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Maximize Utility subject to R≤1: A Simple Price-Theory Approach to Covid-19 Lockdown and Reopening Policy

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Maximize Utility subject to $R \leq 1$: A Simple Price-Theory Approach to Covid-19 Lockdown and Reopening Policy

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Abstract

This paper presents a simple price-theory approach to Covid-19 lockdown and reopening policy. The key idea is to conceptualize $R \leq 1$ as a constraint, allowing traditional economic and societal goals to be the policy objective, all within a simple static optimization framework. This approach yields two main insights. First, the $R \leq 1$ constraint imposes a *disease-transmission budget* on society. Society should optimally spend this budget on the activities with the highest ratio of utility to disease-transmission risk, dropping activities with too low a ratio of utility to risk. Second, masks, tests, and other simple interventions increase activities' utility-to-risk ratios, and hence *expand how much activity society can engage in and utility society can achieve while staying within the $R \leq 1$ budget.* A simple numerical example, based on estimates from the medical literature for R_0 and the efficacy of facemasks and complementary measures, suggests the potential gains are enormous. Overall, the formulation provides economics language for a policy middle ground between society-wide lockdown and ignore-the-virus, and a new infectious threat response paradigm alongside "eradicate" and "minimize".

*An earlier version of the ideas in this paper circulated on April 1, 2020 under the title "$R < 1$ as an Economic Constraint: Can We 'Expand the Frontier' in the Fight Against Covid-19?" The goal of the April 1st draft was to influence thinking about Covid-19 lockdown and reopening policy in real time. This draft has more formal detail and discussion of related literature in the hopes of clarifying the argument and building academic knowledge that may prove useful in the future. The April 1 draft and conference presentations on this work from April and May 2020 are available on the author's website. I thank Jason Abaluck, Nikhil Agarwal, Mohammad Akbarpour, Matteo Aquilina, Emma Berndt, Judy Chevalier, John Cochrane, Michael Droste, Amy Finkelstein, Joshua Gans, Austan Goolsbee, Sam Hanson, Ralph Koijen, Scott Kominers, Shengwu Li, Yueran Ma, Neale Mahoney, Emi Nakamura, Emily Oster, James Stock, Adi Sunderam, Chad Syverson, Sarah Taubman and Heidi Williams for helpful discussions and feedback. I thank Ethan Che, Jiahao Chen, Jia Wan, and Zizhe Xia for research assistance. I thank the University of Chicago Booth School of Business for research support. I am not a medical expert but rather am an economist with expertise in market design, an engineering-oriented branch of microeconomics. I try to be explicit throughout this paper where I am using my own non-expert understanding of medical facts. Contact: eric.budish@chicagobooth.edu.

†This paper is dedicated to my partners in lockdown and in life: Emma, Nathan and Jacob.

1 Introduction

This paper suggests that it may be useful to conceptualize $R \leq 1$ as a policy constraint, allowing traditional economic and societal goals to then be the objective function in a simple static optimization problem. Informally,

$$\begin{aligned} &\text{maximize Social Welfare} \\ &\quad \text{subject to} \\ &\text{Standard Economic Constraints} \\ &R \leq 1\text{: Reduce the Covid-19 Average Transmission Rate to Below 1} \end{aligned} \tag{1}$$

As is widely understood, diseases with average transmission rates above 1 eventually infect huge numbers of people, whereas diseases with average transmission rates below 1 gradually dissipate. An $R \leq 1$ constraint is therefore a simple way to incorporate the dynamics of the health problem society faces — in particular, the risk of exponential growth of the number of infections and deaths — into familiar static optimization.

This simple approach perhaps bridges the usual economic approach and the health policy perspective. Health policy experts, especially in the initial response to the Covid-19 crisis in Feb-March 2020, often seemed to have a constrained optimization problem in the back of their heads like

$$\begin{aligned} &\text{minimize Spread of Covid-19} \\ &\quad \text{subject to} \\ &\text{Keep Society Functioning} \end{aligned} \tag{2}$$

Superficially, formulation (1) looks very different from (2). But, because of the $R \leq 1$ constraint, (1) will reasonably *approximate* the medical objective in (2). At the same time, by having traditional economic and societal goals as the objective function, (1) leads to very different policy implications. In particular, formulation (2) inevitably leads to a large societal lockdown — it does not allow for *any* unnecessary risk. Whereas, formulation (1) seeks to allow as much socially-valuable activity as possible subject to $R \leq 1$.

This approach to the problem yields two main insights. First, the $R \leq 1$ constraint imposes a *disease-transmission budget* on society. This paper's simple model suggests that society should optimally spend this budget by ranking activities by their *ratio of utility to disease-transmission risk* — in effect, the activity's social welfare "bang for buck" per unit of virus risk. This part of the analysis is presented in Section 3.

Second, facemasks, Romer mass testing, six feet of social distance, and other related ideas can be conceptualized as reducing the transmission risk of activities — their "cost" in terms of the $R \leq 1$ risk budget — at relatively low cost to utility as compared to lockdown. Such interventions thus *expand the frontier* of how much social welfare can be achieved while keeping within the disease constraint; they improve society's bang for buck per unit of virus risk. Section 4 models these ideas formally. Section 5 presents a detailed numerical example, using estimates from the medical literature for R_0 and the efficacy of facemasks and complementary measures, and uniform distributions for utility and risk.

The numerical example highlights just how valuable simple interventions can be. If $R_0 = 2.5$ and utility and risk are uniformly distributed, then without simple interventions society has to drop fully 45% of activities, which together constitute 27% of pre-virus utility and 60% of risk, to get to $R \leq 1$. This is a severe societal lockdown. Whereas if simple interventions can reduce transmission risk by 50% — which is in line with many estimates for the efficacy of facemasks and complementary measures, and is likely conservative if better testing is considered as well — then society can maintain nearly 90% of its pre-virus activities (dropping only those with the very poorest utility-to-risk ratios), while still achieving $R \leq 1$.

Remark on Related Literature: Static vs. Dynamic Models. Whereas this paper takes a static approach treating traditional economic goals as the objective and $R \leq 1$ as a constraint, there is now a much larger and more influential literature in economics that builds dynamic models with both traditional economic goals and health as objectives, incorporating disease dynamics as a contraint. Early examples include Alvarez, Argente and Lippi (2020), Eichenbaum, Rebelo and Trabandt (2020), Farboodi, Jarosch and Shimer (2020), Jones, Philippon and Venkateswaran (2020), and Acemoglu et al. (2020).

At some level a dynamic approach is "more correct". However, to get tractability, the dynamic models all make important simplifying assumptions, and for this reason I think static and dynamic approaches may be complementary. To be precise, a dynamic model seems necessary for analyzing time-varying lockdown policies or optimal time-varying paths for R, whereas a static approach may allow for a richer analysis of the optimal policy to achieve a given level of R, with $R \leq 1$ the centrally important case. The specific modeling elements incorporated in my analysis that are abstracted from in the dynamic models cited above are (i) heterogeneous activities that vary in their ratio of utility to risk, and (ii) "masks" that can reduce activities' riskiness at relatively low cost to utility, and hence can increase the utility-to-risk ratio.[1]

[1] By contrast, for example, in Alvarez, Argente and Lippi (2020) agents are either (i) "in lockdown" and not

I would also like to underscore the following point. If a combination of (i) simple low-cost interventions, such as masks, tests, social distance, etc., and (ii) dropping activities with the poorest utility-to-risk ratios, such as crowded bars, large indoor gatherings, etc., allows society to get to $R \leq 1$ relatively cheaply — where we understand "cheaply" in relation to either the massive societal and economic costs of lockdown, or the massive mortality cost of the virus spreading unchecked — then the optimal dynamics become a lot less interesting and important (in a good way). Roughly speaking: just get to $R \leq 1$ and stay there until a vaccine is available.[2]

Remark on the Context for this Draft and the April 1st Draft. I circulated an initial draft of this paper on April 1st, 2020, in the midst of the national lockdown. My goal at the time was to influence thinking about reopening policy in two related ways.

First, to focus attention on $R \leq 1$, as opposed to "minimize", as the appropriate policy goal from a health perspective. If eradication is infeasible, a policy that avoids exponential growth approximates the goal of avoiding as many infections as possible, while allowing for other policy considerations besides the virus (economic, societal, and other aspects of health). A lot of the public-health messaging at the time ("Stay Home. Stay Safe.") suggested that society's *only* focus should be containing the virus. I suspect history will look back at that messaging as a mistake, draining public goodwill and compliance energy. Again, if eradication were feasible, this would be a different story.

Second, to try to encourage an engineering mindset towards achieving $R \leq 1$ at least economic cost. Lockdown is not a very creative policy. Given our knowledge of the way the virus spreads, there are many more creative ways to limit risk while allowing people to live some approximation of their pre-lockdown lives. This point also complements the first one: once the goal is not to minimize but rather to prevent exponential growth, interventions need not be perfect to be valuable.[3]

Whether the April draft succeeded in some modest way is hard to know. Clearly, there has been a lot more public attention on R and on masks and social distance as an alternative to

productive, or (ii) "not in lockdown", produce homogeneous output w, and have just as many contacts and just as much infectiousness as in a society that is completely unaware of the virus. This setup thus assumes that the only way to achieve $R \leq 1$ is with a widescale lockdown.

[2]For research on the economics of accelerating vaccine development, please see Athey et al. (2020). If the stock of infections is large, the social planner may optimally wish to invest in reducing the stock of infections first, with R significantly less than 1, before then transitioning to $R \leq 1$; see further discussion in Section 2 and Section 6.1. If the stock of infections is small and the time horizon until a vaccine is short, then there may not be much quantitative difference between $R \leq 1$ and R slightly larger than 1; see Figure 1 for intuition.

[3]Approximations, non-standard constraints, and an engineering attitude are all common in my home field of economics, market design. See Roth's (2002) famous manifesto on "The Economist as Engineer", as well as my own more modest methodological contribution on the do's and don'ts of non-standard objectives and constraints, Budish (2012).

lockdown. At the same time, I think it is safe to say that we have not managed as a society to agree that our goal is to maximize societal well-being subject to keeping the virus contained.[4]

The purpose of the present draft is to present the April draft's argument more completely, with additional formal detail and literature discussion. It seems possible that this will be a useful input to policy thinking in the current pandemic, or possibly it will be useful for future pandemics. At the very least, I feel it is important to clarify to other academics what my argument was.

2 Why $R \leq 1$?

Importance of $R \leq 1$ in the SIR Model. Figure 1 illustrates the importance of the $R \leq 1$ threshold in the standard SIR epidemiological model.[5] Each line represents a different number of initial infections, ranging from 1,000 to 10 million. The horizontal axis of each panel varies R_0 ("R-nought"), the initial average transmission rate.[6] More precisely, what varies along the horizontal axis is the SIR model's β parameter (which represents the rate of infectiousness), with the γ parameter (where $\frac{1}{\gamma}$ represents the duration of infectiousness) held fixed, and with R_0 defined according to $R_0 \equiv \frac{\beta}{\gamma}$. The vertical axis then depicts the *cumulative* number of infections and deaths over a 12 month period, using a relatively conservative infection fatality rate of 0.7%.[7] That is, rather than focus on the dynamic path of the virus over time, the figure just plots the total number of infections and deaths.

Focus first on the middle and right of the figure. What this shows is that if the transmission rate is high enough — roughly, $R_0 = 1.5$ or higher — there will be in excess of 200 million infections and 1 million deaths in the United States, essentially irrespective of the initial seed of infections. This is because of exponential growth. Now look right around $R_0 \approx 1$. As is widely understood, $R_0 = 1$ is the critical threshold in the SIR model, above which the number of

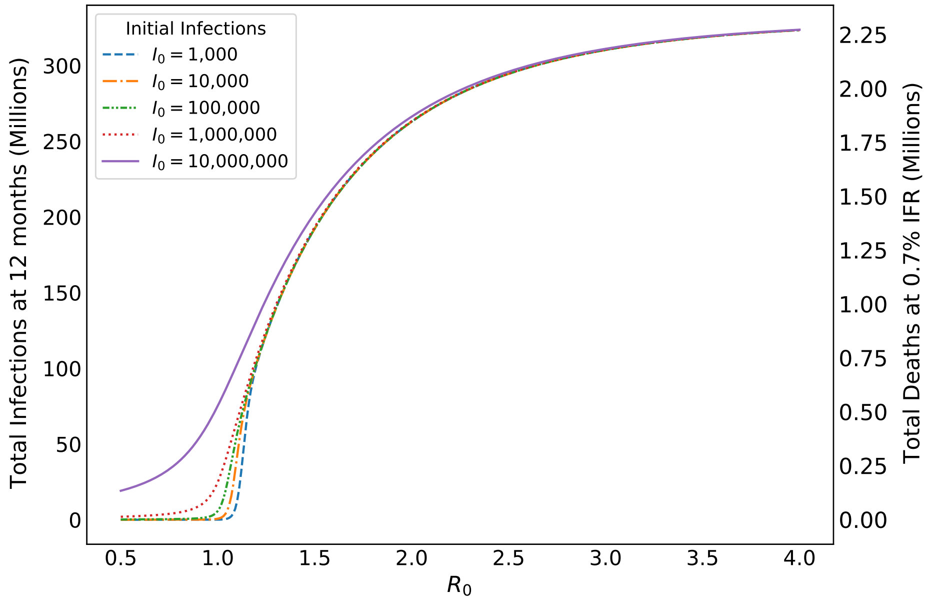
[4] To say the least.

[5] See Atkeson (2020) and Stock (2020) for helpful introductions to the SIR model written for economists. See Avery et al. (2020) for an excellent survey that discusses additional modeling approaches and open questions.

[6] A note on notation: throughout this paper I will mostly use the notation "R", without any subscripts or arguments, to refer to the average transmission rate of Covid-19 at a moment in time as a function of any interventions, behavioral changes, or accumulating herd immunity. This is sometimes called R_e (e for "effective"), R_t (with t for evolution over time), or $\hat{R}$. I reserve the notation R_0 ("R-nought") to refer specifically to either (i) the initial transmission rate of Covid-19, or (ii) output from the standard SIR model with initial conditions $R_0 \equiv \frac{\beta}{\gamma}$.

[7] The 0.7% figure is based on the CDC's Pandemic Planning Scenarios Current Best Estimate, most recently updated Sept 10, 2020 (Centers for Disease Control and Prevention, 2020). The CDC estimate provides IFR estimates by age group which I turn into an overall IFR using Census Bureau data for population by age group. The CDC's optimistic scenario generates an IFR of 0.4% and its pessimistic scenario generates an IFR of 1.3%. The influential Imperial College modeling team used an IFR of 0.9% in Ferguson et al. (2020) and has estimated the IFR to be in the range 0.9%-1.26% for Europe (Flaxman et al., 2020). Economics papers such as Fernández-Villaverde and Jones (2020) and Stock (2020) emphasize the econometric difficulty of identifying the true number of underlying infections and hence the IFR.

Figure 1: **Cumulative Infections and Deaths as a Function of R_0 in the Standard SIR Model (United States, 12 months)**



Note: Output is based on the standard SIR model. Each line depicts a different initial infection seed. The γ parameter is fixed throughout at $\frac{1}{5}$, which represents a duration of infectiousness of 5 days. (The figure is similar if other reasonable values of γ are used instead). The β parameter, which represents the rate of infectiousness, is varied such that $R_0 = \frac{\beta}{\gamma}$ is the value depicted along the horizontal axis. The vertical axis depicts the cumulative number of infections and deaths in the United States over a 12-month period as a function of $R_0 = \frac{\beta}{\gamma}$, based on an infection fatality rate of 0.7% (per CDC estimates) and a population of 330 million.

cumulative infections starts to grow quickly. For the bottom 3 lines — initial seeds of 1,000 to 100,000, which may represent the stock of infections in the United States as of late February or early-mid March 2020[8] — if $R_0 < 1$ then the cumulative number of infections is hard to discern from zero on the figure, whereas even by $R_0 = 1.2$ or so there will be about 100 million infections and over half a million deaths. If the initial seed is larger, such as 1 million or 10 million — which may represent an intervention that occurs "late", after a sustained period of exponential growth — then there is more significant of a difference between $R_0 = 1$ and R_0 meaningfully less than 1, but $R_0 \approx 1$ remains a critical point beyond which cumulative infections grow even more quickly. I will return to this scenario of high stock of infections later in this section and then in more detail in Section 6.1.

Is $R \leq 1$ Possible? In a word, yes. Many countries around the world, and many US states, have achieved average transmission rates $R \leq 1$ for sustained periods of time.

[8]The first confirmed case of community spread in the United States was announced on Feb 27th. In the first week of March, there were about 250 confirmed cases. In the week leading to March 15th, there were about 3000 confirmed cases. It is widely known that actual cases meaningfully exceed reported cases, especially early in the crisis when testing was especially poor. See Stock (2020) and Stock et al. (2020).

Most estimates for the initial average transmission rate R_0 of Covid-19 — in a society that is not even aware that the virus is spreading, let alone engaged in policy interventions — are between $2.0 - 4.0$. The CDC's current best estimate is 2.5 and scenario range is $2.0 - 4.0$, while the influential Imperial College study (Ferguson et al., 2020), which reportedly influenced lockdown policy decisions, used a baseline R_0 of 2.4 and a range of $2.0 - 2.6$.[9] This initial transmission rate depends on both the properties of the virus (e.g., how is it transmitted) and behavior in the society that the virus is embedded in (e.g., how often do people behave in ways that risk transmission). For comparison, the initial transmission rate R_0 of standard flu is around 1.5, whereas for mumps and measles it is greater than 10.

The subsequent transmission rate R can be lower than the initial transmission rate R_0 for two basic reasons. First, even with no change in societal behavior, R declines over time from the initial R_0 as more and more people have already been infected: this is what generates the well-known hump-shaped dynamic disease curves in the SIR model, made famous by the "flatten the curve" graphic, where there is a phase of exponential growth and then at some point so much of society has already been infected that the transmission rate transitions to $R \leq 1$. This decline is known as herd immunity. However, given Covid-19 estimated mortality rates, achieving $R \leq 1$ via herd immunity would come at considerable cost in lives lost.[10]

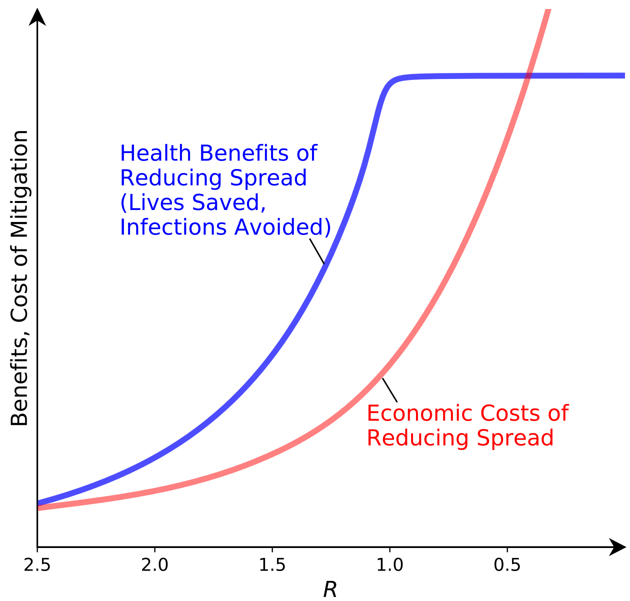
Second, R can be lower than the initial R_0 because of changes in behavior and policy interventions. This includes everything from severe lockdowns, to more targeted activity bans (e.g., large indoor gatherings), to relatively simple interventions such as masks, protective gloves, hand-washing, stay-home-if-sick, and six feet of social distance (whether mandated or encouraged), to more widespread testing and contact tracing. In the notation of the SIR model, these all either reduce the infectiousness rate β or the infectiousness duration $\frac{1}{\gamma}$. Intuitively, if the initial R_0 is around 2.5, then if these kinds of behavior changes and policy interventions can reduce the spread by around 60% (i.e., $\frac{2.5-1.0}{2.5} = 0.6$), then we can achieve $R \leq 1$. In the pessimistic scenario where R_0 is around 4.0, a 75% reduction in spread is required to achieve $R \leq 1$.

As emphasized in the April 1st draft of this paper, we know a lot about how Covid-19 spreads (and we know even more now). It therefore does not seem crazy to think that we can engineer a 60-

[9] The CDC's Pandemic Planning Scenarios webpage provides citations to several studies in the medical literature that inform their current best estimate of 2.5 as well as their scenario range of 2.0 to 4.0. The key difficulty in estimating R_0 is that the data on the time-series of infections are poor, especially early in the pandemic when testing was highly variable (Stock, 2020). Fernández-Villaverde and Jones (2020) provide R_0 estimates across a wide range of cities and countries using mortality data; their highest point estimate is 2.7 for New York City, and their estimate for the United States overall is 2.02 (Table 1).

[10] For example, if $R_0 = 2.5$ and the infection fatality rate is 0.7%, then herd immunity in the United States would occur at on the order of 200 million infections, corresponding to 1.4 million deaths. Even if R_0 is just 1.3, herd immunity would require on the order of 75 million infections and about 500,000 deaths. The "flatten the curve" graphic seems to have implicitly viewed meaningful reduction in R as achievable but $R \leq 1$ as impossible.

Figure 2: **Why $R \leq 1$ May be Optimal Policy: Basic Price-Theory Intuition**



Note: The blue line depicts the same information as the $I_0 = 100,000$ case of Figure 1, but with both axes flipped as described in the text. The red line depicts a convex cost curve; as emphasized in the text, the convex shape of the cost curve is microfounded in Sections 3 and 4 but its rate of change and its location relative to the blue line are both unknown. Both curves are depicted under the assumption that R_0 without any interventions or behavior changes is 2.5, per the CDC's current best estimate.

75% reduction in transmission without a severe societal lockdown. Several estimates suggest that facemasks and complementary measures alone could reduce transmission rates by over 50%. Paul Romer has shown that widespread random testing could reduce the transmission rate dramatically as well. See detailed discussion of both masks and tests in Section 4.3.

This paper's model will allow for a combination of (i) simple interventions such as masks and tests, and (ii) dropping activities with especially poor utility-to-risk profiles, to achieve the risk reduction necessary to achieve $R \leq 1$.

Is $R \leq 1$ Optimal? Price-Theory Intuition. Figure 2 depicts a simple graphical illustration of why $R \leq 1$ may be an appropriate target for economic policy. The blue line, labeled "Health Benefits of Reducing Spread", depicts the same information as in Figure 1 but with *both axes flipped*: on the horizontal axis, further to the right now means *lower* R as opposed to higher, and on the vertical axis, higher now represents the number of people *not* infected or dead as opposed to the number infected or dead. The red line, labeled "Economic Costs of Reducing Spread", represents the economic cost of reducing the spread of the virus, e.g., by reducing economic activity or utilizing facemasks. The theory in Sections 3 and 4 will microfound that this cost curve is increasing and convex, but I emphasize that both the level of curvature, and the level of the cost curve relative to the benefits curve, are both unknown.[11]

Optimal policy maximizes the difference of benefits less costs. The reason that $R \leq 1$ may be an optimal policy target is that the benefits curve has a "kink" at $R = 1$ — the health benefits of lowering R increase at an increasing rate until $R = 1$, at which point the curve not only becomes concave but essentially flat.[12] Therefore, a policy that targets $R \leq 1$ reaps almost all of the health benefits of mitigation — and, particularly the steeply increasing part as R approaches 1 from above, representing the gain from avoiding exponential growth — without incurring further, increasing, economic mitigation costs to go even further into the concave and essentially flat part of the health benefits curve.

Is $R \leq 1$ enough? If the initial stock of infections is high, then there will be a quantitatively large difference in the number of ultimate infections between the case of R just less than 1 versus R significantly less than 1. For example, as depicted in Figure 1, in the case of an initial stock of 1 million infections R of 0.99 leads to around 22 million cumulative infections, whereas R of 0.5 leads to around 2 million cumulative infections.

Thus, if the stock of infections at the time of policy intervention is sufficiently high — which represents a policy intervention that is "late", after a meaningful period of exponential growth — then it may be socially optimal to aim for R meaningfully less than 1, or to adopt a dynamic policy in which R is at first meaningfully less than 1, and then gradually constraints are relaxed.

Additionally, there is uncertainty about how various interventions and behavior changes affect R. This uncertainty can also be a reason to at first aim for R meaningfully less than 1, and then gradually relax.

In either case, this paper's simple price-theory formulation remains useful but might need to be implemented with R constraints that vary over time; i.e., initially with $R \leq x$ for some $x < 1$, and then transitioning to a steady state of $R \leq 1$. This dynamic thinking about R corresponds to the idea of "The Hammer and the Dance" in an influential Medium post of Pueyo (2020), and also can be helpful for interpreting various reopening "roadmaps" such as the Gottlieb et al. (2020) "Road Map to Reopening".

We will return to this issue in Section 6.1.

[11]Section 6.1 will discuss evidence in Goolsbee and Syverson (2020) and behavioral SIR models that suggest that the economic cost of reducing spread is potentially *decreasing* in the region to the left of $R = 1.0$, because of the economic damage caused by fear of contracting the virus if the stock of infections grows large, which will occur if $R > 1$. These points only enhance the case for $R \leq 1$ as an optimal policy target.

[12]The figure depicts the blue benefits curve for the case of $I_0 = 100,000$. The kink is even more stark for the cases of $I_0 = 10,000$, and $I_0 = 1,000$. The transition from the convex part of the curve to the concave part is more gradual for the case of $I_0 = 1$ million and especially $I_0 = 10$ million, i.e., for a very large current stock of infections.

Is $R \leq 1$ too much? The emphasis on $R \leq 1$ implicitly assumes that the mortality rate of the virus and disease burden for non-lethal cases are high relative to the costs of reducing the virus's spread. This assumption could be false, either overall or for a meaningful subset of the population (e.g., relatively young and without underlying conditions).

If this assumption is false for the population overall then the ideas in this paper are not very useful. The case in which this assumption is true for some sub-populations but false for others (along the lines of Acemoglu et al. (2020)) will be discussed in detail in Section 6.2.

3 Initial Model (without Simple Interventions)

Society chooses a vector of activities $x \in X = [0,1]^n$. Each activity i has traditional economic benefits and costs, denoted v_i and c_i, and a disease-transmission risk denoted r_i. Initially, the v_i's and c_i's ignore the existence of the virus; that is, v_i and c_i represent activity i's benefits and costs in the world pre-coronavirus, with $v_i > c_i$ for all i. The disease-transmission risk r_i represents the activity's expected contribution to transmission of the coronavirus. For simplicity, benefits, costs, and risk are each linear in activities. Formally, activity vector $x \in X$ yields traditional economic benefits minus costs of $\sum_i x_i(v_i - c_i)$, and an effective reproduction rate of the virus of $\sum_i x_i r_i$.[13]

3.1 Three Approaches to the Problem

Myopic Utilitarian Objective ("Ignore the Virus"). A society that is unaware of or ignores the virus solves the program

$$\max_{x \in X} \sum_{i=1}^{n} x_i(v_i - c_i) \tag{3}$$

Society thus fully engages in all activities. This yields myopic utility (i.e., ignoring the growth of the virus) of $\sum_i (v_i - c_i)$ and a virus reproduction rate of $\sum_i r_i$. Define

$$U_0 \equiv \sum_i (v_i - c_i)$$
$$R_0 \equiv \sum_i r_i$$

as the utility level and reproduction rate in a society that fully engages in its typical range of activities. U_0 thus represents the level of social welfare in pre-virus society and R_0 thus represents

[13] A simple way to incorporate diminishing returns into the model is to have activities come in multiple units with decreasing $v - c$ and/or increasing r over units. The April 1st draft of this paper had more flexible concave benefits and convex costs, but this additional generality does not add much insight and complicates notation and exposition.

the reproduction rate in a society that is both fully open and that does not take even the simplest virus precautions.

Pure Medical Objective ("Minimize the Virus"). A society whose only goal is to minimize the spread of the virus solves the program

$$\min_{x \in X} \sum_{i=1}^{n} x_i r_i \tag{4}$$

Society thus engages only in activities with zero disease-transmission risk, i.e., with $r_i = 0$. Program (4) could be augmented to capture the idea that there is a minimal required set of essential activities, by adding a constraint $x \geq \underline{x}$, where $\underline{x}$ denotes this societal minimum set of activities.

Maximize Utility subject to $R \leq 1$ (Proposed Middle Ground). This paper suggests that a useful formulation of society's problem is:

$$\max_{x \in X} \sum_{i=1}^{n} x_i (v_i - c_i) \tag{5}$$

$$\text{subject to}$$

$$\sum_{i=1}^{n} x_i r_i \leq 1$$

The objective in program (5) is the same economic objective as in program (3), but the constraint $\sum_{i=1}^{n} x_i r_i \leq 1$ encodes the health objective that the virus is contained. As shown above in Figure 1, a society that achieves $R \leq 1$ starting from a time when the stock of infections is still relatively small *approximates* the pure medical objective in (4). Whereas, if $R > 1$, the number of infections grows exponentially, eventually leading to a large number of cumulative infections and deaths. Additionally, if $R > 1$, then as the number of infections grows fear of the virus may directly lower activity utilities relative to their pre-virus levels (see Section 6.1 for this extension).

3.2 Maximize Utility subject to $R \leq 1$: Solution

Let

$$\rho_i = \frac{v_i - c_i}{r_i} \tag{6}$$

denote the ratio of benefits minus costs to disease-transmission risk for each activity i. That is, ρ_i represents activity i's *net utility per unit of disease-transmission risk.* For activities with $r_i = 0$ define $\rho_i = \infty$.

The optimal solution to (5) is found by choosing activities in descending order of their ρ_i ratios until the disease-transmission constraint is reached. Intuitively, $R \leq 1$ is like a *disease-transmission budget constraint*, and r_i is the *risk price* of activity i. The way to maximize utility subject to the transmission budget constraint is to choose the activities with the highest utility $v_i - c_i$ per unit of virus risk r_i, i.e., the activities with the highest ratios ρ_i. This is the standard logic of the divisible-goods version of the knapsack problem in operations research.

Formally, let ρ_i^* denote the threshold at which the budget constraint is reached when choosing activities in descending order of ρ_i, i.e., the solution to

$$\rho^* = \inf \left\{ \rho_i : \sum_{j:\rho_j > \rho_i} r_j \leq 1 \right\} \tag{7}$$

The optimal choice of the activity vector x^* is then:

$$x_i^* = \begin{cases} 1 & \text{if } \rho_i > \rho^* \\ q & \text{if } \rho_i = \rho^* \\ 0 & \text{if } \rho_i < \rho^*. \end{cases}$$

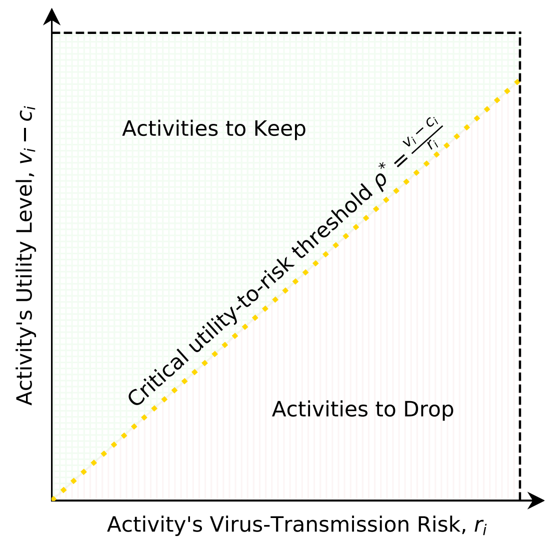
where $q := \left(1 - \sum_{j:\rho_j > \rho^*} r_j\right) / \left(\sum_{j:\rho_j = \rho^*} r_j\right)$ is a constant defined to exhaust the risk budget given that all activities with $\rho_i > \rho^*$ are done in full and all activities with $\rho_i < \rho^*$ are fully dropped.

Which Activities to Keep and Drop: Graphical Depiction. Figure 3 presents a simple graphical depiction of the optimal solution to (5) in terms of which activities the social planner keeps and drops.

The vertical axis represents the utility of an activity, $v_i - c_i$. The horizontal axis represents the virus-transmission risk of an activity, r_i. The diagonal line represents the criticial threshold ρ^* for the ratio of utility to virus-transmission risk. All activities above the diagonal line are kept (hatched green), and all activities below the diagonal line are dropped (striped red). Intuitively, activities above the diagonal line have enough “bang for buck” — enough utility per unit of virus-transmission risk — to be included in the optimum. Activities below the diagonal line are too expensive in virus-transmission terms to be worth their utility gains.

Notice that activities with very high risk can be included in the optimum solution if their utility is sufficiently high. And, conversely, some activities with relatively low risk should be dropped if their utility is sufficiently low. The key for the optimum is to sort activities not by their absolute level of risk, but by their utility per unit of risk.

Figure 3: **Which Activities to Keep and Drop**



Note: This figure illustrates the optimal mix of activities to maximize societal utility subject to $R \leq 1$. The diagonal line depicts the critical threshold ρ^* for the ratio of utility-to-risk. Activities with utility-to-risk ratios above ρ^* should optimally be kept and activities with utility-to-risk ratios below ρ^* should optimally be dropped. The placement of the line is illustrative and is based on the numerical example in Section 5 with $R_0 = 2.5$, no simple interventions, and uniform distributions of utility and risk.

Convex Cost of Mitigation. For any $K \in [0, R_0]$, let $U_{R \leq K}$ be the utility in the optimal solution to program:

$$\max_{x \in X} \sum_{i=1}^{n} x_i (v_i - c_i)$$

subject to

$$\sum_{i=1}^{n} x_i r_i \leq K$$

This is the same program as (5) but with a disease-transmission budget of K instead of 1. Define the cost of mitigation from R_0 to K by $c(R_0 - K) = U_0 - U_{R \leq K}$. It is straightforward to show that this cost of mitigation is increasing and convex. Intuitively, one has to drop more and more attractive activities the larger is the desired reduction in transmission, with attractiveness defined in terms of the utility-to-risk ratio $\rho_i = \frac{v_i - c_i}{r_i}$. Please see Appendix A.1 for a proof.

This result provides a simple microfoundation for the "Economic Costs of Reducing Spread" curve in Figure 2. I reiterate, however, that we do not know the level of the curve relative to the health benefits curve, nor do we know its rate of curvature. Some example cost curves will be simulated in Section 5, after we add simple interventions to the model in Section 4

4 Simple Interventions (Masks, Tests, etc.)

As I emphasized in the April 1st draft of this paper: (i) R_0 as measured, of around 2.5-3.0, describes the growth of Covid-19 in a population that is unaware of the virus, so is not taking even the most basic precautions against its spread; (ii) we know a lot about how Covid-19 spreads (and have learned even more over time); and thus (iii) it therefore stands to reason that some relatively simple behavior changes and policy interventions — the April 1st draft emphasized facemasks, gloves, social distancing practices, handwashing, stay-home-if-sick, and testing — might be able to reduce R meaningfully.[14] R could then be reduced even further by dropping some activities. As I wrote in the April 1st draft:

> ...we should try to be creative about combinations of policies and interventions that together bring R below 1, without an indefinite period of significant harm to the economy and society. If R_0 were 10 this would seem helpless but with R_0 on the order of 2.5-3, with a relatively clear understanding of how the disease does and doesn't spread, and with several empirical examples to date of countries bringing their $R < 1$, it seems achievable. (Budish 2020, pg. 7)

4.1 Adding Simple Interventions to the Model

We can incorporate this idea of simple, low-cost interventions into the model as follows. For each activity i there is a "mask" technology that:

- Weakly reduces the economic benefit of the activity from v_i to $v_i^m \leq v_i$
- Weakly increases the economic cost of the activity from c_i to $c_i^m \geq c_i$
- Weakly reduces the disease-transmission risk of the activity from r_i to $r_i^m \leq r_i$

The terminology "masks" is meant to represent the various kinds of simple interventions described above, the set of which may further evolve over time as our understanding continues to improve. The reason I use the term "masks" rather than "non-pharmaceutical interventions" (NPIs) is that the term NPIs has come to include both the simple interventions of the sort I have in mind here as well as severe lockdowns (Ferguson et al., 2020).

The first two assumptions capture that masks at least weakly reduce the benefits and increase the costs of activities relative to pre-virus utility levels. On the benefits side: facemasks are uncomfortable, physical distance reduces joy, etc. On the costs side: it is costly to erect glass

[14] I will review the evidence that has accumulated in support of masks, tests, and related simple interventions below in Section 4.3.

partitions between customers and check-out clerks, factory lines are less efficient with physical distance, tests cost time and money, etc.

The last assumption captures the whole point of masks, which is to reduce the riskiness of activities. Masks therefore allow society to engage in more activity subject to any given risk budget.

4.2 Optimal Masks

In this section I provide a simple necessary condition for it to be optimal to adopt simple interventions ("masks") for a given activity, and then provide a formula that describes the optimal such interventions.

Let $\rho_i^m = \frac{v_i^m - c_i^m}{r_i^m}$ denote activity i's utility-to-risk ratio with masks, analogous to ρ_i as defined in (6) without masks. A <u>necessary</u> condition for it to be optimal to use the masked version of activity i is that masks improve the activity's utility-to-risk ratio:

$$\rho_i^m \geq \rho_i \tag{8}$$

Intuitively, masks must increase society's "bang per buck" per unit of virus risk, allowing the social risk budget to be stretched further. See Appendix A.2.2 for a formal statement and proof.

Interestingly, this condition is not quite sufficient. To see why, consider the following simple example. There are two activities. Activity 1 has parameters $v_1 - c_1 = 1$, $r_1 = 1$ without masks and $v_1^m - c_1^m = 0.7$, $r_1^m = 0.5$ with masks. Activity 2 has parameters $v_2 - c_2 = 0.1$, $r_2 = 1$ without masks and $v_2^m - c_2^m = 0.1$, $r_2^m = 0.5$ with masks. The optimum without masks is to do just activity 1 (exactly exhausting the risk budget) whereas the optimum with masks is to do both activities in full. Yet, utility is higher without masks than with masks. What drives this example is that the utility harm of masks to the socially valuable activity (Activity 1) is relatively large, and the risk budget that is freed up is then spent on a much lower value activity (Activity 2). Thus, in this example, even though masks allow for more activity in total, welfare is lower.

The example suggests that for mask adoption to increase welfare, the marginal activities that are enabled by mask adoption must be of sufficiently high value relative to the utility harm of masks. Define $\Delta r_i = r_i - r_i^m$ and $\Delta u_i = (v_i - c_i) - (v_i^m - c_i^m)$. A <u>sufficient</u> condition for it to be optimal to use the masked version of activity i is that the necessary condition (8) holds, and, additionally:

$$\underline{\rho}_i^* \geq \frac{\Delta u_i}{\Delta r_i} \tag{9}$$

where $\underline{\rho}_i^*$ denotes the utility-to-risk ratio of the marginal activity if a mask is adopted for activity i, taken as a lower bound over potential mask policies for activities other than i. The right-hand-side of (9) represents the cost of freeing up additional risk budget by masking activity i: the numerator is the utility harm of the mask, the denominator is the amount of risk budget that is freed up. The condition thus requires that the marginal use of the freed-up risk budget is guaranteed to be high enough to justify the utility cost of the mask.[15] See Appendix A.2.3 for a formal statement and proof, as well as a version of the sufficient condition that applies to masking a set of activities.

Now suppose that we can flexibly design the set of simple interventions for activity i. What is the optimal such set of interventions? Assume that society's marginal utility-to-risk ratio ρ^* is exogenous to the mask policy of activity i. Let $\{m_i^1, \ldots, m_i^K\}$ denote the set of potential mask policies for activity i. The optimal mask policy for activity i is the choice from this set that maximizes:

$$\underbrace{\Delta r_i}_{\substack{\text{risk reduction} \\ \text{from mask}}} \cdot \underbrace{\rho^*}_{\substack{\text{marginal value} \\ \text{of risk budget}}} - \underbrace{\Delta u_i}_{\substack{\text{utility harm} \\ \text{of mask}}} \tag{10}$$

In words: the optimal mask for activity i maximizes its risk reduction (Δr_i) times the marginal societal value of this additional risk budget (ρ^*) minus the utility harm of the mask itself (Δu_i). While intuitive, notice that the optimal mask for activity i is not necessarily the one that maximizes the *ratio* of utilty-to-risk. Eliminating the last epsilon of risk is great for the ratio of utility-to-risk but only has a small benefit for societal utility. This may not be worth it if the utility cost of eliminating this last epsilon of risk is high. Instead, (10) tells us that the optimal masks are those that achieve large *absolute* reductions of the quantity of risk, at small absolute harm to utility.

4.3 Evidence on Simple Interventions

This sub-section briefly reviews the empirical evidence on simple interventions. This evidence, in conjunction with data on the economic harm from the lockdown, suggests that the economic case for adopting simple interventions is overwhelming for activities with any non-trivial amount of risk r_i.

[15] To see the relationship between the necessary condition (8) and the sufficient condition (9), rearrange (8) as $\rho_i^m \geq \frac{\Delta u_i}{\Delta r_i}$. (This takes several algebraic manipulations, see Appendix A.2.4). The difference is ρ_i^m on the left-hand-side in this manipulated version of the necessary condition, in place of $\underline{\rho}_i^*$ on the left-hand-side of the sufficient condition. Intuively, while the sufficient condition requires that the *marginal* use of risk budget is higher than the cost of freeing up this risk budget, the necessary condition requires that an inframarginal use of risk budget is higher. Since this manipulated version of the necessary condition is not very interpretable, I prefer to use $\rho_i^m \geq \rho_i$ as the necessary condition.

Medical Evidence on Facemasks. Significant evidence has accumulated since early 2020 on the efficacy of facemasks and other simple interventions. Three helpful starting points for economics readers are Abaluck et al. (2020*b*), Chu et al. (2020), and Howard et al. (2020).

Abaluck et al. (2020*b*) reviews the medical evidence available at the time (the paper was first circulated April 1, 2020) and conducts regression analyses using case prevalence and deaths as outcome variables and mask-use norms as an explanatory variable. Their evidence points to reductions in the growth rate of prevalence and deaths on the order of 40-60%, which translates to a reduction in R on the order of 20-45%.[16] Subsequent versions of this regression approach include Hatzius, Struyven and Rosenberg (2020) and Leffler et al. (2020); these studies look at mask-use mandates as opposed to (or in addition to) norms and find somewhat larger effects. The Hatzius, Struyven and Rosenberg (2020) study finds reductions in growth rates of about 60%, and the Leffler et al. (2020) study reports a reduction in growth rates of 75%. The Abaluck et al. (2020*b*) paper also does cost-benefit analysis of the case for adopting masks. The paper finds the cost-benefit case is overwhelming even under very conservative assumptions.[17]

Chu et al. (2020) provide a detailed meta-study of the medical literature on masks. In health care settings masks are estimated to have a "relative risk" of 0.30, which corresponds to a 70% reduction in R.[18] In non health care settings their estimate is a relative risk of 0.56 which corresponds to a 44% reduction in R.

Howard et al. (2020) provide a wider survey of the literature on masks, helpfully organizing it into several distinct types of evidence. There are direct observational studies (e.g., comparing prevalence in hospitals with and without widespread mask use, as in studies surveyed by Chu et al. (2020)); ecological studies (e.g., comparing prevalence rates across geographies with different levels of mask use, as in the regression studies mentioned above); source control studies (e.g., direct measurement of various kinds of masks' abilities to block various kinds of infectious particles);[19] and model-based approaches (e.g., using source control estimates to then simulate population spread). Across all of these different kinds of evidence, the reductions are very large. Their

[16] At the higher end, if R_0 without masks is 4.0 and the reduction in the growth rate with mask norms is 60%, the implied R with mask norms is approximately $4.0 - 60\%(4.0 - 1.0) = 2.2$, which is a reduction from the original R_0 of $1.8/4.0 = 45\%$. At the lower end, if R_0 without masks is 2.0 and the reduction is 40%, the implied R with mask norms is $2.0 - 40\%(2.0 - 1.0) = 1.6$ which is a reduction from the original R_0 of 20%.

[17] Specifically, Abaluck et al. (2020*b*) report gains of $3000-$6000 per cloth mask under the assumption that masks reduce risk by only 10% — which is significantly more conservative than their own estimates, which have been subsequently reinforced by other studies — and also focusing only on the benefits from the value of lives saved, ignoring any additional economic benefits.

[18] In fact in the four studies they survey that are specific to Covid-19 the reduction is even more dramatic, with point estimates of relative risk of 0.03-0.04, which corresponds to a 96-97% reduction in R. However these studies have important limitations that the authors discuss in detail (see Figure 4 and the surrounding text).

[19] In laboratory environments, masks are estimated to block as much as 99.98% of viral droplets (N95 masks), with 95% reduction possible for the cloth masks that are widely available (Ma et al. (2020); see also Konda et al. (2020) for related evidence).

summary conclusion is that

> "near universal adoption of non-medical masks when out in public, in combination with complementary public health measures could successfully reduce R_e (effective-R) to below 1." Howard et al (2020, pg. 11)

In their article they discuss estimates of the initial reproduction rate R_0 in the range of 2.4-3.9, so their conclusion that masks and complementary public health measures could reduce R to less than 1 corresponds to a 58%-74% reduction in transmission.

There is not yet an RCT study of masks, though there is at least one in the field (Abaluck et al., 2020*a*). Still, the preponderance of evidence from various sorts of empirical studies, combined with common sense conceptual understanding based on how the virus is known to spread, all point to reductions that are significant, perhaps on the order of 50% or more in conjunction with complementary measures.

Romer Mass Testing. Paul Romer, in a series of blog posts, conducted simple disease-transmission simulations ("dots in a box") under three scenarios: (i) myopic ignore-the-virus behavior, (ii) lock down ("isolating at random"), and (iii) frequent random testing ("isolating based on test results"). The key point from the simulations is that large-scale random testing of a population, with temporary isolation of those who test positive, can achieve the same reduction in disease-transmission as a lockdown at much lower economic cost (Romer, 2020*a*,*b*,*c*). A follow-up paper then formalized the reduction in R that is possible from population-scale testing, under various assumptions about frequency, accuracy, speed, and targeting. Under optimistic assumptions population-scale testing easily achieves $R \leq 1$ on its own. Even under relatively pessimistic assumptions, population-scale testing can help achieve $R \leq 1$ in conjunction with other measures (see especially Figure 3 of Taipale, Romer and Linnarsson, 2020).

In the language of this paper's model, Romer's population-scale tests are a mask that reduces the disease-transmission risk of any particular activity, i.e., lowers risk from r_i to r_i^m. The more frequent, accurate, etc., are the tests, the larger is the reduction. Contact tracing can be an important complement to the random tests: if a random test detects that Alice is positive only after she has been infectious for some time, the random test may not prevent her infection of Bob, but with contact tracing it might be possible to detect Bob's infection before he infects Carol. Like with tests, contact tracing need not be perfect to be helpful.

Testing that is frequent, accurate and fast enough for the relevant population for activity i can reduce r_i arbitrarily close to 0, but at cost $c_i^m - c_i$ that increases with the frequency and expense of the test.[20] This paper's model suggests that tests are therefore likely to be of especially high

[20]See Droste, Stock and Atkeson (2020) for a recent detailed cost-benefit analysis of testing that finds that

social value for activities that are both high pre-virus utility $v_i - c_i$ and high risk r_i: the activities in the top-right corner of Figure 3. Additionally, tests are likely to be especially socially valuable for activities where facemasks and other cheaper interventions either are not sufficiently effective (i.e., $r_i^m - r_i$ is small) or are too harmful to utility (i.e., $v_i - v_i^m$ is large). Congregate settings such as nursing homes may be an example of the former (see Chen, Chevalier and Long (2020)); film and television production is an obvious example of the latter.

An intellectually interesting question is what is the optimal mix of masks, tests, etc., to achieve $R \leq 1$ at least social cost. Practically, however, it is important to keep in mind that all of these interventions are very cheap relative to the social cost of lockdown, as I will discuss next.

Shadow Cost of the Lockdown Constraint. In the three month period from mid-March 2020 to mid-June 2020, there were nearly 42 million unemployment claims in the United States (versus about 3 million in the three months prior). US GDP contracted at 31.7% on an annualized basis in Q2 2020, with a drop in global GDP from pre-lockdown peak to trough (April 10th, 2020) of about 20%.[21]

To put it plainly: the shadow cost of the lockdown constraint was enormous.

4.4 Price-Theory Intuition

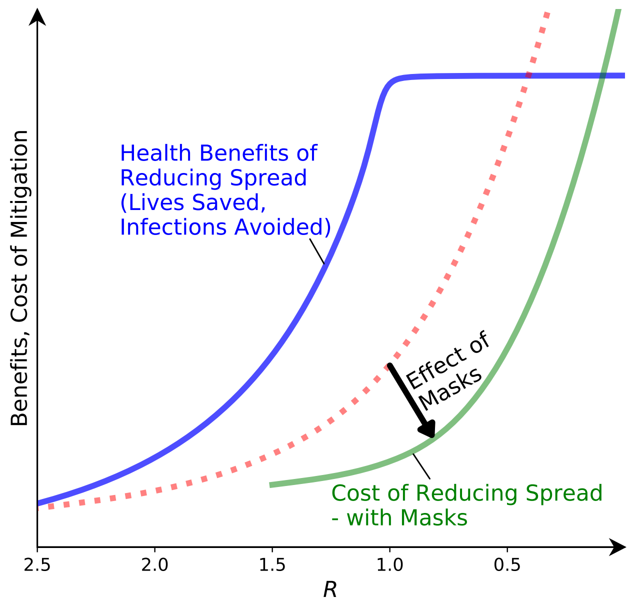
Figure 4 illustrates the price-theory intuition for the effect of masks on optimal policy. Under the conditions described above, masks increase the social welfare that is achievable for a given level of disease-transmission risk. This is equivalent to *masks reducing the economic cost of virus mitigation.* The figure illustrates a case where, without masks, mitigation to $R \leq 1$ is optimal but expensive. With masks, mitigation to $R \leq 1$ remains optimal but is significantly less expensive. Specific numerical examples of mitigation cost curves, and how they are impacted by masks, will be provided in Section 5.

The figure may represent a society that is initially in lockdown, at significant economic expense, and then transitions to reopening with simple interventions such as masks and tests that can help keep $R \leq 1$. This was my hope for the United States's reopening strategy when I circulated the first version of this paper on April 1st, 2020.

benefits far exceed costs over a wide range of testing scenarios.

[21] The unemployment figure is from the BLS via https://oui.doleta.gov/unemploy/claims.asp for the weeks ending March 21st 2020 through June 13th 2020. The Q2 2020 US GDP figure is from the BEA's revised estimate released Aug 27th 2020 available via https://www.bea.gov/news. The peak-to-trough figure for global GDP is from The Economist (2020) via Goldman Sachs. See also Chetty et al. (2020) and https://opportunityinsights.org/tracker-resources/ for real-time data on the economic impacts of Covid-19.

Figure 4: **Simple Interventions Reduce the Cost of Mitigation**



Note: This figure illustrates the effect of simple interventions, denoted "masks" in the model, on the economic cost of mitigation. The solid-blue line and dotted-red line are the same as in Figure 2. The solid-green line illustrates the lowering of the economic cost of mitigation from masks. The reduction is illustrative and is based on the numerical example from Section 5 under the assumptions that masks reduce risk by 40% and harm utility by 5%.

5 Detailed Numerical Example: the Value of Simple Interventions

This section presents a detailed numerical example in two steps. First, it considers the baseline model of Section 3 and depicts the optimal activity mix and cost of mitigation curve, in a simple numerical environment using uniform distributions and empirical evidence on R_0. Second, I add simple interventions (masks, tests, social distance, etc.) to the example. This yields the most important results of the section: magnitudes for just how economically valuable simple interventions are, given what we know about their efficacy and R_0.

5.1 Numerical Example: Initial Model without Masks

Define activity i's utility $u_i = v_i - c_i$, i.e., (net) utility equals benefits less costs. Let activities' utilities and risks be jointly uniformly distributed on $[0, \bar{u}] \times [0, \bar{r}]$. If society engages in all activities, i.e., society engages in Program (3) above, then myopic utility and disease-transmission are given by:

$$U_0 = \int_0^{\bar{r}} \int_0^{\bar{u}} u_i \frac{1}{\bar{u}\bar{r}} du dr = \frac{\bar{u}}{2}$$

$$R_0 = \int_0^{\bar{r}} \int_0^{\bar{u}} r_i \frac{1}{\bar{u}\bar{r}} dudr = \frac{\bar{r}}{2}$$

Without loss of generality, set $\bar{u} = 1$. The relation $\bar{r} = 2R_0$ allows us to express $\bar{r}$ based on a parameter choice for R_0, which can be based on empirical evidence.

Now consider the constrained problem in which society chooses activities to maximize utility subject to a constraint on R; particular attention will be given to the constraint $R \leq 1$. Formally, for any $K \in [0, R_0]$, society solves

$$\max_{x(\cdot)} \int_0^{2R_0} \int_0^1 x(u,r) \cdot u_i \frac{1}{2R_0} dudr \tag{11}$$
$$\text{subject to}$$
$$\int_0^{2R_0} \int_0^1 x(u,r) \cdot r_i \frac{1}{2R_0} dudr \leq K$$

The analysis in Section 3 shows that the optimal solution to (11) is characterized by a utility-to-risk threshold ρ^*, such that all activities with utility-to-risk ratio above ρ^* are included and all activities with utility-to-risk ratio below ρ^* are dropped. Appendix A.3 obtains ρ^* for this example in closed form.

Table 1 describes the features of the optimal solution to achieve $R \leq 1$, for values of R_0 ranging from 2.0 to 4.0. Let me highlight the results with $R_0 = 2.5$, the CDC's current best estimate. In this case, to achieve $R \leq 1$ requires dropping 45% of activities, that together constitute 60% of risk and 27% of social utility. Society keeps the other 55% of activities, leaving it with just 73% of pre-virus utility. Even though society keeps the activities with the highest utility-to-risk ratio, the welfare cost is significant: over one-quarter of social welfare. This is a numerical illustration of what we mean by lockdown.

Even at the CDC's optimistic scenario, $R_0 = 2.0$, achieving $R \leq 1$ requires dropping 38% of activities constituting 19% of social welfare. At the CDC's pessimistic scenario, $R_0 = 4.0$, achieving $R \leq 1$ requires dropping 57% of activities constituting 42% of social welfare.

Remark: Super-Spreader Activities. A limitation of the uniform-distribution assumption is that it does not allow for super-spreader activities — a small mass of activities with particularly large r_i. Intuitively, if super-spreader activities are incorporated into the example, then (i) fewer activities will need to be dropped to get to $R \leq 1$, but (ii) whether social utility is higher or lower than in the uniform example depends on whether the super-spreader activities are high or low utility. If there is a mass of super-spreader activities in the bottom-right of Figure 3 (low u_i,

Table 1: **Optimal Solution to Achieve $R \leq 1$, without Simple Interventions**

	Value of R_0				
	2.0	2.5	3.0	3.5	4.0
To Achieve $R \leq 1$:					
% Activities Dropped	37.5	45.0	50.0	53.7	56.7
% Pre-Virus Utility Dropped	18.8	27.0	33.3	38.3	42.3
Relative to Pre-Virus Economy:					
% Activities Kept	62.5	55.0	50.0	46.3	43.3
% Utility Kept	81.2	73.0	66.7	61.7	57.7

Note: Please see the text of Section 5.1 for description of the numerical example without simple interventions.

high r_i), then utility will be comparatively higher than in the uniform case, whereas if there is a mass of super-spreader activities in the top-right of Figure 3 (high u_i, high r_i), then utility will be comparatively lower than in the uniform case.

5.2 Adding Masks to the Example

Now add simple interventions to the example, which as in Section 4 I refer to as "masks". For simplicity, assume that masks reduce risk by a uniform percentage across activities, denoted γ_r, and similarly reduce utility by a uniform percentage across activities, denoted γ_u. Thus activity i with original utility u_i and risk r_i has masked-utility of $(1-\gamma_u)u_i$ and masked-risk of $(1-\gamma_r)r_i$. With this assumption we still have a joint uniform distribution of utility and risk, only now on $[0,(1-\gamma_u)\bar{u}] \times [0,(1-\gamma_r)\bar{r}]$. Thus all of the same math from above goes through analogously.

Table 2 summarizes the analysis. The Panels vary the assumption for R_0: Panel A utilizes the CDC's current best estimate of $R_0 = 2.5$, Panel B utilizes the CDC's pessimistic scenario of $R_0 = 4.0$, and Panel C utilizes the CDC's optimistic scenario of $R_0 = 2.0$. The columns of each panel vary mask efficacy from 30%-70%. The studies summarized in Section 4.3 suggest that 30% is a conservative figure for masks alone, 50% may be an appropriate estimate for the use of masks in conjunction with complementary measures such as social distancing, and that 70% may be either an optimistic estimate for masks and related measures, or represent a combination of these kinds of measures in conjunction with Romer mass testing.

Focus first on the 50% efficacy column of Panel A. Simple interventions reduce average transmission from $R_0 = 2.5$ to $R = 1.25$ without any reduction in activity levels. Thus, to achieve $R \leq 1$ requires a further 20% reduction of risk. This can be accomplished by dropping the 15% of activities with the worst utility-to-risk ratios, which together constitute just 3% of pre-virus

Table 2: **Optimal Solution to Achieve $R \leq 1$: Large Effect of Simple Interventions**

Panel A: Main R_0 Scenario

		Mask Efficacy				
	No Masks	30%	40%	50%	60%	70%
R if all activities are kept	2.50	1.75	1.50	1.25	1.00	0.75
To achieve $R \leq 1$:						
% Activities Dropped	45.0	32.1	25.0	15.0	0.0	0.0
% Pre-Virus Utility Dropped	27.0	13.8	8.3	3.0	0.0	0.0
Society % of Pre-Virus Utility:						
if Masks Reduce Utility by 0%	73.0	86.2	91.7	97.0	100.0	100.0
if Masks Reduce Utility by 10%	N/A	77.6	82.5	87.3	90.0	90.0

Panel B: High R_0 Scenario

		Mask Efficacy				
	No Masks	30%	40%	50%	60%	70%
R if all activities are kept	4.00	2.80	2.40	2.00	1.60	1.20
To achieve $R \leq 1$:						
% Activities Dropped	56.7	48.2	43.7	37.5	28.1	12.5
% Pre-Virus Utility Dropped	42.3	31.0	25.5	18.8	10.5	2.1
Society % of Pre-Virus Utility:						
if Masks Reduce Utility by 0%	57.7	69.0	74.5	81.2	89.5	97.9
if Masks Reduce Utility by 10%	N/A	62.1	67.0	73.1	80.5	88.1

Panel C: Low R_0 Scenario

		Mask Efficacy				
	No Masks	30%	40%	50%	60%	70%
R if all activities are kept	2.00	1.40	1.20	1.00	0.80	0.60
To achieve $R \leq 1$:						
% Activities Dropped	37.5	21.4	12.5	0.0	0.0	0.0
% Pre-Virus Utility Dropped	18.8	6.1	2.1	0.0	0.0	0.0
Society % of Pre-Virus Utility:						
if Masks Reduce Utility by 0%	81.2	93.9	97.9	100.0	100.0	100.0
if Masks Reduce Utility by 10%	N/A	84.5	88.1	90.0	90.0	90.0

Note: Please see the text of Section 5.1-5.2 for description of the numerical example with simple interventions ("masks"). The three R_0 scenarios are based on the CDC's current best estimate, pessimistic scenario, and optimistic scenario, respectively.

utility. Society maintains activities that together constitute 97% of pre-virus utility. If masks reduce the utility of the maintained activities by 10%, i.e., $\gamma_u = 10\%$, then society's total utility is 87.3% of pre-virus utility levels.

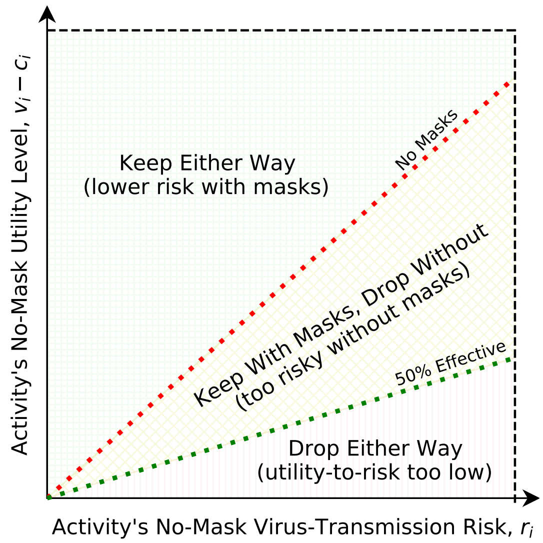
If simple interventions are sufficiently effective, such interventions alone can reduce R to less than 1 without any reduction in activity levels. This occurs if $R_0(1 - \gamma_r) \leq 1$; for example, if $R_0 = 2.5$ and the reduction in risk is at least 60%. If we think beyond the uniform-distribution example to include a small mass of super-spreader activities, as discussed earlier, then the more likely practical conclusion is that society would only have to drop super-spreader activities, keeping everything else.

In the pessimistic scenario of $R_0 = 4.0$, simple interventions can have an especially dramatic effect on social utility. Without interventions, society has to drop over 50% of activity and over 40% of utility to reach $R \leq 1$. If simple interventions are 60% effective, society can drop 28% of activity constituting 10% of utility. If simple interventions are 70% effective, society can drop just 12.5% of activity constituting just 2% of pre-virus utility.

Figure 5 illustrates how masks affect the optimal activity mix. The top-left region (green squares) depicts activities that are included in the optimum whether or not masks are utilized. If masks are utilized, these activities are lower risk, and society then optimally spends this freed-up risk budget on the yellow-hatched region labeled "Keep With Masks, Drop Without". These are the activities that, without masks, are too risky per unit of utility, but with masks can be included in the optimum. The bottom right red-striped region depicts activities that are optimally dropped even with masks; these are the activities with the lowest utility-to-risk ratios. The figure depicts the dividing line for the case of $R_0 = 2.5$ and 50% mask effectiveness. The higher is mask effectiveness, the smaller is this striped region, vanishing to zero if the risk reduction is 60% or greater.

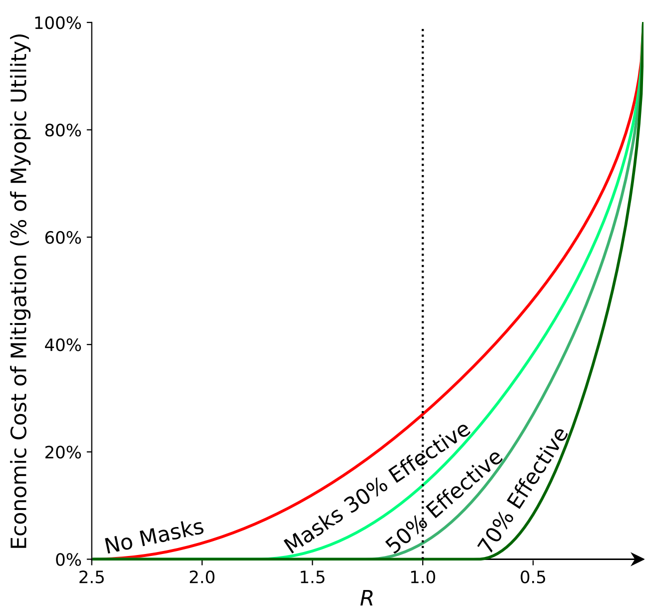
Figure 6 illustrates the effect of masks on the cost of mitigation curves. This figure complements the illustrative price theory diagram provided earlier, Figure 4. Focus first on the 50% effectiveness line. Masks get us from $R_0 = 2.5$ to $R = 1.25$ without dropping any activities. Then, the cost curve begins increasing as activities are dropped, but at first this increase is slow because society drops only those activities with the poorest utility-to-risk profiles. For this reason, society can get all the way to $R = 1$ very cheaply. If masks are 30% effective, the cost is meaningfully higher but still lower than without masks. If masks are 70% effective, society can get all the way to $R = 1$ for free. Yet, even in this optimistic case, a policy of "minimize the virus" remains very expensive — the costs get arbitrarily high as society drops more and more activities with non-zero risk.

Figure 5: **Effect of Simple Interventions on the Mix of Activities to Keep and Drop**



Note: This figure illustrates how simple interventions, denoted "masks" in the model, expand the set of optimal set of activities used to maximize societal utility subject to $R \leq 1$. The diagonal line labeled "No Masks" depicts the critical threshold ρ^* for the ratio of utility-to-risk if there are no masks; this is the same line as in Figure 3 and is based on the uniform-distribution example with $R_0 = 2.5$. Without masks, activities above the "No Masks" line should optimally be kept and activities below this line should optimally be dropped. The diagonal line labeled "50% Effective" depicts how the mix of activities expands if masks are adopted and they uniformly reduce risk by 50%. Activities with utility and un-masked risk in the yellow hatched region, above the "50% Effective" line but below the "No Masks" line, can be included in the optimum with masks whereas they are dropped without masks.

Figure 6: **Effect of Masks on the Economic Cost of Mitigation**



Note: This figure presents the economic cost of mitigation curve in the numerical example with no masks and with masks of varying effectiveness. The horizontal axis represents the level of mitigation and the vertical axis is the cost depicted as a percentage of myopic utility U_0 as defined above. These curves correspond to the illustrative cost of mitigation curves presented in price-theory diagrams Figure 2 (without masks) and Figure 4 (with masks).

6 Discussion: Is $R \leq 1$ Enough? Too Much?

6.1 Is $R \leq 1$ Enough? High Stock of Infections and Fear of the Virus

In the initial model, $u_i = v_i - c_i$ represents the private utility of an activity, ignoring any virus considerations. The virus enters the analysis through r_i, the risk of transmission; the $R \leq 1$ constraint is a way of capturing the negative externalities of privately beneficial behavior that risks spreading the virus.

Clearly, if the level of infections in an area is high enough, this will directly affect the private utility of many kinds of activities because of the fear of catching the virus. Goolsbee and Syverson (2020) provide empirical evidence on the quantitative importance of this channel.

This idea can be captured in the model by letting $u_i^{exposed}$ denote the utility from activity i if perceived exposure to the virus is high, and assuming that $u_i > u_i^{exposed}$ for all i. If $u_i^{exposed} < 0$, individuals wil drop the activities on their own, even without any kind of formal ban, as documented by Goolsbee and Syverson (2020). Several papers elaborating what are now called Behavioral SIR models make this point, and several suggest that, since $u_i^{exposed}$ seems likely to decrease monotonically with the stock of infections, R may automatically equilibrate to around 1.[22]

Figure 7 depicts the high stock of infections case graphically. Panel A depicts the original myopic cost of mitigation curve and the modified cost curve with a high stock of infections. The cost curve in the high stock of infections case is worse than the original because the utility of any activity i that is kept is lower with a high stock of infections, i.e., $u_i^{exposed} < u_i$. The Behavioral SIR models suggest furthermore that the points to the left of $R = 1$ on the original mitigation cost curve are illusory — if the social planner chooses one of those points, the stock of infections will rise, and we will end up instead on the high stock of infections curve which is worse.

Panel B adds two additional curves to the figure: the original cost curve with masks, and a curve that represents the case of both masks and a high stock of infections. This latter curve is worse for the same reason as above; the private utility of any activity that is kept is lower with a high stock of infections than without, even if masks are utilized in both scenarios. That is, the curves assume that $u_i^m > u_i^{m,exposed}$, where u_i^m denotes the private utility of activity i with masks in the original case, and $u_i^{m,exposed}$ denotes the private utility of activity i with masks if the stock of infections is high.

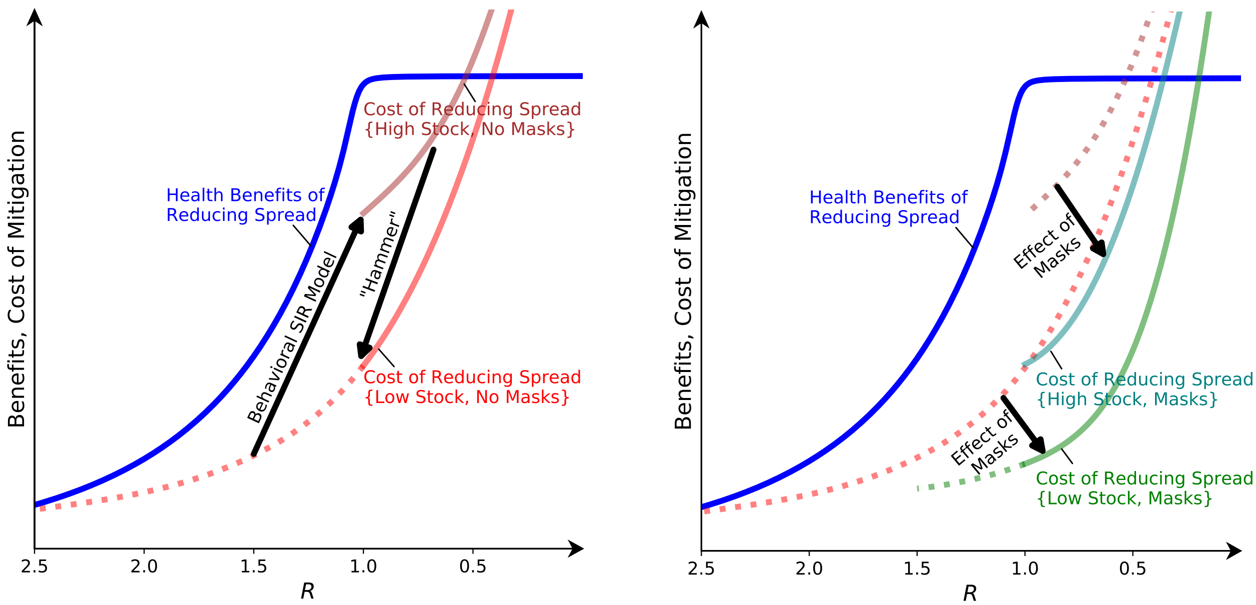
A society with a high stock of infections faces a more complicated dynamic problem than

[22] See Cochrane (2020), Toxvaerd (2020), and Keppo et al. (2020) for models with this equilibration feature. See Atkeson, Kopecky and Zha (2020) for related stylized empirical facts and see Gans (2020) for an article that both surveys this literature and suggests directions for future modeling.

Figure 7: **Effect of High Stock of Infections (“Fear of the Virus”) on the Economic Cost of Mitigation**

Panel A: Fear of the Virus

Panel B: Fear + Masks



Note: Panel A illustrates the effect of a high stock of infections (“fear of the virus”) without masks. The blue line and the red line are exactly as in Figure 2. The gray line represents the economic cost of mitigation in the case of a high stock of infections ({High Stock, No Masks}). The figure depicts that a high stock of infections increases the cost of mitigation due to fear of catching the virus. The red line is dotted to the left of $R = 1.0$ to represent that if $R > 1$ the stock of infections will grow and society will thus transition from the low stock of infections case to the high stock of infections case with higher costs. The transition arrows labeled “Behavioral SIR Model” and “Hammer” are described in the text. Panel B illustrates the effect of masks on the economic cost of mitigation in both the high-stock and low-stock cases ({High Stock, Masks} and {Low Stock, Masks}). In both panels, the effects of fear of the virus and of masks are illustrative. The curves are based on the numerical example from Section 5 assuming that fear of the virus reduces utility by 20% if the stock of infections is high (both with and without masks) and that masks reduce risk by 40%.

a society that acts before the stock of infections grows. In particular, a society with a high stock of infections may wish to first invest in significantly reducing the stock of infections (i.e., R significantly lower than 1), before then transitioning to a steady state with $R \leq 1$ once the stock is sufficiently low. This is the thesis of the “Hammer and the Dance” argument of Pueyo (2020), formalized in sophisticated dynamic models by Farboodi, Jarosch and Shimer (2020) and Assenza et al. (2020).[23] In the simplified graphical depiction here, society may choose a point further up the high-stock-of-infections cost curve than is statically optimal, in order to then transition to the low-stock-of-infections cost curve and adopt the $R \leq 1$ strategy going forward.

One additional point that bears emphasis is that if $R > 1$ is not a real choice in the longer run

[23] In the Farboodi, Jarosch and Shimer (2020) analysis, if the initial stock of infections is high, the optimal policy is first to choose R significantly lower than 1 to reduce the stock of infections, and then transition to a steady state with R actually a tiny bit larger than 1. Intuitively, starting from a low enough stock of infections, $R = 1.01$ does not lead to a large number of infections and deaths in the next twelve months, and in their model there is a first-order gain from relaxing the constraint from, say, $R = 0.99$ to $R = 1.01$.

anyways, as in the behavioral SIR models and as is consistent with the Goolsbee and Syverson (2020) evidence, then society would have been better served to treat $R \leq 1$ as a constraint from the outset, and then maximize utility subject to this constraint. Why not start on and stay on the best cost curve (with masks and a low stock of infections) rather than spend significant time on the worst one?

6.2 Is $R \leq 1$ Too Much? Acemoglu et al (2020) Young/Old Argument

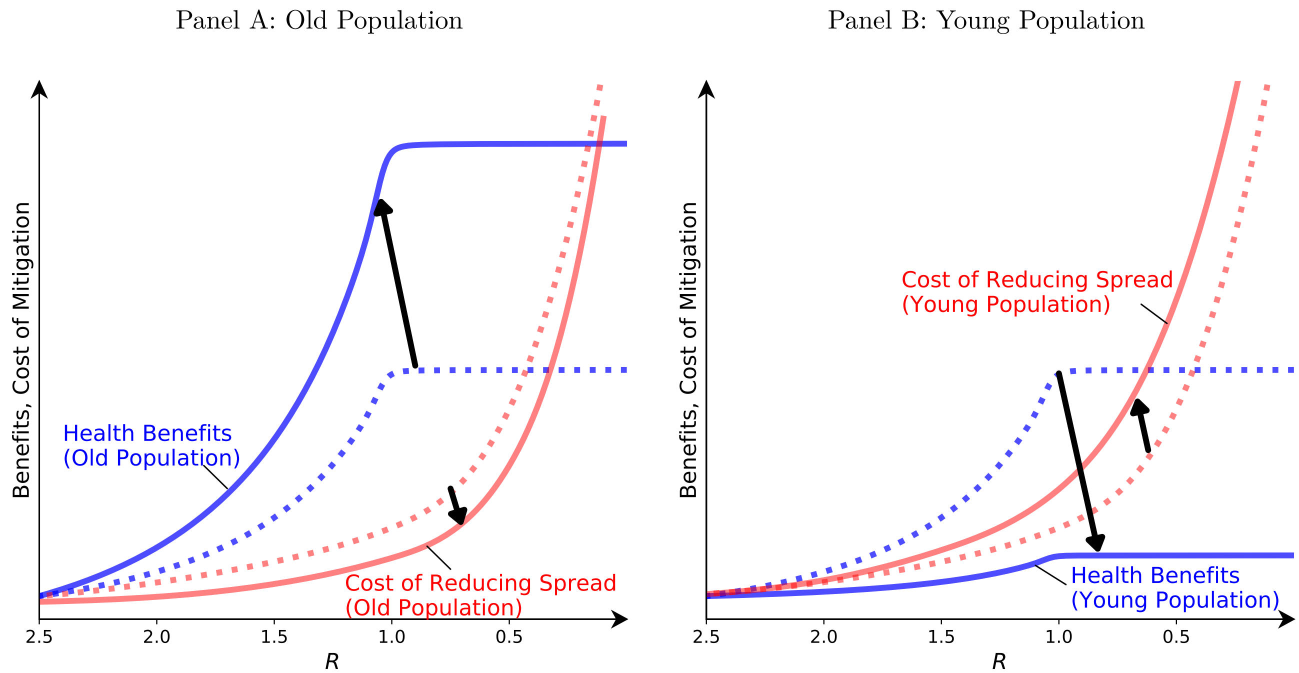
In an important and influential paper, Acemoglu et al. (2020) analyze a model with optimal dynamic lockdowns that can distinguish among three types of people: young, middle-aged, and old. Their analysis is motivated by the medical fact that the virus is significantly more dangerous for the old than for the young — the infection fatality rate for adults over 75 may be as much as 50-100 times as high as that for adults in their 20's — as well as the economic fact that the economic cost of restricting mobility is especially high for people in school and in the work force. The authors conclude that the optimal policy involves stricter lockdowns for the old than for the young.

Figure 8 translates the Acemoglu et al. (2020) argument into the simple price-theory diagram from earlier. As depicted, the optimal policy favors $R \leq 1$ overwhelmingly for the old (Panel A), whereas for the young the benefits of $R \leq 1$ are not worth the economic costs (Panel B). The Acemoglu et al. (2020) model does not consider masks, non-lockdown forms of social distance, or other similar means of reducing risk in case an infected and susceptible interact (similarly to many of the other dynamic SIR models cited in the introduction).[24] It does consider testing and contact tracing in detail (Section 5.3), but only at levels that are insufficient on their own to get to $R \leq 1$. The optimal calibrated policy with such moderately-effective testing and no masks involves a strict lockdown for the old, and either a modest lockdown for the young and middle-aged (see their Figure 5.14) or, if there is enough social separation between age groups, no lockdown for the young and middle-aged (see their Figure 5.15). In terms of the infection rate, the optimum involves $R \leq 1$ for the old but $R > 1$ for the young and middle-aged, as depicted here in Figure 8.

Presumably, if masks or other non-lockdown methods of reducing infectiousness were incorporated into the Acemoglu et al. (2020) model and sufficiently effective alongside tests, society

[24]Formally, in the Acemoglu et al. (2020) model individuals in each group j are either "in lockdown" or not, and when in lockdown their productivity is lower by a parameter ζ_j that "captures the relative productivity of home vs. market production." The main calibrations then set $\zeta_j = 0.70$ "so that workers on average lose 70% of their productivity when under lockdown." The main calibrations do not consider the possibility that workers could be closer to fully productive without incurring all of the virus-transmission risk of pre-lockdown society in case an infected and susceptible interact — i.e., there is no role for facemasks, non-lockdown forms of social distance, etc. When the model considers testing and contact tracing in detail (Section 5.3), it finds that this intervention is very effective.

Figure 8: **Depiction of the Acemoglu, Chernozhukov, Werning and Whinston "Young-Old" Argument**



Note: This figure illustrates the Acemoglu et al. (2020) young-old argument. In both panels, the dotted blue line represents the health benefits of reducing the spread for the full population and the dotted red line represents the economic cost of mitigation for the full population; these are rescaled versions of the curves in Figure 2. Panel A illustrates the curves for the Old population; as Acemoglu et al. (2020) describe, the economic costs of mitigation are lower and the health benefits of mitigation are significantly higher. Panel B illustrates the curves for the Young population; as Acemoglu et al. (2020) describe, the economic costs of mitigation are higher and the health benefits of mitigation are significantly lower. The placement of the curves is illustrative and is meant to help describe the Acemoglu et al. (2020) argument. See the text of Section 6.2 for additional discussion.

would choose $R \leq 1$ for the young and middle-aged as well, especially if there is socially-valuable interaction between these lower-risk populations and more vulnerable populations. Whereas, the optimum would involve $R > 1$ for the sufficiently young and healthy if (i) the disease burden for this population is sufficiently low; (ii) the interventions required to achieve $R \leq 1$ for this population are sufficiently harmful to utility relative to this disease burden; and (iii) cross-infection of the old or vulnerable could be sufficiently controlled.

7 Conclusion: A New Play in the Pandemic Playbook?

This paper has suggested that "maximize utility subject to $R \leq 1$" may be a useful formulation of the Covid-19 lockdown and reopening policy problem. That is, treat the prevention of exponential growth of the virus as a constraint, and then place traditional economic and societal goals as the objective.

There are four features of Covid-19, relative to other past pandemics, that together make this formulation potentially appropriate:

1. Mortality / morbidity cost high: Covid-19 is sufficiently lethal and harmful that $R \leq 1$ is a desirable policy goal even at meaningful expense.

   [In the language of this paper's analysis: the curve "health benefits of reducing spread" in Figures 2 and 4 is sufficiently high relative to the costs of mitigation to justify a policy response.]

2. Eradication likely not feasible: Covid-19 had already spread relatively widely by the time of policy intervention in many countries, making eradication an unrealistic goal for many countries.

   [Mitigation requires simple interventions or dropping activities, until a vaccine or effective treatment is ready, as opposed to a one-time fixed cost of eradication.]

3. $R \leq 1$ feasible with modestly expensive measures: with an initial R_0 in the ballpark of 2.0-4.0, and a fast understanding of how the virus spreads, medical experts quickly converged upon a suite of public-health responses that together could achieve $R \leq 1$. As Atul Gawande put it: "we have learned in hospitals where we've been going to work every day in the pandemic and have avoided infections, that if you have *hygiene, distancing, mandatory masks, and screen everybody for symptoms so that they stay home and get tested, that shuts the virus down*."[25] These interventions are not free, but they are trivial relative to either the 2 million U.S. deaths forecast in the Imperial College model under an "ignore the virus" policy (Ferguson et al., 2020), or the 40 million jobs lost during the lockdown period (see Section 4.3).

   [The $R \leq 1$ point on the "economic cost of reducing spread" curve with simple interventions, Figure 4, is relatively low.]

4. Minimize unboundedly expensive: when eradication of an infectious threat is not feasible, the second-best from a public health perspective is to minimize its spread. For example Dr. Michael Osterholm writes: "As epidemiologists, we have two goals. The first is to prevent. When that is not possible, the second is to minimize ..." (Osterholm and Olshaker, 2020, pg. 26). As emphasized in the introduction, however, the minimize objective makes it difficult to think about tradeoffs *if the interventions themselves are very expensive.* This latter point

[25] The quote is from an interview with David Axelrod recorded on July 9, 2020. Emphasis added. See also Gawande's March 21, 2020 New Yorker article "Keeping the Coronavirus from Infecting Health-Care Workers" which makes similar points based on the experiences of Singapore and Hong Kong. This article is a useful reminder of how early on medical experts understood how to contain the virus without lockdown.

bears emphasis. In the HIV pandemic, the minimize strategy entailed widespread public education about safe sex, condom distribution, needle exchange, etc. — but no public health expert recommended literally trying to minimize the spread of the virus by banning sex (or banning non-monogamous sex, etc.). In the Covid-19 pandemic, however, the minimize strategy has included not just the kinds of interventions that are relatively cheap like virus-risk education, masks, testing, etc., but also much more severe and expensive measures which have included banning many forms of social and commercial contact.

[The "economic cost of reducing spread" curve grows very steeply as R gets lower than lower than 1.0, even if simple interventions are utilized, because of the increasingly-valuable activities that have to be dropped.]

A New Play in the Infectious-Threat Playbook? Figure 9 contrasts this paper's analysis of Covid-19 (Panels A and B) with several other infectious-threat scenarios — past plays in the epidemiology playbook.

Panel C depicts the case where it is "optimal to ignore" a particular infectious threat, because the health benefits of reducing the spread are too low to justify a policy response.

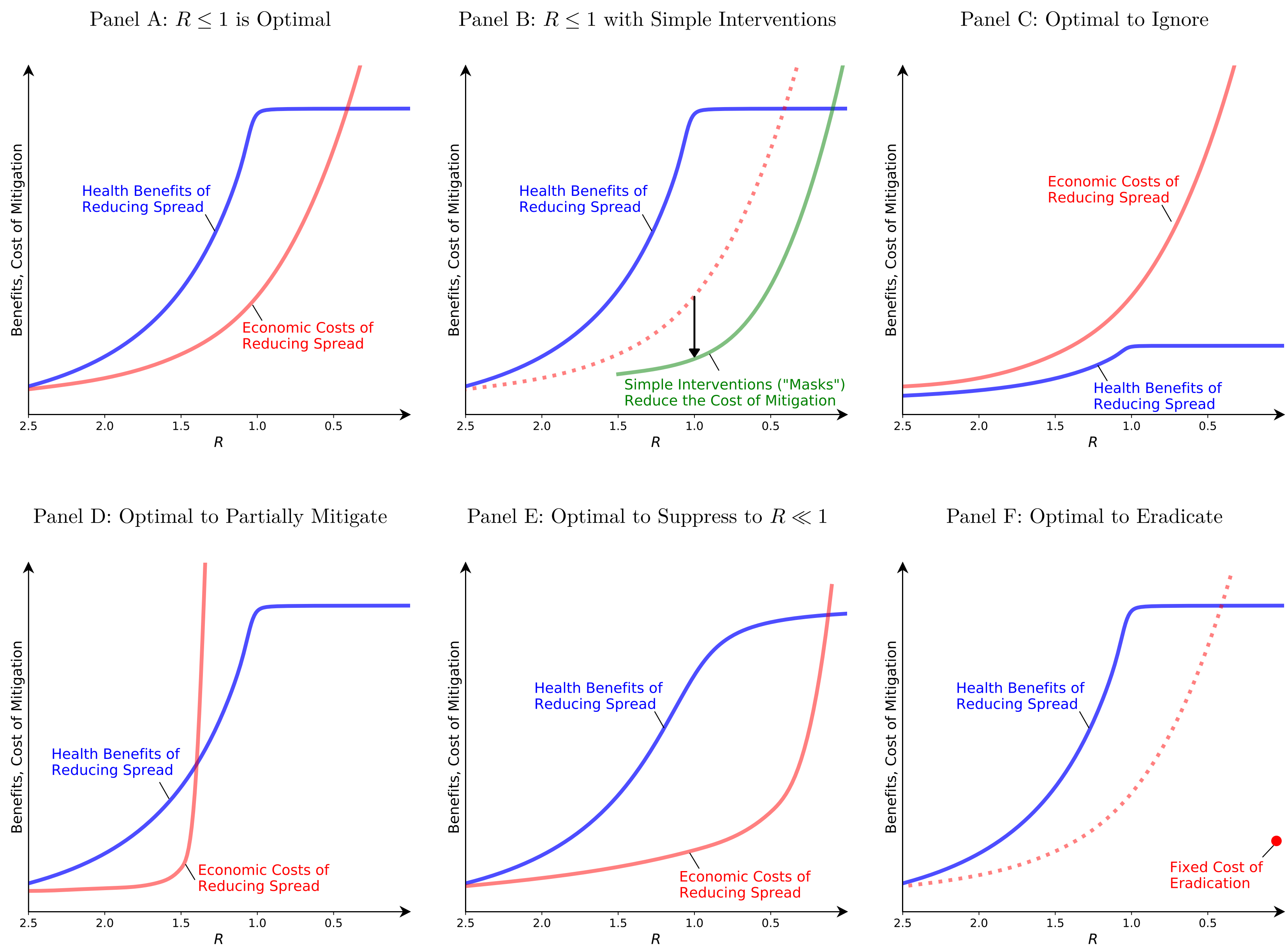
Panel D depicts the case where it is "optimal to partially mitigate" to $R > 1$. The cost curve is fairly flat up to a point (about $R = 1.5$ in the figure, purely for conceptual illustration), and then increases steeply upwards. This represents a scenario in which there are some interventions that are both cheap and effective — "no brainers" to do as fully as possible — but then it is impossible or prohibitively expensive to do more. With Covid-19, relatively cheap measures have been proven to be enough to get to $R \leq 1$, whereas, for example, for HIV there was no comparable set of interventions that was enough.

Panel E depicts the case where it is "optimal to suppress to R significantly less than 1". This requires two things. First, that the cost curve remains relatively flat beyond $R = 1$ — for example, simple interventions or targeted activity bans are sufficiently effective. Second, that the benefits curve is still meaningfully increasing beyond $R = 1$, because the stock of infections is sufficiently high.[26]

Panel F depicts the case where it is "optimal to eradicate". Costs of eradication can be viewed as a one-time fixed cost as opposed to an ongoing flow cost of reducing spread via interventions or activity bans. Eradication is optimal if this one-time fixed cost is sufficiently low, as depicted in Panel F.

[26]The curve in Panel F depicts the data from Figure 1 with an initial stock of infections of 10 million. Alternatively, one could generate a benefits curve that is meaningfully increasing beyond $R = 1$ with a less-severe stock of infections if the hospital system is stretched beyond capacity, which would increase the IFR until infections are brought down.

Figure 9: **Infectious-Threat Playbook**



Note: The blue line of Panels A, B, C, D, and F depicts the same information as the $I_0 = 100,000$ case of Figure 1, but with both axes flipped as described in the text of Section 2. The blue line of panel E depicts the same information as the $I_0 = 10$ million case of Figure 1, with both axes flipped. All cost curves are conceptual and illustrative. Please see the text of Section 7 for discussion of each case.

The public-health instinct expressed by Dr. Osterholm to eradicate if feasible, and otherwise minimize, is a useful heuristic in the scenarios depicted in Panels D, E, and F. Specifically, if we understand "eradicate if feasible" to mean feasible at a fathomable fixed cost, and "minimize" to mean doing all of the cheap interventions, on the relatively flat part of the mitigation curve, as fully as possible, then this heuristic gets to the optimal policy response in each of Panels D, E, and F.

However, if the costs of interventions being considered grow large enough — as they clearly have done in the Covid-19 response — then this instinct may lead to sub-optimal policy. This is the case depicted in Panels A and B and that has been emphasized throughout this paper.

It therefore seems that Covid-19 required a novel play in the epidemiological playbook: maximize societal utility subject to $R \leq 1$. That is, get to $R \leq 1$ as efficiently as possible, using either simple interventions or targeted bans of activities with particularly poor utility-to-risk.

I want to close by emphasizing that this paper, at most, puts economics language on a formulation that many medical experts have converged on as well. There were also several other economists who, early in the pandemic, seemed to have in mind that getting to $R \leq 1$ at the lowest harm to societal utility is the appropriate formulation of the policy problem.[27]

[27] Examples include John Cochrane (see especially his blog posts of March 29th, April 8th and April 14th), my colleague Austan Goolsbee (who said early on that the "First Law of Virus Economics" is to contain the virus), and Paul Romer whose work on mass random testing is discussed extensively above. I imagine that many other economists have had some version of "maximize utility subject to $R \leq 1$" implicit in their thinking about the Covid-19 response. I hope this paper is useful nonetheless.

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A Theory Appendix

A.1 Economic Cost of Mitigation is Convex and Increasing

For any $K \in [0, R_0]$, let $U_{R \leq K}$ be the utility in the optimal solution to program:

$$\max_{x \in X} \sum_{i=1}^{n} x_i(v_i - c_i)$$

$$\text{subject to}$$

$$\sum_{i=1}^{n} x_i r_i \leq K$$

Define $U_0 = U_{R \leq R_0}$, that is, U_0 is (myopic) social utility without any disease-transmission constraint. Define the cost of mitigation from R_0 to $R_0 - \delta$ by $c(\delta) = U_0 - U_{R \leq R_0 - \delta}$. In this subsection we prove that the cost of mitigation is increasing and convex in the quantity of mitigation δ :

Proposition 1. *The cost of mitigation from R_0 to $R_0 - \delta$, denoted by $c(\delta) = U_0 - U_{R \leq R_0 - \delta}$, is increasing and convex in δ.*

Proof. We start the proof with the following lemma:

Lemma 1. $\frac{dU_{R \leq K}}{dK} = \rho_+^*(K)$*, where* $\rho_+^*(K)$ *is defined as:*

$$\rho_+^*(K) = \begin{cases} sup\left\{\rho_i : \sum_{j:\rho_j \geq \rho_i} r_j > K\right\}, & K < R_0 \\ 0, & K = R_0 \end{cases}$$

Intuitively, $\rho_+^*(K)$ is the utility-to-risk ratio of the marginal activity affected by an infinitesimal increase in the disease-transmission budget K when $K < R_0$. $\rho_+^*(K)$ is weakly positive and monotonically decreasing in K over the range $[0, R_0]$.

Proof. Consider a small increase in the disease-transmission budget, Δ, and take the limit as $\Delta \to 0$. When $K = R_0$, additional budget does not translate into more utility since all activities can already be performed in full. Thus, $\frac{dU_{R \leq K}}{dK}|_{K=R_0} = 0$. When $K < R_0$, the optimal policy spends disease-transmission budget in descending order of the utility-to-risk ratio ρ_i. Therefore, we have:

$$\frac{dU_{R \leq K}}{dK} = \lim_{\Delta \to 0} \frac{U_{R \leq K+\Delta} - U_{R \leq K}}{\Delta} = \lim_{\Delta \to 0} \frac{\Delta \cdot \rho_+^*(K)}{\Delta} = \rho_+^*(K).$$

We now show that $\rho_+^*(K)$ is monotonically decreasing in K over the range $[0, R_0]$. For any $K_1 < K_2$ in the range $[0, R_0)$, by the definition of $\rho_+^*(K)$ and set inclusion, we have:

$$\rho_+^*(K_2) = \sup\left\{\rho_i : \sum_{j:\rho_j \geq \rho_i} r_j > K_2\right\} \leq \sup\left\{\rho_i : \sum_{j:\rho_j \geq \rho_i} r_j > K_1\right\} = \rho_+^*(K_1).$$

Therefore, $\rho_+^*(K)$ is monotonically decreasing in K on $[0, R_0)$. Since ρ_i is weakly positive for all i and we have defined $\rho_+^*(R_0) = 0$, we thus have that $\rho_+^*(K)$ is weakly positive and monotonically decreasing in K on $[0, R_0]$, as required. □

Next, take the derivative of $c(\delta)$:

$$\begin{aligned}\frac{dc(\delta)}{d\delta} &= \frac{dU_0}{d\delta} - \frac{dU_{R\leq R_0-\delta}}{d\delta} \\ &= 0 - \rho_+^*(R_0-\delta)\cdot(-1) = \rho_+^*(R_0-\delta)\end{aligned}$$

By Lemma 1, $\rho_+^*(K)$ is monotonically decreasing with K, so $\rho_+^*(R_0-\delta)$ is monotonically increasing with δ. Therefore, $c(\delta)$ is convex. Since $\rho_+^*(R_0-\delta)$ is weakly positive by construction, $c(\delta)$ is increasing as well. □

A.2 Support for Section 4: Model with Simple Interventions

A.2.1 Formal Statement of the Model with Masks

As described in the text of Section 4, for each activity i there is an original version with parameters (v_i, c_i, r_i) and a "masked" version with parameters (v_i^m, c_i^m, r_i^m). The term "masks" is meant to cover simple low-cost interventions including facemasks as well as tests, social distance, hand-washing, stay-home-if-sick, etc. Throughout we assume that masks weakly reduce activity benefits, weakly increase activity costs, and weakly reduce activity risk: $v_i^m \leq v_i$, $c_i^m \geq c_i$, and $r_i^m \leq r_i$.

The social planner now chooses both a mask policy vector $m \in M = \{0,1\}^n$, where $m_i \in \{0,1\}$ denotes whether or not masks are utilized for activity i, as well as the activity vector $x \in X = [0,1]^n$. The formal optimization problem, for any $K \in [0, R_0]$, is now:

$$\begin{gathered}\max_{x\in X, m\in M} \sum_{i=1}^n x_i\left[(1-m_i)(v_i-c_i)+m_i(v_i^m-c_i^m)\right] \\ \text{subject to} \\ \sum_{i=1}^n x_i\left[(1-m_i)r_i+m_i r_i^m\right] \leq K.\end{gathered} \tag{12}$$

To simplify notation, define

$$\begin{aligned}U(x,m) &:= \sum_{i=1}^n x_i\left[(1-m_i)(v_i-c_i)+m_i(v_i^m-c_i^m)\right] \\ R(x,m) &:= \sum_{i=1}^n x_i\left[(1-m_i)r_i+m_i r_i^m\right]\end{aligned}$$

We can now rewrite Problem (12) as

$$\begin{gathered}\max_{x\in X, m\in M} U(x,m) \\ \text{subject to} \\ R(x,m) \leq K.\end{gathered} \tag{13}$$

Let x^* and m^* denote the optimal activity vector and mask policy. Given a specific mask policy m, let $x^*(m)$ denote the optimal activity vector holding fixed that mask policy, and let $OPT(m)$ denote the associated utility at mask profile m and activity vector $x^*(m)$.

A.2.2 Statement and Proof of Necessary Condition for Mask Adoption

Proposition 2. *(Necessary Condition, Activity i Mask Adoption) Let x^*, m^* be an optimal solution to Problem* (13)*. If an activity i satisfies $m_i^* = 1$ and $x_i^* > 0$, then $\rho_i^m \geq \rho_i$.*

Proof. Suppose towards a contradiction that $\rho_i^m < \rho_i$. In the optimum x^* activity i creates a total risk cost of $x_i^* r_i^m$. Fix x_{-i}^* and m_{-i}^* but now instead choose $m_i = 0$ and choose $x_i' = x_i^* \cdot \frac{r_i^m}{r_i} \leq x_i^*$. In words, x_i' spends the same risk budget on activity i as in the optimum x_i^* but now on the un-masked version. Note that r_i^m must be strictly positive because $r_i^m = 0$ would imply $\rho_i^m = \infty$ which contradicts $\rho_i^m < \rho_i$; since $r_i^m \leq r_i$ by the definition of masks r_i must be strictly positive as well.

This new policy is feasible since the total risk cost remains unchanged and $x_i' \in [0, 1]$ is feasible. Denote $u_i = v_i - c_i$ and $u_i^m = v_i^m - c_i^m$. We have:

$$\begin{aligned} x_i' u_i &= x_i^* \cdot \frac{r_i^m}{r_i} \cdot u_i \\ &= x_i^* \cdot r_i^m \cdot \rho_i \\ &> x_i^* \cdot r_i^m \cdot \rho_i^m \\ &= x_i^* u_i^m, \end{aligned}$$

where the inequality follows from the presumption that $\rho_i^m < \rho_i$, the assumption in the proposition statement that $x_i^* > 0$, and $r_i^m > 0$ as noted above. Hence utility is higher using the un-masked version of activity i than using the masked version of activity i. Contradiction. ☐

A.2.3 Statement and Proof of Sufficient Condition for Mask Adoption

Let ρ_m^* denote the utility-to-risk ratio of the marginal activity (as defined in (7)) when the mask policy vector is m. Formally,

$$\rho_m^* = \sup \left\{ \tilde{\rho} : \sum_{j : \rho_j^m \geq \tilde{\rho}} m_j r_j^m + \sum_{j : \rho_j \geq \tilde{\rho}} (1 - m_j) r_j > 1 \right\}.$$

Similarly, for any mask policy vector m and feasible activity vector x, we can define $\bar{\rho}(m, x)$ as the best utility-to-risk ratio from activities not fully performed in x:

$$\bar{\rho}(m, x) = \sup \left\{ m_i \rho_i^m + (1 - m_i) \rho_i \mid i : x_i < 1 \right\}.$$

We can also view $\bar{\rho}(m, x)$ as the marginal utility-to-risk ratio at x because it is the ratio of the best activity to add to activity vector x if the risk budget increases by a small amount.

Note that $\bar{\rho}(m, x^*(m)) = \rho_m^*$ by construction.

We will state two versions of the sufficient condition: the first for a single activity i and the second for a set of activities J.

Proposition 3. *(Sufficient Condition, Activity i Mask Adoption). Suppose activity i satisfies the necessary condition, and define*

$$\underline{\rho}_i^* = \min_{m_{-i} \in M_{-i}} \rho_{(m_i = 1, m_{-i})}^*.$$

If the risk constraint binds at the optimal activity vector under the full-mask policy $m = \mathbf{1}$, and it holds that

$$\underline{\rho}_i^* \geq \frac{\Delta u_i}{\Delta r_i},$$

then $OPT(m_i = 1, m_{-i}) \geq OPT(m_i = 0, m_{-i})$ for any m_{-i}. In words, it is optimal to adopt the mask for activity i under any mask policy for the other activities m_{-i}.

Proof. Consider an arbitrary m_{-i}. To simplify the notation, write $m_0 = (m_i = 0, m_{-i})$ and $m_1 = (m_i = 1, m_{-i})$. We want to show that $OPT(m_1) \geq OPT(m_0)$. If $x_i^*(m_0) = 0$, then masking activity i cannot hurt utility. Therefore assume $x_i^*(m_0) > 0$, that is, activity i is used in the optimum if it is not masked. We construct a feasible activity vector $\tilde{x}$ under mask policy m_1 which we show leads to weakly greater utility. In words, $\tilde{x}$ starts with activity vector $x^*(m_0)$, and then optimally spends the extra risk budget that is freed up by masking activity i. More formally, let $\tilde{x}$ be the optimal activity vector under mask policy m_1 under the additional constraint that $\tilde{x}_j \geq x_j^*(m_0)$ for all activities j. We prove the result with the following steps:

$$
\begin{aligned}
OPT(m_1) &\geq U(\tilde{x}, m_1) && (14)\\
&= U(x^*(m_0), m_1) + U(\tilde{x} - x^*(m_0), m_1) && (15)\\
&= \underbrace{U(x^*(m_0), m_0)}_{\text{Utility from } x^*(m_0)} - \underbrace{x_i^*(m_0) \cdot \Delta u_i}_{\substack{\text{Cost of masking}\\ \text{activity } i}} + \underbrace{U(\tilde{x} - x^*(m_0), m_1)}_{\substack{\text{additional utility from}\\ \text{spending the freed-up budget}}} && (16)\\
&= OPT(m_0) - x_i^*(m_0) \cdot \Delta u_i + x_i^*(m_0) \cdot \Delta r_i \cdot \frac{U(\tilde{x} - x^*(m_0), m_1)}{x_i^*(m_0) \cdot \Delta r_i} && (17)\\
&\geq OPT(m_0) - x_i^*(m_0) \cdot \Delta u_i + x_i^*(m_0) \cdot \Delta r_i \cdot \bar{\rho}(m_1, \tilde{x}) && (18)\\
&\geq OPT(m_0) - x_i^*(m_0) \cdot \Delta u_i + x_i^*(m_0) \cdot \Delta r_i \cdot \rho_{m_1}^* && (19)\\
&= OPT(m_0) + x_i^*(m_0) \cdot (\rho_{m_1}^* \cdot \Delta r_i - \Delta u_i) && (20)\\
&\geq OPT(m_0) + x_i^*(m_0) \cdot (\underline{\rho}_i^* \cdot \Delta r_i - \Delta u_i) && (21)\\
&\geq OPT(m_0). && (22)
\end{aligned}
$$

Step (14) follows from the definition of optimality. Step (15) adds and substracts the vector $x^*(m_0)$ which by construction is weakly smaller in all elements than $\tilde{x}$. Step (16) explicitly breaks out the utility cost of adopting a mask for activity i from the utility under the optimal activity vector if i is not masked. Step (17) notes that the first element is the optimal utility under the original mask policy m_0, and multiplies and divides the last term by $x_i^*(m_0) \cdot \Delta r_i$.

Step (18) is a key step in the argument. In the term $\frac{U(\tilde{x} - x^*(m_0), m_1)}{x_i^*(m_0) \cdot \Delta r_i}$, the denominator is the amount of risk budget freed up by masking activity i, and the numerator is the utility that is gained by spending this freed-up risk budget to go from activity vector $x^*(m_0)$ to activity vector $\tilde{x}$. Thus the ratio is the average utility-to-risk ratio of the additional expenditure. Because this freed-up budget is spent optimally over activities not in $x^*(m_0)$ — that is, in descending order of utility-to-risk — this average must be larger than the utility-to-risk ratio of the marginal activity left undone (or not done in full) under $\tilde{x}$, namely $\bar{\rho}(m_1, \tilde{x})$. Step (19) then follows by noting $\rho_{m_1}^* \leq \bar{\rho}(m_1, \tilde{x})$ by construction. In words, the optimal activity vector $x^*(m_1)$ at mask policy m_1, without the constraint that $\tilde{x}_j \geq x_j^*(m_0)$, must have a weakly lower marginal utility-to-risk than the activity vector with the constraint, because the optimum consumes in strict descending order of utility-to-risk under m_1 whereas the constrained policy might not.

Step (20) rearranges terms. Step (21) follows from the definition of $\underline{\rho}_i^*$. Step (22) follows from the assumption in the statement of the proposition. Stringing these all together gives $OPT(m_1) \geq OPT(m_0)$, as required. □

We now state the sufficient condition for a set of activities.

Proposition 4. *(Sufficient Condition, Mask Adoption for a Set of Activities). Suppose all activities i in set J satisfy the necessary condition, and define*

$$\rho_J^* = \min_{m_{-J} \in M_{-J}} \rho^*_{(m_J=\mathbf{1}, m_{-J})}.$$

If the risk constraint binds at the optimal activity vector under the full-mask policy $m = \mathbf{1}$, and it holds for all activities i in set J that

$$\rho_J^* \geq \frac{\Delta u_i}{\Delta r_i} \cdot,$$

then $OPT(m_J = \mathbf{1}, m_{-J}) \geq OPT(m'_J, m_{-J})$ for any m_{-J} and any m'_J. In words, it is optimal to adopt masks for all activities in set J under any mask policy for the other activities m_{-J}.

The proof for Proposition 4 is essentially identical to that for Proposition 3 and is omitted.

A.2.4 Mathematical Relationship Between Necessary and Sufficient Conditions.

The relationship between the necessary and sufficient conditions can be seen as follows:

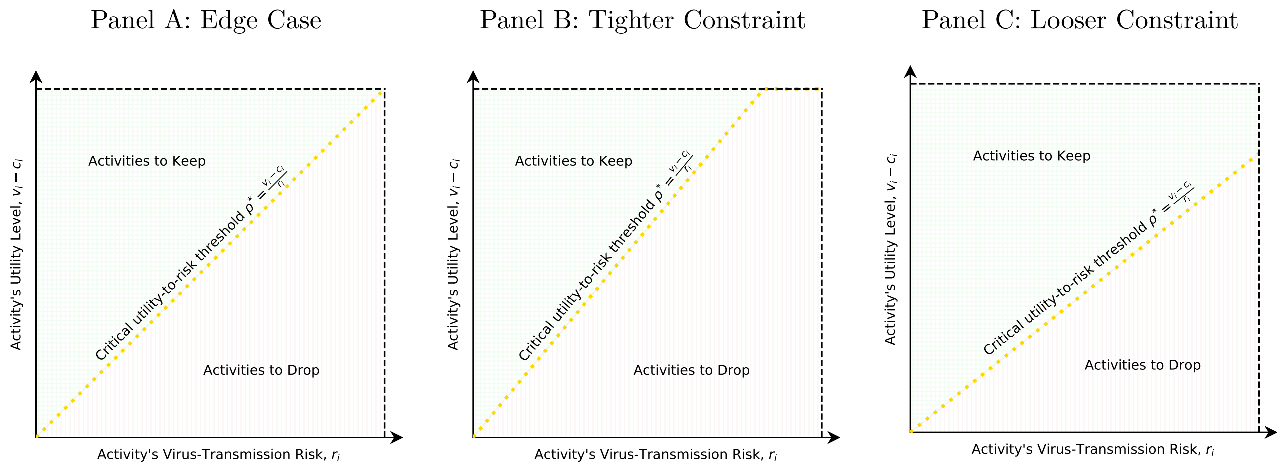
$$\begin{aligned}
\rho_i^m \geq \rho_i &\Leftrightarrow \frac{u_i^m}{r_i^m} \geq \frac{u_i}{r_i} \\
&\Leftrightarrow u_i^m r_i \geq u_i r_i^m \\
&\Leftrightarrow u_i^m r_i - u_i^m r_i^m \geq u_i r_i^m - u_i^m r_i^m \\
&\Leftrightarrow (r_i - r_i^m) u_i^m \geq (u_i - u_i^m) r_i^m \\
&\Leftrightarrow (r_i - r_i^m) \cdot \frac{u_i^m}{r_i^m} \geq u_i - u_i^m \\
&\Leftrightarrow \rho_i^m \geq \frac{\Delta u_i}{\Delta r_i}.
\end{aligned}$$

Thus, while the sufficient condition requires that the *marginal activity's* utility-to-risk ratio exceeds $\frac{\Delta u_i}{\Delta r_i}$, the utility-cost per unit of freed-up risk budget, the necessary condition requires that the *activity's own* utility-to-risk ratio exceeds $\frac{\Delta u_i}{\Delta r_i}$. If included in the optimum, the activity is by definition weakly inframarginal, thus the necessary condition is less demanding than the sufficient one.

A.3 Characterization of ρ^* for Numerical Example

As in the theory of Section 3, the optimal policy is characterized by a threshold strategy ρ^* such that any activity with $\rho \geq \rho^*$ is done in full, and any activity with $\rho < \rho^*$ is completely dropped. Since the optimal policy must fully spend the disease-transmission budget, the problem is solved by finding the ρ^* such that the total risk cost from all activities above the line $u_i = \rho^* r_i$ equals the targeted level $K \in [0, R_0]$. Depending on the targeted value K, we may have 3 different cases as depicted in Figure 10.

Figure 10: **Optimal Threshold Illustration**



Notes: This figure depicts the optimal utility-to-cost ratio threshold under 3 different cases. For each panel, the vertical axis represents the net utility of activities and the horizontal axis represents the transmission costs. The XY-plane represents the space of all activities. Panel A shows the edge case where the targeted disease transmission $K = R_0/3$. In this case, the transmission costs from all activities above the diagonal integrate to K. Panel B shows the case of a tighter constraint when $K < R_0/3$. Panel C shows the case of a looser constraint when $K > R_0/3$.

Case 1: Edge Case. We first focus on the edge case depicted in Figure 10 Panel A where the costs from all activities above the diagonal sum exactly to the targeted level K. In this case, $\rho^* = \frac{1}{2R_0}$. We now calculate the total risk costs from all activities in the green hatched area above the diagonal

$$\begin{aligned}
\int_0^{2R_0} \int_{\rho^* r}^{1} \frac{r}{2R_0} du dr &= \int_0^{2R_0} \frac{1}{2R_0}(1 - \rho^* r) r dr \\
&= \frac{1}{2R_0} \cdot (\frac{1}{2}r^2 - \frac{1}{3}\rho^* r^3)\Big|_0^{2R_0} \\
&= \frac{1}{2R_0} \cdot (\frac{1}{2} \cdot 4R_0^2 - \frac{1}{3} \cdot \frac{1}{2R_0} \cdot 8R_0^3) \\
&= \frac{R_0}{3}
\end{aligned}$$

Therefore, when $K = \frac{R_0}{3}$, the total risk costs from all activities in the green hatched area above the diagonal exactly equal the disease transmission budget.

Case 2: Tighter Constraint When $K < R_0/3$, we fill up the transmission budget with a smaller triangle (Figure 10 Panel B). Line $u_i = \rho^* r_i$ intersects the square at point $(1/\rho^*, 1)$. The total transmission cost of activities allowed is given by:

$$\begin{aligned}
\int_0^{1/\rho^*}\int_{\rho^* r}^{1}\frac{r}{2R_0}du dr &= \int_0^{1/\rho^*}\frac{1}{2R_0}(1-\rho^* r)r dr \\
&= \frac{1}{2R_0}\cdot(\frac{1}{2}r^2-\frac{1}{3}\rho^* r^3)\Big|_0^{1/\rho^*} \\
&= \frac{1}{2R_0}\cdot(\frac{1}{2}\cdot\frac{1}{\rho^{*2}}-\frac{1}{3}\rho^*\cdot\frac{1}{\rho^{*3}}) \\
&= \frac{1}{12R_0\rho^{*2}}
\end{aligned}$$

The total cost should equal the targeted level K,

$$\frac{1}{12R_0\rho^{*2}} = K \implies \rho^* = \frac{1}{\sqrt{12R_0K}}.$$

Case 3: Looser Constraint When $X > R_0/3$, we need to fill up the transmission budget with activities from the green hatched trapezoid region (Figure 10 Panel C). Line $u_i = \rho^* r_i$ intersects the square at point $(2R_0, 2R_0\rho^*)$. The total cost from activities in the green hatched trapezoid is given by the myopic total transmission cost R_0 minus the cost from the red striped triangle:

$$\begin{aligned}
R_0 - \int_0^{2R_0}\int_0^{r\rho^*}\frac{r}{2R_0}du dr &= R_0 - \int_0^{2R_0}\rho^* r^2\cdot\frac{1}{2R_0}dr \\
&= R_0 - \frac{1}{2R_0}\cdot\rho^*\cdot\frac{1}{3}r^3\Big|_0^{2R_0} \\
&= R_0 - \frac{1}{2R_0}\cdot\rho^*\cdot\frac{1}{3}\cdot 8R_0^3 \\
&= R_0 - \frac{4}{3}\rho^* R_0^2
\end{aligned}$$

At the targeted transmission level K,

$$R_0 - \frac{4}{3}\rho^* R_0^2 = K \implies \rho^* = \frac{3(R_0-K)}{4R_0^2}.$$

Bringing these three cases together we have:

$$\rho^* = \begin{cases} \frac{3(R_0-K)}{4R_0^2} & \text{if } K \geq \frac{R_0}{3}, \\ \frac{1}{\sqrt{12R_0K}} & \text{if } K \leq \frac{R_0}{3}. \end{cases}$$

□