects of monetary policy on risk premia in an exchange economy with segmented markets and in a model of banking, respectively.\textsuperscript{3} We instead study these effects operating through the revaluation of heterogeneous agents’ balance sheets in a conventional New Keynesian setting.

Indeed, our paper most directly builds on prior work focused on risk premia in New Keynesian economies. We clarify the sense in which Bernanke, Gertler, and Gilchrist (1999) served as a seminal HANK model focused on heterogeneity in MPRs rather than MPCs.\textsuperscript{4} As we demonstrate, however, heterogeneity in MPRs need not rely on market segmentation, justifying its relevance even in markets which may not be intermediated by specialists. In relating movements in the risk premium to the

\textsuperscript{2}In recent work, Panageas (2020) studies the common implications of these models, and Toda and Walsh (2020) emphasize the role of portfolio heterogeneity in redistribution, as in our analysis.  
\textsuperscript{3}More recently, Bhandari, Evans, and Golosov (2019), Chen and Phelan (2021), and Coimbra and Rey (2021) study monetary policy and risk premia in models with segmentation and intermediation.  
\textsuperscript{4}In Bernanke et al. (1999), households can only trade bonds while entrepreneurs can trade bonds and capital. In equilibrium, households have a zero MPR while entrepreneurs have a positive MPR. Changes in net worth across these agents thus affects credit spreads and economic activity.
real economy, we make use of the insight in Ilut and Schneider (2014), Caballero and Farhi (2018), and Caballero and Simsek (2020) that an increase in the risk premium will induce a recession if the safe interest rate does not sufficiently fall in response.\footnote{While these authors make this point in the case of a time-varying price of risk (as in our model), a similar result obtains with a time-varying quantity of risk as in Fernandez-Villaverde, Guerron-Quintana, Kuester, and Rubio-Ramirez (2015), Basu and Bundick (2017), and DiTella (2020).} We build especially on the latter two papers, as well as Brunnermeier and Sannikov (2012, 2016), in emphasizing the effects of heterogeneity in asset valuations on risk premia. Relative to these papers, we explore the importance of such heterogeneity for monetary transmission in a calibration to the U.S. economy.\footnote{In recent complementary work, Pflueger and Rinaldi (2021) study monetary transmission and risk premia in a representative agent New Keynesian model augmented with consumption habits.}

Like all of these papers, our analysis also provides a theoretical counterpart to the large empirical literature studying links between risky asset prices and real activity. Focusing first on stock prices, the evidence in support of the $q$-theory of investment has been mixed, and causal estimates of stock prices on consumption are complicated by the fact that they may simply be forecasting other determinants of consumption. Recently, Pflueger, Siriwardane, and Sunderam (2020) and Chodorow-Reich, Nenov, and Simsek (2021) have employed cross-sectional identification strategies to overcome these challenges, finding evidence in support of the cost-of-capital and consumption wealth mechanisms in our model. More broadly interpreting our model as studying the effect of monetary policy on risky claims on capital, there is substantial evidence that spreads on risky corporate debt predict real activity (e.g., Gilchrist and Zakrajsek (2012) and Lopez-Salido, Stein, and Zakrajsek (2017)).

Outline  In section 2 we characterize our main insights in a two-period environment. In section 3 we compare empirical evidence on the equity market response to monetary policy shocks to the quantitative predictions of our model enriched to the infinite horizon and calibrated to the U.S. economy. Finally, in section 4 we conclude.

## 2 Analytical insights in a two-period environment

We first characterize our main conceptual insights in a two-period environment allowing us to obtain simple analytical results. Heterogeneity in risk aversion induces heterogeneity in household portfolios. An expansionary monetary policy shock lowers
the risk premium on capital by redistributing to relatively risk tolerant households. A reduction in the risk premium amplifies the stimulus to investment, consumption, and output. These results are robust to heterogeneity in rules-of-thumb, portfolio constraints, background risk, or beliefs. More generally, they hold whenever levered households, whose relative wealth rises upon a monetary easing, have high propensities to save in capital relative to bonds — i.e., high MPRs.

2.1 Environment

There are two periods, 0 and 1. To isolate the key mechanisms, we make a number of parametric assumptions which are relaxed later in the paper.

**Households** A unit measure of households indexed by $i \in [0,1]$ have Epstein-Zin preferences over consumption in each period $\{c^i_0, c^i_1\}$ and labor supply $\ell_0$

$$\log v^i_0 = (1 - \beta) \log c^i_0 - \frac{\ell_0^{1+1/\theta}}{1 + 1/\theta} + \beta \log \left( \mathbb{E}_0 \left[ (c^i_1)^{1-\gamma} \right] \right)^{\frac{1}{1-\gamma}},$$  \hspace{1cm} (1)

with a unitary intertemporal elasticity of substitution, discount factor $\beta$, relative risk aversion $\gamma^i$, (dis)utility of labor $\bar{\theta}$, and Frisch elasticity $\theta$. Labor in period 0 is not indexed by $i$ because (as we describe below) households supply the same amount. In period 1 production only uses capital and thus there is no labor supplied.

In addition to consuming and supplying labor, the household chooses its position in a nominal bond $B^i_0$ and in capital $k^i_0$ subject to the resource constraints

$$P_0 c^i_0 + B^i_0 + Q_0 k^i_0 \leq W_0 \ell_0 + (1 + i_{-1}) B^i_{-1} + (\Pi_0 + (1 - \delta_0) Q_0) k^i_{-1}, \hspace{1cm} (2)$$

$$P_1 c^i_1 \leq (1 + i_0) B^i_0 + \Pi_1 k^i_0. \hspace{1cm} (3)$$

$B^i_{-1}$ and $k^i_{-1}$ are its endowments in these same assets. The consumption good trades at $P_t$ units of the nominal unit of account (“dollars”) at $t$, the household earns a wage $W_0$ dollars in period 0, one dollar in bonds purchased at $t$ yields $1 + i_t$ dollars at $t + 1$, and one unit of capital purchased for $Q_t$ dollars at $t$ yields a dividend $\Pi_t$ dollars at $t + 1$ plus $(1 - \delta_{t+1}) Q_{t+1}$ dollars at $t + 1$. Capital fully depreciates after period 1 ($\delta_1 = 1$).

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7Following Woodford (2003), we model the economy at the cashless limit.
Supply-side  The nominal wage is rigid at its level set the previous period

\[ W_0 = W_{-1}. \quad (4) \]

Each household is willing to supply the labor demanded of it from firms, appealing to households’ market power in the labor market which we spell out later in the paper.

In period 0, the representative producer hires \( \ell_0 \) units of labor and rents \( k_{-1} \) units of capital from households to produce the final good with TFP of one. It also uses \( \left( \frac{k_0}{k_{-1}} \right)^{x^*} x_0 \) units of the consumption good to produce \( x_0 \) new capital sold to households, where \( x^* \) indexes adjustment costs and it takes \( k_0 \) as given. The producer thus earns

\[ \Pi_0 k_{-1} = P_0 \ell_0^{1-\alpha} k_{-1}^\alpha - W_0 \ell_0 + Q_0 x_0 - P_0 \left( \frac{k_0}{k_{-1}} \right)^{x^*} x_0. \quad (5) \]

In period 1, the producer rents \( k_0 \) units of capital and has TFP \( \exp(\epsilon_1^*) \), so it earns

\[ \Pi_1 k_0 = P_1 \exp(\epsilon_1^*) k_0^\alpha. \quad (6) \]

Future TFP is uncertain in period 0, following

\[ \epsilon_1^* \sim N \left( -\frac{1}{2} \sigma^2, \sigma^2 \right). \quad (7) \]

Policy  The government sets monetary policy \( \{i_0, P_1\} \) by committing to \( P_1 = P_0 \), eliminating inflation risk in the nominal bond,\(^8\) and following the Taylor rule

\[ 1 + i_0 = (1 + \bar{i}) \left( \frac{P_0}{P_{-1}} \right)^{\phi} \exp(\epsilon_0^m) \quad (8) \]

with reference price \( P_{-1} \), where \( \epsilon_0^m \) is the shock of interest. It follows that the real interest rate between periods 0 and 1 is\(^9\)

\[ 1 + r_1 \equiv (1 + i_0) \frac{P_0}{P_1} = (1 + \bar{i}) \left( \frac{P_0}{P_{-1}} \right)^{\phi} \exp(\epsilon_0^m). \]

\(^8\)It is straightforward to allow \( P_1 = P_0 \exp(\iota \epsilon_1^*) \) for \( \iota \neq 0 \), so that there is inflation risk in the nominal bond. Our quantitative analysis in the next section features inflation risk.

\(^9\)Between periods \( t \) and \( t + 1 \) we denote \( i_t \) the nominal interest rate known in period \( t \) and \( r_{t+1} \) the realized real interest rate depending on the price level in period \( t + 1 \).
Market clearing  Market clearing in goods is
\[ \int_0^1 c_i^0 di + \left( \frac{k_0}{k_{-1}} \right) \chi^x x_0 = \ell_0^{1-\alpha} k_{-1}^\alpha, \quad \int_0^1 c_i^1 di = \exp(\epsilon_i^z) k_0^\alpha, \] (9)
in the capital rental market is
\[ \int_0^1 k_{i-1}^i di = k_{-1}, \quad \int_0^1 k_0^i di = k_0, \] (10)
in the capital claims market is
\[ (1 - \delta_0) \int_0^1 k_{i-1}^i di + x_0 = \int_0^1 k_0^i di, \] (11)
and in bonds is
\[ \int_0^1 B_0^i di = 0. \] (12)

Equilibrium  Given the state variables \{W_{-1}, P_{-1}, \{B_{i-1}^i, k_{i-1}^i\}, i_{-1}, \epsilon_0^m\} and a stochastic process for \(\epsilon_1^z\) in (7), the definition of equilibrium is then standard:

**Definition 1.** An equilibrium is a set of prices and policies such that: (i) each household \(i\) chooses \(\{c_i^0, B_i^0, k_i^0, c_i^1\}\) to maximize (1) subject to (2)-(3), (ii) wages are rigid as in (4), (iii) the representative producer chooses \(\{\ell_0, x_0\}\) to maximize profits (5) and earns profits (6), (iv) the government sets \(\{i_0, P_1\}\) according to \(P_1 = P_0\) and (8), and (v) the goods, capital, and bond markets clear according to (9)-(12).

We now characterize the comparative statics of this economy with respect to a monetary policy shock \(\epsilon_0^m\) in a sequence of three main propositions. Each result builds on the last, and each makes use of only a few equilibrium conditions.

2.2 Monetary policy, redistribution, and the risk premium

We first provide a general result characterizing the effect of a monetary policy shock on the expected excess return on capital.

We need to know each household’s desired portfolio in capital. Define \(i\)’s real savings
\[ a_i^0 = b_i^0 + q_0 k_i^0, \]
and portfolio share in capital
\[ \omega_i^0 \equiv \frac{q_0 k_i^0}{a_i^0}, \]
where we use lower-case to denote the real analogs to the nominal variables introduced earlier. Let \( 1 + r_k^1 \) denote the gross real returns on capital
\[ 1 + r_k^1 \equiv \frac{\Pi_1 P_0}{Q_0 p_1} = \frac{\pi_1}{q_0}. \]

Then \( i \)'s optimality condition for \( \omega_i^0 \) is given by
\[ \mathbb{E}_0 \left[ (c_i^1) - \gamma_i^i (r_k^1 - r_1) \right] = 0. \] (13)

Taking a Taylor approximation of the expression inside the expectation up to second order in the excess log return, it follows that the optimal portfolio share in capital approximately satisfies
\[ \omega_i^0 \approx \frac{1}{\gamma_i} \mathbb{E}_0 \log(1 + r_k^1) - \log(1 + r_1) + \frac{1}{2} \sigma^2. \] (14)

Given a positive risk premium, more risk tolerant households choose a larger portfolio share in capital. This is the only approximation we use in the results which follow.

Simply by aggregating (14) and making use of the asset market clearing conditions (10) and (12), we obtain the first result of the paper, the proof of which (along with all other proofs) is in appendix A:

**Proposition 1.** The risk premium on capital is
\[ \mathbb{E}_0 \log(1 + r_k^1) - \log(1 + r_1) + \frac{1}{2} \sigma^2 = \gamma \sigma^2, \]
where
\[ \gamma \equiv \left( \int_0^1 a_i^0 \left[ \int_0^1 a_0' d\gamma \right] \frac{1}{\gamma^i} \right)^{-1}. \]

The change in the risk premium in response to a monetary shock is
\[ \frac{d \left[ \mathbb{E}_0 \log(1 + r_k^1) - \log(1 + r_1) \right]}{de_0^m} = \gamma \sigma^2 \int_0^1 \frac{d \left[ a_i^0 \int_0^1 a_0' d\gamma \right]}{de_0^m} (1 - \omega_i^0) \, di. \] (15)
Hence, a monetary policy shock affects the risk premium if it redistributes across households with heterogeneous desired portfolios. If monetary policy does not redistribute \( \left( \frac{d\left[ a_0^i / \int_0^1 a_0^i di \right]}{d\epsilon_0^n} = 0 \text{ for all } i \right) \) or households have identical desired portfolios \( (\omega_0^i = 1 \text{ for all } i) \), there is no effect on the risk premium. Away from this case, redistributing wealth to households with relatively high desired portfolios in capital lowers the risk premium. Intuitively, such redistribution raises the relative demand for capital, lowering the required excess return to clear asset markets.

### 2.3 Risk premium and the real economy

We now characterize why a change in the risk premium is relevant for the real economy.

The link between investment and the risk premium is due to the relation between the expected return to capital and investment. Indeed, optimal investment solving (5) and equilibrium dividends in (6) together imply that the expected return on capital is given by

\[
E_0 \log(1 + r^k_1) = \log \alpha + E_0 \log z_1 + \chi^x \log k_{-1} - (1 - \alpha + \chi^x) \log k_0. \tag{16}
\]

Hence, investment is declining in the expected return to capital.

The link between consumption and the risk premium operates through household wealth, both directly through the value of capital holdings and indirectly through the effect of investment on the level of production and thus disposable income. Indeed, household \( i \)'s optimal choice of consumption is given by

\[
c_0^i = (1 - \beta)n_0^i(w_0\ell_0, P_0, \pi_0, q_0),
\]

where we collect \( i \)'s wealth as a function of non-predetermined variables in

\[
n_0^i(w_0\ell_0, P_0, \pi_0, q_0) \equiv w_0\ell_0 + \frac{1}{P_0}(1 + i_{-1})B_{-1}^i + (\pi_0 + (1 - \delta_0)q_0)k_{-1}^i. \tag{17}
\]

Aggregating and making use of firms’ resource constraint (5) and the market clearing conditions (9)-(12), we thus obtain:
Proposition 2. The change in investment in response to a monetary shock is

\[
\frac{dk_0}{d\epsilon_m^0} = -\frac{k_0}{1 - \alpha + \chi^x} \left[ d \left[ E_0 \log(1 + r_k^1) - \log(1 + r_1) \right] \frac{d}{d\epsilon_m^0} + d \log(1 + r_1) \right].
\] (18)

The change in consumption \(c_0 \equiv \int_0^1 c_i^0 di\) in response to a monetary shock is

\[
\frac{dc_0}{d\epsilon_m^0} = \frac{1 - \beta}{\beta} q_0(1 + \chi^x) \frac{dk_0}{d\epsilon_m^0}.
\]

The change in output \(y_0 \equiv \ell_0^{1-\alpha} k_0^{\alpha} \) in response to a monetary shock is

\[
\frac{dy_0}{d\epsilon_m^0} = \frac{dc_0}{d\epsilon_m^0} + q_0 \left( 1 + \chi^x \frac{x_0}{k_0} \right) \frac{dk_0}{d\epsilon_m^0}.
\] (19)

Thus, conditional on the real interest rate, a decline in the risk premium is associated with an increase in investment. The increase in investment stimulates aggregate demand and thus production. Consumption rises both because of the increase in the value of capital holdings and because of the rise in disposable income induced by higher production. This feeds back to further stimulate aggregate demand and thus production.\(^{10}\) These results apply to the case of a monetary policy shock the broader insights of Caballero and Simsek (2020) linking risk premia and the real economy.

2.4 Monetary transmission via the risk premium

We now sign the transmission of a monetary policy shock via the risk premium.

The relevant measure of redistribution toward household \(i\) in Proposition 1 is the change in its savings share. Since agents share the same marginal propensity to save (\(\beta\)), this is equal to the change in its wealth share

\[
\frac{d \left[ a_0^i / \int_0^1 a_0^{i'} di' \right]}{d\epsilon_m^0} = \frac{d \left[ n_0^i / \int_0^1 n_0^{i'} di' \right]}{d\epsilon_m^0}.
\] (20)

\(^{10}\)These effects parallel the real effects of a decline in the real interest rate, holding fixed the risk premium. Indeed, as Proposition 2 makes clear, holding fixed the risk premium, a decline in the real interest rate must lead to a fall in the real return on capital as agents rebalance into capital. The associated rise in investment and household wealth stimulates aggregate demand and thus output.
Given (17) and defining $n_0 \equiv \int_0^1 n^i_0 di$, the change in its wealth share is in turn

$$d \left[ \frac{n^i_0}{\int_0^1 n^i_0 di} \right] = \frac{1}{n_0} \left[ -1 + \frac{i-1}{i_{-1}} B^i_{-1} \frac{d \log P_0}{d\epsilon^m_0} + \left( k_{-1} - \frac{n^i_0}{n_0} k_{-1} \right) \left( \frac{d\pi_0}{d\epsilon^m_0} + \left( 1 - \delta \right) \frac{dq_0}{d\epsilon^m_0} \right) \right].$$ (21)

Hence, in this setting there are three channels through which wealth is redistributed on impact of a monetary policy shock: via inflation (which redistributes towards nominal borrowers) or via an increase in profits or the price of capital (which redistribute towards those with a disproportionate claim on capital). These heterogeneous exposures to a monetary shock have been previously exposited in the HANK literature, as by Auclert (2019). Propositions 1 and 2 imply that it is their covariance with desired portfolio shares which matters for transmission through risk premia.

When agents’ initial endowments are consistent with their desired portfolios in period 0 — as would be the case in the steady-state of an infinite horizon model — and they start with same initial levels of wealth, we can sharply sign these effects:

**Proposition 3.** Suppose agents differ in risk aversion $\{\gamma^i\}$; their initial endowments are consistent with their desired portfolio in period 0 ($\{\omega^i_{-1} = \omega^i_0\}$); and they have the same initial levels of wealth. Then:

- a cut in the nominal interest rate lowers the risk premium, and
- the resulting stimulus to investment, consumption, and output are larger than a representative agent economy starting from the same aggregate allocation.

Intuitively, relatively risk tolerant agents finance levered positions in capital by borrowing in nominal bonds.$^{11}$ A cut in the nominal interest rate generates inflation, an increase in profits, and an increase in the price of capital, redistributing wealth to these agents. Proposition 1 implies that this lowers the risk premium. At least given a conventional Taylor rule, the endogenous response of the real interest rate is not sufficiently strong to overturn the amplification characterized in Proposition 2.

$^{11}$Recall that leverage here reflects the net economic exposure households have in capital and bonds. In practice, this requires accounting for the balance sheets of firms in which households own equity and the balance sheet of the government, which we do in our quantitative analysis in section 3. For instance, risk tolerant households may simply own equity in firms and risk averse households may simply own nominal claims. The former would be levered through the balance sheets of the firms in which they invest (which must be issuing the nominal claims to the risk averse).
2.5 Other sources of heterogeneity

The preceding results do not rely on heterogeneity in risk aversion alone; they also apply when there is heterogeneity in portfolios arising from other primitives.

**Binding constraints or rules-of-thumb**  Suppose a measure of households are not at an interior optimum in their portfolio choice because of the additional constraint

\[ q_0 k_0^i = \omega^i a_0^i, \]

reflecting either a binding leverage constraint or a rule-of-thumb in portfolios. When \( \omega^i = 0 \) in particular, this means the household cannot participate in the capital market. Such constraints are consistent with prior asset pricing models with segmented markets or rules-of-thumb as well as macro models of the financial accelerator.

**Background risk**  Suppose households are subject to idiosyncratic risk beyond the aggregate risk already described: their capital chosen in period 0 is subject to a shock \( \epsilon_1^i \) in period 1, modeled as a multiplicative change in the efficiency units of capital. \( \epsilon_1^i \) is iid across households and independent of the aggregate TFP shock \( \epsilon_1^z \), and \( \eta^i \) controls the degree of background risk according to

\[
\log \epsilon_1^i \sim N \left( -\frac{1}{2} \eta^i \sigma^2, \eta^i \sigma^2 \right).
\]

This environment captures features of the large literatures in macroeconomics and finance with entrepreneurial income risk.

**Subjective beliefs**  Suppose household \( i \) believes that TFP follows

\[
\epsilon_1^z \sim N \left( -\frac{1}{2} \zeta^i \sigma^2, \zeta^i \sigma^2 \right)
\]

even though the objective (true) probability distribution remains described by (7). As in the large literature on belief disagreements, households with \( \zeta^i > 1 \) are “pessimists” and households with \( \zeta^i < 1 \) are “optimists”.

We can then prove:
Proposition 4. Suppose households differ in risk aversion \( \gamma^i \), being constrained and (among those that are) constraints \( \omega^i \), background risk \( \eta^i \), and beliefs \( \varsigma^i \). Further suppose that their endowments are identical to their choices in period 0 and they are otherwise identical. Then we obtain the same results as in Proposition 3.

Intuitively, in this more general environment a household’s portfolio share in capital is falling in risk aversion \( \gamma^i \), background risk \( \eta^i \), and pessimism \( \varsigma^i \), and rising in the leverage constraint or rule-of-thumb \( \omega^i \) (if applicable). Regardless of these underlying drivers, so long as households enter period 0 with endowments reflecting these same portfolios, it will be the case that an expansionary monetary policy shock redistributes to those wishing to hold relatively more capital. Thus, an expansionary shock lowers the risk premium, amplifying the stimulus to the real economy.

2.6 Exposures and the marginal propensity to take risk

The robustness of these results derives from the tight link between households’ exposures to a monetary policy shock and their marginal portfolio choices given a dollar of income. In a more general environment, we now demonstrate that it is the covariance between the two which governs the effects of such a shock on the risk premium.

Consider how a household’s optimal portfolio changes with an additional dollar of income. Let the capital, bond, and total savings policy functions solving each household’s micro-level optimization problem be given by \( k^i_0(\cdot) \), \( b^i_0(\cdot) \), and \( a^i_0(\cdot) \), respectively. Their arguments are the household’s wealth \( n^i_0 \) and all other aggregates which the household takes as given, such as the real interest rate \( r_1 \) and price of capital \( q_0 \). Then:

**Definition 2.** Household \( i \)'s marginal propensity to take risk (MPR) is

\[
mpr^i_0 \equiv \frac{q_0 \partial k^i_0}{\partial n^i_0} / \frac{\partial a^i_0}{\partial n^i_0}.
\]

The MPR summarizes the household’s marginal portfolio choice in capital. It captures a dimension of behavior in principle orthogonal to the marginal propensity to consume emphasized in prior work. Note that the following results also hold when inflation risk renders the nominal bond risky; we give the MPR its name because under any realistic calibration the payoff on capital is more risky than on bonds.
In the environment studied in the prior subsections, households’ marginal and equilibrium portfolios are identical ($\text{mpr}_{i0}^i = \omega_{i0}^i$). This is no longer the case if households have a non-unitary elasticity of intertemporal substitution or supply labor in period 1.\textsuperscript{12} We can still obtain analytical results in this more general environment, however, by studying the limit as aggregate risk falls to zero. In doing so, we apply techniques developed by Devereux and Sutherland (2011) in the context of open-economy macroeconomics to the present environment and our particular statistics of interest.\textsuperscript{13} Letting variables with bars denote values at the point of approximation without aggregate risk, and returning to the case without portfolio constraints, rules-of-thumb, background risk, and belief differences for simplicity, we obtain:

**Proposition 5.** At the limit of zero aggregate risk, i’s portfolio share in capital is

\[
\omega_{i0}^i \equiv \bar{q}_{i0}^i \bar{k}_{i0}^i = \left( \frac{\bar{c}_1^i}{(1 + \bar{r}_1^i) \bar{a}_0^i} \right) \frac{\bar{\gamma}^i}{\gamma^i} - \frac{\bar{w}_1}{(1 + \bar{r}_1^i) \bar{a}_0^i},
\]

(22)

and its MPR is

\[
\text{mpr}_{i0}^i = \frac{\bar{\gamma}^i}{\gamma^i},
\]

(23)

where

\[
\bar{\gamma} = \left[ \int_0^1 \int_0^1 \frac{\bar{c}_1^i}{\bar{c}_1^i} dr' \gamma^{i'} \right]^{-1}.
\]

(24)

is the harmonic average of risk aversion weighted by households’ future consumption.

This proposition naturally generalizes (14). It remains that a household’s portfolio share in capital and its MPR are higher the less risk averse it is relative to other households in the economy.\textsuperscript{14} Nonetheless, the portfolio share and MPR are no longer the same: a household’s portfolio share in capital depends not only on risk aversion but also its motive to hedge labor income also subject to TFP shocks, captured by the last term in (22). This hedging motive is irrelevant on the margin.

\textsuperscript{12}The results which follow assume that households supply an identical (unitary) amount of labor in period 1, but this is easily generalized.

\textsuperscript{13}In particular, a second-order approximation to optimal portfolio choice and the method of undetermined coefficients implies households’ limiting portfolios. A similar approach to the partial derivatives of households’ first-order conditions with respect to $n_{i0}^i$ implies their limiting MPRs.

\textsuperscript{14}Even though we are asking how the individual household allocates wealth both in equilibrium and when given a marginal dollar, the risk aversion of all other households is relevant because this controls the prices faced by the household in general equilibrium.
The distinction between portfolios and MPRs is useful in clarifying their roles in a generalization of Proposition 1, our final analytical result of the paper. Approximating households’ optimal portfolio choice (13) and the asset market clearing conditions (10) and (12) around the point with zero aggregate risk, and denoting with hats log/level deviations from this point, we obtain:

**Proposition 6.** Up to third order in the perturbation parameters $\{\sigma, \hat{\epsilon}_1, \hat{\epsilon}_0^m\}$,

$$\mathbb{E}_0 \hat{r}_1^k - \hat{r}_1 + \frac{1}{2} \sigma^2 = \tilde{\gamma} \sigma^2 + \left[ \tilde{\gamma} \int_0^1 \frac{c_i^l / \int_0^1 c_i^{l'} di'}{de_0^m} (1 - mpr_i^0) di \right] \hat{\epsilon}_0^m \sigma^2. \quad (25)$$

Hence, a monetary policy shock will lower the risk premium if it redistributes wealth to households with relatively high MPRs. This decouples and clarifies the respective role of portfolios and MPRs. Portfolios — more precisely, those which households enter the period with — govern how wealth redistributes on impact of a monetary policy shock, and are contained in $\frac{d[c_i^l / \int_0^1 c_i^{l'} di']}{de_0^m}$. MPRs govern how agents allocate the change in wealth on the margin. We will thus focus on heterogeneity in both portfolios and MPRs in our quantitative results, to which we now turn.

3 Quantitative relevance in the infinite horizon

We first revisit the empirical evidence on the equity premium response to monetary policy shocks which poses a challenge to workhorse models where risk premia barely move. We then calibrate our model to match standard “macro” moments as well as novel “micro” moments from the Survey of Consumer Finances which discipline the cross-sectional heterogeneity in MPRs and exposures to monetary policy. In response to an unexpected monetary easing in our model economy, wealth endogenously redistributes to relatively high MPR households, rationalizing the equity premium response found in the data and amplifying the stimulus in real activity.

3.1 Empirical effects of monetary policy shocks in U.S. data

The effects of an unexpected shock to monetary policy have been the subject of a large literature in empirical macroeconomics. In response to an unexpected loosening, the
price level rises and production expands, consistent with workhorse New Keynesian models. But, as found in Bernanke and Kuttner (2005) and a number of subsequent papers using asset pricing data, the evidence further suggests that risk premia fall.\textsuperscript{15,16}

We refresh the findings in Bernanke and Kuttner (2005) using the structural vector autoregression instrumental variables (SVAR-IV) approach in Gertler and Karadi (2015). Using monthly data from July 1979 through June 2012, we first run a six-variable, six-lag VAR using the 1-year Treasury yield, CPI, industrial production, S&P 500 return relative to the 1-month T-bill, 1-month T-bill return relative to the change in CPI, and smoothed dividend-price ratio on the S&P 500.\textsuperscript{17,18} Over January 1991 through June 2012 we then instrument the residuals in the 1-year Treasury yield (the \textit{monetary policy indicator}) with an external instrument: policy surprises constructed using the current Fed Funds futures contract on FOMC days aggregated to the month level from Gertler and Karadi (2015). The identification assumptions are that the exogenous variation in the monetary policy indicator in the VAR are due to the structural monetary shock and that the instrument is correlated with this structural shock but not the five others. With a first-stage F statistic of 14.4, this instrument is strong according to the threshold of Stock, Wright, and Yogo (2002).

We then plot the impulse responses to a negative monetary policy shock using this instrument in Figure 1. Since the structural monetary policy shock is not observed, its magnitude should be interpreted through the lens of the approximately 0.2\textit{pp} decrease in the 1-year yield on impact. Consistent with the wider literature, industrial production and the price level rise, and the real interest rate falls. Excess returns rise by 1.9\textit{pp} on impact; given the comparatively tiny decline in the real interest rate, this means the real return on the stock market is also approximately 1.9\textit{pp}. Notably, excess returns are small and negative in the months which follow, consistent with a

\begin{itemize}
\item \textsuperscript{15}This effect on risk premia may co-exist with the revelation of information, a channel studied by Nakamura and Steinsson (2018) and others. The analysis of Jarocinski and Karadi (2020) implies that by confounding “pure” monetary policy shocks with such information shocks, our estimates may understate the increase in the stock market following a pure monetary easing.
\item \textsuperscript{16}In addition to this literature, there is also evidence that changes in the monetary policy rule affect risk premia. For instance, using a regime-switching model Bianchi, Lettau, and Ludvigson (2022) find that a more dovish monetary policy rule is associated with a lower equity premium.
\item \textsuperscript{17}The series for the 1-year Treasury yield, CPI, and industrial production are taken from the dataset provided by Gertler and Karadi (2015). The remaining series are from CRSP.
\item \textsuperscript{18}The smoothed dividend-price ratio is the 3-month moving average of dividends divided by the price of the stock at the end of the month, value-weighted over the S&P 500. We linearly detrend this series given changes in corporate payout policy over the sample period.
\end{itemize}
Figure 1: effects of 1 SD monetary shock

Notes: 90% confidence interval at each horizon is computed using the wild bootstrap with 10,000 iterations, following Mertens and Ravn (2013) and Gertler and Karadi (2015).

decline in the equity premium and the fall in the dividend/price ratio.

Following Bernanke and Kuttner (2005), we can decompose the 1.9 pp real return on the stock market into news about higher dividend growth, lower real risk-free discount rates, and lower future excess returns using a Campbell-Shiller decomposition:

\[
(\text{real stock return})_t - \mathbb{E}_{t-1}[(\text{real stock return})_t] = (\mathbb{E}_t - \mathbb{E}_{t-1}) \sum_{j=0}^{\infty} \kappa^j \Delta(\text{dividends})_{t+j} \\
- (\mathbb{E}_t - \mathbb{E}_{t-1}) \sum_{j=1}^{\infty} \kappa^j (\text{real rate})_{t+j} - (\mathbb{E}_t - \mathbb{E}_{t-1}) \sum_{j=1}^{\infty} \kappa^j (\text{excess return})_{t+j},
\]

where \( \kappa = \frac{1}{1 + dp} \) and \( dp \) is the steady-state dividend yield. Using the SVAR-IV to compute the revised expectations in real rates and excess returns given the monetary shock, we obtain the decomposition in Table I.\(^1\) 1.1 pp (59%) of the initial return on the stock market is due to news about lower future excess returns, 0.1 pp (8%) is due to news about lower future risk-free rates, and 0.6 pp (33%) is due to news about

\(^1\)As in Bernanke and Kuttner (2005), we use our VAR to compute \( (\text{excess return})_t - \mathbb{E}_{t-1}[(\text{excess return})_t], (\mathbb{E}_t - \mathbb{E}_{t-1}) \sum_{j=1}^{\infty} \kappa^j (\text{real rate})_{t+j}, \) and \( (\mathbb{E}_t - \mathbb{E}_{t-1}) \sum_{j=1}^{\infty} \kappa^j (\text{excess return})_{t+j}, \) and we assign to dividend growth the residual implied by (26).
Table I: Campbell-Shiller decomposition following 1 SD monetary shock

<table>
<thead>
<tr>
<th></th>
<th>$pp$</th>
<th>As share of effect on real stock return</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real stock return</td>
<td>1.92</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>[1.54, 2.59]</td>
</tr>
<tr>
<td>Dividend growth news</td>
<td>0.64</td>
<td>33%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[-0.25, 1.48]</td>
</tr>
<tr>
<td>− Future real rate news</td>
<td>0.15</td>
<td>8%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[-0.12, 0.39]</td>
</tr>
<tr>
<td>− Future excess return news</td>
<td>1.13</td>
<td>59%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[0.37, 2.27]</td>
</tr>
</tbody>
</table>

Notes: decomposition in (26) uses $\kappa = 0.9962$ following Campbell and Ammer (1993). 90% confidence interval in brackets is computed using the wild bootstrap with 10,000 iterations, following Mertens and Ravn (2013) and Gertler and Karadi (2015).

higher dividend growth. Accounting for estimation uncertainty, at least 19% and potentially all of the return on the stock market is due to news about lower future excess returns, validating the original message from Bernanke and Kuttner (2005).

The important role of the risk premium in explaining the return on the stock market is robust to details of the estimation approach. In appendix B.1 we change the number of lags used in the VAR; change the sample periods over which the VAR and/or first-stage is estimated; add variables to the VAR; and use an alternative instrument for the monetary policy shock. Across these cases we confirm the message of the baseline estimates above: in response to a monetary policy shock which reduces the 1-year Treasury yield by approximately 0.2 $pp$, real stock returns rise by 1.5-3.1 $pp$, and news about future excess returns explains 35%-85% of this increase.

The dimensionality reduction offered by a VAR enables us to generate the long-horizon forecasts needed for the Campbell-Shiller decomposition, unlike a local projection. As noted by Stock and Watson (2018), we can test the assumption of invertibility implicit in the SVAR-IV both by assessing whether lagged values of the instrument have forecasting power when included in the VAR and by comparing the estimated impulse responses to those obtained using a local projection with instrumental variables (LP-IV). We show in appendix B.1 that both of these tests fail to reject the null hypothesis that invertibility in our application is satisfied.\textsuperscript{20}

\textsuperscript{20}As an alternative approach, a recursive VAR ordering the IV first in the VAR is robust to
Finally, augmenting our VAR with cross-sectional data corroborates the redistributive mechanism through which our model rationalizes the risk premium response to a monetary shock. In appendix B.1, we construct two measures of the relative wealth of agents relatively more exposed to the stock market: a total return index of high beta hedge funds relative to low beta hedge funds, and an analogous index for mutual funds. These measure the relative wealth of a household continually (re-)invested in high beta funds relative to low beta funds. On impact of a monetary easing, we find that the relative return of high beta funds rises on impact and then falls thereafter. This is consistent with the wealth share of relatively risk tolerant investors rising, since risk sharing calls for the wealth of risk tolerant investors to load more on the return on capital — i.e., to have a higher beta.

Objectives in the remainder of paper The rest of the paper enriches the model from section 2 and studies a calibration to the U.S. economy matching micro evidence on portfolio heterogeneity and conventional macro moments on asset prices and business cycles. We first ask whether redistribution in such an environment can quantitatively rationalize the estimated stock market response to a monetary policy shock. We then use the model to quantify the implications for the real economy.

3.2 Infinite horizon environment

We first outline the environment, building on that from section 2.1. We describe the main changes here and present the complete environment in appendix C.

3.2.1 Household preferences and constraints

To ensure a stationary wealth distribution despite permanent differences in risk aversion, we assume a perpetual youth structure in which each household dies at rate $\xi$ and has no bequest motive. This implies an underlying wealth distribution among households having a particular coefficient of relative risk aversion $\gamma^i$. Appendix C non-invertibility (Plagborg-Moller and Wolf (2021)), while estimation using a VAR still means that we can implement (26). While the impulse responses using this approach are noisier than our baseline using the SVAR-IV, the point estimates imply that 74% of the increase in the stock market following a monetary shock is due to news about lower future excess returns. The recursive approach is closely related to the identification strategy used by Paul (2020) in recent work also finding that expansionary monetary policy raises the stock market in part by lowering future excess returns.
proves the existence of a representative household at each $\gamma^i$.\footnote{So that the model permits aggregation into representative households despite the existence of non-traded labor income, we allow households to trade claims to a labor endowment with other households having the same risk aversion, as further described in appendix C. This approach extends that in Lenel (2020) to a setting with endogenous labor supply and production.}

Representative household $i$ maximizes a generalization of (1)

$$v_t^i = \left(1 - \beta \right) \left( c_t^i \Phi \left( \int_0^1 \ell_t^i(j) dj \right) \right)^{1-1/\psi} + \beta \mathbb{E}_t \left[ (\mu_{t,t+1}^i v_{t+1}^i)^{1-\gamma^i} \right]^{1-1/\psi} \right)^{1/1-1/\psi}$$

(27)

with disutility of labor each period

$$\Phi(\ell_t^i) = \left(1 + \frac{1}{\psi} - 1\right) \theta \left( \frac{(\ell_t^i)^{1+1/\theta}}{1 + 1/\theta} \right)^{\frac{1/\psi}{1+1/\theta}},$$

(28)

consistent with balanced growth as in Shimer (2010) and Trabandt and Uhlig (2011). $\mu_{t,t+1}^i$ reflects the wealth of households of type $i$ at $t+1$ who were also alive at $t$, relative to the average household of type $i$ at $t+1$. It is characterized in appendix C.

We assume each household is comprised of a measure one of workers $j$ supplying a different variety, allowing us to accommodate wage stickiness in the usual way. In particular, each household pays Rotemberg (1982) wage adjustment costs for each $j$

$$AC_t^W(j) = \frac{\chi^W}{2} W_t \ell_t \left( \frac{W_t(j)}{W_{t-1}(j) \exp(\varphi_t)} - 1 \right)^2,$$

(29)

where $\chi^W$ controls the magnitude of adjustment costs, $W_t \ell_t$ is the economy-wide wage bill, and $\exp(\varphi_t)$ in the reference wage is an adjustment for rare disasters described below. These adjustment costs are not indexed by $i$ because there is a common wage for each variety supplied by households, as described below. We further assume these costs are paid to the government and rebated back to households so that they only affect the allocation via the dynamics of wages.

Finally, we assume households also face a lower bound on capital

$$k_t^i \geq k_t^z,$$

(30)

where $z_t$ is productivity, discussed below. Such a constraint captures components of capital which households hold for reasons beyond financial returns, such as housing.
3.2.2 Supply-side

A union representing each variety \( j \) chooses \( W_t(j) \) and \( \ell_t(j) \) to maximize the utilitarian social welfare of union members given the allocation rule

\[
\ell_t^j(j) = \phi^j \ell_t(j),
\]

(31)

satisfying \( \int_0^1 \phi^j \, dj = 1 \). A representative labor packer purchases varieties and combines them to produce a CES aggregate with elasticity of substitution \( \epsilon \)

\[
\ell_t = \left[ \int_0^1 \ell_t(j)^{(\epsilon-1)/\epsilon} \right]^{\epsilon/(\epsilon-1)}
\]

(32)

which it then sells at \( W_t \), earning

\[
W_t \ell_t - \int_0^1 W_t(j) \ell_t(j) \, dj.
\]

(33)

A representative producer then purchases the labor aggregate and rents capital, and it uses consumption goods to produce new capital goods sold to households.

3.2.3 Aggregate productivity

We now assume that productivity \( z_t \) follows a unit root process

\[
\log z_t = \log z_{t-1} + \epsilon_t + \varphi_t,
\]

(34)

where \( \epsilon_t \) is an iid shock from a Normal distribution with mean zero and standard deviation \( \sigma^z \), \( \varphi_t \) is a rare disaster equal to zero with probability \( 1 - p_t \) and \( \varphi < 0 \) with probability \( p_t \), and \( p_t \) follows an AR(1) process

\[
\log p_t - \log \bar{p} = \rho (\log p_{t-1} - \log \bar{p}) + \epsilon^p_t,
\]

(35)

where \( \epsilon^p_t \) is an iid shock from a Normal distribution with mean zero and standard deviation \( \sigma^p \). Following Barro (2006), Gourio (2012), and Wachter (2013), we introduce the disaster with time-varying probability to help match the level of the equity
premium and volatility of returns. So that the dynamics upon a rare disaster are well-behaved, we assume that the disaster destroys capital and reduces the reference wage in households’ wage adjustment costs in proportion to the decline in productivity. The first assumption implies that aggregate output is

\[ y_t \equiv (z_t \ell_t)^{1-\alpha} (k_{t-1} \exp(\varphi_t))^{\alpha}, \]  

(36)

where productivity is now labor-augmenting and thus consistent with balanced growth.

### 3.2.4 Monetary and fiscal policy

Finally, monetary policy is now characterized by the Taylor rule (8) each period

\[ 1 + i_t = (1 + \bar{i}) \left( \frac{P_t}{P_{t-1}} \right)^\phi m_t, \]  

(37)

where policy shocks follow an AR(1) process

\[ \log m_t = \rho^m \log m_{t-1} + \epsilon_t^m, \]  

(38)

where \( \epsilon_t^m \) is an iid shock from a Normal distribution with mean zero and standard deviation \( \sigma^m \).

Fiscal policy is characterized by three elements. First, the government subsidizes workers’ labor income at a constant rate \( \frac{1}{\epsilon-1} \) rebated back to each household, eliminating the average wage markup in the usual way. Second, the government participates in the bond market financed by lump-sum taxes in which household \( i \) pays a share \( \nu^i \). Given the latter assumption (and that households face no constraints in the bond market) the government bond position has no effect on the equilibrium allocation, so we assume it is a constant real value relative to productivity: \( B^g_t/(P_t z_t) = b^g \). Its only purpose is to make measured portfolios in model and data comparable. Third, the government collects the wealth of dying households and endows it to newborn households. We describe the rule employed when doing so in the next subsection.

\[ ^{22} \text{Following Bianchi, Ilut, and Schneider (2018), financial frictions on firms together with uncertainty shocks on operating cost could further improve the model on this dimension. Following Guvenen (2009) and Garleanu and Panageas (2015), heterogeneity in the intertemporal elasticity of substitution could also help lower the volatility of the real interest rate relative to excess returns.} \]
3.2.5 Equilibrium and model solution

The definition of equilibrium naturally generalizes Definition 1.

We solve the model globally using numerical methods. Given this, we limit the heterogeneity across households to make the computation tractable. We divide the continuum of households into three groups \{a, b, c\} within which households have identical preferences. The index \(i\) now refers to the representative household of each group. The fraction of households belonging to group \(i\) is denoted \(\lambda^i\), where \(\sum_i \lambda^i = 1\).

We solve a stationary transformation of the economy obtained by dividing all real variables except labor by \(z_t\) and nominal variables by \(P_t z_t\). In the transformed economy we obtain a recursive representation of the equilibrium in which the aggregate state in period \(t\) is given by the monetary policy state variable \(m_t\), disaster probability \(p_t\), scaled aggregate capital \(k_{t-1}/(z_{t-1} \exp(\epsilon^i_t))\), scaled prior period’s real wage \(w_{t-1}/(z_{t-1} \exp(\epsilon^i_t))\), and wealth shares \(\{s^i_t\}\) of any two groups. Assuming that the government endows newborn households of each group with a share \(\bar{s}^i\) of dying households’ wealth, these wealth shares follow

\[
s^i_t = \lambda^i (1 - \xi) \frac{(1 + i_{t-1})(B^i_{t-1} + \nu^i B^g_{t-1}) + (\Pi_t + (1 - \delta)Q_t)k^i_{t-1} \exp(\varphi_t)}{\Pi_t + (1 - \delta)Q_t} + \xi \bar{s}^i_t. \tag{39}
\]

Productivity shocks inclusive of disasters only govern the transition across states, but do not separately enter the state space itself.

We solve the model using sparse grids as described in Judd, Maliar, Maliar, and Valero (2014). When forming expectations, we use Gauss-Hermite quadrature and interpolate with Chebyshev polynomials for states off the grid. The stochastic equilibrium is determined through backward iteration, while dampening the updating of asset prices and individuals’ expectations over the dynamics of the aggregate states.

The code is written in Fortran and parallelized using OpenMP, so that convergence can be achieved in a few minutes on a standard desktop computer.

3.3 Parameterization, first moments, and second moments

We now parameterize the model to match micro moments informing the heterogeneity across groups as well as macro moments regarding the business cycle and asset prices.
3.3.1 Micro: the distribution of wealth, labor income, and portfolios

We seek to match the distribution of wealth, labor income, and financial portfolios in U.S. data, giving us confidence in the model’s MPRs and exposures to a monetary shock. We proceed in three steps with the 2016 Survey of Consumer Finances (SCF).

First, we decompose each household’s wealth ($A^i$) into claims on the economy’s capital stock ($Qk^i$, in positive net supply) and nominal claims ($B^i$, in zero net supply accounting for the government and rest of the world). We describe this procedure in detail in appendix B.2 and provide a broad overview here. We first add estimates of defined benefit pension wealth for each household since this is the major component of household net worth which is excluded from the SCF.\textsuperscript{23} We then proceed by line item to allocate how much household wealth is held in nominal claims versus claims on capital.\textsuperscript{24} In the same spirit as Doepke and Schneider (2006), the key step in doing this is to account for the implicit leverage households have on capital through publicly-traded and privately-held businesses. The aggregate leverage implicit in these equity claims must be consistent with that of the business sectors in the Financial Accounts. We parameterize the dispersion in leverage in these claims to match evidence on the dispersion in households’ expected rates of return.

Second, we stratify households by their wealth to labor income {$\frac{A^i}{W^L}$} and capital portfolio share {$\frac{Qk^i}{A^i}$}, defining our three groups. We sort households on these variables based on Proposition 5, which demonstrated that the capital portfolio share is informative about households’ risk aversion and thus MPR only after properly accounting for their non-traded exposure to aggregate risk through labor income.\textsuperscript{25} Group $a$ corresponds to households with high wealth to labor income and a high capital portfolio share, group $b$ corresponds to households with high wealth to labor income but a low capital portfolio share, and group $c$ corresponds to households with low wealth to labor income. We define “high” wealth to labor income as households

\textsuperscript{23}We use the estimates of Sabelhaus and Volz (2019) described further in the appendix.

\textsuperscript{24}We note in particular that we treat DB pension entitlements as a nominal asset of households, under the interpretation that households have a fixed claim on the pension sponsor which is then the residual claimant on the investment portfolio. In contrast, DC pension assets, as with other mutual fund assets, are decomposed into nominal claims and claims on capital as described here.

\textsuperscript{25}We sort households by a measure of their capital portfolio share after excluding from both the numerator and denominator assets and liabilities associated with the primary residence and vehicles, even though for each group we report and target the capital portfolio share accounting for all assets and liabilities. We sort households on the former measure since households’ decisions regarding their primary residence and consumer durables may reflect considerations beyond risk and return.
Table II: heterogeneity in wealth to labor income and the capital portfolio share

<table>
<thead>
<tr>
<th>Group</th>
<th>Share households</th>
<th>( \frac{W^i}{A^i} )</th>
<th>Share households</th>
<th>( \frac{Q^i}{A^i} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \geq p90 )</td>
<td>( \geq p60 )</td>
<td>( \leq p90 )</td>
<td>( \geq p60 )</td>
<td>( \leq p90 )</td>
</tr>
<tr>
<td>Group a</td>
<td>Share households: 4%</td>
<td>( \sum_{i \in a} W^i / \sum_i W^i ): 3%</td>
<td>( \sum_{i \in a} A^i / \sum_i A^i ): 18%</td>
<td>( \sum_{i \in a} Q^i / \sum_{i \in a} A^i ): 2.0</td>
</tr>
<tr>
<td>Group b</td>
<td>Share households: 36%</td>
<td>( \sum_{i \in b} W^i / \sum_i W^i ): 14%</td>
<td>( \sum_{i \in b} A^i / \sum_i A^i ): 58%</td>
<td>( \sum_{i \in b} Q^i / \sum_{i \in b} A^i ): 0.5</td>
</tr>
<tr>
<td>Group c</td>
<td>Share households: 60%</td>
<td>( \sum_{i \in c} W^i / \sum_i W^i ): 83%</td>
<td>( \sum_{i \in c} A^i / \sum_i A^i ): 23%</td>
<td>( \sum_{i \in c} Q^i / \sum_{i \in c} A^i ): 1.1</td>
</tr>
</tbody>
</table>

Notes: observations are weighted by SCF sample weights.

above the 60th percentile of this measure, and a “high” capital portfolio share as households above the 90th percentile of this measure.

Third, we summarize the labor income, wealth, and financial portfolios of these three groups, provided in Table II. Group a households earn 3% of labor income, hold 18% of wealth, and have an aggregate capital portfolio share of 2.0. Group b households earn only 14% of labor income, hold 58% of wealth, and have an aggregate capital portfolio share of 0.5. Group c households earn 83% of labor income, hold only 23% of wealth, and have an aggregate capital portfolio share of 1.1. To better understand the nature of households in each group, in Table III we first project an indicator for the household having private business wealth on households’ group indicator. Households in group a are especially more likely to have private business wealth. We then project an indicator for the household head being older than 54 and out of the labor force, together capturing a retired household head, on households’ group indicator. Households in group b are especially more likely to be retired.

\[\text{26 The 90th percentile strikes a balance between capturing the tail of levered households and maintaining a large enough sample size. In appendix D.2, we consider the robustness of our results to raising this cutoff to the 99th percentile. In this case, a households hold less wealth but are much more levered, and in fact have approximately the wealth share and leverage of the broker-dealer and hedge fund sectors, admitting an intermediary asset pricing interpretation to this calibration.}\]
Table III: indicators for private business wealth or being retired on group indicators

<table>
<thead>
<tr>
<th></th>
<th>( {hbus^t = 1} )</th>
<th>( {age^t &gt; 54, lf^t = 0} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 1 {i \in a} )</td>
<td>0.37 (0.03)</td>
<td>0.37 (0.03)</td>
</tr>
<tr>
<td>( 1 {i \in b} )</td>
<td>0.05 (0.01)</td>
<td>0.55 (0.01)</td>
</tr>
<tr>
<td>Observations</td>
<td>6,229</td>
<td>6,229</td>
</tr>
<tr>
<td>Adj ( R^2 )</td>
<td>0.05</td>
<td>0.37</td>
</tr>
</tbody>
</table>

Notes: observations are weighted by SCF sample weights and standard errors adjust for imputation and sampling variability following Pence (2015). Each specification includes a constant term (not shown), capturing the baseline probability of holding private business wealth or being retired among households in group \( c \).

In appendix B.2, we apply the exact same approach as above to stratify households in 2007 using the 2007-2009 SCF panel. We then exploit the panel structure of this survey to follow households through 2009. Among other findings, we document that households’ portfolio share in capital are very persistent across these two years, both at the group and individual levels. This validates our calibration approach of matching the cross-sectional data using permanent differences in households’ risk preferences.

3.3.2 Macro: business cycle dynamics and asset prices

We also calibrate the model to match standard macro moments regarding the business cycle and asset prices. In terms of the business cycle, we use NIPA data on consumption of non-durables and services as well as investment in durables and capital, together with the time series of the working age population from the BLS, to estimate the volatilities of quarterly per capita growth rates in those series over Q3 1979 to Q2 2012 (consistent with our sample period for the VAR). In terms of asset prices, we use the data from CRSP described earlier to estimate the annualized average real interest rate and excess return on the S&P 500 over July 1979 - June 2012. We further estimate the second moments of expected returns using our VAR. We compute analogous moments in our model assuming that an equity claim (with return \( r_e \)) is a levered claim to capital with a debt to equity ratio of 0.5.\(^{27}\)

\(^{27}\)This ratio is commonly used in the literature (e.g., Barro (2006)). It also implies assets to equity of 1.5, very close to our estimate of 1.6 for public equities in appendix B.2.
3.3.3 Parameterization

A model period corresponds to one quarter. After setting a subset of parameters in accordance with the literature, we calibrate the remaining parameters to be consistent with the macro and micro moments described above. All stochastic properties of the model are estimated using a simulation where no disasters are realized in sample.\footnote{We make this choice since we compare the model to post-World War II data.}

**Externally set parameters** A subset of model parameters summarized in Table IV are set externally. Among the model’s preference parameters, we set $\psi$ to 0.8. We note that this parameter controls both the intertemporal elasticity of substitution in consumption as well as the complementarity between consumption and labor. A value less than one is consistent with evidence on the consumption responses to changes in interest rates as well as consumption-labor complementarity.\footnote{See, for instance, Barsky, Juster, Kimball, and Shapiro (1997), Hall (2009), and Shimer (2010).} The Frisch elasticity of labor supply is set to $\theta = 1$, roughly consistent with the micro evidence for aggregate hours surveyed in Chetty, Guren, Manoli, and Weber (2011). The three types have measure $\lambda^a = 4\%$, $\lambda^b = 36\%$, and $\lambda^c = 60\%$ and the labor allocation rule features $\phi^a = 3\%/\lambda^a$, $\phi^b = 14\%/\lambda^b$, and $\phi^c = 83\%/\lambda^c$, consistent with Table II. Households die with probability $\xi = 1\%$, implying an expected horizon of 25 years, consistent with households transitioning across groups through the life cycle.

On the production side, we choose standard values of $\alpha = 0.33$ for the capital share of production and $\delta = 2.5\%$ for the quarterly depreciation rate. We set $\bar{p}$ so that, accounting for Jensen’s inequality and the calibrated volatility and persistence $\sigma^p$ and $\rho^p$ below, the average disaster probability $p = 0.5\%$. This follows Barro (2006) and implies that a disaster shock is expected to occur every 50 years. The depth of the disaster is set to $\varphi = -15\%$, consistent with the estimates of Nakamura, Steinsson, Barro, and Ursua (2013) who account for the recovery after a disaster. We choose an elasticity of substitution across worker varieties $\epsilon = 10$ and Rotemberg wage adjustment costs of $\chi^W = 150$, together implying a Calvo (1983)-equivalent frequency of wage adjustment between 4 and 5 quarters, consistent with the evidence in Grigsby, Hurst, and Yildirmaz (2021).

Finally, in terms of policy, we set the Taylor coefficient on inflation to the standard value $\phi = 1.5$. Monetary policy shocks have a standard deviation of $\sigma^m = 0.25\%/4$ with zero persistence. The share of lump-sum taxes financing government debt paid...
<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi$</td>
<td>IES</td>
<td>0.8</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Frisch elasticity</td>
<td>1</td>
</tr>
<tr>
<td>$\lambda^a$</td>
<td>measure of a households</td>
<td>4%</td>
</tr>
<tr>
<td>$\lambda^b$</td>
<td>measure of b households</td>
<td>36%</td>
</tr>
<tr>
<td>$\phi^a$</td>
<td>labor a households</td>
<td>3%/1</td>
</tr>
<tr>
<td>$\phi^b$</td>
<td>labor b households</td>
<td>14%/1</td>
</tr>
<tr>
<td>$\xi$</td>
<td>death probability</td>
<td>1%</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>1 - labor share</td>
<td>0.33</td>
</tr>
<tr>
<td>$\delta$</td>
<td>depreciation rate</td>
<td>2.5%</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>elast. of subs. across workers</td>
<td>10</td>
</tr>
<tr>
<td>$\chi^W$</td>
<td>Rotemberg wage adj costs</td>
<td>150</td>
</tr>
<tr>
<td>$\bar{p}$</td>
<td>disaster probability</td>
<td>0.45%</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>disaster shock</td>
<td>-15%</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Taylor coeff. on inflation</td>
<td>1.5</td>
</tr>
<tr>
<td>$\sigma^m$</td>
<td>std. dev. MP shock</td>
<td>0.25%/4</td>
</tr>
<tr>
<td>$\rho^m$</td>
<td>persistence MP shock</td>
<td>0</td>
</tr>
<tr>
<td>$\lambda^a\nu^a$</td>
<td>a share of taxes to finance $-B^a$</td>
<td>18%</td>
</tr>
<tr>
<td>$\lambda^c\nu^c$</td>
<td>c share of taxes to finance $-B^c$</td>
<td>23%</td>
</tr>
</tbody>
</table>

Table IV: externally set parameters

by group $i$ ($\lambda^i\nu^i$) is equal to their wealth share in Table II.

**Calibrated parameters** We calibrate the remaining parameters to target the macro and micro moments described above. Table V reports in each line a parameter choice and moment in model and data that this parameter is closely linked to.

We first match the first and second moments of quantities and returns. The standard deviation of the productivity shock $\sigma^z$ is set to 0.55% to match quarterly consumption growth volatility of 0.5%. The capital adjustment cost is set to $\chi^z = 3.5$ to target the volatility of investment growth. Due to the precautionary savings motive, $\beta = 0.98$ is high enough to match the low annualized real rate observed in the data. We set $\gamma^b = 25.5$ to target the annualized excess return on equity. The standard deviation $\sigma^p$ and persistence $\rho^p$ in the disaster probability process target the standard deviation and autocorrelation of the annualized expected real rate from our VAR.
Table V: targeted moments and calibrated parameters

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
<th>Moment</th>
<th>Target</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma^2$ std. dev. prod.</td>
<td>0.55%</td>
<td>$\sigma(\Delta \log c)$</td>
<td>0.5%</td>
<td>0.6%</td>
</tr>
<tr>
<td>$\chi^x$ capital adj cost</td>
<td>3.5</td>
<td>$\sigma(\Delta \log x)$</td>
<td>2.1%</td>
<td>2.0%</td>
</tr>
<tr>
<td>$\beta$ discount factor</td>
<td>0.98</td>
<td>$4r_{t+1}$</td>
<td>1.3%</td>
<td>1.5%</td>
</tr>
<tr>
<td>$\gamma^b$ RRA b</td>
<td>25.5</td>
<td>$4[r_{t+1} - r_{t+1}]$</td>
<td>7.3%</td>
<td>7.0%</td>
</tr>
<tr>
<td>$\sigma^p$ std. dev. log dis. prob.</td>
<td>0.47</td>
<td>$\sigma(4E_{t+1})$</td>
<td>2.2%</td>
<td>2.2%</td>
</tr>
<tr>
<td>$\rho^p$ persist. log dis. prob.</td>
<td>0.8</td>
<td>$\rho(4E_{t+1})$</td>
<td>0.79</td>
<td>0.75</td>
</tr>
<tr>
<td>$\gamma^a$ RRA a</td>
<td>10</td>
<td>$qk^a/a^a$</td>
<td>2.0</td>
<td>2.3</td>
</tr>
<tr>
<td>$\overline{k}$ lower bound $k^i$</td>
<td>10</td>
<td>$qk^c/a^c$</td>
<td>1.1</td>
<td>0.9</td>
</tr>
<tr>
<td>$\xi a^a$ newborn endowment a</td>
<td>0%</td>
<td>$\lambda^a a^a / \sum_i \lambda^i a^i$</td>
<td>18%</td>
<td>21%</td>
</tr>
<tr>
<td>$\xi c^c$ newborn endowment c</td>
<td>-0.25%</td>
<td>$\lambda^c a^c / \sum_i \lambda^i a^i$</td>
<td>23%</td>
<td>23%</td>
</tr>
<tr>
<td>$b^g$ real value govt bonds</td>
<td>-2.7</td>
<td>$-\sum_i \lambda^i b^i / \sum_i \lambda^i a^i$</td>
<td>-10%</td>
<td>-10%</td>
</tr>
</tbody>
</table>

Notes: targeted business cycle moments are from Q3/79-Q2/12 NIPA and targeted asset pricing moments are from 7/79-6/12 data underlying the VAR. The model assumes a debt/equity ratio of 0.5 on a stock market claim. The first and second moments in the model are estimated over 50,000 quarters after a burn-in period of 5,000 quarters, with no disaster realizations in sample. The disutilities of labor $\{\bar{\theta}^a, \bar{\theta}^b, \bar{\theta}^c\}$ are jointly set to $\{0.64, 2.89, 0.43\}$ so that the average labor wedge is zero for each group and $\ell = 1$, where the latter is a convenient normalization. The Taylor rule intercept $\bar{i}$ is set to 0.3% to target zero average inflation.

We next match the micro heterogeneity in portfolios and wealth. We set $\gamma^a = 10 < \gamma^b$ to target $a$ households’ capital portfolio share. $\gamma^c$ and $\overline{k}$ are difficult to separately calibrate: the relatively high ratio of labor income to wealth among group $c$ households means that they would endogenously choose to hedge this exposure to productivity shocks by holding a lower position in capital, consistent with Proposition 5, and are thus more likely to be constrained by (30). We set $\gamma^c$ equal to the (population-weighted) harmonic mean of $\gamma^a$ and $\gamma^b$ and calibrate $\overline{k}$ to target the capital portfolio share of $c$ households in the data. The initial endowments of newborns are chosen to target the measured wealth shares of the three groups. Matching these wealth shares requires that only $b$ households receive a positive endowment when born.31

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30 We view this as a realistic description of the data, given that $\overline{k}$ is meant to capture components of the economy’s capital stock which households hold for reasons beyond their financial returns. In the SCF, 51% of the aggregate capital held by group $c$ households is in their primary residence and vehicles, while the same ratio is only 35% for group $b$ households and 7% for group $a$ households.

31 This is consistent with the fact that in the SCF, total federal transfers less taxes (estimated using NBER Taxsim) is higher for $b$ households than $a$ or $c$ households. We note that a negative
$\sigma(\Delta \log y)$ & 0.8% & 0.9% \\
$\sigma(\Delta \log \ell)$ & 0.8% & 0.8% \\
$\sigma(d/p)$ & 0.2% & 0.2% \\

$\sum \lambda^i \text{mpr}^i \approx 0.2$ & 0.3 \\
mpr$^a$ & 1.9 \\
mpr$^b$ & 0.7 \\
mpr$^c$ & 0.0 \\

$\sum \lambda^i \text{mpc}^i \approx 0.2$ & 0.02 \\
mpc$^a$ & 0.02 \\
mpc$^b$ & 0.02 \\
mpc$^c$ & 0.02 \\

Table VI: untargeted macro and micro moments 

Notes: see notes accompanying Table V on construction of moments in data and model.

Finally, we set $b^g$ so that the aggregate bond position of households relative to total wealth is 10%, as implied by Table II. We set the disutilities of labor so that the average labor wedge is zero for each group and $\ell = 1$, the latter a convenient normalization. We set the Taylor rule intercept $\tilde{i}$ so that average inflation is zero.

### 3.3.4 Untargeted moments

Table VI reports the values of several untargeted moments and their empirical counterparts. In terms of macro moments, the model closely matches the quarterly volatilities of output growth, employment growth, and the smoothed dividend price ratio.\(^{32,33}\) Related to the latter, the model generates a quarterly volatility of annualized expected excess equity returns of 2.8%, which accounts for more than half of the volatility estimated in the data by the studies surveyed in Duarte and Rosa (2015).

Turning to micro moments, we first consider MPRs. The model generates het-

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31

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\(^{32}\)When computing the smoothed dividend price ratio, we smooth dividends over 12 months rather than over 3 months as in the VAR. This is meant to more accurately compare our model (which features no dividend adjustment costs) with the data.

\(^{33}\)While we emphasize monetary shocks in the main text given that monetary transmission is our focus in this paper, these shocks contribute little to aggregate fluctuations. Appendix D.1 studies productivity shocks and changes in disaster probabilities, the drivers of fluctuations in our model.
Heterogeneity in quarterly MPRs consistent with Proposition 5 in the analytical results. Group \(a\) households are the most risk tolerant and have the highest MPR, borrowing $0.9 for every $1 of marginal net worth to invest in capital. Group \(b\) and \(c\) households have higher levels of risk aversion and correspondingly lower MPRs. As noted above, group \(c\) households have a higher ratio of labor income to wealth and thus are endogenously constrained by (30). Hence, on the margin their average MPR is zero.

Quasi-experimental evidence is consistent with the MPRs in our calibration. Weighting by the fraction of households, the average MPR in our model is 0.3. Using data on Norwegian lottery winners, Fagereng, Holm, and Natvik (2021) estimate an average marginal propensity to save in risky assets relative to save overall of 0.14.\(^{34}\) Using data on Swedish lottery winners, Briggs, Cesarini, Lindqvist, and Ostling (2015) estimate an analogous ratio of 0.15.\(^{35}\) These imply an MPR of roughly 0.2 after accounting for reasonable estimates of the leverage of firms in which households invest.\(^{36}\) MPRs further rise with wealth per household in our calibration — recalling that wealth per household is highest among \(a\) households and then \(b\) households — consistent with evidence from these studies.\(^{37}\) While the range in estimated MPRs in these studies is smaller than that in our model, estimated MPRs based on lotteries may underestimate the relevant statistic for households in groups \(a\) and \(b\) of our model. As lottery winnings are paid out as cash or riskless deposits, the estimated MPR may understate the MPR in response to dividends or capital gains, more relevant for the balance sheet revaluation among the wealthy (\(a\) and \(b\) households) emphasized in this paper.\(^{38}\) Among owners of private businesses, overrepresented in these households, the estimated MPR may particularly understate their true MPR because investment in private businesses is not included in the definition of (traded) risky assets.

The model’s heterogeneity in MPRs contrasts with its implied MPCs, which are

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\(^{34}\)In their Table 6, the average marginal propensity to save in stocks, bonds, and mutual funds is 0.058 and the marginal propensity to save in these assets, deposits, or repay debt is 0.407.

\(^{35}\)In their Table B.8, the average marginal propensity to save in risky assets is 0.085 and the marginal propensity to save in these assets, safe assets, bank accounts, or repay debt is 0.58.

\(^{36}\)With firm leverage of 1.6 estimated in appendix B.2, these estimates imply an MPR of 0.22-0.24.

\(^{37}\)Table 5(B) of Fagereng et al. (2021) demonstrates that the MPR of households in the lowest quartile of wealth is below that of others. Figure 3 of Briggs et al. (2015) demonstrates that the MPR of households in the bottom half of the age-adjusted wealth distribution is below that of others.

\(^{38}\)Using the Panel Study of Income Dynamics, Brunnermeier and Nagel (2008) document significant inertia in financial portfolios, with a negative change in the risky share after receiving one dollar of cash or deposits but an increase in the risky share after receiving one dollar of unexpected returns on risky assets. In recent work, Fagereng, Holm, Moll, and Natvik (2019) also find evidence that households “save by holding” on to nearly 100% of assets experiencing capital gains.
essentially identical across agents. This is an intentional implication of our model environment which features no idiosyncratic labor income risk nor heterogeneity in discount factors, allowing us to focus on the consequences of heterogeneity in portfolio choice alone. Unsurprisingly, the model further generates an average quarterly MPC which is an order of magnitude lower than that typically estimated in the data. We expect that adding additional features to our model which raise the average MPC would only amplify the real consequences of movements in the risk premium.\textsuperscript{39}

We finally note that, consistent with our analytical results, it is the exposures and MPRs in our calibration which are essential for the effects of a monetary shock — not the precise microfoundation. In appendix D.3, we consider an alternative environment in which households have identical risk aversion but are exposed to heterogeneous amounts of idiosyncratic risk in their return on capital, as in the environment with background risk described in section 2.5 and the broader literature on entrepreneurship in macroeconomic models. To match the same data on portfolios, a households are calibrated to be exposed to less idiosyncratic risk than other agents; equivalently, their risk-adjusted returns are highest. The resulting MPRs are very similar to the baseline model, and the quantitative results which follow are robust.

3.4 Impulse responses to a monetary policy shock

We now simulate the effects of a negative shock to the nominal interest rate. We demonstrate that our model can rationalize the stock market responses to a monetary policy shock in the data. The effects of monetary policy on the risk premium on capital amplify the transmission to the real economy by 1.3-1.4 times.

3.4.1 Model versus RANK

Figures 2, 3, and 4 compare the impulse responses to those in a counterfactual representative agent New Keynesian (RANK) economy. In the latter, we set $\gamma^i = 19$ for all groups, equal to the wealth-weighted harmonic mean of risk aversion in the model.

We choose the shock $\epsilon_0^{\text{m}}$ in our model to generate a 0.2\textit{pp} reduction in the 1-year nominal yield, consistent with section 3.1. We obtain this yield by computing, in each state, the price that each household would pay for a 1-year nominal bond. We then

\textsuperscript{39}This is suggested by the Keynesian cross in Proposition 2 and recent analyses of investment and asset prices in HANK models such as Auclert et al. (2020) and Caramp and Silva (2021).
set the price to that of the highest-valuation household. Importantly, we re-calibrate $\epsilon_0^m$ in the RANK economy to match this same decline in the 1-year yield.

Figure 2 summarizes the effect of the monetary policy shock on expected returns. The first panel reports the change in the yield on the 1-year nominal bond. The second and third panels depict the resulting change in the expected real interest rate and the expected excess returns on capital. The latter is clear: the risk premium declines substantially and persistently in the model, unlike RANK. The former is more nuanced: the expected real interest rate initially declines by more relative to RANK, but in the subsequent quarters exceeds that in RANK. This is because we need to simulate a more negative $\epsilon_0^m$ in the model to match the same decline in the 1-year Treasury, since monetary policy endogenously tightens in subsequent quarters in response to the stimulus from lower risk premia. For this reason, the results which follow are similar if we calibrate the shock in RANK to minimize the absolute value difference between the expected real interest rate path versus the model. Following Proposition 2, we can thus interpret differences in the macro dynamics between the model and RANK as arising from the differing risk premium responses.

Figure 3 demonstrates that redistribution drives the decline in the risk premium in our model. The first panel of the first row demonstrates that realized excess returns on capital are substantially positive on impact, followed by small negative returns in the quarters which follow — consistent with the initial decline in expected excess returns and the empirical pattern estimated in Figure 1. The positive excess return on impact follows from each of the channels characterized in section 2.4: unexpected inflation...
which lowers the realized real interest rate, shown in the third panel; a higher price of capital, shown in the first panel of the second row; and higher short-run profits due to lower real wages and higher employment in this sticky wage environment, shown in the second and third panels of this row. Together these forces redistribute to the high MPR households who hold levered claims on capital, evident from their financial wealth share shown in the second panel of the first row. The persistence in their wealth share drives the persistent decline in expected excess returns.

Figure 4 examines the consequences for policy transmission to the real economy. The impact effects on investment, consumption, and output are 1.3-1.4 times larger versus the RANK economy. Moreover, the stimulus in our model remains persistently higher than the RANK economy despite the endogenous tightening of monetary policy in the model because the risk premium falls by more than the risk-free rate rises. These patterns are consistent with our discussion of Proposition 2.

Quantitatively, the price and quantity effects of the monetary policy shock in

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40While the response of inflation is more immediate in the model than estimated in the data in Figure 1, in practice much nominal debt has longer duration than the one period assumed in the model. Hence, we conjecture that more sluggish price inflation would not change the redistribution of wealth much if the model was also enriched to feature longer duration nominal debt.
our model are consistent with the empirical estimates even though these were not targeted in the calibration. First, the impact effect on excess returns of 1.2pp is only slightly lower than the 1.9pp increase estimated in Figure 1. Second and crucially, a Campbell-Shiller decomposition on the model impulse responses matches the role of news about lower future excess returns in driving the initial stock market return in the data. We summarize this in Table VII. The model contrasts starkly with the RANK economy, where essentially none of the transmission to the stock market operates though news about future excess returns. Third, the peak output stimulus in the model of 0.7pp is comparable to the peak industrial production stimulus estimated in Figure 1, giving us confidence in the model’s real predictions.41

3.4.2 Decomposing redistribution and its consequences

We can further use households’ policy functions in capital to decompose the sources of redistribution and its consequences for capital accumulation. As clarified in Proposition 2, it is the amplification in capital accumulation which also underlies the amplification in consumption and thus overall output in our model.

In particular, given the policy function \( k^i(n^i, \epsilon^m, \Theta) \) in the recursive representation of the economy in which \( n^i \) denotes the agent’s net worth, \( \epsilon^m \) denotes the value of the monetary shock, and \( \Theta \) denotes all other pre-determined state variables, we can decompose the elasticity of a household’s capital holdings to a monetary shock into

\[
\frac{d \log k^i}{d \epsilon^m} = \left( \frac{n^i}{qk^i} \right) \left( \frac{\partial k^i}{\partial n^i} \right) \left( \frac{d \log n^i}{d \epsilon^m} \right) + \frac{\partial \log k^i}{\partial \epsilon^m}, \tag{40}
\]

41We conjecture that adding features such as investment adjustment costs could better match the hump-shapes estimated in the data, following Christiano, Eichenbaum, and Evans (2005).
Table VII: Campbell-Shiller decomposition after monetary shock

<table>
<thead>
<tr>
<th>% Real stock return</th>
<th>Data [90% CI]</th>
<th>Model</th>
<th>RANK</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dividend growth news</td>
<td>33% [-13%, 71%]</td>
<td>52%</td>
<td>65%</td>
</tr>
<tr>
<td>- Future real rate news</td>
<td>8% [-6%, 21%]</td>
<td>16%</td>
<td>35%</td>
</tr>
<tr>
<td>- Future excess return news</td>
<td>59% [19%, 108%]</td>
<td>32%</td>
<td>0%</td>
</tr>
</tbody>
</table>

Notes: estimates from data correspond to Table I. Comparable estimates obtained in the model assuming a debt/equity ratio of 0.5 on a stock market claim.

where the first term on the right-hand side summarizes the response due to the change in the household’s wealth and the second summarizes the response to the changes in prices and future state variables. The elasticity of the household’s wealth is in turn

\[
\frac{d \log n^i}{de^m} = \frac{1}{n^i} \frac{dw^i}{de^m} \frac{1 + \frac{i - 1}{n} B^i + \nu B^g}{n} \frac{d \log P}{de^m} + \frac{k^i}{n} \left( \frac{d \pi}{de^m} + (1 - \delta) \frac{dq}{de^m} \right) + \frac{1}{n^i} \frac{dt^i}{de^m},
\]

which implies that the change in group i’s relative wealth is

\[
\frac{d (\lambda n^i / n)}{de^m} = \lambda \left( \frac{dw^i}{de^m} - n^i \frac{dw^i}{n^i de^m} \right) - \frac{\lambda^i}{n} \frac{i - 1}{P} \left( B^i + \nu B^g \right) \frac{d \log P}{de^m}
\]

\[
+ \frac{\lambda^i}{n} \left( k^i - n^i \frac{k^i}{n} \right) \left( \frac{d \pi}{de^m} + (1 - \delta) \frac{dq}{de^m} \right) + \frac{\lambda^i}{n} \frac{dt^i}{de^m}, \tag{41}
\]

generalizing (21). The first term on the right-hand side summarizes the contribution of labor income, the second and third the contribution of financial wealth, and the fourth the contribution of transfers.\(^{42}\) Similar decompositions have been employed by Luetticke (2021) and other papers in the HANK literature to quantify the effects of redistribution on macroeconomic aggregates.

Using (40), Table VIII decomposes capital accumulation in the first period of the model simulation depicted in Figures 2-4.\(^{43}\) There are two main takeaways. First, the redistribution towards a households drives the equilibrium increase in capital accumulation: \(\left( \frac{n^a}{q^a} \right) \left( \frac{\partial k^a}{\partial n^a} \right) (d \log n^a) \) accounts for most of \(d \log k^a\), and the latter in turn accounts for the aggregate capital accumulation. Second, the redistribution towards

\(^{42}\)We have already accounted for the transfers to households arising from the revaluation of government debt in the term involving \(B^i_{-1} + \nu B^g_{-1}\), so that the transfers \(t^i\) here only refer to those from household birth and death given the model’s perpetual youth structure.

\(^{43}\)We multiply (40) by the size of the monetary shock simulated in Figures 2-4.
a households occurs at the expense of b households, implying that the redistribution which matters for these effects is among the wealthy who hold heterogeneous portfolios. In light of Table III, this also accords well with the view that the losers from a monetary expansion are wealthy retirees, as in Doepke and Schneider (2006).

Using (41), Table IX further clarifies the sources of redistribution towards a households. The baseline parameterization indicates that the balance sheet revaluation towards these households via debt deflation, higher profits on capital, and a higher price of capital together account for virtually all of their increase in relative wealth by 25bp. Redistribution via labor income and government transfers (as part of the model’s perpetual youth structure) are small and in fact redistribute away from these households. Using alternative parameterizations, the remaining columns of Table IX further illuminate the model primitives governing redistribution via balance sheet revaluation. Each column only changes a single parameter from our baseline and simulates the same monetary policy shock.

The second column reports the results for an economy in which monetary policy shocks are persistent, setting $\rho^m = 0.75$, demonstrating the importance of redistribution through debt deflation. In that case, a monetary policy shock induces a stronger response of inflation relative to the baseline, as can be seen in the third row. The increase in the wealth share of a households thus increases to 39bp in the first row.

---

**Table VIII: decomposing capital accumulation on impact of shock**

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d \log k^i$</td>
<td>78bp</td>
<td>-188bp</td>
<td>0bp</td>
</tr>
<tr>
<td>$n^i/(qk^i)$</td>
<td>0.4</td>
<td>2.8</td>
<td>1.0</td>
</tr>
<tr>
<td>$q\partial k^i/\partial n^i$</td>
<td>1.9</td>
<td>0.7</td>
<td>0.0</td>
</tr>
<tr>
<td>$d \log n^i$</td>
<td>135bp</td>
<td>-25bp</td>
<td>35bp</td>
</tr>
<tr>
<td>$\partial \log k^i$</td>
<td>-24bp</td>
<td>-137bp</td>
<td>0bp</td>
</tr>
</tbody>
</table>

Notes: decomposition of $d \log k^i$ depicted in Figures 2-4 uses (40) evaluated at 1,000 different points drawn from ergodic distribution of the state space and multiplies this identity by the size of the simulated monetary shock. bp denotes basis points (0.01%).

---

44 We again multiply by the size of the monetary shock simulated in Figures 2-4. Note $\frac{\Lambda_s n^a}{n}$ is the wealth share inclusive of labor income, whereas $s^a$ in Figure 3 is the financial wealth share alone.

45 We note that almost all of the redistribution through debt deflation owes to $B^a$ (disciplined by the debt of these households in the SCF directly on balance sheet or via firm equity), rather than the indirect exposure through the government $\nu^a B^g$. This is because $B^a$ is more than 15 times $\nu^a B^g$. 
Table IX: decomposing wealth redistribution to a households on impact of shock

<table>
<thead>
<tr>
<th>Expression</th>
<th>Base</th>
<th>$\rho^m = 0.75$</th>
<th>$\chi^x = 0$</th>
<th>$\chi^W = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d(\lambda^a n^a/n)$</td>
<td>25bp</td>
<td>39bp</td>
<td>23bp</td>
<td>21bp</td>
</tr>
<tr>
<td>$(\lambda^a/n)(d[w^a] - (n^a/n)d[w\ell])$</td>
<td></td>
<td>$-1bp$</td>
<td>$-1bp$</td>
<td>$-1bp$</td>
</tr>
<tr>
<td>$(\lambda^a/n)(-(1+i_{-1})/P)(B^a - \nu^a B^a_{-1})d\log P$</td>
<td></td>
<td>20bp</td>
<td>33bp</td>
<td>23bp</td>
</tr>
<tr>
<td>$(\lambda^a/n)(k^a_{-1} - (n^a/n)k_{-1})d\pi$</td>
<td></td>
<td>1bp</td>
<td>1bp</td>
<td>1bp</td>
</tr>
<tr>
<td>$(\lambda^a/n)(k^a_{-1} - (n^a/n)k_{-1})(1-\delta)dq$</td>
<td></td>
<td>6bp</td>
<td>6bp</td>
<td>0bp</td>
</tr>
<tr>
<td>$(\lambda^a/n)dt^a$</td>
<td></td>
<td>$-1bp$</td>
<td>$-1bp$</td>
<td>$-1bp$</td>
</tr>
</tbody>
</table>

Notes: decomposition of $d(\lambda^a n^a/n)$ depicted in Figures 2-4 uses (41) evaluated at 1,000 different points drawn from ergodic distribution of the state space and multiplies this identity by the size of the simulated monetary shock. bp denotes basis points (0.01%).

The third column eliminates the capital adjustment cost by setting $\chi^x = 0$, mitigating the redistribution through asset prices. In that case a monetary policy shock has no effect on the price of capital and therefore reduces the unexpected return on capital, as reported in the fifth row. There is a countervailing effect of a larger inflation response in the third row: the smaller adjustment cost amplifies capital accumulation which in turn amplifies the response in the labor market. Nonetheless, the increase in the wealth share of a households falls to 23bp in the first row.

The fourth column eliminates nominal wage rigidity by setting $\chi^W = 0$, demonstrating the role of changes in profit income in inducing redistribution across households. When wage rigidity is zero, the decline in the real wage and the stimulus to employment is essentially eliminated. It follows that the change in profits is negligible in the fourth row, which renders the change in the price of capital negligible in the fifth row. The increase in the wealth share of a households falls to 21bp even though the redistribution through debt deflation is again amplified in the third row.

4 Conclusion

In this paper we revisit monetary transmission in a New Keynesian environment with heterogeneous propensities to bear risk. An expansionary monetary policy shock lowers the risk premium if it redistributes to households with high MPRs. Heterogeneity in risk aversion, portfolio constraints, rules of thumb, background risk, or beliefs im-
ply redistribution in this way. In a calibration matching heterogeneity in the U.S. economy, this mechanism rationalizes the stock market responses to monetary policy which have eluded existing frameworks and amplifies its real effects.

The framework of this paper can be further developed along a number of dimensions. First, it seems fruitful to synthesize our perspective emphasizing assets’ exposure to aggregate risk with the existing HANK literature emphasizing asset liquidity: in such a setting, an investor’s MPR out of liquid versus illiquid wealth will differ, likely a better match to the micro data. Second, while we have focused for concreteness on the equity premium, a natural question is the extent to which our insights can explain the broader effects of monetary policy across asset classes, as in the Treasury market or foreign exchange market. Third, while our analysis has focused on the conditional responses to monetary policy shocks, it would be useful to examine the model’s implied comovements between real activity and risk premia when featuring a richer set of business cycle shocks calibrated to the data. We leave these questions for future work.

References


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A Proofs of analytical results

A.1 Proposition 1

Proof. Multiplying both sides of (14) by \( a_0^i \), integrating over all \( i \), and then using asset market clearing \( \int_0^1 a_0^i \omega_0^i di = \int_0^1 a_0^i di \), we obtain the claimed expression for the risk premium on capital. Differentiating with respect to \( \epsilon_0^m \), we have that

\[
\frac{d}{d\epsilon_0^m} \left[ \mathbb{E}_0 \log(1 + r_k^1) - \log(1 + r_1) \right] = \sigma^2 \frac{d\gamma}{d\epsilon_0^m} = \gamma \sigma^2 \int_0^1 \frac{a_0^i / \int_0^1 a_0^i' di'}{\epsilon_0^m} (1 - \omega_0^i) di,
\]

as claimed, where the second equality uses (14) and asset market clearing. \( \square \)

A.2 Proposition 2

Proof. The result for \( \frac{dk_0}{d\epsilon_0^m} \) follows from differentiating (16). Now differentiating household \( i \)'s consumption policy function and (17) and integrating over all \( i \) yields

\[
\frac{dc_0}{d\epsilon_0^m} = (1 - \beta) \left[ \frac{dy_0}{d\epsilon_0^m} + \frac{(1 - \delta_0)q_0 k_{-1}}{k_0} \chi x \frac{dk_0}{d\epsilon_0^m} \right],
\]

where we have used firms’ flow of funds, asset market clearing, and equilibrium investment. Differentiating goods market clearing and using equilibrium investment yields

\[
\frac{dy_0}{d\epsilon_0^m} = \frac{dc_0}{d\epsilon_0^m} + q_0 \left( 1 + \frac{x_0}{k_0} \chi \right) \frac{dk_0}{d\epsilon_0^m}.
\]

Combining these and using capital accumulation yields the result for \( \frac{dc_0}{d\epsilon_0^m} \). \( \square \)

A.3 Proposition 3

Proof. The Taylor rule and Fisher equation imply

\[
\frac{d \log P_0}{d\epsilon_0^m} = \frac{1}{\phi} \frac{d \log(1 + r_1)}{d\epsilon_0^m} - \frac{1}{\phi}.
\]

(A.1)
Combining with equilibrium labor demand, the production function, and a rigid nominal wage in period 0 yields

\[
\frac{1}{y_0} \frac{dy_0}{d\epsilon_0} = \frac{1}{\alpha} \frac{1}{\phi} \frac{d\log(1 + r_1)}{d\epsilon_0} = \frac{1}{\alpha} \frac{1}{\phi}. \tag{A.2}
\]

By (15), (18), and (20), it follows that

\[
\frac{dk_0}{d\epsilon_0} = -\frac{k_0}{1 - \alpha + \chi^x} \left[ -\gamma \sigma^2 \frac{1}{n_0} Cov^i \left( \frac{dn^i_0}{d\epsilon_0}, \omega_0 \right) \right] + \frac{d\log(1 + r_1)}{d\epsilon_0}, \tag{A.3}
\]

where \(Cov^i\) denotes a cross-sectional covariance. Combining this with (21), (A.1),

\[
\frac{1}{\pi_0} \frac{d\pi_0}{d\epsilon_0} = \frac{1}{y_0} \frac{dy_0}{d\epsilon_0}, \tag{A.4}
\]

implied by equilibrium profits, and

\[
\frac{1}{q_0} \frac{dq_0}{d\epsilon_0} = \chi^x \frac{1}{k_0} \frac{dk_0}{d\epsilon_0}, \tag{A.5}
\]

implied by equilibrium investment, we have that

\[
Cov^i \left( \frac{dn^i_0}{d\epsilon_0}, \omega_0^i \right) = \frac{1}{1 - \gamma \sigma^2(1 - \delta_0) \frac{q_0}{n_0} \frac{\chi^x}{1 - \alpha + \chi^x} Cov^i(k_{-1}^i, \omega_0^i) \times \left[ -Cov^i \left( \frac{1 + i^i_{-1}B_{-1}}{P_0}, \omega_0 \right) \left( \frac{1}{\phi} \frac{d\log(1 + r_1)}{d\epsilon_0} - \frac{1}{\phi} \right) \right] + \left( \frac{\pi_0}{y_0} \frac{dy_0}{d\epsilon_0} - (1 - \delta_0)q_0 \frac{\chi^x}{1 - \alpha + \chi^x} \frac{d\log(1 + r_1)}{d\epsilon_0} \right) Cov^i \left( k_{-1}^i, \omega_0^i \right) \right] \tag{A.6}
\]

Finally, combining (A.3) and (19) yields

\[
\frac{dy_0}{d\epsilon_0} = \left[ \gamma \sigma^2 Cov^i \left( \frac{dn^i_0}{d\epsilon_0}, \omega_0^i \right) - \frac{\gamma^2 \sigma^2}{\alpha} Cov^i \left( 1 + \frac{1}{\phi} \frac{d\log(1 + r_1)}{d\epsilon_0}, \omega_0 \right) \right] \times \left( \frac{1 + \chi^x(1 - \beta(1 - \delta_0)k_{-1}/k_0)}{1 - \alpha + \chi^x} \right). \tag{A.7}
\]

With heterogeneity in \(\gamma^i\) and the assumption that households’ initial endowments are consistent with the portfolios (14), we have \(Cov^i \left( B_{-1}^i, \omega_0^i \right) < 0\) and \(Cov^i \left( k_{-1}^i, \omega_0^i \right) > 0\). Straightforward algebra using the system of equations (A.2),
(A.6), and (A.7) then implies

\[
\frac{dy_0}{d\epsilon_0^m} < \frac{dy_0}{d\epsilon_0^m}\big|_{\text{RANK} < 0},
\]

\[
Cov^i\left(\frac{dn_0^i}{d\epsilon_0^m}, \omega_0^i\right) < Cov^i\left(\frac{dn_0^i}{d\epsilon_0^m}, \omega_0^i\right)\big|_{\text{RANK} = 0},
\]

\[
\frac{d\log(1 + r_1)}{d\epsilon_0^m} < \frac{d\log(1 + r_1)}{d\epsilon_0^m}\big|_{\text{RANK} \in (0, 1)},
\]

provided that heterogeneity is not too extreme (and thus the initial equilibrium is stable). We can thus conclude from (A.1), (A.3), (A.4), and (A.5) that a monetary tightening generates deflation, a fall in profits, and a fall in the price of capital; that it generates a rise in the risk premium only in the model with heterogeneity; and that the real effects of a monetary shock are amplified in the model with heterogeneity. \(\square\)

### A.4 Proposition 4

**Proof.** For a household up against a portfolio constraint or following a rule-of-thumb, \(\omega_0^i\) is simply implied by the constraint or rule-of-thumb. For all other households, a Taylor approximation of (13) yields

\[
\omega_0^i \approx \frac{1}{\gamma^i(\varsigma^i + \eta^i)} \frac{E_0 \log(1 + r_{1,k}^i) - \log(1 + r_1) + \frac{1}{2}(\varsigma^i + \eta^i)\sigma^2}{\sigma^2}, \tag{A.8}
\]

where \(E_0^i\) is the household’s subjective expectation and \(r_{1,k}^i\) is the return on capital accounting for the idiosyncratic component of its return. Given the distributional assumptions on \(\eta^i\) and \(\varsigma^i\), we have that

\[
E_0^i \log(1 + r_{1,k}^i) - \log(1 + r_1) + \frac{1}{2}(\varsigma^i + \eta^i)\sigma^2 = E_0 \log(1 + r_1^k) - \log(1 + r_1) + \frac{1}{2}\sigma^2, \tag{A.9}
\]

where \(E_0\) denotes the objective expectation and \(r_1^k\) the return on capital without idiosyncratic risk (equivalently, aggregating over idiosyncratic risk). Then multiplying both sides of (A.8) by \(a_0^i\), integrating over all unconstrained households which we denote as \(i \notin C\), and using asset market clearing, it is straightforward to show

\[
E_0 \log(1 + r_1^k) - \log(1 + r_1) + \frac{1}{2}\sigma^2 = \gamma\sigma^2
\]
where now
\[
\gamma \equiv \left(1 - \int_{i\in C} a_i^0 (1 - \omega_i^0) di \right) \left(\int_{i\notin C} \int_{i\notin C} a_i^0 \omega_i^0 d\gamma(\varsigma^i + \eta^i) \right)^{-1}.
\]
Differentiating and again using market clearing, (A.8) for unconstrained households, and (A.9) yields
\[
\frac{d}{d\epsilon_m} \left[ E_0 \log(1 + r^k_1) - \log(1 + r^k_1) \right] = \gamma \left(\int_{i=0}^{1} a_i^0 di \right) \sigma^2 \int_{i=0}^{1} \frac{d}{d\epsilon_m} \left[ a_i^0 / \int_{i=0}^{1} a_i^0 di \right] (1 - \omega_i^0) di.
\]
The remaining arguments in Propositions 2 and 3 are unchanged.

A.5 Proposition 5

Proof. We first characterize households’ portfolio share in capital in the limit of zero aggregate risk. Up to second-order, (13) implies
\[
E_0 \hat{r}_1 - \hat{r}_1 + \frac{1}{2} \sigma^2 = \gamma^i \hat{c}_{1z}^i \sigma^2 + o(|| \cdot ||^3)
\]
given the first-order approximation of \( \hat{c}_1 \) in terms of the states
\[
\hat{c}_1 = \delta_{z1}^i \hat{c}_1^z + \delta_{m0}^i \hat{c}_0^m + o(|| \cdot ||^2)
\]
with coefficients \( \delta. \) Anticipating the result in Proposition 6, it follows that
\[
\delta_{z1}^i = \frac{\bar{\gamma}}{\gamma^i}.
\]
Approximating up to first order the period 1 resource constraint and equilibrium wages and profits, the method of undetermined coefficients implies
\[
\delta_{z1}^i = \frac{\bar{w}_1 + \bar{\pi}_1 \bar{k}_0^i}{\bar{c}_1^i}.
\]
Substituting in (A.11) and re-arranging, we can conclude that
\[
\frac{\bar{q}_0 \bar{k}_0^i}{a_0^i} = \frac{\bar{\pi}_1 \bar{k}_0^i}{(1 + r_1) a_0^i} = \frac{\bar{c}_1^i}{(1 + \bar{r}_1) a_0^i} \frac{\bar{\gamma}}{\gamma^i} - \frac{\bar{w}_1}{(1 + \bar{r}_1) a_0^i}.
\]
where the first equality uses $\bar{q}_0 = \frac{\bar{q}_1}{1+\bar{r}_0}$ absent aggregate risk.

We now characterize households’ marginal responses to a unit of income in the limit of zero aggregate risk. Differentiating and combining households’ optimal portfolio choice condition and period 1 resource constraint yields

\[ 0 = \mathbb{E}_0 \left( c_1^i \right)^{-\gamma} \frac{\gamma^i}{c_1^i} (r_1^k - r_1) \left( 1 + r_1 \right) \frac{\partial b_0^i}{\partial n_0^i} + \pi_1 \frac{\partial k_0^i}{\partial n_0^i}. \]  

(A.13)

A second-order approximation then implies

\[ 0 = \left( 1 + \bar{r}_1 \right) \frac{\partial b_0^i}{\partial n_0^i} + \pi_1 \frac{\partial k_0^i}{\partial n_0^i} \left( \frac{\gamma + 1}{c_1^i} (\bar{w}_1 + \pi_1 \bar{k}_0^i) \sigma^2 + \pi_1 \frac{\partial k_0^i}{\partial n_0^i} \sigma^2 + o(||\cdot||^3) \right). \]  

(A.14)

Again anticipating the result in Proposition 6 and using (A.11) and (A.12), it follows from (A.14) that

\[ \bar{q}_0 \frac{\partial k_0^i}{\partial n_0^i} = \frac{\gamma}{\gamma^i} \frac{\partial a_0^i}{\partial n_0^i} \]

using $\bar{q}_0 = \frac{\bar{q}_1}{1+\bar{r}_0}$ and $\bar{q}_0 \frac{\partial k_0^i}{\partial n_0^i} = \frac{\partial a_0^i}{\partial n_0^i}$. The expression for $\text{mpr}_0^i \equiv \bar{q}_0 \frac{\partial k_0^i}{\partial n_0^i}$ follows.

A.6 Proposition 6

Proof. We first derive the result up to second order. Multiplying both sides of (A.10) by $\frac{\bar{w}_1}{\bar{r}_1}$, integrating over all households $i$, and making use of the market clearing conditions which imply that

\[ \int_0^1 \bar{c}_1^i di = \int_0^1 (\bar{w}_1 + \pi_1 \bar{k}_0^i) di, \]

we obtain

\[ \mathbb{E}_0 \bar{r}_1^k - \bar{r}_1 + \frac{1}{2} \sigma^2 = \left( \frac{\bar{c}_1}{\int_0^1 \bar{c}_1^i di} \right)^{-1} \sigma^2 + o(||\cdot||^3), \]  

(A.15)

defining $\gamma$ as in the claim.

We now derive the result up to third order. The third-order approximation of
optimal portfolio choice for household $i$ is

$$
\mathbb{E}_0 \hat{r}_1^k - \hat{r}_1 + \frac{1}{2} \sigma^2 =
\gamma^i \delta_{z_1}^c \sigma^2 + \left[ \gamma^i \left( \delta_{m_0}^c \gamma^i + \delta_{m_0}^c \right) - \left( \gamma^i \right)^2 \delta_{m_0}^c \delta_{m_0}^c + \gamma^i \delta_{z_1}^c \delta_{m_0} - \gamma^i \delta_{m_0}^c \right] \hat{\epsilon}_0^m \sigma^2 + o(\| \cdot \|^4).
$$

given the second-order expansion of $c_1^i$ in terms of the underlying states

$$
c_1^i = \delta_{m_0}^c \epsilon_0^m + \delta_{z_1}^c \epsilon_1 + \frac{1}{2} \delta_{m_0}^c (\epsilon_0^m)^2 + \delta_{z_1}^c \epsilon_0^m \epsilon_1 + \frac{1}{2} \delta_{z_1}^c (\epsilon_1^z)^2 + \frac{1}{2} \delta_{z_1}^c \sigma^2 + o(\| \cdot \|^3).
$$

Making use of (A.11) substantially simplifies this to

$$
\mathbb{E}_0 \hat{r}_1^k - \hat{r}_1 + \frac{1}{2} \sigma^2 = \gamma^i \delta_{z_1}^c \sigma^2 + \gamma^i \delta_{m_0}^c \hat{\epsilon}_0^m \sigma^2 + o(\| \cdot \|^4).
$$

(A.16)

Again multiplying both sides by $\frac{\epsilon_0^m}{\gamma^i}$, integrating over all households $i$, and making use of the market clearing conditions, we obtain

$$
\mathbb{E}_0 \hat{r}_1^k - \hat{r}_1 + \frac{1}{2} \sigma^2 = \gamma \sigma^2 + \frac{\gamma^i}{\int_0^1 \bar{c}_1 \delta_{m_0}^c \delta_{m_0}^c \, di} \left( \int_0^1 \bar{c}_1^i \delta_{m_0}^c \, di \right) \hat{\epsilon}_0^m \sigma^2 + o(\| \cdot \|^4).
$$

It remains to further characterize the coefficient on $\hat{\epsilon}_0^m \sigma^2$ in closed form. Taking a second-order approximation of the period 1 resource constraint and equilibrium wages and profits, the method of undetermined coefficients implies

$$
c_1^i \delta_{m_0}^c + c_1^i \delta_{m_0}^c \delta_{z_1}^c = \alpha \bar{w}_1 \delta_{m_0}^k + \bar{\pi}_1 \delta_{m_0}^k - (1 - \alpha) \bar{\pi}_1 \bar{k}_1 \delta_{m_0}^k.
$$

(A.17)

It follows that

$$
\int_0^1 c_1^i \delta_{m_0}^c \, di = - \int_0^1 c_1^i \delta_{m_0}^c \delta_{z_1}^c \, di + \bar{\pi}_1 \int_0^1 \delta_{m_0}^k \, di,
$$

using market clearing and the definition of equilibrium wages and profits.1 Further using a first-order approximation to capital claims market clearing and goods market clearing implies

$$
\int_0^1 c_1^i \delta_{m_0}^c \, di = \int_0^1 c_1^i \delta_{m_0}^c \left( 1 - \delta_{z_1}^c \right) \, di = \int_0^1 c_1^i \delta_{m_0}^c \left( 1 - \frac{\gamma^i}{\gamma^i} \right) \, di,
$$

1We linearize rather than log-linearize with respect to $\{k_0^2, b_0, a_0^2\}$ since these may be negative.
where the second equality uses (A.11). Recall from Proposition 5 that $\bar{\gamma}_i \equiv \frac{\bar{\gamma}}{\gamma}$. Since the definition of $\bar{\gamma}$ implies

$$\int_0^1 \bar{c}_i^j \delta_{\bar{c}_{i0}}^j \left(1 - \frac{\bar{\gamma}^i}{\gamma^i}\right) di = \int_0^1 \left(\bar{c}_i^j \delta_{i0}^j - \frac{\bar{c}_i^j}{\int_0^1 \bar{c}_i^{j'} di'} \int_0^1 \bar{c}_i^{j'} \delta_{i0}^{j'} di'\right) \left(1 - \frac{\bar{\gamma}^i}{\gamma^i}\right) di$$

and

$$\frac{d \left[ \bar{c}_i^j / \int_0^1 \bar{c}_i^j \right]}{d \bar{\epsilon}_0^m} = \frac{1}{\int_0^1 \bar{c}_i^{j'} di'} \left(\bar{c}_i^j \delta_{i0}^j - \frac{\bar{c}_i^j}{\int_0^1 \bar{c}_i^{j'} di'} \int_0^1 \bar{c}_i^{j'} \delta_{i0}^{j'} di'\right),$$

we obtain the coefficient on $\epsilon_0^m \sigma^2$ given in the claim. □

B Empirical appendix

B.1 The effect of monetary shocks

B.1.1 Robustness to details of estimation approach

We first demonstrate that the broad messages of our baseline estimates of the effects of a monetary policy shock in section 3.1 are robust to details of the estimation.

Given a monetary policy shock, Table A.I summarizes the impact effect on the 1-year Treasury yield, the impact effect on the real S&P 500 return (implied by the real rate and excess return), and the share of the latter accounted for by news about future excess returns in the Campbell-Shiller decomposition (26). First, we find that the baseline results using 6 lags in the VAR are little affected if 4-8 lags are used instead. Second, we find that the results are broadly robust to using the same January 1991 - June 2012 period for both the VAR and IV regressions, or limiting the analysis of monetary policy shocks to the first half of the IV sample alone (January 1991 - September 2001). The expansionary monetary policy shock lowers the stock market when using the second half of the IV sample alone (October 2001 - June 2012), but we note that the instrument is weak over this sub-sample (having a first-stage $F$ statistic of 4.7, not shown). Third, we find that news about future excess returns tends to be, if anything, even more important when adding other variables included in the analyses of Bernanke and Kuttner (2005) and Gertler and Karadi (2015) on which we build. Finally, we find similar results when using as the instrument the three-month ahead Fed Funds futures contract instead of the current contract.
<table>
<thead>
<tr>
<th></th>
<th>1-year Treasury yield (pp)</th>
<th>Real stock return (pp)</th>
<th>Share future excess return news (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Baseline</strong></td>
<td>-0.22</td>
<td>1.92</td>
<td>59%</td>
</tr>
<tr>
<td><strong>Number of lags in VAR</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>-0.21</td>
<td>1.85</td>
<td>52%</td>
</tr>
<tr>
<td>5</td>
<td>-0.22</td>
<td>1.79</td>
<td>54%</td>
</tr>
<tr>
<td>7</td>
<td>-0.23</td>
<td>1.86</td>
<td>62%</td>
</tr>
<tr>
<td>8</td>
<td>-0.23</td>
<td>1.90</td>
<td>56%</td>
</tr>
<tr>
<td><strong>Sample periods</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VAR: 1/91-6/12, IV: 1/91-6/12</td>
<td>-0.14</td>
<td>1.51</td>
<td>36%</td>
</tr>
<tr>
<td>VAR: 7/79-6/12, IV: 1/91-9/01</td>
<td>-0.21</td>
<td>3.07</td>
<td>49%</td>
</tr>
<tr>
<td>VAR: 7/79-6/12, IV: 10/01-6/12</td>
<td>-0.17</td>
<td>-2.08</td>
<td>37%</td>
</tr>
<tr>
<td><strong>Variable added to VAR</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Excess bond premium</td>
<td>-0.21</td>
<td>2.21</td>
<td>83%</td>
</tr>
<tr>
<td>Mortgage spread</td>
<td>-0.24</td>
<td>1.53</td>
<td>54%</td>
</tr>
<tr>
<td>3-month commercial paper spread</td>
<td>-0.20</td>
<td>2.13</td>
<td>65%</td>
</tr>
<tr>
<td>5-year Treasury rate</td>
<td>-0.17</td>
<td>1.61</td>
<td>78%</td>
</tr>
<tr>
<td>10-year Treasury rate</td>
<td>-0.17</td>
<td>1.56</td>
<td>76%</td>
</tr>
<tr>
<td>Term spread</td>
<td>-0.21</td>
<td>2.04</td>
<td>64%</td>
</tr>
<tr>
<td>Relative bill rate</td>
<td>-0.18</td>
<td>2.55</td>
<td>70%</td>
</tr>
<tr>
<td>Change in 3-month Treasury rate</td>
<td>-0.19</td>
<td>2.25</td>
<td>65%</td>
</tr>
<tr>
<td>3-month ahead FF as IV</td>
<td>-0.21</td>
<td>2.21</td>
<td>65%</td>
</tr>
</tbody>
</table>

Table A.I: robustness of 1 SD monetary shock on returns and components

Notes: series for the Gilchrist and Zakrajsek (2012) excess bond premium, mortgage spread, 3-month commercial paper spread, 5-year Treasury rate, and 10-year Treasury rate are taken from the dataset provided by Gertler and Karadi (2015). The term spread (10-year Treasury rate less 1-month Treasury yield), relative bill rate (difference between the 3-month Treasury rate and its 12-month moving average), and change in the 3-month Treasury rate are constructed using CRSP.

**B.1.2 Testing invertibility and comparing SVAR-IV and LP-IV**

We now demonstrate that the assumption of invertibility implicit in our SVAR-IV is validated by statistical tests suggested in the literature.

Stock and Watson (2018) propose a Hausman-type test statistic of the null hypothesis that invertibility is satisfied by comparing the impulse response at horizon $h$ for a given variable under the SVAR-IV and LP-IV. We implement the LP-IV by projecting each outcome variable $h$ months ahead on the 1-year Treasury yield, instrumenting for the latter using the Fed Funds futures surprise also used in our baseline...
Table A.II: tests of invertibility assumed in the VAR

<table>
<thead>
<tr>
<th></th>
<th>1-yr Treasury</th>
<th>CPI</th>
<th>Industrial production</th>
<th>1-mo real rate</th>
<th>1-mo excess return</th>
<th>Dividend/price</th>
</tr>
</thead>
<tbody>
<tr>
<td>SW [2018] test</td>
<td>0.45</td>
<td>0.61</td>
<td>0.97</td>
<td>0.80</td>
<td>0.68</td>
<td>0.76</td>
</tr>
<tr>
<td>Granger test</td>
<td>0.07</td>
<td>0.16</td>
<td>0.88</td>
<td>0.13</td>
<td>0.41</td>
<td>0.93</td>
</tr>
</tbody>
</table>

Notes: the first row is the bootstrapped p-value for the null hypothesis that the SVAR-IV and LP-IV impulse responses are the same 1, 13, 25, and 37 months after shock, using the test statistic provided in Stock and Watson (2018). We construct the variance matrix needed for this statistic using the 10,000 iterations of the wild bootstrap used to construct confidence intervals for our SVAR-IV estimates in the main text. The second row is the p-value for the null hypothesis that the coefficients on 6 lags of the instrument are jointly equal to zero when added to the VAR.

SVAR-IV. The first row of Table A.II summarizes the p-value for this test in our setting jointly applied at horizons $h \in \{1, 13, 25, 37\}$ for each variable, demonstrating that we cannot reject the null at standard significance levels.

Stock and Watson (2018) also recommend the use of the complementary Granger causality test in Forni and Gambetti (2014): if invertibility is satisfied, lagged values of the instrument should not have predictive power given the variables included in the VAR. We include 6 lags of our instrument in the VAR and construct an F statistic associated with the null hypothesis that these coefficients are jointly zero for each variable in the VAR. We again cannot reject the null at standard significance levels.

**B.1.3 Adding relative wealth responses to the VAR**

We finally augment our VAR with two measures of the relative wealth of agents with heterogeneous exposures to capital. The wealth of agents more exposed to the stock market rises on impact of a monetary easing, consistent with the model mechanisms.

We construct the relative wealth measures using monthly data on the returns of hedge funds and mutual funds through which households invest. In each case, we have an unbalanced panel of data on returns $r_{ft}$ and assets $A_{ft}$ for funds indexed by $f$ and months in time indexed by $t$; the data sources are described further below. For each fund, we project the time-series of its monthly return on the S&P 500 return relative

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2Following Stock and Watson (2018), to make this specification comparable with the SVAR-IV and further improve the precision of estimates, we include 6 lags of each of the variables included in the VAR as controls. Moreover, given the serial correlation of the instrument discussed in Ramey (2016) and Stock and Watson (2018), we include a lag of the instrument as an additional control.
to the 1-month Treasury return. The estimated coefficient is the fund’s estimated beta $\beta_f$. We sort funds by their beta, compute the median fund beta $\beta_{p50}$, and then define the average monthly return (weighted by the prior month’s assets) for funds with a beta above or below the median, $r_{t}^{high}$ and $r_{t}^{low}$, respectively. Finally, we define the relative total return index relative to an initial date 0 as

$$relwealth_t = \sum_{s=0}^{t} r_{s}^{high} - \sum_{s=0}^{t} r_{s}^{low}.$$  

This is a measure of the relative wealth of a household which continually reinvests in high beta funds versus a household which continually reinvests in low beta funds. We obtain one such measure for hedge funds and another for mutual funds.

Our source of hedge fund data is the Lipper TASS database from June 1990 through June 2012. Following Getmansky, Lee, and Lo (2015), we focus on funds that provide monthly data; we define the monthly fund return as the ratio of its NAV to its prior month NAV if these measures are available, and as the self-reported return if they are not; we only keep observations for which the monthly fund return is between -100 and 200; and we only keep observations for which the monthly fund return is not equal to the last two observations. We further only keep observations with reported estimated assets which are not equal to the previous observation, and only keep funds with at least 120 observations meeting the criteria described above. We are left with 733 hedge funds in the sample.

Our source of mutual fund data is the CRSP Survivorship-Bias-Free Mutual Fund database from June 1990 through June 2012. We keep only observations with non-missing returns and prior period total net assets, and funds with at least 120 observations meeting these criteria. We are left with 13,011 mutual funds in the sample.

Having constructed the relative wealth measures in this way, we add both to our VAR. We estimate the VAR and the IV regressions using the January 1991 - June 2012 period. Figure A.1 displays the estimated responses of both relative wealth measures to a monetary easing which results in a roughly 0.2pp decline in the 1-year Treasury yield; the estimated responses of the other variables in the VAR are similar to the baseline results presented in the main text and are thus excluded for brevity. In both cases a household (continually re-)invested in high beta funds would experience an increase in relative wealth on impact of a monetary easing, after which its wealth
share would decline.\footnote{The larger response of the relative wealth measure using mutual funds is consistent with the fact that we estimate a larger dispersion in betas in the sample of mutual funds than hedge funds.} This result is not mechanical: while the hedge funds and mutual funds are sorted based on their unconditional betas over the sample period, these measures of relative reported returns do not otherwise use any information on market returns, and their conditional response to a monetary easing need not have exhibited the same qualitative pattern as the excess S&P 500 return — but they do.

\section*{B.2 Micro moments from the SCF}

\subsection*{B.2.1 Construction of household portfolios}

We now provide supplemental details on our measurement of household portfolios using the 2016 SCF described in section 3.3. We proceed in three steps.

First, to the SCF data we add an estimate of defined benefit pension wealth for each household, since this is not included in the SCF measure of net worth. We use the estimates of Sabelhaus and Volz (2019).

Second, we proceed by line item to allocate how much household wealth is in directly held claims on capital, indirect claims on capital through business equity, or nominal claims. Direct claims on capital are non-financial assets (vehicles, primary residence, residential real estate excluding the primary residence, non-residential real estate, and other miscellaneous non-financial assets). Indirect claims on capital through business equity come in two forms: publicly traded stocks or privately-owned businesses. We assume the following line items reported in the SCF summary extract

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure.png}
\caption{effects of a 1 SD monetary shock on relative wealth measures}
\end{figure}

Notes: see notes accompanying Figure 1 in main text.
include stocks: stock mutual funds, other mutual funds, and directly held stocks, all of which we assume are fully invested in stocks; combination mutual funds, 50% of which we assume are invested in stocks; and savings accounts that may be invested in stocks and are included in transaction accounts (such as 529 or state-sponsored education accounts), other managed assets, and quasi-liquid retirement assets, for which we use the self-reported fraction of these accounts invested in stocks. We assume the remaining portion of these line items not invested in stocks, as well as all other line items not mentioned above, are purely nominal assets or liabilities.

Third, we account for households’ leverage via indirect claims on capital. In particular, if household $i$ owns $1$ in equity in a firm with leverage $lev^i$, then we assign the household $Qk^i = lev^i$ and $B^i = 1 - lev^i$. We assume that the leverage of publicly traded stocks and private businesses owned by household $i$ are

$$lev_{public}^i = lev_{public}^i \epsilon^i,$$
$$lev_{private}^i = lev_{private}^i \epsilon^i,$$

respectively, where the idiosyncratic component $\epsilon^i$ is drawn from a $\Gamma(\theta^{-1}, \theta)$ distribution having mean one. $lev_{public}$ thus reflects the aggregate leverage of the household sector in publicly traded stocks; $lev_{private}$ reflects the aggregate leverage of the household sector in private businesses; and $\theta$ controls the dispersion of household leverage in these claims. The $\Gamma$ distribution is right-skewed, which accords well with the literature studying portfolio and return heterogeneity discussed further below.

We use the Financial Accounts (FA) to discipline $lev_{public}$ and $lev_{private}$. $lev_{public}$ equals the net leverage of the consolidated nonfinancial corporate sector (FA table S.5.a) and financial business sector (FA table S.6.a), net of the central bank (FA table S.61.a), government DB pension funds (FA tables L.119.b and L.120.b), all defined contribution (DC) pension funds (FA tables L.118.c, L.119.c, and L.120.c), and mutual funds (FA table L.122). $lev_{private}$ equals the net leverage of the consolidated nonfinancial noncorporate sector (FA table S.4.a) and non-profit sector (FA table B.101.n). We compute net leverage by dividing the aggregate position in capital by

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4Our approach here follows the construction of the EQUITY variable in the summary extract.
5We exclude the central bank and government DB pension funds because we model these as part of the government sector (the latter consistent with our interpretation of DB pensions in footnote 24). We exclude DC pension funds and mutual funds because these are pass-through entities whose assets have already been folded into that of households using our approach described so far.
net equity issued to other sectors. Capital is given by total assets net of nominal assets and equity assets. Net equity issued to other sectors is given by equity liabilities plus net worth net of equity assets. Using the Q2 2019 release of the FA, the resulting measures of 2016 leverage we obtain are $lev_{public} = 1.6$ and $lev_{private} = 1.1$.

With these measures of aggregate leverage alone, we can decompose the $104,721bn$ in total U.S. household net worth ($\sum_i A_i$) into $11,228bn$ in nominal claims ($\sum_i B^i$) and $93,492bn$ in capital ($\sum_i Qk^i$).⁶

We finally use recent evidence on the heterogeneity in households’ expected returns on wealth to discipline $\theta$. Using granular data on the portfolios of the universe of Swedish households, Bach, Calvet, and Sodini (2020) construct household-specific measures of expected excess returns. Over the 2000-2007 period, the cross-sectional standard deviation in expected excess returns on gross assets was 31% of the expected excess returns of the global market.⁷ We choose $\theta = 1.05$ so that the implied cross-sectional standard deviation in leverage on assets in our SCF sample equals 31% of the aggregate leverage in public equities estimated above ($lev_{public} = 1.6$). The fact that a positive value of $\theta$ is needed to match the evidence on return heterogeneity is consistent with broader results from the literature that households in fact do not hold identical, diversified equity portfolios.⁸ Nonetheless, even when $\theta \to 0$, there is heterogeneity in capital portfolio shares (and thus expected returns) because of households’ heterogeneous portfolios across nominal claims, capital, and equity.

B.2.2 Application to the 2007-2009 SCF panel

Using the 2007-2009 SCF panel, we can apply the exact same methodology as described in the prior subsection to characterize households’ portfolios and sort them into three groups as of 2007. In this subsection, we follow these households over the next two years using this survey’s unique panel structure, demonstrating that the evolution of portfolios is broadly consistent with the mechanisms in our model.

Rows 1-4 of Table A.III summarize households by group as of 2007. The moments

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⁶We have validated that this aggregate balance sheet is consistent with market clearing in nominal claims after accounting for the balance sheets of the government and rest of the world.

⁷In their Table 8, these authors report a cross-sectional standard deviation in expected excess returns of 1.81%. In their section I.D, they report a long-run (1983-2016) average of the global market excess return of 5.8%. The ratio between these is 31%. We use this evidence because of the absence of comparable data in the United States with exhaustive coverage of households’ wealth.

⁸It is also consistent with some households investing in equity through levered investment intermediaries such as hedge funds and private equity, which cannot be identified in the SCF.
Table A.III: heterogeneity and persistence in the 2007-2009 SCF panel

<table>
<thead>
<tr>
<th>Row</th>
<th>Expression</th>
<th>Group (g)</th>
<th>( a )</th>
<th>( b )</th>
<th>( c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Share households</td>
<td></td>
<td>4%</td>
<td>35%</td>
<td>61%</td>
</tr>
<tr>
<td>2</td>
<td>( \sum_{i \in g} W_{2007} \ell_{2007} / \sum_i W_{2007} \ell_{2007} )</td>
<td>4%</td>
<td>21%</td>
<td>75%</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>( \sum_{i \in g} A_i^{2007} / \sum_i A_i^{2007} )</td>
<td>16%</td>
<td>63%</td>
<td>20%</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>( \sum_{i \in g} Q_i^{2007} k_i^{2007} / \sum_i A_i^{2007} )</td>
<td>1.9</td>
<td>0.6</td>
<td>1.2</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>( \sum_{i \in g} A_i^{2009} / \sum_i A_i^{2007} - \sum_{i \in g} A_i^{2007} / \sum_i A_i^{2007} )</td>
<td>-1.7%</td>
<td>-0.6%</td>
<td>+2.2%</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>( \sum_{i \in g} Q_i^{2009} k_i^{2009} / \sum_i A_i^{2009} )</td>
<td>1.8</td>
<td>0.5</td>
<td>1.2</td>
<td></td>
</tr>
</tbody>
</table>

Notes: observations are weighted by SCF sample weights. Construction of capital and bond positions implicit in business equity use \( \text{lev}_{\text{public},2007} = 1.4, \text{lev}_{\text{public},2009} = 1.3, \text{lev}_{\text{private},2007} = 1.1, \) and \( \text{lev}_{\text{private},2009} = 1.2 \) obtained using the Financial Accounts for 2007 and 2009. \( \theta \) is calibrated so that expected return heterogeneity in 2007 is as in Bach et al. (2020), and \( \epsilon \) is assumed the same in 2007 and 2009 for each household.

Row 5 reports the change in the aggregate wealth share of households in each group between 2007 and 2009. The key message is that \( a \) households experienced a decline in their wealth share over these two years, consistent with the decline in risky asset prices over the 2007-2009 period and the fact that these households were levered in such claims. The redistribution away from \( a \) households on impact of a decline in the price of capital is indeed a key mechanism at play in the quantitative model.

Row 6 reports the aggregate capital portfolio share in 2009 by group. The key message is that it is very close to its counterpart in 2007; portfolio heterogeneity across groups is quite persistent. This holds even within each group: at the individual household level, projecting the capital portfolio share in 2009 on a constant and its 2007 value implies a coefficient of 0.94 on the latter.\(^9\) This is consistent with the permanent differences in risk tolerance across households in the quantitative model.

We note that the redistribution away from \( a \) households and the persistence in capital portfolio shares holds even if we exclude all assets and liabilities involving vehicles and housing, thereby focusing on capital held in business equity alone.

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\(^9\)As portfolio shares can become very large when net worth is close to zero, we run this regression only on (the more than 97% of) households with a 2007 capital portfolio share between -10 and 10.
C Infinite horizon environment

C.1 Environment

We now outline in more detail the environment studied in section 3.

Households The unit measure of households is now organized into three groups \( i \in \{a, b, c\} \) with measures \( \{\lambda^i\} \) such that \( \sum_i \lambda^i = 1 \). Household \( i \) in group \( i(\iota) \) is comprised of a continuum of members \( j \in [0, 1] \) supplying a differentiated variety of labor, with full consumption insurance within the household. The household has Epstein-Zin preferences

\[
v_t^\iota = \left(1 - \beta\right) \left(c_t^\iota \Phi \left(\int_0^1 \bar{\ell}_t(j) dj / \bar{\ell}_t\right)\right)^{1-1/\psi} + \beta \mathbb{E}_t \left(\left(v_{t+1}^\iota\right)^{1-\gamma}\right)^{1-1/\psi} + \beta \mathbb{E}_t \left(\left(v_{t+1}^\iota\right)^{1-\gamma}\right)^{1-1/\psi},
\]

where \( \bar{\ell}_t \) denotes the household’s labor endowment. In equilibrium the representative household of each group will hold a unitary labor endowment, so that these preferences simplify to (27) as given in the main text. Assuming that the household was alive the previous period, it faces the resource constraint

\[
P_t c_t^\iota + B_t^\iota + Q_t k_t^\iota + Q_t^\bar{\ell}(\iota) \bar{\ell}_t - (1 - \tau) \int_0^1 W_t(j) \ell_t(j) dj - \int_0^1 AC_t W(j) dj + (1 + i_{\iota-1}) B_{\iota-1} + (\Pi_t + (1 - \delta) Q_t) k_{\iota-1} \exp(\varphi_t) + Q_t^\bar{\ell}(\iota) \bar{\ell}_{\iota-1} + T_t^\iota,
\]

where the cost of setting the wage for member \( j \) is (29), and \( Q_t^\bar{\ell}(\iota) \) is the price of the labor endowment for households in group \( i \). It will be convenient to define the household’s share of its group’s aggregate financial wealth inclusive of labor endowment,

\[
\mu_t^\iota = \frac{(1 + i_{\iota-1}) B_{\iota-1} + (\Pi_t + (1 - \delta) Q_t) k_{\iota-1} \exp(\varphi_t) + Q_t^\bar{\ell}(\iota) \bar{\ell}_{\iota-1}}{\int_{t' = i(\iota')} = i(\iota) \left[(1 + i_{\iota-1}) B_{\iota-1} + (\Pi_t + (1 - \delta) Q_t) k_{\iota-1} \exp(\varphi_t) + Q_t^\bar{\ell}(\iota') \bar{\ell}_{\iota-1}\right] d\iota'}.
\]

Finally, households further face the capital constraint (30). In our calibration, this constraint will (almost always) only bind for \( c \) households.

Supply-side A union represents each labor variety \( j \) across households. Each period, it chooses \( W_t(j), \ell_t(j) \) to maximize the utilitarian social welfare of union mem-

A15
bers subject to the allocation rule
\[ \ell_t^i(j) = \frac{\bar{\ell}_t^i}{\int_{\nu' = i(i')} \ell_t^{j(i')} \rho^{j(i')} \ell_t(j)}. \]

That is, within group \( i \), labor is allocated across households in proportion to their labor endowments. The labor packer combines varieties supplied by the union, earning profits each period (33) given (32). The representative producer hires \( \ell_t \) units of the labor aggregator in period \( t \) and combines it with \( k_{t-1} \exp(\varphi_t) \) units of capital rented from households. It further uses \( \left( \frac{k_t}{k_{t-1} \exp(\varphi_t)} \right)^{\chi_x} x_t \) units of the consumption good to produce \( x_t \) new capital goods, where it again takes \( k_t \) as given. It thus earns profits
\[ \Pi_t k_{t-1} \exp(\varphi_t) = P_t (z_t \ell_t)^{1-\alpha} (k_{t-1} \exp(\varphi_t))^\alpha - W_t \ell_t + Q_t x_t - P_t \left( \frac{k_t}{k_{t-1} \exp(\varphi_t)} \right)^{\chi_x} x_t. \]

Productivity follows (34).

Policy The government follows a standard Taylor rule (37) where monetary policy shocks \( m_t \) follow (38). The government sets \( \tau = -\frac{1}{\epsilon - 1} \) and the transfers
\[ T_t^i = \int_0^1 AC_t^W(j) dj + \tau \int_0^1 W_t(j) \ell_t^i(j) dj + \mu_t^i \nu^{j(i)} ((1 + i_t - 1) B_t^g - B_i^g). \]

Within group \( i \), the government rebates the proceeds from its bond market trading in proportion to households’ wealth.\(^{10}\) As described in the main text, the government further collects the wealth of dying households and endows it to newborns.

Market clearing Market clearing in goods each period is now
\[ \int_i c_t^i + \left( \frac{k_t}{k_{t-1} \exp(\varphi_t)} \right)^{\chi_x} x_t = (z_t \ell_t)^{1-\alpha} (k_{t-1} \exp(\varphi_t))^\alpha, \]
in labor is
\[ \left[ \int_0^1 \ell_t(j)^{(\epsilon-1)/\epsilon} dj \right]^{\epsilon/(\epsilon - 1)} = \ell_t, \]

\(^{10}\)We assume, however, that households do not internalize the dependence of their rebate on their wealth in their decisions, preserving the neutrality of government participation in the bond market.
in the capital rental market is
\[ \int k^t_{t-1} = k_{t-1}, \]
in the capital claims market is
\[ (1 - \delta) \int k^t_{t-1} \exp(\varphi_t) + x_t = \int k^t_t, \]
in bonds is
\[ \int B^t_t + B^b_t = 0, \]
and in the labor endowment is
\[ \int \bar{\ell}^t_t = 1, \quad \int \bar{\ell}^a_t = 1, \quad \int \bar{\ell}^b_t = 1. \]

**Equilibrium** Given initial states \{W_{-1}, \{B^a_{-1}, k^a_{-1}\}, i_{-1}, z_0, \varphi_0, p_0, m_0\} and the stochastic processes (34)-(38), the equilibrium naturally generalizes Definition 1. Since labor varieties and unions \( j \) are symmetric, \( \ell_t(j) = \ell_t \) and going forward we drop \( j \).

**C.2 Aggregation into representative households**

Within each group of households \( i \in \{a, b, c\} \), we can then prove the homogeneity of households’ policies in wealth, implying a representative household of each group.

**Proposition C.1.** Households’ optimal policies satisfy

\[ v^i_t = \mu^i_tv^i_t, \quad c^i_t = \mu^i_tc^i_t, \quad B^i_t = \mu^i_tB^i_t, \quad k^i_t = \mu^i_tk^i_t, \quad \bar{\ell}^i_t = \mu^i_t, \]

where the variables with an \( i \in \{a, b, c\} \) superscript correspond to those of a representative household endowed with aggregate group-specific wealth. Furthermore, the evolution of wealth shares is identical across households in a given group, conditional on surviving through the next period:

\[ \frac{\mu^i_{t+1}}{\mu^i_t} = \mu^{i(i)}_{t,t+1}. \quad (A.18) \]

This result follows from households’ ability to trade labor endowments with other households in the same group, the assumed labor allocation rule, the assumed lump-sum transfers, and the assumed endowments. We exclude the proof for brevity.
C.3 Re-scaled economy

The results of the prior subsection allow us to summarize the equilibrium in terms of the first-order conditions of the three representative households $i \in \{a, b, c\}$, their resource constraints, and the market clearing conditions. Scaling by the price level $P_t$ and permanent level of productivity $z_t$, we obtain a stationary transformation of the economy which we can numerically solve over the state variables

$$\left\{ \frac{k_{t-1}}{z_t \exp(\epsilon_{z_t})}, \frac{W_{t-1}}{P_t z_{t-1} \exp(\epsilon_{z_t})}, s^a_i, s^c_i, p_t, m_t \right\},$$

where $s^i_t$ denotes the financial wealth share of group $i$.

D Additional quantitative results

D.1 Impulse responses to other shocks

We now characterize the impulse responses to other shocks in section 3.

In Figure A.2, we summarize the effects of a two standard deviation increase in
productivity in both the model and counterfactual RANK economies. In the first row, the first panel demonstrates that the central bank following a standard Taylor rule will cut the nominal interest rate (in response to the price deflation induced by this shock). The second and third panels demonstrate that the expected real interest rate and the expected excess returns on capital decline following the shock. The first two panels of the second row demonstrate that redistribution drives the decline in the risk premium in our model: as in the case of a negative monetary policy shock, realized excess returns on capital are substantially positive on impact, and this raises the wealth share of high MPR \( a \) households who hold levered claims on capital. The third panel in this row demonstrates that output rises, as expected. The difference between the output responses in the model and RANK is minimal, arising from the endogenous tightening of monetary policy in the model in response to the stimulus from lower risk premia.

In Figure A.3, we summarize the effects of a two standard deviation increase in disaster risk in both the model and RANK economies. In the first row, the first panel demonstrates that the central bank following a standard Taylor rule will cut the nominal interest rate (again in response to the price deflation induced by this shock). The second and third panels demonstrate that the expected real interest rate and the expected excess returns on capital decline following the shock. The first two panels of the second row demonstrate that redistribution drives the decline in the risk premium in our model: as in the case of a negative monetary policy shock, realized excess returns on capital are substantially positive on impact, and this raises the wealth share of high MPR \( a \) households who hold levered claims on capital. The third panel in this row demonstrates that output rises, as expected. The difference between the output responses in the model and RANK is minimal, arising from the endogenous tightening of monetary policy in the model in response to the stimulus from lower risk premia.
shock, reflecting the increase in precautionary saving). The second panel demonstrates that the expected real interest rate declines following the shock. The third panel demonstrates that expected excess returns rise following the shock, reflecting both the persistent increase in the quantity of risk and transitory redistribution of wealth away from the relatively risk tolerant. The latter is absent in the RANK economy. The first two panels in the second row rationalize the dynamics of the wealth distribution in the model with heterogeneity: realized excess returns on capital are negative on impact and positive in the quarters which follow, so a households lose in relative wealth on impact but then recoup these losses. The final panel demonstrates that output falls on impact of the increase in uncertainty — despite the fact that the households’ intertemporal elasticity of substitution is less than one. This is consistent with the effects of uncertainty shocks in New Keynesian models and it is for this reason that the model-implied equity premium is countercyclical.

D.2 Alternative calibration of wealth and leverage

We now present the results in an alternative calibration in which a households hold a smaller fraction of the economy’s wealth but are more levered and thus have a higher MPR. Because these forces offset in determining the effects of a monetary policy shock on the risk premium, the net effects are comparable to our baseline results.

Formally, we change the cutoff in the capital portfolio share between group a and b agents. Figure A.4 summarizes the fraction of labor income, fraction of wealth, and ratio of aggregate capital to wealth for groups a and b as we vary the cutoff.
(the moments for c households are unaffected). As is evident, a higher cutoff means that group a households are more levered, but conversely have a lower share of total wealth. Our baseline calibration employed a cutoff at the 90th percentile.

In this subsection we instead employ a cutoff at the 99th percentile. The targeted moments are the right-most points in Figure A.4: a agents are now only 0.4% of agents, earn even less labor income than that, own 2% of wealth, and have a capital portfolio share of 4.4; b agents meanwhile earn 17% of labor income, own 75% of wealth, and have a capital portfolio share of 0.8. These targets mean that we can interpret our a agents as capturing highly levered sectors such as security broker-dealers and hedge funds, which issue nominal claims to households (the asset-rich b and asset-poor c) to hold capital. Indeed, the estimates in He and Krishnamurthy (2013) imply that these sectors own 3% of U.S. wealth and have leverage of 4.5, comparable to the 2% wealth share and capital portfolio share of 4.4 obtained here.

Given these new targets, we maintain the same externally set parameters as in Table IV except that we set $\lambda^a = 0.4\%$; $\lambda^b = 39.6\%$; $\phi^a = 0$ (so a agents do not supply any labor, consistent with their interpretation as levered institutions); $\phi^b = 17\%/\lambda^b$; and $\nu^a = 2\%/\lambda^a$. We further set the death probability to $\xi = 4\%$ so that the solution is numerically stable given a agents’ high targeted capital portfolio share.

Table A.IV: targeted moments and calibrated parameters, alternative calibration

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
<th>Moment</th>
<th>Target</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma^z$ std. dev. prod.</td>
<td>0.55%</td>
<td>$\sigma(\Delta \log c)$</td>
<td>0.5%</td>
<td>0.6%</td>
</tr>
<tr>
<td>$\chi^x$ capital adj cost</td>
<td>3.5</td>
<td>$\sigma(\Delta \log x)$</td>
<td>2.1%</td>
<td>2.0%</td>
</tr>
<tr>
<td>$\beta$ discount factor</td>
<td>0.984</td>
<td>$4r_{+1}$</td>
<td>1.3%</td>
<td>1.1%</td>
</tr>
<tr>
<td>$\gamma^b$ RRA b</td>
<td>21</td>
<td>$4[r_{+1}^c - r_{+1}]$</td>
<td>7.3%</td>
<td>7.3%</td>
</tr>
<tr>
<td>$\sigma^p$ std. dev. log dis. prob.</td>
<td>0.47</td>
<td>$\sigma(4E_{r_{+1}})$</td>
<td>2.2%</td>
<td>2.2%</td>
</tr>
<tr>
<td>$\rho^p$ persist. log dis. prob.</td>
<td>0.8</td>
<td>$\rho(4E_{r_{+1}})$</td>
<td>0.79</td>
<td>0.74</td>
</tr>
<tr>
<td>$\gamma^a$ RRA a</td>
<td>2.5</td>
<td>$qk^a/a^a$</td>
<td>4.4</td>
<td>4.8</td>
</tr>
<tr>
<td>$k$ lower bound $k^i$</td>
<td>10</td>
<td>$qk^c/a^c$</td>
<td>1.1</td>
<td>1.0</td>
</tr>
<tr>
<td>$\xi^a$ newborn endowment a</td>
<td>0%</td>
<td>$\lambda^a a^a / \sum_i \lambda^i a^i$</td>
<td>2%</td>
<td>3%</td>
</tr>
<tr>
<td>$\xi^c$ newborn endowment c</td>
<td>0.9%</td>
<td>$\lambda^c a^c / \sum_i \lambda^i a^i$</td>
<td>23%</td>
<td>24%</td>
</tr>
<tr>
<td>$b^g$ real value govt bonds</td>
<td>-2.7</td>
<td>$-\sum_i \lambda^i b^i / \sum_i \lambda^i a^i$</td>
<td>-10%</td>
<td>-10%</td>
</tr>
</tbody>
</table>

Notes: see notes accompanying Table V in main text. The disutilities of labor $\{\bar{\theta}^a, \bar{\theta}^b, \bar{\theta}^c\}$ are jointly set to $\{0.00, 2.47, 0.44\}$ so that the average labor wedge is zero for each group and $\ell = 1$.
Figure A.5 compares the effects of a monetary policy shock in this environment to a counterfactual RANK economy in which $\gamma^i = 18$ for all groups. Table A.V presents the Campbell-Shiller decomposition of the stock market return following the shock. As is evident, it remains the case that a substantial share of the stock market return is due to news about future excess returns. And again, the impact effect on output is roughly 1.3 times larger than in the RANK economy.
<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
<th>Moment</th>
<th>Target</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma^z )</td>
<td>0.55%</td>
<td>( \sigma(\Delta \log c) )</td>
<td>0.5%</td>
<td>0.5%</td>
</tr>
<tr>
<td>( \chi^x )</td>
<td>3.5</td>
<td>( \sigma(\Delta \log x) )</td>
<td>2.1%</td>
<td>2.1%</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.98</td>
<td>4r_{t+1}</td>
<td>1.3%</td>
<td>1.4%</td>
</tr>
<tr>
<td>( \gamma^a = \gamma^b = \gamma^c )</td>
<td>11</td>
<td>4 \left[ r_{t+1}^a - r_{t+1} \right]</td>
<td>7.3%</td>
<td>7.4%</td>
</tr>
<tr>
<td>( \sigma^p )</td>
<td>1.20</td>
<td>( \sigma(4\bar{E}r_{t+1}) )</td>
<td>2.2%</td>
<td>2.2%</td>
</tr>
<tr>
<td>( \rho^p )</td>
<td>0.8</td>
<td>( \rho(4\bar{E}r_{t+1}) )</td>
<td>0.79</td>
<td>0.70</td>
</tr>
<tr>
<td>( \eta^b )</td>
<td>0.001</td>
<td>( qk^a/a^a )</td>
<td>2.0</td>
<td>2.5</td>
</tr>
<tr>
<td>( k )</td>
<td>10</td>
<td>( qk^c/a^c )</td>
<td>1.1</td>
<td>0.9</td>
</tr>
<tr>
<td>( \xi a_{sa} )</td>
<td>-0.02%</td>
<td>( \lambda^a a^a/\sum \lambda^i a^i )</td>
<td>18%</td>
<td>22%</td>
</tr>
<tr>
<td>( \xi a_{sc} )</td>
<td>-0.15%</td>
<td>( \lambda^c a^c/\sum \lambda^i a^i )</td>
<td>23%</td>
<td>24%</td>
</tr>
<tr>
<td>( b^g )</td>
<td>-2.6</td>
<td>( -\sum \lambda^i b^i/\sum \lambda^i a^i )</td>
<td>-10%</td>
<td>-10%</td>
</tr>
</tbody>
</table>

Table A.VI: targeted moments and calibrated parameters, idiosyncratic risk

Notes: see notes accompanying Table V in main text. The disutilities of labor \( \{ \bar{\theta}^a, \bar{\theta}^b, \bar{\theta}^c \} \) are jointly set to \( \{0.64, 2.89, 0.43\} \) so that the average labor wedge is zero for each group and \( \ell = 1 \).

D.3 Alternative microfoundation of portfolio heterogeneity

We now present the results in an alternative environment which microfounds differences in portfolios and MPRs without differences in risk aversion. Instead, we assume households experience idiosyncratic returns on capital, and the volatility of these returns differs across groups, consistent with one of the alternative forms of heterogeneity considered in section 2.5. Our quantitative results are again robust.

Formally, we assume that households’ choice of capital \( k^i_t \) in period \( t \) is subject to an idiosyncratic quality shock at \( t+1 \), turning into \( \epsilon^i_{t+1} k^i_t \) units of capital which can be rented in the spot market to firms. The quality shock is an \textit{iid} shock from a lognormal distribution, the variance of which is the group-specific parameter \( \eta^i \):

\[
\log \epsilon^i_{t+1} \sim N \left( -\frac{1}{2} \eta^i, \eta^i \right).
\]

In equilibrium, this affects households’ portfolio choice, but since the quality shock has a mean value of one the representative household still holds \( k^i_t \) units of capital and none of the supply-side conditions or aggregate resource constraints are affected.\(^{11}\)

\(^{11}\)As described earlier in this appendix, to obtain aggregation within groups we allow households
Figure A.6: responses to negative monetary policy shock, idiosyncratic risk

Notes: see notes accompanying Figure 2 in main text.

<table>
<thead>
<tr>
<th>% Real stock return</th>
<th>Data [90% CI]</th>
<th>Model</th>
<th>RANK</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dividend growth news</td>
<td>33% [-13%,71%]</td>
<td>51%</td>
<td>65%</td>
</tr>
<tr>
<td>−Future real rate news</td>
<td>8% [-6%,21%]</td>
<td>20%</td>
<td>35%</td>
</tr>
<tr>
<td>−Future excess return news</td>
<td>59% [19%,108%]</td>
<td>28%</td>
<td>1%</td>
</tr>
</tbody>
</table>

Table A.VII: decomposition after monetary shock, idiosyncratic risk

Notes: see notes accompanying Table A.V in main text.

To rationalize their high leverage, a households are now calibrated to be those with the smallest volatility of idiosyncratic returns; equivalently, they have the highest risk-adjusted returns. Formally, we maintain the same externally set parameters as in Table IV and set $\eta^a = 0$. We then re-calibrate the parameters in Table A.VI assuming that households share the same level of risk aversion $\gamma$ calibrated to match the level of the equity premium, and calibrating $\eta^b$ and $k$ to match households’ capital portfolio shares. Taken together, this environment builds on a large literature to trade a claim on a labor endowment with other households in the same group. So that the valuation of the labor endowment behaves similarly as in the model with heterogeneity in risk aversion, we also assume the idiosyncratic quality shock applies to the labor endowment claim.
on entrepreneurship in macroeconomic models emphasizing idiosyncratic risk and entrepreneurs as those with relatively good investment ideas.

Figure A.6 compares the effects of a monetary policy shock in this environment to a counterfactual RANK economy in which $\eta^i$ is identical across groups. Table A.VII presents the Campbell-Shiller decomposition of the stock market return following the shock. It remains the case that a substantial share of the stock market return is due to news about future excess returns. And again, the impact effect on output is roughly 1.3 times larger than in the RANK economy.

References


