Uncertainty and Decision-Making During a Crisis: How to Make Policy Decisions in the COVID-19 Context?

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Policymaking during a pandemic can be extremely challenging. As COVID-19 is a new disease and its global impacts are unprecedented, decisions need to be made in a highly uncertain, complex and rapidly changing environment. In such a context, in which human lives and the economy are at stake, we argue that using ideas and constructs from modern decision theory, even informally, will make policymaking more a responsible and transparent process.

Keywords
model uncertainty, ambiguity, robustness, decision rules

1. Introduction

The coronavirus disease 2019 (COVID-19) pandemic exposes clear decision problems faced by governments and international organisations. Policymakers are charged with taking actions to protect their population from the disease, whilst lacking reliable information on the virus and

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its transmission mechanisms, on the effectiveness of possible actions, and their (direct and indirect) health and socio-economic consequences. The rational policy decision would combine the best available scientific evidence—typically provided by expert opinions and modelling studies—but in an uncertain and rapidly changing environment, the pertinent evidence is highly fluid, making it challenging to produce scientifically-grounded predictions of the outcomes of alternative courses of action.

Given this context, a great deal of attention has been paid to how policymakers have handled uncertainty in the COVID-19 response (Lazzerini and Putoto 2020; Chater 2020; Anderson et al. 2020; Emanuel et al. 2020; Hansen 2020). Policymakers have been confronted with very different views on the potential outbreak scenarios stemming from divergent experts' assessments or differing modelling predictions. In the face of such uncertainty, policymakers may respond by attempting to balance the alternative perspectives, or they may fully embrace one without a concern that this can vastly misrepresent our underlying knowledge base (Johnson-Laird 2010). This tendency to lock on to a single narrative—or more generally, this inability to handle uncertainty—may result in overlooking valuable insights from alternative sources, and thus in misinterpreting the state of COVID-19 outbreak, potentially leading to suboptimal decisions with possible disastrous consequences (Chater 2020; Cancryn 2020; Rucker et al. 2020).

In this paper, we argue that insights from decision theory provide a valuable way to frame policy challenges and ambitions. Even if the decision theory constructs are ultimately used only informally in practice, they provide a valuable guide for policymaking that is both transparent and copes with the severe uncertainty in sensible ways. First, we outline a framework to understand and guide decision-making under uncertainty in the context of the COVID-19 pandemic. Second, we show how formal decision rules could be used to guide policymaking and illustrate their use with the example of school closures. The decision rules we present allow policymakers to recognise that they do not know which of the many potential scenarios is ‘correct’ and to act accordingly by making precautionary and robust decisions, which remain valid for a wide range of futures and keep options open (Lempert and Collins 2007). Third, we discuss new directions to define a more transparent approach for communicating the degree of certainty in scientific findings and knowledge, which is particularly relevant to decision makers managing pandemics.

2. Decision under uncertainty

2.1. The policymaker’s problem(s)

The decision-making problem faced by a high-level government policymaker during a crisis like the COVID-19 pandemic is not trivial. In a first stage, when a new infectious disease appears, the policymaker may attempt to contain the outbreak by taking early actions to control onwards transmission (e.g. isolation of confirmed and suspected cases, contact tracing). If this containment phase is unsuccessful, the policymaker faces a second-stage decision problem, which consists in determining the appropriate level, timing and duration of interventions to mitigate the course of clinical infection, such as banning mass gatherings, closing schools and more extreme ‘lockdown’ restrictions.

While these measures are expected to reduce the burden of the pandemic by lowering the peak incidence, they also impose extra costs on society. For instance, at a more personal
level, they may have adverse impacts on mental health, domestic abuse, and job loss. Moreover, there are societal losses due to the immediate reduced economic activity coupled with a potentially prolonged recession as well as adverse impacts on longer term health and social gradients. Policymakers must thus promptly cope with a complex and multi-faceted picture of direct and indirect, proximal and distal, health and socio-economic trade-offs. If, in the acute phase of the pandemic, the trade-off between reducing mortality and morbidity and its associated socioeconomic consequences may seem relatively straightforward, in general most trade-offs are difficult and costly. How should the policymaker decide when and how to bring in, and then relax, constraints and measures in a way that is justifiable, not just from a health and economic perspective, but politically? The answer critically depends on the prioritisation and balance of potentially conflicting objectives (Hollingsworth et al. 2011).

2.2. Scientific evidence and the role of modelling

Scientific knowledge is foundational to the prevention, management and treatment of global outbreaks. Some of this evidence can be summarised in pandemic preparedness and response plans (at both international and national levels), or might be directly obtained from panels of scientists with expertise in relevant areas of research, such as epidemiologists, infectious disease modellers, and social scientists. An important part of the scientific evidence comes from quantitative models (Morgan 2019). Quantitative models are abstract representations of reality that provide a logically consistent way to organise thinking about the relationships among variables of interest. They combine what is known in general, with what is known about the current outbreak, to produce predictions to help guide policy decisions (Den Boon et al. 2019). Models have then been used to understand some of the key features of the virus that we need to know in order to decide how to respond to it. Ultimately, models are particularly useful to project the health and economic consequences of different public health interventions and to guide decisions on resource allocations (Holmdahl and Buckee 2020) (more details on the various types of models used are provided in S1 Appendix).

2.3. Uncertainty

Decisions within a pandemic context have to be made under an overwhelming time pressure and amid high scientific uncertainty, with minimal quality evidence, and potential disagreements among experts and models. This is specifically the case for the current pandemic as COVID-19 is a new disease. For example, at the early stage of the pandemic, there is uncertainty about the basic characteristics of the virus, such as its transmissibility, severity and natural history (Anderson et al. 2020; Hellewell et al. 2020; R. Li et al. 2020). This translates into uncertainty about the system dynamics, which in turn makes the consequences of alternative policy interventions, such as school closures or wearing masks in public, uncertain. At a later stage of the pandemic, information overload becomes an issue, making it more difficult for the decision maker to identify useful and good-quality evidence. The consequence is that, given the many uncertainties they are built on, no single model can be truly predictive in the context of an outbreak management strategy. Yet, if their results are used as insights providing potential quantitative stories, among alternative ones, models can offer guidance to policymakers by helping them to understand the fragments of information available, uncover what might really be going on, and eventually help determine the appropriate policy response. The distinction between three layers of uncertainty—uncertainty within models, across models and about models—can help the policymaker understand the
extent of the problem (Hansen 2014; Marinacci 2015; Hansen and Marinacci 2016; Aydogan et al. 2018).

**Uncertainty within models**

Uncertainty *within* models reflects the standard notion of risk: uncertain outcomes with known probabilities. Models may include random shocks or impulses with prespecified distributions. It is the modelling counterpart to flipping coins or rolling dice in which we have full confidence in the probability assessment.

**Uncertainty across models**

Uncertainty *across* models encompasses both the unknown parameters for a family of models or more discrete modelling differences in specification. Thus, it relates to unknown inputs needed to construct fully specified probability models. In the COVID-19 context, this corresponds for example to the uncertainty of some model parameters, such as how much transmission occurs in different age groups or how infectious people can be before they have symptoms. Existing data, if available and reliable, can help calibrate these model inputs. Another challenge for the policymaker is that there is a proliferation of modelling groups, researchers and experts in various disciplines (epidemiology of course, but also economics and other social sciences). Each of them provides forecasts and projections about the evolution of the disease and/or its socioeconomic consequences. This uncertainty *across* models and their consequent predictions may be difficult to handle by policymakers, especially as one approach is not necessarily superior to another but simply adds another perspective (S.-L. Li et al. 2017). There is no single ‘view.’

Analysis of this form of uncertainty is typically the focal point of statistical approaches. Bayesian analyses, for instance, confront this via the use of subjective probabilities, whereas robust Bayesians explore sensitivity to prior inputs. Decision theory explores the ramifications of subjective uncertainty, as there might be substantial variation in the recommendations *across* different models and experts, reflecting different specific choices and assumptions regarding modelling type and structure.

**Uncertainty about models**

Finally, as models are, by design, simplifications of more complex phenomena, they are necessarily misspecified, at least along some dimensions. For instance, they might not mention certain variables that matter, which modelers are or are not aware of, or they may be limited in the scope of functional relationships considered, unknown forms of specification and measurement errors, and so forth. In consequence, there is also uncertainty *about* the assumptions and structures of the models themselves (e.g., how do they depict behaviours among the variables of interest), and it might sometimes be challenging, even for experts, to assess the merits and limits of alternative models and predictions*. This is what we mean in our reference to uncertainty *about* models.

3. How to make rational decisions under uncertainty?

Having characterised the elements of the decision problem under uncertainty (see Figure 1), the question remains how to make the best possible decision? In other words, how should the

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* Note that another way to see this additional layer of uncertainty is as uncertainty over predictions of alternative models that have not been developed yet.
policymaker proceed to aggregate the different (and usually conflicting) scientific findings, model results and expert opinions, which are themselves uncertain by construction and by lack of reliable data, and ultimately determine policy? This is where insights from modern decision theory are of value. It proposes a formal approach, taking the form of normative guidelines and “rules,” to help policymakers make the best, i.e. most rational decision under uncertainty.

![Decision Problem under Uncertainty](image)

**Figure 1:** Overview of the decision problem under uncertainty

### 3.1. How can formal decision rules be useful?

The formal decision rules proposed by decision theorists are powerful, mathematically-founded† tools that relate theoretical constructs and choice procedures to presumably observable data. Making a decision based on such rules is equivalent to complying implicitly to a set of general consistency conditions or principles governing human behaviour. During a crisis like the COVID-19 pandemic, using decision theory as a formal guide to policymaking will lend credibility by ensuring that the resulting actions are coherent and defensible. To illustrate how decision theory can be acting as a coherence test (Itzhak Gilboa and Samuelson 2020), imagine the case of a policymaker trying to determine what is the best response to give in the current pandemic. The decision makers can make up their mind by whatever mix of

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† Typically, each of these rules results from an axiomatisation (i.e., an equivalence result taking the form of a theorem that relates a theoretical description of decision-making to conditions on observable data (Itzhak Gilboa et al. 2019)).
intuition, expert advice, imitation, and quantitative model results they have available, and then check their judgement by asking whether they can justify the decision using a formal decision rule. Conceptually, it can be seen as a form of dialogue between the policymakers and decision theory, in which an attempt to justify a tentative decision helps to clarify the problem and, perhaps, leads to a different decision. Used this way, formal decision rules may constitute a huge help to the policymakers in clarifying the problem they are dealing with, testing their intuition, avoiding reasoning mistakes, and avoiding potential pitfalls that we know from psychology (e.g., confirmation bias, optimism bias, representativeness heuristic, prospect theory, etc.) (Cairney and Kwiatkowski 2017).

It is likely that, at some point, committees will be investigating how decisions were taken during the crisis and policymakers will be held to account for their actions, for example about the way lockdown measures were implemented and lifted. A formal decision model can be an important part to defend one’s choice and generate ex post justifiability. For example, a policymaker who had to decide which neighborhoods to keep under lockdown and which not will have to be able to explain how such decisions have been made to citizens who might think they have not been treated fairly.

3.2. Which decision rule to follow?

As decision theory does not propose a single, but a variety of different, normative rules to make a decision under uncertainty, the question that naturally arises is which rule to follow? The answer to this question ultimately depends on the policymakers’ characteristics, e.g., which conditions or behavioural principles they want to comply with, or how prudent they want the policy to be. In order to demonstrate how distinct quantitative model outputs can be combined and used in formal decision rules, and what are the resulting recommendations in terms of policy responses, we present a simple example of a decision problem of school closures during the COVID-19 pandemic in Figure 2 (details are provided in S2 Appendix). The policymaker’s problem consists in finding the right balance between protecting health and preventing economic and social disruptions by choosing whether and for how long to keep schools closed, given the scarce scientific evidence and the disagreement that may exist across model projections.

Decision rules primarily differ in how they handle probabilities. According to the Bayesian view, which holds that any source of uncertainty can be quantified probabilistically, the policymaker should always have well-defined probabilities about the impacts of the measures taken. If they rely on quantitative model outputs or expert advice to obtain different estimates, then they should attach a well-defined probability weight to each of them and compute an average. Thus, in the absence of objective probabilities, the decision makers have their own subjective probabilities that they use to guide decisions.

However, it may not always be rational to follow this approach (Itzhak Gilboa, Postlewaite, and Schmeidler 2008; 2009; 2012; Itzhak Gilboa and Marinacci 2013), which is limited because of its inability to make the distinction between epistemic uncertainty (due to limited knowledge or ignorance) and aleatory uncertainty (due to the intrinsic randomness in the world). In the response to COVID-19, the Bayesian approach requires the policymaker to express probabilistic beliefs (about the impact of a policy, about the correctness of a given model, etc), without being told which probability it makes sense to adopt, nor being allowed to say “I don’t know”. In view of the disagreements that may exist across different model outputs, or expert
Case study: Decisions about school closures and their length during COVID-19 pandemic

In this case study, we explore the problem of a policymaker having to make a decision about school closures and their length during the COVID-19 pandemic, and illustrate the difference of policy prescribed by different decision rules.

1. Context: By end of April 2020, 191 countries had implemented national school closures in response to the COVID-19 pandemic (United Nations Educational Scientific and Cultural Organization 2020). Yet the effectiveness of such a measure is highly uncertain, due to the lack of data on the relative contribution of school closures to transmission control, and conflicting modelling results (Viner et al. 2020).

2. The policymaker’s problem: Decisions about closures and their length involve a series of trade-offs. The policy assessment thus involves weighing benefits and costs of alternative courses of action. On the one hand, school closures can slow the pandemic and its impact by reducing child-child transmission, thus delaying the pandemic peak that overwhelms health care services, and therefore ultimately reducing morbidity and associated mortality. If this is the case, such interventions bring clear health benefits for the society and avoid unsustainable demands on the health system. On the other hand, school closure can have high direct and indirect health and socio-economic costs. For example, they may increase child-adult transmission, reduce the availability of healthcare and key workers to work and thus reduce the capacity of healthcare (Brooks et al. 2020; Bayham and Fenichel 2020). Economic costs of lengthy school closures are also high (Sadique, Adams, and Edmunds 2008; Keogh-Brown et al. 2010; Lempel, Epstein, and Hammond 2009), generated for example through absenteeism by working parents, loss of education, etc.

3. Uncertainty: The evidence supporting national closure of schools in the COVID-19 pandemic context is weak. In particular, evidence of COVID-19 transmission through child-to-child contact or through schools is not available at the time of decision (Viner et al. 2020). As a consequence, it is unclear whether school closures are effective in the COVID-19 pandemic (Bayham and Fenichel 2020).

4. The formal decision problem

4.1 Setup

In this example, the policy action is to choose whether to and how long to close school for. The consequence includes both the benefits of the action (lives saved, reduction in future cases and beds needed, etc.) and its costs (reduction in education for children, health care workers and other key workers not working, lives cost due to changes in disease dynamics, etc.). The consequence also depends on the realisation of a state of the environment. For example, the way the number of deaths, beds, and cases are affected by school closures depends on the biology of the virus and baseline transmission dynamics, which are outside the decision maker’s control. The consequence function then relates actions and states to consequences. It can for example be the net benefit (benefit - costs) of school closures, expressed in monetary terms.

4.2 Model uncertainty

As there is no evidence supporting school closures at the early stage of the COVID-19 pandemic, policymakers rely on different epidemiological model projections and/or the advice of experts to assess the effectiveness of the measure. For example, imagine three different projections:
- The first projection (Model 1) is based on the only evidence we have, which is the one coming from influenza outbreaks for which the majority of transmission is between children (Mangtani and others 2014). Closing schools is thus the biggest contributor to reducing $R_t$ to below 1 and it may be the only intervention that could do so. In this case the benefit is proportional to the duration of school closure.
- Alternatively, the second projection (Model 2) relies on some previous coronavirus outbreaks, for which evidence suggests minimal transmission between children (Wong et al. 2003). Here, $R_t$ cannot be reduced below 1, school closures do not affect the size of the epidemic, and therefore do not bring any benefits.
- The third scenario (Model 3) projects that some child to child transmission happens so that closing schools contributes to reducing $R_t$ to below 1 and reduces the size of the epidemic. However, this only works in combination with other measures (Prem et al. 2020). Without it, $R_t$ would be above 1, but an isolated measure school closure does not have such a big effect (Ferguson et al. 2020). Under this scenario, the effectiveness of school closures is important at the beginning but declines as time goes on.

Assuming there is no uncertainty regarding the costs of school closures, the collection of potential models characterizing a combination of health/economy environment, $M$ consists of three elements.

4.3 Policy objective

In view of the scarce evidence concerning the use and effectiveness of school closures during the COVID-19 pandemic, as well as the disagreement that may exist across model projections and/or expert opinions, policymakers have to find the right balance between protecting health and preventing economic and social disruptions. Choosing the appropriate length of this apparently common-sense measure may be exceptionally challenging as lengthy school closures bear very high costs and can therefore substantially reduce any benefit to health systems and populations, whereas earlier relaxation of the measure increases the risk that transmission surged again, leading to a second peak.

Preferences The choice made by the policymaker ultimately depends on her preferences, such as the degree to which she likes/dislikes uncertainty (Berger and Bossetti 2020). These preferences are represented numerically via a decision rule $V$. The value $V(a)$ attained by selecting an action $a$ may be interpreted in welfare terms.

Optimum Before making a decision, the policymaker knows all the elements of the decision problem. After the decision, she only observes the consequence of the intervention chosen. In formal terms, the objective of the policymaker is to select the action $a$ (in our case, the duration of school closures) that is optimal according to her preferences, in the sense that it is preferred to any other action available.

4.4 Illustration

The marginal benefit (MB) of school closure is constant and positive (model 1), null (model 2), or positive and decreasing with the duration of the measure (model 3). In contrast, the marginal cost (MC) is increasing. It is desirable to maintain the school closed as long as the MB outweighs the MC. So, if the ‘true’ MB was known, it would be easy to find the optimal duration of school closure: 0 if model 2 is the correct one, 10 if it is model 3, and 20 in the case of model 1. Yet, in reality, there is a lot of uncertainty.

5. Decision rules and optimal choices

To address the epistemic uncertainty across models, the policymaker may follow different decision rules. These rules differ on whether the policymaker may quantify her belief about which is the correct model. If it exists, we let $\mu(m)$ be the policymaker’s subjective belief that $m$ is the true model.

5.1 Subjective Expected utility rule

5.2 Smooth ambiguity rule

5.4 Multiple priors rule

opinions, another path may be to acknowledge one’s own ignorance and to relax the assumption that we can associate precise probabilities to any event. Modern decision theory proposes decision rules in line with this non-Bayesian approach. These rules, which are fully compatible with normative interpretations, could be particularly useful to design robust policies in this COVID-19 pandemic context, as detailed in Figure 2.
Expected utility has long been the standard way to consider rational decision making under uncertainty (Savage 1954). We assume that a utility function $u$ translates economic monetary consequences into utility levels. This function captures attitudes towards uncertainty within models. For each action $a$ and each model $m$, we can compute the expected reward associated with a given action: $R(a,m) = \sum u(\mu, z)m(z)$. Given that different models exist, an expected reward is considered each possible model. They are averaged according to the policymaker’s beliefs. The subjective expected utility has long been the standard way to frame the discussion of policies using insights from decision theory. This could clarify the tradeoffs and encourage a more sanguine response to the uncertainties that are present when making the decision maker consider only the model providing the lowest expected reward. This is the case when the “worst” health/economy model is considered (i.e. model 1). Here, prior probabilities do not play any role when choosing the optimal policy. The maxmin criterion is $V_{\min}(a) = \min_m R(a,m)$. In our example, the optimal policy is to keep the school closed for 20 weeks.

The smooth ambiguity criterion (Klibanoff, Marinacci, and Mukerji 2005) proposes another way to distinguish attitudes toward uncertainty within and across models. It takes the form $V_{\phi}(a) = \sum_\phi R(a, m)\mu(m)$, where the concavity of $\phi$ reflects uncertainty aversion (i.e. being more averse to uncertainty across models than $u$ within models). In our example, if the prior distribution is uniform, and $\phi$ is logarithmic, the optimal policy is $a=12$ weeks.

5.3 Maxmin rule
The maxmin rule (Wald 1950) is an extremely cautious rule that makes the decision maker consider only the model providing the lowest expected reward. This is the case when the “worst” health/economy model is considered (i.e. model 1). Here, prior probabilities do not play any role when choosing the optimal policy. The maxmin criterion is $V_{\phi}(a) = \min_m R(a,m)$, where the concavity of $\phi$ reflects uncertainty aversion (i.e. being more averse to uncertainty across models than $u$ within models). In our example, the optimal policy is $a=12$ weeks.

During a period of crisis, policymakers, who make decisions on behalf of others, may be required to provide a protocol that suggests a decision theoretic model supporting their decisions. Decision theory can contribute to pandemic response by providing a way to organise a large amount of potentially conflicting scientific knowledge and providing rules for evaluating response options and turning them into concrete decision-making. In practical terms, one way to make sure that the policy options on the table are in line with formal decision rules could be achieved by having a decision analyst in the group of advisors to nurture a dialogue and better frame the discussion of policies using insights from decision theory. This could clarify the tradeoffs and encourage a more sanguine response to the uncertainties that are present when assessing the alternative courses of action and result in an improved policy outcome (Itzhak Gilboa and Samuelson 2020; Itzhak Gilboa et al. 2018).

An alternative approach (I. Gilboa and Schneider 1989) relaxes the assumption that the policymaker can quantify uncertainty across models through a single probability distribution $\mu$. Instead, because she does not have sufficient information, the policymaker may have multiple priors over the different models. The multiple priors decision rule is $V_{\mu}(a) = \min_\delta \sum_m R(a,m)\mu(m)$, where $\delta$ is the set of priors. In contrast with the maximin rule the least favorable model, this criterion considers the least favorable among all the subjective expected utilities determined by each prior $\mu$. In our example, a particular prior distribution may be the uniform that gives equal weights, $\mu(m) = 1/3$, to all the possible models, while another prior may not consider model 2 to be plausible (in which case, some $\mu(2)=0$). In this case, the multiple prior rule leads to an optimal policy of 13 weeks.

4. Discussion
This case study exemplifies that looking across model projections (some of which represent “best guesses” while others represent “reasonable worst-case” possibilities) can lead to significantly different optimal policies. Overall, our point is that when exploring alternative courses of action, policymakers are necessarily unsure of the consequences. In this context, sticking to the Bayesian paradigm may not only require substantive expertise (in weighting the pros and cons of alternative models), but also overshadow the policymaker’s reaction to the variability that may exist across models.

In this example, the decision problem setup has been deliberately kept of minimal complexity in order to focus on the decision theory aspects. In particular, the set of actions is here limited to a single intervention (the duration of the school closure), while in reality the decision problem would, of course, require a much higher dimensional space (e.g., selective local closures, school dismissal, etc), the interaction with other social distancing measures, or the ability to integrate start and stop times. For expositional simplicity, we also abstracted from concerns about model misspecification, while recognising this to be an integral part of how decision makers should view the alternative models or perspectives that they confronted.

\footnote{As these projections are typically premised on “reasonable” bounds in terms of their model inputs.}
To make the decision-making process under uncertainty more efficient, we also suggest acknowledging and communicating the various uncertainties in a transparent way (Manski 2019). For example, illustrating, quantifying, and discussing the various sources of uncertainty may help policymakers better understand the potential impact of their choices. This implies that modellers provide all information that is needed to reconstruct the analysis, including information about the choices of model structures, assumptions, and parameter values. Moreover, the way uncertainty around these choices is reflected into the model results needs to be accurately communicated, for example through systematically reporting uncertainty boundaries around the estimates provided (World Health Organization and other 2019). It is then the role of scientific and policy advisors to synthesise all this information (Cairney and Kwiatkowski 2017), coming from diverse sources across different disciplines, possibly of different quality, and to help policymakers to turn it into actionable information for decisions, while making sure the complete range of uncertainty (including within and across models) is clearly reported and understood properly (Spiegelhalter, Pearson, and Short 2011; Bosetti et al. 2017).

One possible way to go is to enhance standardisation by developing and adopting common metrics for evaluating and communicating the degree of certainty in key findings. While several approaches have been proposed, insights could, for example, be gained from the virtues and the shortcomings of the reports of the Intergovernmental Panel on Climate Change (IPCC) (Mastrandrea et al. 2010). Another way is to develop further communication and collaboration between model developers and decision makers to improve the quality and utility of models and the decisions they support (Rivers et al. 2019).

Finally, while policymakers are responsible for making decisions, they are also responsible for communicating to professionals and to the public. The way individuals respond to advice and measures selected is as important as government actions, if not more (Anderson et al. 2020). Communication should thus be an essential part of the policy response to uncertainty. In particular, government communication strategies to keep the public informed of what we (do not (World Health Organization and others 2017)) know should balance the costs and benefits of revealing information (how much, and in what form) (Aikman et al. 2011).

As government strategies have been extensively debated in the media and models have become more scrutinised, one lesson learned from the COVID-19 management experience may be that policymakers and experts must increase the transparency of their approaches. Using the constructs from decision theory in policymaking, even in an informal way, will help ensure prudent navigation through the uncertainty that pervades this and possibly future pandemics. Being open about the true degree of uncertainty surrounding the scientific evidence used to guide policy choices, and allowing for the assumptions of the models used, or for the decision-making process itself to be challenged is a valuable way of retaining public trust (Fiske and Dupree 2014). At the same time, it is important to counteract what is too often displayed by self-described experts who seek to influence policymakers and the public by projecting a false pretense of knowledge.

Materials

S1 Appendix. Supplement on the role of modelling.

S2 Appendix. Methods supplement to Figure 2.
References


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S1 Appendix – Supplement on the role of modeling

The main purpose of projection models is to assess what is likely to happen to an outcome if policy interventions—either independently or in combination—were put in place. In the COVID-19 pandemic, the models help guide decision-making by giving one potential answer to the question “what is likely to happen, if we do X, Y and/or Z?”. Projection models typically first estimate a model in which no intervention is implemented. This scenario defines a benchmark against which the benefits and costs of different intervention scenarios can be estimated.

Health-oriented models

Epidemiological projection models are used to provide disease control options to the decision makers. Two approaches to modelling may be distinguished: process-based models and curve-fitting approaches.

Process-based models

Process-based models try to capture the mechanisms by which diseases spread. To be useful, these models need to be appropriately designed for the questions being addressed and allow sufficient complexity to capture the essence of the mechanisms of transmission and policy interventions (Ferguson et al. 2003). A popular model is the SEIR model, in which each person is considered as either susceptible, exposed, infected or recovered. The SEIR model defines groups or compartments and mathematical rules by which the number of individuals move between each group. Transitions between model compartments are typically estimated using mathematical (differential) equations.

Agent-based models (ABM) are an alternative popular modelling approach, which instead explicitly models individual agents by assigning them characteristics and probabilities of acting in certain ways according to those characteristics. ABM outcomes then represent the aggregation of these modeled processes.

Both modelling approaches allow for different assumptions about the nature of the processes modelled, these can be either deterministic or stochastic. In deterministic processes, the transitions between compartments of an SEIR or the choice of an ABM do not allow for random variation, that is, the model will provide a single output. Stochastic approaches allow for random variation in parameter values such as R0 and therefore, provide multiple outputs per model. The ranges of these outputs are then presented with values that occur more frequently considered to be more likely.

Different research groups have used different versions of these models. For example, a widely used model from the team at Imperial College London uses a combination of stochastic compartmental models and ABMs (Ferguson et al. 2020). The model developed by the LSHTM-CMMID team, which also informs policy in the UK, is an age-structured SEIR model (Davies et al. 2020).
Curve-fitting approaches

The second type of model does not attempt to characterise the underlying transmission process. Instead, curve-fitting approaches attempt to specify a set of mathematical expressions that best fit the shape of the epidemic growth curve. Once a curve is obtained, it can be used to provide projections under different intervention scenarios. Curve fitting approaches can hugely vary in terms of complexity, and be parametric or non-parametric. The model developed by the Institute for Health Metrics and Evaluation is an example of curve-fitting approach. These types of models can lead to wildly inappropriate projections as they do not take into account the underlying dynamics of the disease.

Economic-oriented models

As the measures put in place are also largely affecting the economic environment, decision makers are also interested in knowing what are their health and non-health related economic outputs. Different types of projection models are used by economists to assess the effect of interventions on the economy.

Cost-Benefit analysis

In order to study the various trade-offs associated with any policy and ultimately ensure that the best health outcomes are achieved with limited resources, a typical way to proceed is to undertake a Cost-Benefit Analysis (CBA). This implies estimating and monetizing the impact of the policy both in terms of benefits (e.g. mortality reduction, reduced mortality from air pollution, reduction of greenhouse gas emissions, etc) and costs (e.g. decrease of economic activity, but also unemployment, suicide, depression, disruption of the education system, etc) and comparing them. In the case of COVID-19, estimates regarding the number of lives saved associated with different measures generally directly come from epidemiological projection models. Yet one issue that rises when considering health outcomes is how to monetise reduced risks of death and extending lifetimes, in order to compare the benefits to the costs. Three measures of value have been developed to assess the monetary effect of the health aspects. These are Disability Adjusted Life Years (DALYs), Quality Adjusted Life Years (QALYs) and the value of statistical life (VSL) (Greenstone and Nigam 2020; Thunström et al. 2020).

Integrated models

Recently, integrated models emerged, merging the fields of economics and epidemiology by incorporating simplified epidemiological SIR or SIER models of contagion within dynamic economic frameworks. Such models address important policy challenges by explicitly modeling dynamic adjustment paths and endogenous responses to changing incentives. They are used for example to investigate the optimal policy response, or the effectiveness of alternative macroeconomic policies to the economic shocks due to the COVID-19 pandemic (Eichenbaum, Rebelo, and Trabandt 2020; Guerrieri et al. 2020; Alvarez, Argente, and Lippi 2020). Most often, however, uncertainty is not incorporated formally into the modeling but treated ex post, for example by means of sensitivity analyses.
**Computable General Equilibrium models**

Computable General Equilibrium (CGE) models are used to estimate simultaneously what are the potential direct and indirect economic impacts of the COVID-19 outbreak on the economy (e.g., impacts of health on labour supply, health costs on government budgets and consumption impacts on specific sectors) (Keogh-Brown et al. 2020). These are sectoral models of the whole economy (potentially linked to a demographic model), which have been typically widely used in macroeconomic health policy analysis (Smith et al. 2009; Lock et al. 2010). CGE models are usually static (multi-sector) simulation models used for comparative static analysis (they first solve for an equilibrium solution in a baseline scenario and then find a new equilibrium solution after some shock to the initial situation). These models capture the behaviour of different economic agents (e.g., firms, consumers, government, etc) in the economy based on economic theory and specified mathematically as a system of equations which is solved simultaneously.

**Model choice**

Creating a model requires balancing between simplicity and comprehensiveness. Simpler models will likely make strong assumptions such as everyone having an equal chance of being infected, but they are often easier to understand. Complex models will try to incorporate a greater number of effects and interactions in an attempt to better describe reality. This will result in greater difficulties in interpretation and estimation of the models. A further common misnomer is that the more complex a model is the better it will reflect reality, care must be taken not add complexity that does not reflect reality versus a simplified model which does.

**S1 References**


S2 Appendix – Methods supplement to Figure 2 of deciding about school closures and their length during COVID-19 pandemic: formal decision rules

Overview

This Appendix supports the example decision problem presented in Figure 2 of the manuscript "Case study: School closures and their length during COVID-19 pandemic". The purpose of this case study is to show how decision theory could be used in a context where distinct model projections exist. Using a simple example of a decision problem of school closures during the COVID-19 pandemic, we highlight what are the resulting recommendations from different formal decision rules in terms of policy responses.

It is important to note that the decision problem presented in this case study is necessarily simplistic and should be used for demonstrative purposes only.

1 Background

Context By end of April 2020, 191 countries had implemented national school closures in response to the COVID-19 pandemic (United Nations Educational Scientific and Cultural Organization, 2020). Yet the effectiveness of such a measure is highly uncertain, due to the lack of data on the relative contribution of school closures to transmission control, and conflicting modelling results (Viner et al., 2020).

The policymaker’s problem Decisions about closures and their length involve a series of trade-offs. The policy assessment thus involves weighting benefits and costs of alternative courses of action. On the one hand, school closures can slow the pandemic and its impact by reducing child-child transmission, thus delaying the pandemic peak that overwhelms health care services, and therefore ultimately reducing morbidity and associated mortality. If this is the case, such interventions bring clear health benefits for the society and avoid unsustainable demands on the health system. On the other hand, school closures can have high direct and indirect health and socio-economic costs. For example, they may increase child-adult transmission, reduce the ability of healthcare and key workers to work and thus reduce the capacity of healthcare (Brooks et al., 2020; Bayham and Fenichel, 2020). Economic costs of lengthy school closures are also high (Sadique et al., 2008; Lempel et al., 2009; Keogh-Brown et al., 2010), generated for example through absenteeism by working parents, loss of education, etc.
Uncertainty The evidence supporting national closure of schools in the COVID-19 pandemic context was very weak. In particular, evidence of COVID-19 transmission through child-child contact or through schools was not available at the time of decision (Viner et al., 2020). As a consequence, it was unclear whether school closures would be effective in the COVID-19 pandemic (Bayham and Fenichel, 2020).

Framework We use a framework that decomposes uncertainty into distinct layers of analysis: (i) uncertainty within models (also called risk, aleatory uncertainty, or physical uncertainty), (ii) uncertainty across models (also called model ambiguity, or model uncertainty), and (iii) uncertainty about models (also called model misspecification).\footnote{See Arrow (1951); Hansen (2014); Marinacci (2015); Hansen and Marinacci (2016) for a discussion, and Aydogan et al. (2018) for empirical evidence on the distinction between these layers.}

In its colloquial sense, a “model” is generally considered as a stylized (i.e. approximate but tractable) representation of a phenomenon of interest that a natural or social scientist wants to study. Models serve as tools that provide a logically consistent way to organize thinking about the relationships among variables of interest and provide clarity on the implications of those relationships (Mäki, 2011; Beck and Krueger, 2016).

The approach that we follow here is, however, slightly different. We consider a general decision problem in which consequences depend on the states of the environment that are viewed as realizations of an underlying economic or physical generative mechanism (Marinacci, 2015). A model is a probability distribution induced by such a mechanism. It describes states’ variability by combining a structural component based on theoretical knowledge (e.g. economic or physical) and a random component coming from, for example, measurement errors or minor omitted explanatory variables (Koopmans, 1947; Marschak, 1953). We assume that decision makers (DMs) posit a collection of such models.

Uncertainty across model therefore results from the uncertainty about the true underlying mechanism: within the posited collection, there is uncertainty about which model actually governs states’ realizations. However, even after a model is specified, there is still uncertainty within model, i.e. about which specific state will actually obtain; this is the notion of risk typically considered in economics. Finally, the third layer of uncertainty (about models), arises as the true model might not belong to the posited collection of models, reflecting the idea that all posited models have an inherent approximate nature.

2 Making decisions under uncertainty

2.1 The structure of a decision problem

The problem that a DM, in particular a policymaker, faces is to choose an action $a$ within a set $A$ of possible alternative actions, whose consequences $c \in C$ depend on the
realization of a *state of the environment* \( s \in S \) which is outside the DM’s control.

The relationship among consequences, actions and states is described by a consequence function \( \rho : A \times S \to C \), where

\[
c = \rho(a, s)
\]

is the consequence of action \( a \) when state \( s \) obtains. DMs have a (complete and transitive) *preference relation* \( \succcurlyeq \) over actions that describes how they rank the different alternative actions.\(^2\)

The quintet

\[
(A, S, C, \rho, \succcurlyeq)
\]

characterizes the decision problem under uncertainty. The aim of the DM is to select the action \( \hat{a} \) that is *optimal* according to her preference, that is, such that

\[
\hat{a} \succcurlyeq a
\]

for all actions \( a \in A \).

The preference \( \succcurlyeq \) is assumed to admit a numerical representation via a *decision criterion* \( V : A \to \mathbb{R} \), with

\[
a \succcurlyeq b \iff V(a) \geq V(b)
\]

for all actions \( a, b \in A \). This numerical representation permits to formulate the decision problem as an optimization problem

\[
\max_a V(a) \quad \text{sub} \quad a \in A.
\]

Optimal actions \( \hat{a} \) are the solutions of this problem. To find an optimal action thus amounts to solve this optimization problem.

The DM may address, especially in policy problems, state uncertainty through the guise of models. Based on ex ante scientific and socio-economic information, the DM might be able to posit a set of probability models \( M \subseteq \Delta(S) \) describing the likelihoods of the different states. This set of models is taken as a datum of the decision problem, which is now characterized by a sextet

\[
(A, S, C, \rho, \succcurlyeq, M)
\]

It is often assumed, following Wald (1950), that the correct model belongs to the set of models that the DM posits, thus abstracting from model misspecification issues.

\(^2\)As is usual, we write \( a \succcurlyeq b \) if the DM prefers action \( a \) to action \( b \) (i.e., either strictly prefers action \( a \) to action \( b \), \( a \succ b \), or is indifferent between the two, \( a \sim b \)).
3 Decision criteria

The form of the decision criterion $V$ determines the nature of decision problem (2). Next we present different possible criteria.

**Subjective expected utility** The subjective expected utility (SEU) criterion, which dates back to the seminal works of von Neumann and Morgenstern (1947); Wald (1950); Savage (1954) and Marschak and Radner (1972), has typically been the standard way to consider rational decision-making under uncertainty. It has recently been revisited by Cerreia-Vioglio et al. (2013) to accommodate explicitly the presence of uncertainty across models.

Consider the decision problem $(A,S,C,\rho,\succeq,M)$ and assume that a von Neumann-Morgenstern utility function $u : C \rightarrow \mathbb{R}$ translates economic consequences, measured in monetary terms, into utility levels. This function captures attitudes toward risk (i.e. uncertainty within models). The expected reward of action $a$ under model $m$ is:

$$R(a,m) = \sum_s u(\rho(a,s)) m(s). \quad (3)$$

As different models exist and the DM is uncertain about which is the correct one, she evaluates the expected reward of each possible model and aggregates them out by performing a weighted average using her prior probability $\mu$. The SEU criterion is

$$V_{\text{seu}}(a) = \sum_m R(a,m) \mu(m). \quad (4)$$

A DM who adopts this two-stage criterion selects the optimal action $\hat{a}$ by solving the decision problem

$$\max_a V_{\text{seu}}(a) \quad \text{sub } a \in A$$

The optimal action therefore depends on the utility function $u$, a taste component, and on the prior probability $\mu$, an information component. When the prior is uniform, this criterion becomes an average of the expected rewards, with all models equally weighted. In contrast, when a DM posits, possibly wrongly, only a single model $m$ and assigns it full weight – i.e., $\mu(m) = 1$ – the SEU criterion reduces to $V_{\text{eu}}(a) = R(a,m)$. In this case, the DM is dogmatic about a specific model and does not take into account any other model.

**Maxmin** The maxmin criterion

$$V_{\text{maxm}}(a) = \min_m R(a,m) \quad (5)$$
of Wald (1950) focuses on the worst possible model in terms of expected reward, with prior probabilities playing no role.

It is a highly prudential decision criterion that, because of its neglect of prior probabilities, is vulnerable to outliers. Yet, its simplicity makes it a natural worst-case benchmark.

**Smooth ambiguity** The smooth ambiguity criterion, developed by Klibanoff et al. (2005), aims to distinguish attitudes toward uncertainty within and across models. It has the form

\[
V_{smal}(a) = \sum_{m} \phi (R(a,m)) \mu(m) \\
= \sum_{m} v (u^{-1} (R(a,m))) \mu(m)
\]

where \( \phi \equiv v \circ u^{-1} \) represents the attitude toward uncertainty that results from the combination of attitudes toward uncertainty across models (model uncertainty) and within models (risk) represented by \( v \) and \( u \), respectively. Uncertainty aversion results from a higher aversion to model uncertainty than risk. Formally, it corresponds to a concave \( \phi \), with \( v \) more concave than \( u \) (see Marinacci (2015)).

In this two-stage criterion, the DM first addresses risk by evaluating the expected reward of policy \( a \) for each possible model \( m \) and expresses it in monetary terms using a certainty equivalent \( c_m \equiv u^{-1} (R(a,m)) \).

The more averse to risk the DM is, the lower \( c_m \) is. In a second stage, the DM addresses the model uncertainty by evaluating the overall expected reward

\[
\sum_{m} v(c_m) \mu(m)
\]

across all certainty equivalents. When \( u = v \), we are back to the SEU criterion (4). Indeed, this criterion may be rewritten as

\[
V_{seu}(a) = \sum_{m} u (u^{-1} (R(a,m))) \mu(m),
\]

and so assumes the same attitude toward risk and model uncertainty.

The maxmin criterion (5) is a limit case of the smooth decision criterion (6) when aversion to model uncertainty explodes. For example, if \( \phi_\lambda(x) = -e^{-\lambda x} \) and \( \mu(m) > 0 \) for
all \( m \in M \), we have\(^3\)

\[
\lim_{\lambda \to +\infty} \phi^{-1}_{\lambda} \left( \sum_{m} \phi_{\lambda} (R(a,m)) \mu(m) \right) = \min_{m} R(a,m)
\]

The maxmin criterion here corresponds to an extreme aversion to model uncertainty.

**Multiple priors** The multiple prior decision criterion

\[
V_{mp}(a) = \min_{\mu \in C} \sum_{m} R(a,m) \mu(m)
\]

proposed by Gilboa and Schmeidler (1989) considers the least favorable among all the classical subjective expected utilities determined by each prior \( \mu \) in a set \( C \) of priors.

This criterion relaxes the assumption that the DM has enough information to specify a single prior \( \mu \) on models by allowing a set \( C \) of them. This set incorporates both the attitude toward uncertainty across models and an information component. For example, a smaller set may reflect either better information and/or less aversion to uncertainty.

The multiple prior decision criterion generalizes Wald’s maxmin criterion, which is the special, extreme, case of a maximal \( C \) that coincides with the set \( \Delta \) of all possible priors.

A more general \( \alpha \)-version

\[
V_{\alpha-mp}(a) = \alpha \min_{\mu \in C} \sum_{m} R(a,m) \mu(m) + (1 - \alpha) \max_{\mu \in C} \sum_{m} R(a,m) \mu(m),
\]

where both “\( \text{max} \)” and “\( \text{min} \)” appear, has been studied by Ghirardato et al. (2004).

## 4 Making decisions in a pandemic

**States and consequences** With the letter \( R \) we denote a rate of contagion within a given population, i.e., the average number of individuals infected per single case. The baseline rate of contagion, denoted by \( R_0 \), is called basic reproduction number. It applies to a population never exposed to the virus, where everyone is susceptible,\(^4\) and depends on the biology of the virus as well as on the natural (pre-pandemic) socio-economic structure that characterizes the population (Ng and Wen, 2019). The biology of the virus determines its ability to infect (i.e., the probability of infection per interaction) and the duration of infectiousness.\(^5\) The natural socio-economic structure determines the natural social distancing and, through it, the average number of interactions per individual (Dietz, 1993;

\(^3\)As Klibanoff et al. (2005) argue, \( \lambda \) can be interpreted as the coefficient of absolute of ambiguity aversion.

\(^4\)An individual is susceptible if has no immune protection against the virus.

\(^5\)By interaction we mean a contact amenable to virus transmission (in terms of closeness and duration).
Delamater et al., 2019). For instance, natural social distancing might be higher in Northern than in Southern European countries.

These natural factors, biological and socio-economic, determine $R_0$. It is the natural, \textit{ex ante}, rate at which the pandemic progresses, without private and public decisions that respond to it. \textit{Ex post}, after these decisions are put in place and affect the biological and socio-economic factors that determine $R_0$, the relevant rate of contagion becomes the \textit{effective reproduction} (or \textit{reproductive}) number $R_e$ (Wallinga and Lipsitch, 2007). For example, a school closure is a public decision that may increase social distancing (a socio-economic factor), while a diligent use of protective gear is a private decision that may decrease the virus ability to infect (a biological factor).

Here we focus on public decisions, policies, and assume that private ones are subsumed by them.\footnote{A highly non-trivial assumption that, for instance, requires people to use diligently protective gear if asked by local or national authorities.} A policy translates a basic reproduction number $R_0$ into an effective one $R_e$. Yet, how this translation occurs often remains uncertain. For instance, evidence on the effectiveness of school policies for COVID-19 comes from influenza outbreaks, but the ability of children to transmit the disease greatly varies across coronaviruses (Wong et al., 2003).

For this reason, we represent how a policy $a$ maps $R_0$ into $R_e$ via the relation

$$R_e = f(a, R_0, \theta_r, \varepsilon_r), \quad (8)$$

where $\theta_r$ is a structural parameter and $\varepsilon_r$ is a shock.\footnote{Throughout, shocks have zero mean and unit variance.} We assume that $\partial f/\partial a \leq 0$ and $\partial f/\partial R_0 > 0$. For example, if the relation is linear we have

$$R_e = \theta_{r,1}a + \theta_{r,2}R_0 + \varepsilon_r, \quad (9)$$

with $\theta_{r,1} \leq 0$ and $\theta_{r,2} > 0$.\footnote{For simplicity, we allow $R_e$ to be negative (otherwise, we should add constraints that preserve the positivity of $R_e$, something that we prefer to abstract from).} In the baseline scenario without policy intervention – i.e., when $a = 0$ – the effective reproduction number $R_e$ is determined by: (i) the natural proportion $\theta_{r,2}$ of the population that is susceptible, (ii) the basic reproduction number $R_0$ that summarizes the biological and socio-economic factors previously discussed, (iii) a shock $\varepsilon_r$ that accounts for minor omitted variables.

The economic damage $D$, in monetary terms (e.g., loss of GDP), associated with the pandemic is determined by the rate of contagion $R$ via a function

$$D = g(R, \theta_d, \varepsilon_d), \quad (10)$$

where $\theta_d$ is a structural parameter and $\varepsilon_d$ is a shock. This function represents the ability
of health and economic systems to cope with the pandemic. We assume that $\partial g/\partial R > 0$. For example, assuming a quadratic damage function we have:

$$D = \theta_d R^2 + \theta_d R + \varepsilon_d,$$  \hspace{1cm} \text{(11)}

with $2\theta_d R + \theta_d > 0$.

The economic damage $D$ associated with a policy $a$ is then

$$D = g(R, \theta_d, \varepsilon_d) = g(f(a, R_0, \theta_r, \varepsilon_r), \theta_d, \varepsilon_d).$$  \hspace{1cm} \text{(12)}

A policy affects, according to relation $f$, the effective reproduction number and, through it, determines an economic damage according to relation $g$.

In the linear-quadratic example, we have

$$D = \kappa_1 a^2 + \kappa_2 a + \kappa_3,$$  \hspace{1cm} \text{(13)}

where

$$\begin{align*}
\kappa_1 &= \theta_{d,1}\theta_{r,1}^2, \\
\kappa_2 &= \theta_{r,1} (2\theta_{d,1} (\theta_{r,2} R_0 + \varepsilon_r) + \theta_{d,2}) \\
\kappa_3 &= \theta_{d,1} (\theta_{r,2} R_0 + \varepsilon_r)^2 + \theta_{d,2} (\theta_{r,2} R_0 + \varepsilon_r) + \varepsilon_d.
\end{align*}$$  \hspace{1cm} \text{(14-16)}

Policy $a$ has an uncertain implementation cost $C$, represented by a function

$$C = h(a, \theta_c, \varepsilon_c).$$  \hspace{1cm} \text{(17)}

We assume that costs grow more than proportionally, so that $\partial h/\partial a > 0$ and $\partial^2 h/\partial a^2 > 0$ (e.g., the cost of a school closure grows more than proportionally with its duration). For example, a quadratic cost function is

$$C = \theta_{c,1} a^2 + \theta_{c,2} a + \varepsilon_c,$$  \hspace{1cm} \text{(18)}

with $2\theta_{c,1} a + \theta_{c,2} > 0$ and $\theta_{c,1} > 0$.

We assume that the policy maker knows the functional forms of the relations $f$, $g$ and $h$ (e.g., whether they are linear or quadratic) but not their structural parameters. This lack of knowledge, along with that of the basic reproduction number $R_0$ and of the shocks’ value $\varepsilon$, prevents the DM to know the actions’ consequences. States thus have the form

$$s = (R_0, \varepsilon, \theta) \in S$$  \hspace{1cm} \text{(19)}

with both random and structural components. In particular, the vector $\varepsilon = (\varepsilon_r, \varepsilon_d, \varepsilon_c) \in E$
represents the shocks affecting the health and economic systems, while the vector \( \theta = (\theta_r, \theta_d, \theta_c) \in \Theta \) specifies the structural coefficients parametrizing a model population.

If we denote by \( B = -D \) the benefit of policy \( a \) as its ability to reduce the economic damages due to the pandemic, its consequence is the difference \( B - C \) between its benefits and costs. The consequence function is then

\[
\rho (a, \varepsilon, \theta) = -g (f (a, R_0, \theta_r, \varepsilon_r), \theta_d, \varepsilon_d) - h (a, \theta_c, \varepsilon_c) \tag{20}
\]

In the linear-quadratic example, it becomes

\[
\rho (a, \varepsilon, \theta) = - (\kappa_1 + \theta_{c,1}) a^2 - (\kappa_2 + \theta_{c,2}) a - \kappa_3 + \varepsilon_c
\]

**Models and beliefs**  Shocks have the form

\[
\varepsilon_r = \sigma_r w_r \; ; \; \varepsilon_d = \sigma_d w_d \; ; \; \varepsilon_c = \sigma_c w_c
\]

where \( w_r, w_d \) and \( w_c \) are uncorrelated “white noises” with zero mean and unit variance. The vector parameter

\[
\sigma = (\sigma_r, \sigma_d, \sigma_c) \in \Sigma
\]

then specifies the standard deviations of shocks. To ease the analysis, we assume that their distribution \( q_\sigma \) is known, up to their standard deviations \( \sigma \).

We also assume that the distribution \( p_\xi \) of the rate \( R_0 \) is indexed by a parameter \( \xi \in \Xi \) that accounts for different epidemiological views on the quantification of the basic reproduction number. With this, the positive scalar

\[
m (\varepsilon, \theta, R_0)
\]

gives the joint probability of shock \( \varepsilon \), parameter \( \theta \) and rate \( R_0 \) under a posited model \( m \in M \). We adopt the model factorization \( m = q_\sigma \times \delta_\theta \times p_\xi \), that is,\(^9\)

\[
m (\varepsilon, \theta', R_0) = \begin{cases} 
q_\sigma (\varepsilon) p_\xi (R_0) & \text{if } \theta' = \theta \\
0 & \text{else}
\end{cases}
\tag{21}
\]

where \( q_\sigma (\varepsilon) \) is the probability of shock \( \varepsilon \) under the standard deviation specification \( \sigma \), while \( p_\xi (R_0) \) is the probability that \( R_0 \) is the basic reproduction number according to epidemiological view \( \xi \). We can thus index models as

\[
m_{\theta, \sigma, \xi} = q_\sigma \times \delta_\theta \times p_\xi
\]

\(^9\)Here \( \delta_\theta \) is the probability distribution concentrated on \( \theta \), i.e., \( \delta_\theta (\theta) = 1 \) and \( \delta_\theta (\theta') = 0 \) if \( \theta' \neq \theta \).
and denote by \( M = \{ m_{\theta,\sigma,\xi} \} \) the set of models that the policy maker posits.

To address the uncertainty across models, the policy maker has a subjective prior probability distribution \( \mu \) that quantifies beliefs about the correct model. In particular, \( \mu(m) \) is the policy maker subjective belief that \( m \) is the correct model. Because of the factorization, this belief is actually over the values of \( \theta \), \( \sigma \) and \( \xi \) and so has the form \( \mu(\theta,\sigma,\xi) \). A convenient separable form is, with an abuse of notation,

\[
\mu(\theta,\xi) = \mu(\theta,\sigma) \mu(\xi) \tag{22}
\]

For example, consider the set of models \( M = \{ m_{\theta,\sigma,\xi} \} \) that the policy maker posits in the pandemic decision problem. Since models are indexed by the triple \( (\theta, \sigma, \xi) \), we can write the expected reward as

\[
R(a,\theta,\sigma,\xi) = \sum_{\varepsilon, R_0} u(\rho(a,\theta,\varepsilon, R_0)) \cdot m_{\theta,\sigma,\xi}(\varepsilon, R_0)
= \sum_{\varepsilon, R_0} u(\rho(a,\theta,\varepsilon, R_0)) \cdot q_\sigma(\varepsilon) \cdot p_\xi(R_0)
\]

In the SEU case, when combined with the belief (22), we get

\[
V_{\text{seu}}(a) = \sum_{\theta,\xi} R(a,\theta,\sigma,\xi) \cdot \mu(\theta,\sigma,\xi)
= \sum_{\theta} \left( \sum_{\varepsilon, R_0} u(\rho(a,\theta,\varepsilon, R_0)) \cdot q_\sigma(\varepsilon) \cdot \tilde{p}_\xi(R_0) \right) \cdot \mu(\theta,\sigma)
\]

where \( \tilde{p}_\xi(R_0) = \sum_\xi p_\xi(R_0) \cdot \mu(\xi) \).

5 Specific school closure example

In the case study (Fig. 2), the policymaker must decide whether to and how long to close school for. Closing schools is costly (e.g. it increases child-adult transmission, reduces the ability of healthcare and key workers to work and the capacity of healthcare, generates economic costs through absenteeism by working parents, loss of education, etc.), but it helps slow the pandemic and its impact by reducing child-child transmission, thus delaying the pandemic peak that overwhelms health care services, and therefore ultimately reducing morbidity and associated mortality.

Here, we illustrate how different decision rules may be used in this specific example, in which there is only structural uncertainty about the benefits of school closures. The cost function is assumed to be known, so that there are three different models in the set \( M \).

- **Model 1** is based on the evidence coming from influenza outbreaks, for which the
majority of transmission is between children (Mangtani et al., 2014). According to this model, closing schools would be the biggest contributor to reducing \( R_e \) to below 1 and it may be the only intervention that could do so. In this case the benefit would, for example, be proportional to the duration of school closure. In the linear-quadratic example, this would imply that \( \kappa_1 = 0 \) and \( \kappa_2 < 0 \), so that benefits positively depend on the action \( a \): stronger measures reduce the effective reproductive number \( R_e \), and thus the economic damage \( D \) of the pandemic.

- **Model 2**, instead, relies on some previous coronavirus outbreaks, for which evidence suggests minimal transmission between children (Wong et al., 2003). In this case, \( R_e \) cannot not be reduced below 1, school closures do not affect the size of the epidemic, and therefore do not bring any benefits. This is for example the case if \( \kappa_1 = \kappa_2 = 0 \) in the linear-quadratic setup, so that the economic damage, and thus the benefit, are unaffected by the policy action \( a \).

- **Model 3** projects that some child to child transmission happens so that closing schools contributes to reducing \( R_e \) to below 1 and reduces the size of the epidemic. However, this only works in combination with other measures (Prem et al., 2020). Without it, \( R_e \) would be above 1 but as an isolated measure school closure does not have such a big effect (Ferguson et al., 2020). Under this scenario, the effectiveness of school closures is important at the beginning, but declines as time goes on. In the linear-quadratic example, this would imply that \( \kappa_1 > 0 \) and \( \kappa_2 < 0 \).

For simplicity, we assume that there is no uncertainty within models (\( \varepsilon_r = \varepsilon_d = \varepsilon_C = 0 \)).

The illustrative benefit and cost function we used are the following:

- \( B_1(a) = 4a + 20 \) in the case of model 1,
- \( B_2(a) = 100 \) in the case of model 2,
- \( B_3(a) = -0.1a^2 + 4a + 70 \) in the case of model 3,
- \( C(a) = 0.1a^2 + 10 \).

Consider the decision problem \(( A, S, C, \rho, \succ, M \)). In our case, we restrict the action space so that \( A = [0, 20] \). For each of these 3 models \( m_{\theta, \sigma, \xi} \), it is possible to compute the expected reward \( R(a, \theta, \sigma, \xi) \) associated with a school closure policy. The policymaker, however, does not know which is the correct one. The expected reward is, in that sense, itself uncertain because it depends on the values of the different structural parameters used. For each particular model representing the net overall monetary benefits of school policy, it is possible to determine the optimal action to put in place. Table 1 presents their
Table 1: Example of expected rewards and their associated optimal actions with linear utility $u$

<table>
<thead>
<tr>
<th>Model</th>
<th>$R(a, \theta, \sigma, \xi)$</th>
<th>$\hat{a}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 1</td>
<td>$-0.1a^2 + 4a + 10$</td>
<td>20</td>
</tr>
<tr>
<td>Model 2</td>
<td>$-0.1a^2 + 90$</td>
<td>0</td>
</tr>
<tr>
<td>Model 3</td>
<td>$-0.2a^2 + 4a + 60$</td>
<td>10</td>
</tr>
</tbody>
</table>

expected rewards, together with their associated optimal action $\hat{a}$ in the case of linear utility $u$.

We now illustrate what are the differences in optimal school policy among the distinct decision rules that we presented.

If the policymaker considers uncertainty within and across models in the same way, she aggregates the expected rewards by taking a weighted average over them, where the weights represent the degree of belief in each specific model. The decision criterion in this case is the classical SEU criterion (4). For example, under a uniform prior over the possible models, i.e. if $\mu(m) = 1/3$ for all $m$, the optimal decision is a school closure policy $\hat{a}_{seu} = 10$. It therefore means that, putting the same weight on the three different models given by three different sources, or a single model (such as model 3) on which all experts would agree, would for example lead exactly to the same optimal school policy.

Instead, if the policymaker decides to behave extremely precautionary by taking into account only the model providing the lowest expected reward, she only considers model 1 and decides to close schools for the maximum length $\hat{a}_{\text{max}} = 20$. This policymaker is extremely averse to uncertainty across models, and in consequence, uses Wald’s maxmin decision rule consisting in maximizing (5).

Alternatively, if the policy maker is averse to uncertainty in the sense of disliking more uncertainty across than within models but is not as precautionary as a maxmin policymaker, she may follow the smooth ambiguity rule and maximize criterion (6). In such a case, the optimal length of school closures is longer than under expected utility. It approximately corresponds to 12 weeks, when $\phi$ is logarithmic.\(^{10}\)

Finally, if the policymaker has multiple prior probability measures over the models, she can computes the expected utility for each of them, and considers only the one providing the lowest level of subjective expected utility. For example, imagine two distinct priors: the uniform prior, in which the 3 models are weighted equally, and the prior that considers model 2 as implausible, but models 1 and 3 as equally likely (i.e., this prior puts a weight 0 on model 1 and a weight 0.5 over the two other models). The optimal level of cumulative

\(^{10}\)In the case of a logarithmic function $\phi$, the relative ambiguity aversion coefficient is constant and equal to 1.
emissions under the multiple priors model in this situation is also lower than under the subjective expected utility rule. It corresponds to closing schools for approximately 13 weeks. Table 2 summarizes the optimal decisions for each of these decision rules.

Table 2: Example of optimal policies depending on the decision rules followed

<table>
<thead>
<tr>
<th>Decision rules (criterion)</th>
<th>Optimal policy</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{seu}$</td>
<td>$\hat{a}_{seu} = 10$</td>
</tr>
<tr>
<td>$V_{mxm}$</td>
<td>$\hat{a}_{mxm} = 20$</td>
</tr>
<tr>
<td>$V_{smt}$</td>
<td>$\hat{a}_{smt} = 11.65$</td>
</tr>
<tr>
<td>$V_{mp}$</td>
<td>$\hat{a}_{mp} = 13.33$</td>
</tr>
</tbody>
</table>

References


