

Aggregate Bank Capital and Credit Dynamics

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First draft: December, 2014

This version: January, 2016

Abstract

Central banks need a new type of quantitative models for guiding their financial stability decisions. The aim of this paper is to propose such a model. In our model commercial banks finance their loans by deposits and equity, while facing issuance costs when they raise new equity. Because of this financial friction, banks build equity buffers to absorb negative shocks. Aggregate bank capital determines the dynamics of credit. Notably, the equilibrium loan rate is a decreasing function of aggregate capitalization. The competitive equilibrium is constrained inefficient, because banks do not internalize the effect of their individual lending decisions on the future loss-absorbing capacity of the banking sector. In particular, we find that undercapitalized banks lend too much. We show that introducing a minimum capital ratio helps taming excessive lending. Relatively high minimum capital ratios enhance financial stability, whereas mild capital requirements may destabilize the system.

Keywords: macro-model with a banking sector, aggregate bank capital, pecuniary externality, capital requirements

JEL: E21, E32, F44, G21, G28

Acknowledgements: we thank George-Marios Angeletos, Manuel Arellano, Philippe Bacchetta, Bruno Biais, Markus Brunnermeier, Catherine Casamatta, Julien Daubanes, Peter DeMarzo, Jean-Paul Décamps, Sebastian Di Tella, John Geanakoplos, Hans Gersbach, Hendrik Hakenes, Christian Hellwig, Eric Jondeau, Simas Kucinskas, Li (Erica) Xuenan, Michael Magill, Semyon Malamud, Lorian Mancini, David Martinez-Miera, Erwan Morellec, Alistair Milne, Henri Pagès, Bruno Parigi, Greg Phelan, Guillaume Plantin, Martine Quinzii, Raphael Repullo, Yuliy Sannikov, Enrique Sentana, Amit Seru, Jean Tirole and Stéphane Villeneuve, and seminar participants at CEMFI, ETH Zürich, Chicago Booth, Princeton University, Stanford University, Toulouse School of Economics, EPF Lausanne, the University of Nanterre, Yale University, Banque de France, University of Bonn, and the CICF 2015 conference.

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1 Introduction

Central banks need quantitative models for guiding their financial stability decisions. Indeed, in the context of their new macro-prudential responsibilities, they have recently been endowed with powerful regulatory tools. These tools include the setting of capital requirements for all banks, determining capital add-ons for systemic institutions, and deciding when to activate counter-cyclical capital buffers. The problem is that very little is known about the long term impact of these regulations on growth and financial stability. The only quantitative models that central banks currently have at their disposal, the so-called DSGE models, have been designed for very different purposes, namely assessing the short term impact of monetary policy decisions on inflation and economic activity. Until recently, these DSGE models did not even include banks in their representation of the economy. DSGE models are very complex and use very special assumptions, because they have been specifically calibrated to reproduce the short term reaction of prices and employment to movements in central banks' policy rates. It seems therefore clear that a very different kind of model is needed for analyzing the long term impact of macro-prudential policies on bank credit, GDP growth and financial stability. The aim of this paper is to propose such a model.

Building on the recent literature on macro models with financial frictions, we develop a tractable dynamic model where aggregate bank capital determines the dynamics of lending. Though highly stylized, the model is able to generate predictions in line with empirical evidence. Moreover, we show that our model framework is flexible enough to accommodate the analysis of the regulatory policies, by looking at the long-run impact of capital regulation on lending and financial stability.

We consider an economy where firms borrow from banks that are financed by deposits, secured debt and equity. The aggregate supply of bank loans is confronted with the firms' demand for credit, which determines the equilibrium loan rate. Aggregate shocks impact the firms' default probability, which ultimately translates into profits or losses for banks. Banks can continuously adjust their volumes of lending to firms. They also decide when to distribute dividends and when to issue new equity. Equity issuance is subject to deadweight costs, which constitutes the main financial friction in our economy and creates room for the loss-absorbing role of bank capital.¹

In a set-up without the financial friction (i.e., no issuance costs for bank equity) and i.i.d. aggregate shocks, the equilibrium volume of lending and the nominal loan rate would be constant. Furthermore, dividend payment and equity issuance policies would be trivial in this case: Banks would immediately distribute all profits as dividends and would issue new shares to offset losses and honor obligations to depositors. This implies that, in a frictionless world, there would be no need to build up capital buffers and all loans would be entirely financed by debt.

When the financial friction is taken into account, banks' dividend and equity issuance strategies

¹Empirical studies report sizable costs of seasoned equity offerings (see e.g. Lee, Lochhead, Ritter, and Zhao (1996), Hennessy and Whited (2007)). Here we follow the literature (see e.g. Décamps et al. (2011) or Bolton et al. (2011)) by assuming that issuing new equity entails a deadweight cost proportional to the size of the issuance.

become less trivial. We show that there is a unique competitive equilibrium, where all variables of interest are deterministic functions of the aggregate book value of bank equity, which follows a Markov process reflected at two boundaries. Banks issue new shares at the lower boundary, where aggregate book equity of the banking system is depleted. When aggregate bank equity reaches its upper boundary, any further earnings are paid out to shareholders as dividends. Between these boundaries, the changes in banks' equity are only due to their profits and losses. Banks retain earnings in order to increase their loss-absorbing equity buffer that allows them to guarantee the safety of issued debt claims, while avoiding raising new equity too frequently.² The target size of this loss-absorbing buffer is increasing with the magnitude of the financial friction.

We start by exploring the properties of the competitive equilibrium in the “laissez-faire” environment, in which banks face no regulation. Even though all agents are risk neutral, our model generates a positive spread for bank loans. This spread is decreasing in the level of aggregate bank capital. To get an intuition for this result, note that bank equity is more valuable when it is scarce. Therefore, the marginal (or market-to-book) value of equity is higher when total bank equity is lower. Moreover, as profits and losses are positively correlated across banks, each bank anticipates that aggregate bank equity will be lower (higher) in the states of the world where it makes losses (profits). Individual losses are thus amplified by a simultaneous increase in the market-to-book value, whereas individual profits are moderated by a simultaneous decrease in the market-to-book value. As a result, banks only lend to firms when the loan rate incorporates an appropriate premium.

To show that the competitive equilibrium is constrained inefficient, we compare it with the socially optimal allocation. Our analysis reveals two channels of welfare-reducing pecuniary externalities inherent in the model framework. In particular, we find that competitive banks do not internalize the impact of their individual lending decisions on i) the banking system's exposure to aggregate shocks and ii) the profit margin on lending. When the banking system is poorly capitalized, banks lend too much as compared to the socially optimal level, thereby, creating inefficiently high exposure to aggregate risk. Furthermore, as excessive lending put downward pressure on banks' profit margins, it undermines the banking system's ability to accumulate loss absorbing capital through retained earnings. Overexposure to aggregate risk in the states with poor capitalization makes banks effectively more risk-averse when the banking system is well capitalized. As a result, well-capitalized banks lend too little as compared to the social optimum.

As an illustration of the potential of our framework to accommodate the analysis of macroprudential policy tools, we use our model to explore the effects of minimum capital requirements. A standard argument against high capital requirements is that they would reduce lending and growth. By contrast, the proponents of higher capital requirements put emphasis on their positive impact

²The idea that bank equity is needed to guarantee the safety of banks' debt claims is explored in several recent papers. Stein (2012) shows its implication for the design of monetary policy. Hellwig (2015) develops a static general equilibrium model where bank equity is necessary to support the provision of safe and liquid investments to consumers. DeAngelo and Stulz (2014) and Gornall and Strebulaev (2015) argue that, due to the banks' ability to diversify risk, the actual size of this equity buffer may be very small.

on financial stability. To consider the interplay between the aforementioned effects and get some insights into the *long run* consequences of capital regulation, we solve the competitive equilibrium under the regulatory constraint that requires banks to maintain capital ratios above a constant minimum level.

This analysis yields several observations. First, it shows that, when the minimum capital ratio is not too high, the regulatory constraint is binding in poorly-capitalized states and is slack in well-capitalized states. In other words, faced with moderate capital requirements, well-capitalized banks tend to build buffers on top of the required minimum level of capital. By contrast, above some critical level of minimum capital ratios, the regulatory constraint is always binding, so that banks operate without extra capital cushions.

Second, a higher capital ratio translates into a higher loan rate and thus reduces lending for any given level of bank capitalization. Importantly, this effect is present even when the regulatory constraint is slack, because banks anticipate that capital requirements might be binding in the future and require a higher lending premium for precautionary motives. Thus, provided it is not too high, a constant capital ratio mitigates inefficiencies (excessive lending) in poorly capitalized states, but exacerbates inefficiencies (insufficient lending) in well capitalized states. By contrast, for high minimum capital ratios, lending is inefficiently low in all states.

Finally, we look at the impact of a constant minimum capital ratio on financial stability, by analyzing the properties of the probability distribution of aggregate capital across states. Surprisingly, we find that imposing mild capital requirements induces the system to spend a lot of time in the states with low aggregate capitalization, thereby, making it even less stable than in the unregulated set-up. The explanation for this "regulatory instability" is rooted in the interplay between the scale of endogenous volatility in the region where the regulatory constraint is slack and the region where the regulatory constraint is binding. In fact, even a very low minimum capital ratio induces a substantial reduction in lending and, thus, in the endogenous volatility in the constrained region (i.e., in the poorly capitalized states). By contrast, in the unconstrained region (i.e., in the well capitalized states), the endogenous volatility is almost as high as in the unregulated set-up. Thus, the system enters the poorly capitalized states almost as often as in the unregulated set-up, but gets trapped there because of the substantially reduced endogenous volatility. As the minimum capital ratio increases, the differences in the endogenous volatility across states becomes less pronounced. Furthermore, due to higher loan rates, banks benefit from higher expected profits which fosters accumulation of earnings in the banking sector. As a result, the system enters undercapitalized states less frequently and the "trap" disappears. For very high levels of capital requirements, the regulated system becomes even more stable than in the social planner's allocation. However, this "extra" stability comes at the cost of severely reduced lending and output.

Related literature. Our paper pursues the effort of the growing body of the continuous-time macroeconomic models with financial frictions (see e.g. Brunnermeier and Sannikov (2014, 2015),

Di Tella (2015), He and Krishnamurthy (2012, 2013)). Seeking for a better understanding of the transmission mechanisms and the consequences of financial instability, all these papers point out to the key role that balance-sheet constraints and net-worth of financial intermediaries may play in (de)stabilizing the economy in the presence of financing frictions and aggregate shocks. However, the aforementioned works do not explicitly distinguish financial intermediaries from the productive sector (even all of them use "financial intermediaries" as a metaphor for the most productive agents in the economy). We extend this literature by modeling the banking system explicitly, i.e., by separating the production technology from financial intermediation, which enables us to explore the impact of the bank lending channel on financial stability.

A common feature of the above-mentioned papers is the existence of fire-sale externalities in the spirit of Kiyotaki and Moore (1997) and Lorenzoni (2008) that arise when forced asset sales depress prices, triggering amplifying feedback effects. In our model, welfare-reducing pecuniary externalities in the credit market emerge in the absence of costly fire-sales, stemming from the difference between the social and private implied risk aversion with respect to the variations of aggregate bank capital. The main channel of inefficiencies imposed by individual banks' lending decisions is that each bank fails to internalize the impact of its lending choice on the banking system's exposure to aggregate risk, which increases endogenous volatility. An additional channel of inefficiencies is that individual banks do not internalize the impact of their lending decisions on the size of the expected profit margins, which is similar to the effect described in Malherbe (2015). However, whereas in Malherbe (2015), this "margin channel" affects social welfare via the size of the bankruptcy costs for all banks in the economy, in our framework it has the dynamic welfare implications: Namely, expected profit margins affect the ability of banks to accumulate loss-absorbing capital and thus their future capacity to lend.

From a technical perspective, our paper is related to the corporate liquidity management literature (see e.g. Jeanblanc and Shiryaev (1996), Milne and Robertson (1996), Décamps et al. (2011), Bolton et al. (2011, 2013), Hugonnier et Morellec (2015) among others) that places emphasis on the loss-absorbing role of corporate liquid reserves in the presence of financial frictions. In our model, the role of book equity is very similar to the role of liquidity buffers in those models. However, we differentiate from this literature by allowing for the feedback loop between the individual decisions and the dynamics of individual book equity via the general equilibrium mechanism that determines the loan rate and thus affects the expected earnings of banks. As a result, the individual bank's policies are driven by aggregate rather than individual book equity.

Our paper is also linked to the vast literature exploring the welfare effects of bank capital regulation. Most of the literature dealing with this issue is focused on the trade-off between the welfare gains from the mitigation of risk-taking incentives on the one hand³ and welfare losses caused by lower liquidity provision (e.g., Begenau (2015), Van den Heuvel (2008)), lower lending and output

³Bank capital is often viewed as "skin in the game" needed to prevent the opportunistic behaviours of banks' insiders.

(e.g., Nguyễn (2014), Martínez-Miera and Suarez (2014)) on the other hand.⁴ In contrast to the above-mentioned studies, the focus of our model is entirely shifted from the incentive effect of bank capital towards its role of a loss absorbing buffer - the concept that is often put forward by bank regulators. Moreover, the main contribution of our paper is qualitative: we seek to identify the long run effects of capital regulation rather than provide a quantitative guidance on the optimal level of a minimum capital ratio.

More broadly, this paper relates to the literature on credit cycles that has brought forward a number of alternative explanations for their occurrence. Fisher (1933) identified the famous debt deflation mechanism, that has been further formalized by Bernanke et al. (1996) and Kiyotaki and Moore (1997). It attributes the origin of credit cycles to the fluctuations of the prices of collateral. Several studies also place emphasis on the role of financial intermediaries, by pointing out the fact that credit expansion is often accompanied by a loosening of lending standards and "systemic" risk-taking, whereas materialization of risk accumulated on the balance sheets of financial intermediaries leads to the contraction of credit (see e.g. Aikman et al. (2014), Dell’Ariccia and Marquez (2006), Jimenez and Saurina (2006)). In our model, quasi-cyclical lending patterns emerge due to the reflection property of aggregate bank capital that follows from the optimality of “barrier” recapitalization and dividend strategies.

The rest of the paper is organized as follows. Section 2 presents the model. In Section 3 we characterize the competitive equilibrium. In Section 4, we discuss the sources of inefficiencies, comparing the competitive equilibrium with the social planner’s allocation. In Section 5 we introduce capital regulation, analyzing its implications for bank policies and the lending-stability trade-off. Section 6 concludes. All proofs and computational details are gathered in the Appendix. The empirical analysis supporting the key model predictions and various model extensions can be found in the Online Appendix.

2 Model

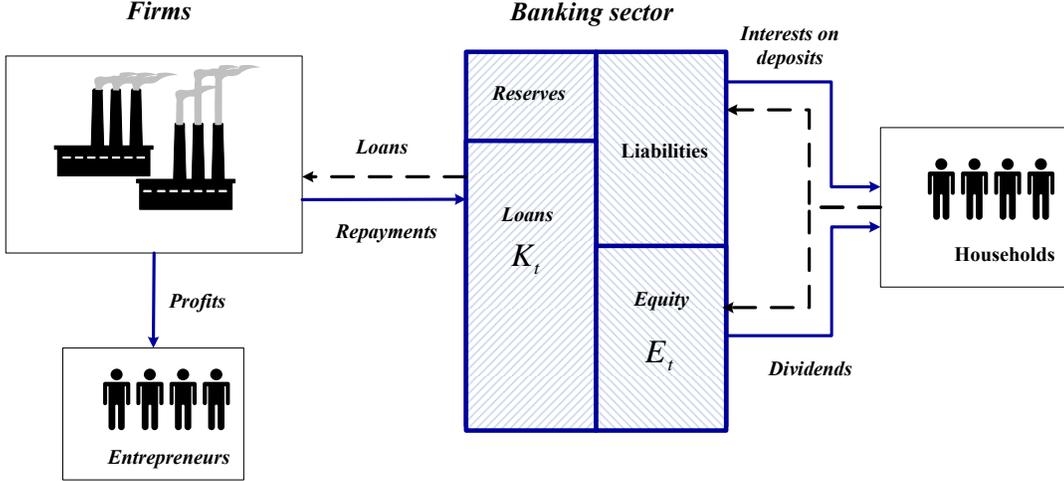
The economy is populated by households, who own and manage banks, and entrepreneurs, who own and manage firms (see Figure 1). All agents are risk-neutral and have the same discount rate ρ . There is one physical good, taken as a numeraire, which can be consumed or invested.

2.1 Economy

Firms. We consider an economy where the volume of bank lending determines the volume of productive investment. Entrepreneurs can consume only positive amounts and cannot save. At

⁴The only exception is the work by De Nicolò et al. (2014) that conducts the analysis of bank risk choices under capital and liquidity regulation in a fully dynamic model, where capital plays the role of a shock absorber, and dividends, retained earnings and equity issuance are modeled under a financial friction captured by a constraint on collateralized debt.

Figure 1: The structure of the economy



any time t each firm is endowed with a project that requires an investment of 1 unit of good and produces xh units of good at time $t+h$. The productivity parameter x is distributed according to a continuous distribution with density function $f(x)$ defined on a bounded support $[0, \bar{R}]$. To finance the investment project, a firm can take up a bank loan, for which it has to repay $1 + R_t h$ (to be determined in equilibrium) at time $t+h$.⁵

For the sake of exposition, we assume that a firm can always repay the interest, while the productive capital (principal) is destroyed with probability $\tilde{p}_t(h)$.⁶ Since the return on investment for a firm with productivity x is always equal to $x - R_t$, a firm asks for a bank loan and invests if and only if $x \geq R_t$. The total demand for bank loans at time t is, thus, given by

$$L(R_t) \equiv \int_{R_t}^{\bar{R}} f(x) dx. \quad (1)$$

Firms are subject to aggregate shocks that affect the probability of productive capital being destroyed. More specifically, firms' default probability $\tilde{p}_t(h)$ is higher under a negative shock and is lower under a positive shock it, i.e.,

$$\tilde{p}_t(h) = \begin{cases} ph - \sigma_0 \sqrt{h}, & \text{with probability } 1/2 \text{ (positive shock)} \\ ph + \sigma_0 \sqrt{h}, & \text{with probability } 1/2 \text{ (negative shock)} \end{cases}, \quad (2)$$

⁵Firms in our model should be thought of as small and medium-sized enterprises (SMEs), which typically rely on bank financing. As is well known, the importance of bank financing varies across countries. For example, according to the TheCityUK research report (October 2013), in EU area, bank loans account for 81% of the long term debt in the real sector, whereas in the U.S. the same ratio amounts to 19%.

⁶The assumption that R is always repaid is, of course, completely inconsequential in the continuous-time limit that we consider throughout the main analysis.

where ph denotes the unconditional probability of default and σ_0 the exposure to aggregate shocks.⁷ Taking the limit for period length $h \rightarrow 0$, net aggregate output follows

$$F(L(R_t))dt - [pdt - \sigma_0 dZ_t]L(R_t), \quad (3)$$

where $\{Z_t, t \geq 0\}$ is a standard Brownian motion and $F(L(R_t))$ denotes the aggregate instantaneous production function:

$$F(L(R_t)) \equiv \int_{R_t}^{\bar{R}} xf(x)dx. \quad (4)$$

Differentiating (4) with respect to R , using (1), immediately yields that at any point in time, the loan demand adjusts so that the marginal product of capital equals marginal costs (i.e., the loan rate):

$$F'(L(R_t)) \equiv R_t.$$

Households. Households are risk-neutral and have a discount rate ρ . As, for instance, in Brunnermeier and Sannikov (2014), they can consume both positive and negative amounts.⁸ Households allocate their wealth between bank equity and bank deposits.⁹ We also assume that households value the payment services provided by liquid deposits and, thus, enjoy additional utility flow of $\lambda(D_t^h)dt$, which is a concave non-decreasing function of the supply of deposits D_t^h .¹⁰ Lifetime utility of households is, therefore, given by:

$$\mathbb{E} \left[\int_0^{+\infty} e^{-\rho t} (dC_t^h + \lambda(D_t^h)) dt \right], \quad (5)$$

where dC_t^h denote the consumption flow at time t .

Provided that $\rho > r_t^d$ (this will be shown in Section 3), it can be shown that the aggregate volume of deposits in the economy is implicitly given by

$$\lambda'(D_t^h) = \rho - r_t^d, \quad (6)$$

where r_t^d is the interest rate on deposits.

Thus, households invest in deposits up to the point where the marginal utility from transaction services equals the difference between households' discount rate and the interest rate on deposits. Intuitively, as households value the transaction services provided by deposits, they are willing to

⁷The extension to heterogeneous default probability is straightforward and would not change our qualitative results.

⁸Negative consumption can be interpreted as disutility from labor to produce additional goods.

⁹The assumption that households cannot invest directly in firms' projects can be justified by technological and informational frictions that are not modeled here explicitly (see e.g. Freixas and Rochet(2008), Chapter 2).

¹⁰See e.g., Diamond and Dybvig (1983), Gorton and Metrick (2010) or Stein (2012).

accept an interest rate below the discount rate.

Banks. At time t , a given bank with $e_t \geq 0$ units of equity chooses the amount of deposits, $d_t \geq 0$, and the volume of lending to firms, $k_t \geq 0$. Banks have access to central bank reserves and refinancing facilities (or, equivalently, to interbank lending) at the exogenously given, constant central bank rate r . Its net position of reserves in the central bank (or/and lending on the interbank lending market) is determined by the following accounting identity:

$$m_t = e_t + d_t - k_t. \quad (7)$$

A bank also chooses the amount of dividends to be distributed to existing shareholders $d\delta_t \geq 0$, and the amount of new equity to be raised, $di_t \geq 0$. Issuing equity entails a proportional (dead-weight) cost γ , which constitutes the main financial friction in our economy. Note that in the presence of this financial friction and aggregate shocks, bank equity will serve the purpose to buffer losses on loans.

Banks supply loans in a perfectly competitive market and, thus, take the loan rate R_t as given. When a bank grants a loan to firms, it bears the risk that the principal will be destroyed, implying that the instantaneous return on lending is given by $(R_t - p)dt - \sigma_0 dZ_t$. Thus, using accounting identity (7), a given bank's book equity evolves according to

$$de_t = re_t dt + k_t[(R_t - p - r)dt - \sigma_0 dZ_t] - d\delta_t + di_t + d_t(r - r_t^d)dt. \quad (8)$$

Similarly, aggregate bank equity E_t , which constitutes the second state variable besides individual bank equity e_t , evolves according to:

$$dE_t = rE_t dt + K_t[(R_t - p - r)dt - \sigma_0 dZ_t] - d\Delta_t + dI_t + D_t(r - r_t^d)dt, \quad (9)$$

where K_t , $d\Delta_t$, dI_t , and D_t denote, respectively, the aggregate volumes of lending, dividends, equity issuance, and deposits at time t . Each bank is run in the interest of its shareholders, implying that lending, deposit, dividend, and recapitalization policies are chosen so as to maximize shareholder value

$$v(e, E) = \max_{k_t, d_t, d\delta_t, di_t} \mathbb{E} \left[\int_0^\tau e^{-\rho t} (d\delta_t - (1 + \gamma)di_t) \mid e_0 = e, E_0 = E \right], \quad (10)$$

where individual and aggregate equity follow (8) and (9) respectively, and $\tau := \inf\{t : e_t < 0\}$ denotes the first time when the book value of bank equity becomes negative, which triggers the bank's default. If it is optimal to inject new equity instead of defaulting, $\tau \equiv \infty$. We assume throughout that this is, in fact, the case and provide the appropriate condition in the proof of Proposition 2. Solving banks' maximization problem constitutes a major part of the equilibrium analysis and is thus referred to Section 3.

2.2 One-period example

Before turning to the analysis of the competitive equilibrium of the dynamic model, it is instructive to briefly consider a one-period problem to illustrate the basic frictions in our framework. In $t = 0$, each bank has an initial endowment of equity e_0 , deposit d , issues new equity i , distributes dividends δ , and grants loans k to firms. Returns are realized, claims are repaid, and consumption takes place at $t = 1$. For a period length normalized to one, firms' default probability is $\tilde{p} \equiv p \mp \sigma_0$. The accounting identity (7) then becomes

$$m = e_0 - \delta + i + d - k, \quad (11)$$

and bank profits at date $t = 1$ can be written as

$$\pi_B = (1 + r)e + k(R - \tilde{p} - r) + d(r - r^d), \quad (12)$$

where $e \equiv e_0 - \delta + i$ denotes an individual bank's equity after recapitalization and dividend distribution. Note that for the deposit market to clear, the deposit rate must coincide with the central bank rate, $r^d \equiv r$, implying that the last term in (12) vanishes. Conditional on the realization of the aggregate shock, a given bank's equity in $t = 1$ is then given by

$$\begin{aligned} e^+ &\equiv (1 + r)e + k[R - r - (p - \sigma_0)], \\ e^- &\equiv (1 + r)e + k[R - r - (p + \sigma_0)]. \end{aligned}$$

To make the non-negativity constraint for equity meaningful in the one-period problem, we assume that interbank lending is fully collateralized.¹¹ Thus, bank capital must be sufficiently high to cover the worst possible loss:

$$e^- \geq 0. \quad (13)$$

Imposing this collateral constraint generates a role for bank capital as a loss absorbing buffer in this static set-up. However, this is merely a short-cut formulation for the endogenous benefits of capital buffers that arise in the dynamic setting.

A competitive equilibrium is characterized by a loan rate R which, by market clearing, determines the aggregate volume of lending $K \equiv L(R)$. Each bank takes the loan rate R as given and chooses dividend policy $\delta \geq 0$, recapitalization policy $i \geq 0$ and the volume of lending $k \geq 0$ so as to

¹¹This condition easily extends to the case when depositors accept some probability of default (Value-at-Risk constraint similar to Adrian and Shin (2010)). An equivalent interpretation is that banks finance themselves by repos, and the lender applies a hair cut equal to the maximum possible value of the asset (the loan portfolio) that is used as collateral.

maximize shareholder value,

$$v = \max_{\delta, i, k} \left\{ \delta - (1 + \gamma)i + \frac{\left(\frac{1}{2}\right) e^+ + \left(\frac{1}{2} + \theta\right) e^-}{1 + \rho} \right\},$$

where θ denotes the Lagrange multiplier associated with the constraint (13).¹² Note that this problem is separable, i.e.,

$$v = e_0 u + \max_{\delta \geq 0} \delta [1 - u] + \max_{i \geq 0} i [u - (1 + \gamma)] + \max_{k \geq 0} k \left[\frac{(R - p - r)(1 + \theta) - \theta \sigma_0}{1 + \rho} \right], \quad (14)$$

where

$$u \equiv \frac{(1 + r)(1 + \theta)}{1 + \rho}. \quad (15)$$

Optimizing with respect to the bank's policies yields the following conditions:

$$1 - u \leq 0 \quad (= \text{if } \delta > 0), \quad (16)$$

$$u - (1 + \gamma) \leq 0 \quad (= \text{if } i > 0), \quad (17)$$

$$-\frac{R - r - p}{R - r - (p + \sigma_0)} \geq \theta \quad (= \text{if } k > 0). \quad (18)$$

Condition (16) shows that $\theta > 0$, which implies that constraint (13) is always binding at both the individual and aggregate level. This determines the loan rate as a function of aggregate bank capitalization, i.e. $R \equiv R(E)$. More specifically, rewriting the binding constraint (13) for aggregate equity yields

$$(1 + r)E + L(R(E)) [R(E) - r - (p + \sigma_0)] = 0. \quad (19)$$

Two implications immediately follow from this expression: i) the equilibrium loan rate $R \equiv R(E)$ is a decreasing function of aggregate capital and ii) the aggregate volume of lending is strictly positive. Hence, equation (18) must hold with equality and pins down the shadow cost function $\theta \equiv \theta(E)$. Substituting (15) through (18) into (A4), we find that the shareholder value function of each bank is proportional to its book equity and explicitly given by

$$v = eu(E) = e \left(\frac{1 + r}{1 + \rho} \right) \left(\frac{-\sigma_0}{R(E) - r - (p + \sigma_0)} \right), \quad (20)$$

where $u \equiv u(E)$ is the market-to-book value of equity. Note that $u(E)$ is a decreasing function of E , implying that bank capital becomes more valuable when it is getting scarce.

Conditions (16) and (17) show that the optimal dividend and recapitalization policies also depend on aggregate capital via the market-to-book value of bank equity. Let E_{max} denote the unique level of aggregate equity such that (16) holds with equality, and E_{min} the one such that (17) holds with

¹²In this one-period model, shareholders receive terminal dividends at the end of the period.

equality. Then, if $E < E_{min}$, shareholders will recapitalize banks by raising in aggregate $E_{min} - E$. Similarly, if $E > E_{max}$, aggregate dividends $E - E_{max}$ are distributed to shareholders. As a result, the market-to-book ratio of the banking sector always belongs to the range $[1, 1 + \gamma]$. The following proposition summarizes our results for this static set-up:

Proposition 1 *The one-period model has a unique competitive equilibrium, where*

- a) *The loan rate $R \equiv R(E)$ is a decreasing function of aggregate capital and it is implicitly given by the binding non-negativity constraint for aggregate capital (19).*
- b) *All banks have the same market-to-book ratio of equity given by (20), which is a decreasing function of aggregate capital.*
- c) *Banks pay dividends when $E \geq E_{max} \equiv u^{-1}(1)$ and recapitalize when $E \leq E_{min} \equiv u^{-1}(1 + \gamma)$.*

Despite the simplistic approach, modelling the need for capital buffers in such a reduced form allows us to illustrate several important features that prevail in the continuous-time equilibrium. First, only the level of aggregate bank capital E matters for banks' policies, whereas the individual banks' sizes do not play any role. Second, banks' recapitalization and dividend policies are of the "barrier type" and are driven by the market-to-book value of equity which, in turn, is strictly decreasing in aggregate bank capital and remains in the interval $[1, 1 + \gamma]$. Finally, the loan rate is decreasing in aggregate bank capital.

3 Competitive Equilibrium

3.1 Solving for the equilibrium

In this section we solve for a Markovian competitive equilibrium, where all aggregate variables are deterministic functions of the single state variable, namely, aggregate bank equity, E_t . In a competitive equilibrium, i) loan and deposit markets must clear; ii) the dynamic of individual and aggregate bank capital is given by (8) and (9), respectively; iii) households and banks maximize (5) and (10), respectively.

Consider first the optimal decision problem of an individual bank that chooses lending $k_t \geq 0$, deposit $d_t \geq 0$, dividend $d\delta_t \geq 0$ and recapitalization $di_t \geq 0$, so as to maximize shareholder value. Making use of the homotheticity property of the equity market value defined in (10), we define the market-to-book value of bank capital,

$$u(E) \equiv \frac{v(e, E)}{e},$$

which can also be interpreted as the marginal value of an individual bank's book equity, e . Note that this allows to work with aggregate bank capital, E , as the single state variable and, thus, to

focus on the capitalization of the banking sector as a whole, instead of the capitalization of each individual bank. By standard dynamic programming arguments, the market-to-book value of a given bank in the economy satisfies the following Bellman equation:

$$\begin{aligned}
(\rho - r)u(E) &= \max_{d\delta \geq 0} \left\{ \frac{d\delta}{e} [1 - u(E)] \right\} - \max_{di \geq 0} \left\{ \frac{di}{e} [1 + \gamma - u(E)] \right\} + \max_{d \geq 0} \left\{ \frac{d}{e} (r - r_t^d) u(E) \right\} \\
&+ \max_{k \geq 0} \left\{ \frac{k}{e} \left[(R(E) - p - r)u(E) + \sigma_0^2 K(E) u'(E) \right] \right\} \\
&+ \left(rE + K(E)[R(E) - p - r] \right) u'(E) + \frac{\sigma_0^2 K^2(E)}{2} u''(E).
\end{aligned} \tag{21}$$

Maximization with respect to the volume of deposits shows that the deposit market clears only if the deposit rate coincides with the central bank rate, i.e. $r_t^d \equiv r$. This implies that an individual bank is indifferent with respect to the volume of deposits. Using (6), it is easy to see that, on the aggregate level, the volume of deposits is constant and can be pinned down from equation $\lambda'(D) = \rho - r$. Moreover, with $r_t^d \equiv r$, the last terms in the equations (8) and (9) describing the dynamics of individual and aggregate equity vanish.

Since banks are competitive, each bank takes the loan rate $R(E)$ as given. For the loan market to clear, the *aggregate* supply of loans has to equal firms' demand for loans in (1) under the prevailing loan rate, i.e. $K(E) = L(R(E))$.¹³ Maximizing (21) with respect to the loan supply of each *individual* bank shows that for lending to be interior, the risk-adjusted loan spread must equal banks' risk-premium:

$$R(E) - p - r = -\frac{u'(E)}{u(E)} \sigma_0^2 L(R(E)). \tag{22}$$

The risk-premium required by any given bank is driven by its implied risk aversion with respect to variation in aggregate capital, $-u'(E)/u(E)$, multiplied by the endogenous volatility of aggregate capital, $\sigma_0^2 L(R(E))$. Since an additional unit of loss absorbing capital in the banking sector is more valuable when it is scarce, the market-to-book ratio $u(E)$ is (weakly) decreasing in aggregate capital (this is shown formally in the proof of Proposition 2). Intuitively, risk-neutral banks become risk-averse with respect to variations in aggregate capital and therefore require a positive risk premium due to the following amplification effect: Upon the realization of a negative aggregate shock, the capitalization of the banking system deteriorates, which drives up the market-to-book value $u(E)$ and, thus, aggravates the impact of the negative shock on each individual bank's shareholder value (recall that $v(e, E) \equiv eu(E)$). Conversely, upon the realization of a positive aggregate shock, the banking system will be better capitalized, which reduces the market to book ratio $u(E)$ and, thus, reduces the impact of the positive shock on each individual bank's shareholder value.

Next, optimization with respect to $d\delta$ and di shows that the optimal dividend and recapitaliza-

¹³This specification is analogous to an economy with constant returns to scale, in which the equilibrium price of any output is only determined by technology (constant marginal cost), whereas the volume of activity is determined by the demand side.

tion policies are of the "barrier" type. That is, each individual bank distributes dividends ($d\delta > 0$) when the level of aggregate bank capital exceeds a critical level E_{max} , that satisfies

$$u(E_{max}) = 1, \quad (23)$$

i.e., the marginal value of equity capital equals the shareholders' marginal value of consumption. Aggregate dividend payments therefore consist of all profits in excess of E_{max} , which causes aggregate capital to be reflected at that point. Moreover, the optimal choice of E_{max} implies that $u'(E_{max}) = 0$, such that at the dividend payout boundary, the required risk-premium in (22) vanishes, i.e.,

$$R(E_{max}) = p + r. \quad (24)$$

That is, as banks make zero expected profits on the marginal loan, all capital in excess of E_{max} is optimally distributed to shareholders rather than invested in the corporate loan market.

Similarly, new equity is issued ($di > 0$) only when aggregate capital reaches a critical threshold E_{min} at which the marginal value of equity equals the total marginal cost of equity issuance, i.e.,

$$u(E_{min}) = 1 + \gamma. \quad (25)$$

At that point, all banks issue new equity to offset further losses on their loan portfolios and, thus, to prevent aggregate equity from falling below E_{min} . Since the market to book ratio $u(E)$ is (weakly) decreasing in aggregate capital, banks recapitalize only if all capital is completely exhausted and, thus, the non-negativity constraint binds on the individual and on the aggregate level.¹⁴ Taken together, aggregate bank equity fluctuates between the reflecting barriers,

$$E_{min} = 0,$$

where all banks recapitalize, and E_{max} , where all banks pay dividends. In the interior region $(0, E_{max})$, banks retain all profits to build up capital buffers and use these buffers to offset losses. For $E \in (0, E_{max})$, the market-to-book value $u(E)$ satisfies

$$(\rho - r)u(E) = \left[rE + L(R(E)) \left(R(E) - p - r \right) \right] u'(E) + \frac{\sigma_0^2 L(R(E))^2}{2} u''(E). \quad (26)$$

¹⁴In the Online Appendix, we show that, in the discrete-time dynamic version of the model, banks are recapitalized at a strictly positive level of E . The reason is that, in the discrete-time dynamic set-up, the non-negativity constraint for capital is binding for low levels of equity both at the individual and at the aggregate level. Accelerating recapitalizations allows shareholders to reduce the "shadow costs" of the binding non-negativity constraint. However, when the length of the time period goes to zero, the region on which the non-negativity constraint is binding shrinks (and collapses to a single point, 0, in the continuous-time limit). It is important to stress, however, that the property $E_{min} = 0$ is *not* a general feature of the continuous-time set-up. In the Online Appendix we show that the recapitalization barrier may depend on the state of the business cycle, similar in spirit to the "market timing" result of Bolton et al. (2013).

The following proposition characterizes the competitive equilibrium.

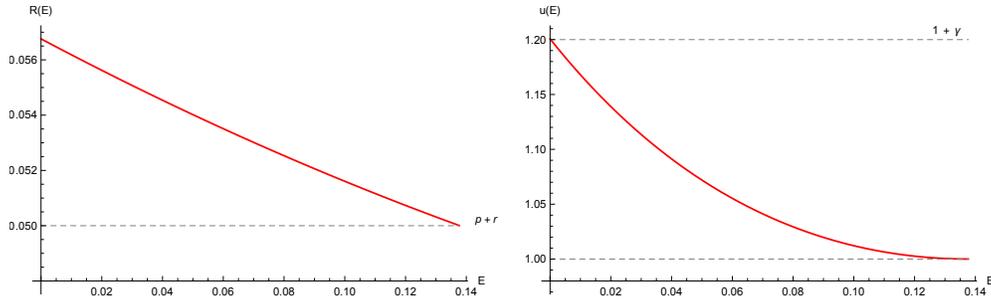
Proposition 2 *There exists a unique Markovian equilibrium, in which aggregate bank capital evolves according to:*

$$dE_t = rE_t dt + L(R(E_t))[(R(E_t) - p - r)dt - \sigma_0 dZ_t]. \quad (27)$$

The equilibrium loan rate $R(E)$ and market-to-book value $u(E)$ satisfy (22) through (26). Banks distribute dividends when E reaches the threshold E_{max} and recapitalize when E reaches 0.

Figure 2 illustrates the typical patterns of the equilibrium loan rate (left panel) and the market-to-book value (right panel).

Figure 2: Loan rate and market-to-book value of equity in the competitive equilibrium



Notes: this figure reports the typical patterns of the loan rate $R(E)$ (left panel) and the market-to-book ratio of equity $u(E)$ (right panel) in the competitive equilibrium. Parameter values: $\rho = 0.05$, $\sigma_0 = 0.05$, $p = 0.02$, $\gamma = 0.2$, $r = 0.03$, $L(R) = \bar{R} - R$, $\bar{R} = 0.17$.

To illustrate the properties of the obtained equilibrium, we use expression (22) to eliminate the market-to-book value (and its derivatives) in the second order differential equation (26). This yields a relatively simple first order differential equation in $R(E)$,

$$R'(E) = - \left(\frac{1}{\sigma_0^2} \right) \frac{2(\rho - r)\sigma_0^2 + [R(E) - p - r]^2 + 2rE[R(E) - p - r]L(R(E))^{-1}}{L(R(E)) - L'(R(E))[R(E) - p - r]}. \quad (28)$$

From (28) it is immediate, that the loan rate $R(E)$ is strictly decreasing in E . That is, as the banking system becomes better capitalized, banks' implied risk-aversion $-u'(E)/u(E)$ decreases which, despite an increase in the endogenous volatility of aggregate capital $\sigma_0^2 L(R(E))$, results in a lower risk-premium in expression (22).¹⁵

Moreover, solving equation (22) subject to boundary condition (23), yields the quasi-explicit expression for the market-to-book value:

$$u(E) = \exp\left(\int_E^{E_{max}} \frac{R(s) - p - r}{\sigma_0^2 L(R(s))} ds \right), \quad (29)$$

¹⁵Since aggregate output in our economy is completely determined by the volume of bank lending, this gives rise to pro-cyclical lending patterns in line with empirical evidence reported, for instance, by Becker and Ivashina (2014).

where $R(E)$ satisfies (28) with the boundary condition $R(E_{max}) = p + r$.

The free boundary E_{max} is then pinned down by boundary condition (25), which transforms to:

$$\int_0^{E_{max}} \frac{R(s) - p - r}{\sigma_0^2 L(R(s))} ds = \ln(1 + \gamma). \quad (30)$$

The fact that $R(E)$ is a decreasing function of E that satisfies the boundary condition $R(E_{max}) = p + r$ implies that the loan rate increases with E_{max} for any level of E . It is then easy to see that the left-hand side of Expression (30) is increasing in E_{max} . This immediately leads to the following result:

Corollary 1 *The target level of aggregate bank capital, E_{max} , is increasing with the magnitude of financial friction γ .*

Intuitively, stronger financial frictions make bank shareholders effectively more risk averse, which induces them to postpone dividend payments to build larger capital buffers. Simultaneously, a higher effective risk aversion implies higher loan rates. Thus, our model suggests that lending and, thereby, output should exhibit more dispersion in the economies with stronger financial frictions.

3.2 Loan rate dynamics and the long-run behavior of the economy

For the special case where the central bank rate r is normalized to zero, the dynamics of the loan rate can be obtained in explicit form. Let $R_t = R(E_t)$ be some twice continuously differentiable function of aggregate capital. Applying Itô's lemma to $R(E_t)$, while using the equilibrium dynamics of $E_t \in (0, E_{max})$ given in (27), yields:

$$dR_t = \underbrace{L(R(E_t)) \left((R(E_t) - p)R'(E_t) + \frac{\sigma_0^2 L(R(E_t))}{2} R''(E_t) \right)}_{\mu(R_t)} dt - \underbrace{\sigma_0 L(R(E_t)) R'(E_t)}_{\sigma(R_t)} dZ_t. \quad (31)$$

Recall that the loan rate satisfies the first-order differential equation (28), which allows us to eliminate its derivatives from expression (31). After some algebra, one can thus obtain the drift and the volatility of R_t in closed form, which is summarized in the following proposition.

Corollary 2 *The loan rate $R_t = R(E_t)$ has explicit dynamics*

$$dR_t = \mu(R_t)dt + \sigma(R_t)dZ_t, \quad p \leq R_t \leq R_{max}, \quad (32)$$

with reflections at both ends of the support. The volatility function is given by

$$\sigma(R) = \frac{2\rho\sigma_0^2 + (R - p)^2}{\sigma_0 \left(1 - (R - p) \frac{L'(R)}{L(R)} \right)}. \quad (33)$$

The drift function is

$$\mu(R) = \frac{\sigma(R)}{2} \left[\frac{\sigma(p) - \sigma(R)}{R - p} - \frac{R - p}{\sigma_0} + \sigma'(R) \right]. \quad (34)$$

The closed-form specifications of the loan rate dynamics allows the explicit analysis of the long-run behavior of the economy. We show below that relying on the results of the *impulse response* analysis that is usually employed to study the long-run dynamics of macro-variables in the traditional marcoeconomic models does not provide a correct picture of the asymptotic behavior of the economy in our framework, as the latter is mainly driven by the endogenous volatility neglected by the impulse response analysis.

Recall that the usual approach to studying the macrodynamics in a DSGE model requires linearizing this model around the deterministic steady-state and perturbing the system by a single unanticipated shock. The equivalent approach in our framework would be to consider a particular trajectory of realization of aggregate shocks such that $dZ_t = 0$ for $t > 0$. Then, the dynamics of the system can be described by the ordinary differential equation (linearization is not needed here):

$$dR_t = \mu(R_t)dt,$$

where the initial shock determines $R_0 > p$.

By using expression (34), it can be shown that $\mu(p) = 0$. Hence, the frictionless loan rate ($R_t = p$) is an equilibrium of the deterministic system. It is *locally* stable when $\mu'(p) < 0$ and is *globally* stable when $\mu(R) < 0$ for all R . After some computations, it can be shown that $\mu'(p) = 2\rho^2\sigma_0^2 \frac{L''(p)}{L(p)}$. Hence, the steady state $R = p$ is locally stable when $L''(p) < 0$. Moreover, it is globally stable when $L''(R) < 0$.

While the outcomes of the impulse response analysis applied to our model might suggest that the economy should remain most of the time at the "deterministic" steady state $R = p$, it is actually never the case in our set-up because of the significant impact of endogenous risk. To demonstrate this, we solve for the ergodic density function of R that shows how frequently each state is visited in the long run.

Let $g(t, R)$ denote the probability density function of R_t . Given the loan rate dynamics defined in Proposition 2, it must satisfy the forward Kolmogorov equation:

$$\frac{\partial g(t, R)}{\partial t} = -\frac{\partial}{\partial R} \left\{ \mu(R)g(t, R) - \frac{1}{2} \frac{\partial}{\partial R} \left[\sigma^2(R)g(t, R) \right] \right\}. \quad (35)$$

Since the process R_t is stationary, we have $\frac{\partial g(t, R)}{\partial t} = 0$ and thus $g(t, R) \equiv g(R)$. Integrating Equation (35) over R yields:

$$\mu(R)g(R) = \frac{1}{2} \frac{\partial}{\partial R} \left[\sigma^2(R)g(R) \right],$$

where the constant of integration is set to zero because of reflection properties of the process. Solving

the above equation by using the change of variable $\hat{g}(R) \equiv \sigma^2(R)g(R)$ ultimately yields:

$$g(R) = \frac{C_0}{\sigma^2(R)} \exp\left(\int_p^{R_{max}} \frac{2\mu(s)}{\sigma^2(s)} ds\right), \quad (36)$$

where the constant C_0 is chosen so as to normalize the solution to 1 over the region $[p, R_{max}]$, i.e. $\int_p^{R_{max}} g(R)dR = 1$.¹⁶

By differentiating the logarithm of the ergodic density defined in (36), we obtain:

$$\frac{g'(R)}{g(R)} = \frac{2\mu(R)}{\sigma^2(R)} - \frac{2\sigma'(R)}{\sigma(R)}. \quad (37)$$

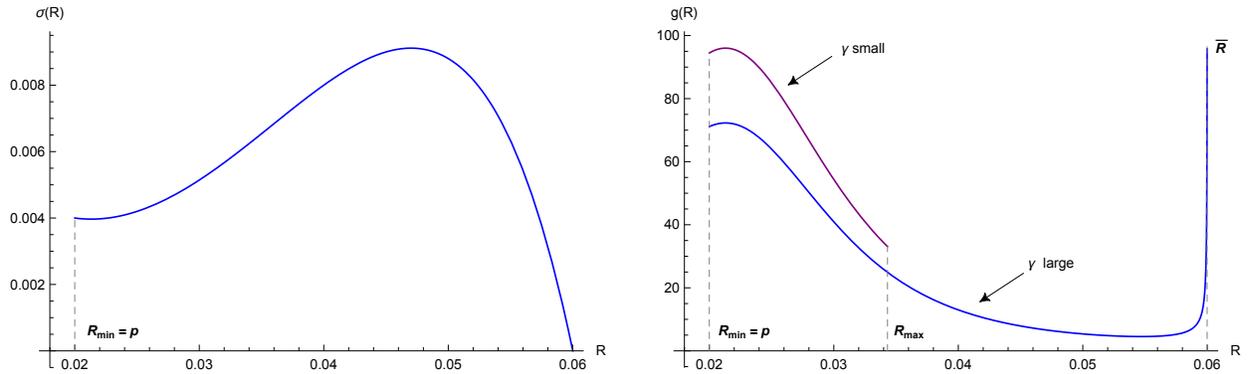
Using the closed-form expressions for $\sigma(R)$ and $\mu(R)$, it can be shown that $\sigma(p) = 2\rho\sigma_0$, $\sigma'(p) = 2\rho\sigma_0 \frac{L'(p)}{L(p)} < 0$ and $\mu(p) = 0$. Hence, $g'(p) > 0$, which means that the state $R = p$ that would correspond to the "deterministic" steady state is definitely *not* the one at which the economy spends most of the time in the stochastic set up.

To get a deeper understanding of the determinants of the long-run behavior of the economy, we resort to the particular specification of the demand for loans:

$$L(R) = \alpha(\bar{R} - R)^\beta, \quad (38)$$

where $\beta \geq 0$, $p < \bar{R}$ and the constant $\alpha \equiv (\bar{R} - p)^{-\beta}$ is chosen so as to normalize the maximum feasible volume of lending (it is attained for $R = p$) to 1.

Figure 3: Volatility and ergodic density of R



Notes: this figure reports the typical patterns of the loan rate volatility (left panel) and the ergodic density (right panel). Parameter values: $\rho = 0.04$, $\sigma_0 = 0.05$, $p = 0.02$, $\beta = 0.5$ and $\bar{R} = 0.06$.

Figure 3 reports the typical patterns of the endogenous volatility $\sigma(R)$ (the left-hand side panel)

¹⁶To ensure that the distribution of R is non-degenerate, it is sufficient to check that $\sigma(R) > 0$ for any $R \in [p, R_{max}]$. From the expression of $\sigma(R)$, it is easy to see that this condition holds for any loan demand specifications such that $L'(R) < 0$ and $L(R) > 0$.

and the ergodic density $g(R)$ (the right-hand side panel) for the above loan demand specification. It shows that the extrema of the ergodic density almost coincide with those of the volatility function, i.e., the economy spends most of the time in the states with the lowest loan rate volatility. Intuitively, the economy can get "trapped" in the states with low loan rate volatility because the endogenous drift is generally too small to move it away from these states. In fact, $\sigma(R)$ turns out to be much larger than $\mu(R)$ for any level of R , so that the volatility impact always dominates the drift impact.¹⁷

Note that functions $\sigma(\cdot)$ and $g(\cdot)$ must be truncated (and, in the case of the ergodic density, rescaled) on $[p, R_{max}]$, where R_{max} depends on the magnitude of the financial friction γ . For the specification of the loan demand function stated in (38) we always have $R_{max} < \bar{R}$. However, R_{max} can be arbitrary close to \bar{R} , which typically happens with very strong financial frictions and low elasticity of credit demand. In that case the economy will spend quite some time in the region where the loan rate is close to R_{max} . This situation can be interpreted as a persistent "credit crunch" that manifests itself via scarce bank equity capital, high loan rates, low volumes of lending and output.¹⁸

3.3 Testable implications

Overall, the analysis conducted in this section delivers two key testable prediction: the equilibrium loan rate and the banks' market-to-book ratio of equity should be decreasing functions of aggregate bank capital. To examine whether these predictions are rejected by the data or not, in the Online Appendix we conduct statistical tests on a data set covering a large panel of publicly traded banks in 43 advanced and emerging market economies for the period 1982-2013. We find that our predictions fit the data extremely well. Table 3 in the Online Appendix shows the results of simple regressions of loan rates and market-to-book ratios of banks equity in three sub-panels: U.S. banks, Advanced countries' (excluding U.S.) banks and Emerging countries' banks. In all cases, the coefficients of Total Bank Equity are negative with p -values indistinguishable from zero.

¹⁷The reason is that the multiplier of $\sigma(R)$ in the expression of $\mu(R)$ (see expression (34)) is very small.

¹⁸This "credit crunch" scenario is reminiscent to the "net worth trap" documented by Brunnermeier and Sannikov (2014).¹⁹ In their model, the economy may fall into a recession because of the inefficient allocation of productive capital between more and less productive agents, which they call "experts" and "households" respectively. This allocation is driven by the dynamics of the equilibrium price of capital, which depends on the fraction of the total net worth in the economy that is held by experts. After experiencing a series of negative shocks on their net worth, experts have to sell capital to less productive households, so that the average productivity in the economy declines. Under a reduced scale of operation, experts may struggle for a long time to rebuild net worth, so that the economy may be stuck in a low output region. In our model, the output in the economy is driven by the volume of credit that entrepreneurs can get from banks, whereas the cost of credit depends on the level of aggregate bank capitalization. When the banking system suffers from a series of adverse aggregate shocks, its loss absorbing capacity deteriorates. As a result, the amplification mechanism working via the market-to-book value becomes more pronounced and bankers thus require a larger lending premium. The productive sector reacts by reducing its demand for credit and the banks have to shrink their scale of operations, which makes it even more difficult to rebuild equity capital.

4 Inefficiencies

In this section we show that the competitive equilibrium is not constrained efficient due to the pecuniary externalities that each bank imposes on its competitors. More precisely, competitive banks do not take into account the effect of their individual lending decisions on the dynamics of aggregate bank equity. This leads to excessive lending when banks are poorly capitalized, implying overexposure of the banking system to aggregate shocks and the erosion of its ability to accumulate earnings. The failure to reduce exposure in bad times in turn aggravates banks' implied risk aversion with respect to variations in aggregate capital and leads to inefficiently low lending in good times (i.e. when the banking system is well capitalized). Thus, our model shows that welfare-reducing pecuniary externalities can emerge in credit markets even if there are no costly fire sales.²⁰

4.1 Welfare in the competitive equilibrium

For simplicity, for the rest of the paper the central bank rate r is normalized to zero. Social welfare in our framework can be computed as the expected present value of firms' profits (which are immediately consumed by entrepreneurs), banks' dividend payments net of capital injections, and the utility flows from holding deposits:

$$W(E) = \mathbb{E} \left[\int_0^{+\infty} e^{-\rho t} \left\{ \pi_F(K) + d\Delta_t - (1 + \gamma) dI_t + \lambda(D)dt \right\} | E_0 = E \right], \quad (39)$$

where $\pi_F(K) \equiv F(K) - KF'(K)$ denotes the aggregate instantaneous firms' profits (i.e., instantaneous output net of the cost of credit).

By using aggregate bank equity E as a state variable, we can apply standard methods to compute the social welfare function. As long as banks neither distribute dividends nor recapitalize, instantaneous utility flows in the economy comprise firms' profits and the utility flow from holding deposits. Therefore, in the region $E \in (0, E_{max})$, the social welfare function, $W(E)$, must satisfy the following differential equation:²¹

$$\rho W(E) = F(K) - KF'(K) + \lambda(D) + K[F'(K) - p]W'(E) + \frac{\sigma_0^2}{2} K^2 W''(E), \quad (40)$$

subject to the boundary conditions:

$$W'(0) = 1 + \gamma, \quad (41)$$

$$W'(E_{max}) = 1. \quad (42)$$

²⁰Examples of macro models with costly fire sales are Brunnermeier and Sannikov (2015), Phelan (2015) and Stein (2012).

²¹For the sake of space, we omit the argument of $K(E)$.

These boundary conditions reflect the fact that when bank recapitalizations occur, the (social) marginal value of bank capital has to equal households' total costs of injecting additional equity. Likewise, when dividend distributions occur, the (social) marginal value of bank capital has to equal households' marginal utility from consumption.

Before solving for the constrained welfare optimum in the next subsection, we first evaluate the social welfare function at the competitive equilibrium in order to get a first grasp of the prevailing distortions. To this end, we take the first derivative of the right-hand side of equation (40) with respect to K . Since we are interested in the impact of a marginal increase in lending on welfare at the competitive equilibrium, we substitute the competitive loan rate from equilibrium condition (22) into the derivative which yields:

$$W' \left[\frac{W''}{W'} - \frac{u'}{u} \right] \sigma_0^2 K + F''(K) K [W' - 1]. \quad (43)$$

The first term in (43) captures an inefficiency stemming from the fact that competitive banks do not take into account how their individual lending decisions affect the exposure of the banking system to aggregate shocks (*“exposure channel”*). The term in square brackets captures the difference between the social planner's, $-W''/W'$, and individual banks', $-u'/u$, implied risk aversion with respect to variations in aggregate capital. The second term in (43) reflects a distributive inefficiency in the competitive equilibrium: An increase in aggregate lending drives down the loan rate ($F''(K) = \partial R / \partial K < 0$). This increases firms' profits but bites into the profits of banks and, thus, impairs banks' ability to accumulate loss absorbing capital in the form of retained earnings (*“margin channel”*).²²

The socially optimal level of lending in the economy is attained when the sum of both terms in expression (43) is zero. Competitive banks lend too little (i.e., welfare could be improved by increasing lending) when this sum is positive and too much (i.e., welfare could be improved by reducing lending) when it is negative. As expression (43) cannot be signed globally without making further assumptions, we proceed by solving for the social planner's allocation and comparing its outcomes with the outcomes of the competitive equilibrium.

4.2 Second best

The second best (social planner's) allocation is characterized by the welfare maximizing lending, dividend and recapitalization policies, subject to the same friction as in the competitive equilibrium (i.e., issuing new capital entails costs γ). The single state variable in the social planner's problem is aggregate bank equity E and the market for bank credit must clear, i.e. $K = L[R(E)]$.

Provided that the welfare function is concave (this has to be verified ex-post), it is immediate

²²The negative effect of competition of banks' profit margins is also acknowledged in Martinez-Miera and Repullo (2010) albeit in an entirely different application.

that optimal dividend and recapitalization policies are of the “barrier type” and thus aggregate equity fluctuates within a bounded support $[E_{min}^{sb}, E_{max}^{sb}]$, where E_{min}^{sb} and E_{max}^{sb} denote the recapitalization and the dividend distribution barriers, respectively. For $E \in [E_{min}^{sb}, E_{max}^{sb}]$, the social welfare function satisfies the following HJB equation:

$$\rho W(E) = \max_{K \geq 0} F(K) - KF'(K) + \lambda(D) + K[F'(K) - p]W'(E) + \frac{\sigma_0^2}{2}K^2W''(E). \quad (44)$$

Taking the first-order condition of the right-hand side of (44) with respect to K , and rearranging terms shows that the second-best loan rate, $R^{sb} \equiv R^{sb}(E)$, satisfies:²³

$$R^{sb} - p = -\frac{W''}{W'}\sigma_0^2K^{sb} + \left(\frac{1}{W'} - 1\right)K^{sb}F''(K^{sb}), \quad (45)$$

where $K^{sb}(E) = L(R^{sb})$.²⁴

Note that the above expression shares some similarities with the expression (22) in the competitive equilibrium, with the notable distinction that the loan rate depends on the *social* implied risk aversion with respect to variation in aggregate equity, $-W''/W'$, whereas in the competitive equilibrium, it is driven by the *private* implied risk aversion $-u'/u$. Moreover, there is an additional term which is always positive or zero, as the (social) marginal value of earnings retained in banks is weakly higher than the marginal value of instantaneously consumed firms' profits, i.e., $W' \geq 1$.

The following proposition summarizes the characterization of the second best allocation.

Proposition 3 *The second best allocation is characterized by the aggregate lending function $K^{sb}(E) = L(R^{sb}(E))$, where $R^{sb}(E)$ is implicitly given by (45). The social welfare function satisfies the ODE*

$$\rho W(E) = F(K^{sb}) - K^{sb}F'(K^{sb}) + \lambda(D) + K^{sb}[F'(K^{sb}) - p]W'(E) + \frac{\sigma_0^2}{2}[K^{sb}]^2W''(E),$$

subject to the boundary conditions $W'(0) = W'(E_{max}^{sb}) - \gamma = 1$. Banks distribute dividends when E_t reaches $E_{max}^{sb} = [W'']^{-1}(0)$ and recapitalize when E_t hits zero.

To illustrate credit market inefficiencies emerging in our model set-up, we resort to the numerical example using a simple linear specification of the demand for loans.²⁵

$$L(R) = \frac{\bar{R} - R}{R - p}.$$

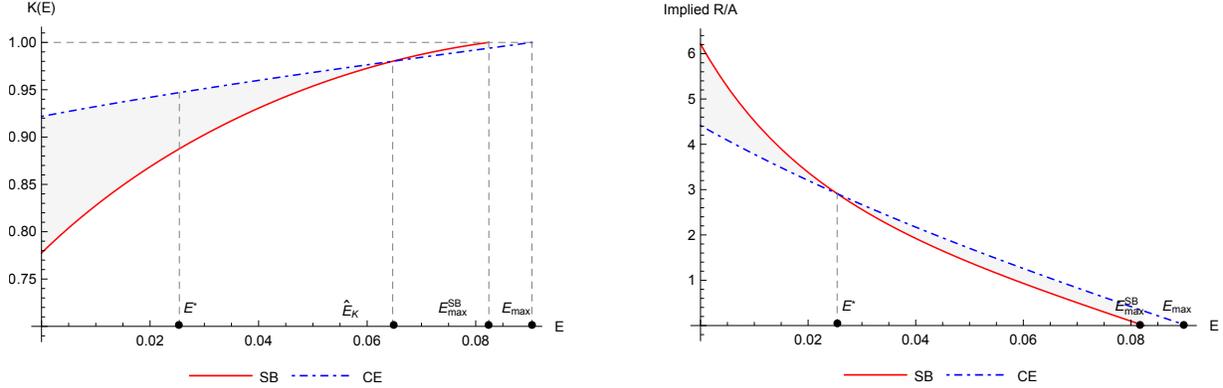
The left panel in Figure 4 shows that in the competitive equilibrium (CE), a poorly-capitalized banking system lends more than under the second best (SB) allocation, which reflects the pecuniary

²³For the sake of space, we abstain from writing the argument of functions $R^{sb}(\cdot)$, $K^{sb}(\cdot)$ and $W(\cdot)$.

²⁴Given that $W'(E_{max}^{sb}) = 1$ and $W''(E_{max}^{sb}) = 0$, it is easy to see that $R^{sb}(E_{max}^{sb}) = p$ like in the competitive equilibrium.

²⁵This specification is obtained from the one introduced in (38) by setting $\beta = 1$.

Figure 4: Credit volume and implied risk aversion under the *linear* loan demand: CE vs SB



Notes: this figure reports the credit volume (left panel) and implied risk-aversion (right panel) in the competitive equilibrium (dash-dotted lines) and the second best allocation (solid lines). Parameter values: $\rho = 0.05$, $\sigma_0 = 0.05$, $p = 0.02$, $\gamma = 0.2$, $\bar{R} = 0.15$.

externality inflicted by each bank on its competitors. The social planner internalizes this externality and reduces the exposure to macro shocks ($\sigma_0 K(E)$) in the states of the world where the implied risk aversion with respect to fluctuations in E is most severe (i.e., close to the recapitalization barrier $E_{min} = 0$). As illustrated in the right panel of Figure 4, this "exposure" externality is reflected by a higher implied risk-aversion of the social planner compared to individual banks (i.e., the first term in (43) is strictly negative) when the system is poorly capitalized, i.e., when $E < E^*$ where E^* is such that $-W''(E^*)/W'(E^*) = -u'(E^*)/u(E^*)$.

Moreover, lower lending in poorly capitalized states of the second best allocation increases banks' profit margins, which makes it easier to rebuild equity buffers after negative shocks. Importantly, this insight remains true even though restricting lending reduces firm profits. The reason is that, in the second best allocation, the social value of banks' profits (which are used to build loss-absorbing equity buffers) is always larger than the social value of firms' profits (which are consumed immediately). Due to this "margin channel", second-best lending remains lower than competitive lending even when the social planner's implied risk-aversion becomes lower than that of banks (namely, this happens on the interval $[E^*, \hat{E}_K]$, where \hat{E}_K is such that $K^{ce}(\hat{E}_K) = K^{sb}(\hat{E}_K)$). This margin channel is captured by the second term in (43), which is strictly negative as long as $W'(E) > 1$.²⁶

Due to the lower risk-exposure and higher expected profits of banks in poorly-capitalized states, the target aggregate equity buffer in the second best allocation is lower than in the competitive equilibrium, i.e. $E_{max}^{sb} < E_{max}^{ce}$. Intuitively the social planner curbs the banking system's exposure

²⁶Recall that, in the competitive equilibrium, $W'(0) = 1 + \gamma$ and thus inequality $W'(E) > 1$ holds for the lower levels of aggregate capitalization. However, when $E \rightarrow E_{max}$, retaining earnings in banks is no longer valuable from the social perspective and $W'(E) < 1$.

to macro shocks when they are most harmful (i.e. when implied risk-aversion is most severe), while individual banks fail to efficiently control aggregate risk. As a result, the critical capital buffer at which the social planner no longer requires a lending premium is lower than the one at which this is the case for individual banks (recall that $R^{ce}(E_{max}^{ce}) = R^{sb}(E_{max}^{sb}) = p$). Hence, if the target aggregate equity buffer E_{max}^{ce} is sufficiently high in the competitive set-up, banks are effectively more risk-averse than the social planner and, thus, lending is inefficiently low in the vicinity of E_{max}^{ce} .²⁷

It is important to stress that restricting lending in the second best allocation is mainly aimed at reducing the exposure of the banking system to aggregate shocks, rather than at increasing banks' profit margins. To emphasize the importance of the *exposure channel* of inefficiencies, consider briefly the case where the demand for loans is *inelastic*, i.e. equal to 1 as long as the loan rate does not exceed reservation rate \bar{R} .²⁸ In second best, $R^{sb}(E) \equiv \bar{R}$, such that firms make zero profits and the *margin channel* of inefficiencies is shut down.²⁹ The quasi-explicit solutions obtained for the inelastic demand for loans allows us to establish the following property:

Proposition 4 *With inelastic demand for loans, when $W''(0) < -(\bar{R} - p)(1 + \gamma)/\sigma_0^2$, there exists a critical level of aggregate capital $\hat{E}_K > 0$, such that $K^{sb}(E) < 1$ when $E \in [0, \hat{E}_K]$ and $K^{sb}(E) = 1$ when $E \in [\hat{E}_K, E_{max}^{sb}]$. When $W''(0) > -(\bar{R} - p)(1 + \gamma)/\sigma_0^2$, $K^{sb}(E) = 1$ for any level of E .*

The above proposition shows that, when the social implied risk aversion in the states with poor bank capitalization is sufficiently high, it is still socially optimal to restrict lending of the undercapitalized banking system, even though the reallocation of profits does not matter. This feature is illustrated in Figure 5 that contrasts the typical patterns of aggregate lending (left panel) and the loan rates (right panel) emerging for the inelastic demand for loans in the competitive equilibrium and the second best allocation.

5 Impact of capital regulation

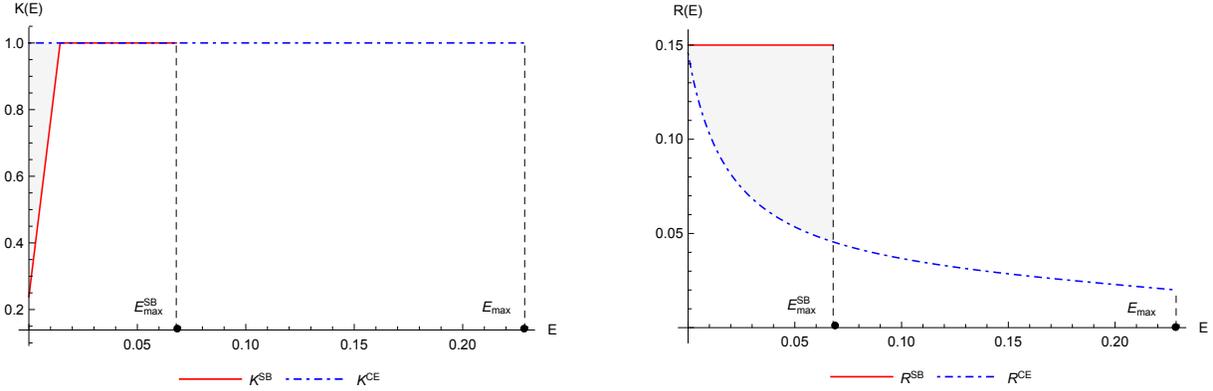
So far we conducted our analysis in the "laissez-faire" environment in which banks face no regulation. Our objective in this section is to study the impact of capital regulation on bank policies and, ultimately, on aggregate output and the stability of the banking system.

²⁷Note that the regulatory restrictions on lending changes the marginal value of bank equity. Namely, in the social planner's allocation, $u(E)$ spikes above $1 + \gamma$ for the low levels of aggregate bank capital. This implies that, if bank shareholders were free to choose the recapitalization barrier, they would recapitalize at a strictly positive level of aggregate capital.

²⁸This specification is obtained from the one introduced in (38) by setting $\beta = 0$.

²⁹This implies that the social welfare function coincides with the value function of a monopolistic bank. This is in contrast to the case with an elastic demand for loans, where firms are making profits and, thus, the socially optimal policies differ from the optimal policies of a monopolistic bank. Namely, in the case with an elastic demand for loans, the monopolistic loan rate is always strictly higher than the second best (and also the competitive) loan rate. Furthermore, with an elastic demand, the optimal dividend barrier of a monopolistic bank is strictly lower than the one corresponding to the second best allocation.

Figure 5: Loan rate and credit volume under *inelastic* loan demand: CE vs SB



Notes: this figure reports the typical patterns of lending (left panel) and the loan rates (right panel) in the competitive equilibrium (dash-dotted lines) and the second best allocation (solid lines). Parameter values: $\rho = 0.02$, $\sigma_0 = 0.05$, $p = 0.02$, $\gamma = 13.5$, $\bar{R} = 0.15$.

5.1 Regulated equilibrium

We consider a requirement for each bank to finance at least fraction Λ of its risky loans by equity, i.e.,

$$e_t \geq \Lambda k_t. \quad (46)$$

Facing a binding capital requirement, banks can either issue new equity or de-lever by reducing lending and debt. We show below that, compared to the unregulated case, banks on the one hand recapitalize earlier (i.e., $E_{min}^\Lambda > 0$) and on the other hand reduce lending when the constraint is binding but, for precautionary reasons, also when it is slack. As in the unregulated case, individual banks choose lending, recapitalization and dividend policies subject to the regulatory constraint to maximize the market value of equity.³⁰

$$v_\Lambda(e, E) \equiv eu_\Lambda(E) = \max_{k_t \leq \frac{e}{\Lambda}, d\delta_t, di_t} \mathbb{E} \left[\int_0^{+\infty} e^{-\rho t} (d\delta_t - (1 + \gamma)di_t) | e_0 = e, E_0 = E \right]. \quad (47)$$

Recall that in the unregulated equilibrium bank lending is always strictly positive and banks recapitalize only when equity is completely depleted. Capital regulation must therefore bind for sufficiently low levels of capital. In this case, lending is determined by the binding constraint (46). If it is slack, the interior level of lending must satisfy an individual rationality condition equivalent to (22) in the unregulated case. In both cases, the relation between equilibrium lending and loan rate is pinned down by the market clearing condition.

³⁰Note that homotheticity is not affected by the considered form of regulation.

Proposition 5 For all $\Lambda \in (0, 1]$, there exists a unique regulated equilibrium, where banks' market to book value of equity satisfies the HJB equation

$$\begin{aligned} \rho u_\Lambda(E) &= L(R(E))(R(E) - p)u'_\Lambda(E) + \frac{\sigma_0^2 L(R(E))^2}{2} u''_\Lambda(E) \\ &+ \frac{E}{\Lambda} [(R(E) - p)u_\Lambda(E) + \sigma_0^2 L(R(E))u'_\Lambda(E)], \end{aligned} \quad (48)$$

for $E \in [E_{min}^\Lambda, E_{max}^\Lambda]$, subject to the boundary conditions $u_\Lambda(E_{max}^\Lambda) = u_\Lambda(E_{min}^\Lambda) - \gamma = 1$ and $u'_\Lambda(E_{max}^\Lambda) = u'_\Lambda(E_{min}^\Lambda) = 0$. Furthermore, there exists a unique threshold E_c^Λ such that

a) for $E \in [E_{min}^\Lambda, E_c^\Lambda]$ the regulatory constraint binds, i.e., $K(E) = E/\Lambda$. The equilibrium loan rate is explicitly given by the market clearing condition:

$$R(E) = L^{-1}(E/\Lambda), \quad (49)$$

where L^{-1} is the inverse function of the loan demand. The evolution of aggregate bank capital is given by:

$$\frac{dE_t}{E_t} = \frac{1}{\Lambda} \left((R(E_t) - p)dt - \sigma_0 dZ_t \right), \quad E \in (E_{min}^\Lambda, E_c^\Lambda).$$

b) for $E_t \in (E_c^\Lambda, E_{max}^\Lambda]$ the regulatory constraint is slack. The equilibrium loan rate satisfies the first-order differential equation³¹

$$R'(E) = -\frac{2\rho\sigma_0^2 + (R - p)^2}{\sigma_0^2 [L(R) - (R - p)L'(R)]},$$

subject to the boundary condition $R(E_c^\Lambda) = L^{-1}[E_c^\Lambda/\Lambda]$.

Finally, there exists a unique threshold $\Lambda^* \in (0, 1)$ such that $E_c^\Lambda = E_{max}^\Lambda$, i.e., the constraint binds for all $E \in [E_{min}^\Lambda, E_{max}^\Lambda]$, when $\Lambda > \Lambda^*$.

Note that, when the constraint is not binding, the equilibrium loan rate is determined by the same condition as in the unregulated equilibrium (see condition (22)), so that the term in square brackets in the second line of HJB (48) vanishes. If the constraint is binding, however, this term is strictly positive and can be interpreted as the shadow costs associated with the constraint.

We turn to the results of our numerical analysis to illustrate the impact of minimum capital regulation on output and stability.³²

³¹This equation is obtained from (28) by setting $r = 0$.

³²In Appendix we provide a detailed description of the computational procedure implemented to solve for the regulated equilibrium.

5.2 Dividend and recapitalization policies

Figure 6 illustrates the impact of the minimum capital ratio on the optimal recapitalization and dividend policies of banks. In contrast to the unregulated case, it is no longer optimal to postpone recapitalization until equity is fully depleted ($E_{min}^\Lambda > 0$). This is an immediate consequence of the fact that banks are allowed to lend at most fraction $1/\Lambda$ of their equity, so that a bank with zero equity and, thus, zero loans, would be permanently out of business. Therefore, as illustrated in the left panel of Figure 6, the recapitalization boundary E_{min}^Λ is strictly increasing in the minimum capital ratio Λ .

When the regulator imposes a higher minimum capital ratio, Λ , the dividend boundary grows at a higher rate than the recapitalization boundary, such that the banking system's maximum loss absorbing capacity, $E_{max}^\Lambda - E_{min}^\Lambda$, expands as well. To understand this effect, note first that, as long as the constraint does not bind globally ($\Lambda \leq \Lambda^*$), banks distribute dividends when the marginal loan is no longer profitable, i.e. the equilibrium loan rate at the dividend boundary E_{max}^Λ (indicated by the dashed line in the right panel of Figure 6) equals the unconditional default rate ($R_{min}^\Lambda = p$). As regulation reduces the supply of bank loans, banks' profit margins increase at any given level of aggregate capital, which in turn drives up E_{max}^Λ . This effect is even more pronounced for sufficiently high minimum capital ratios ($\Lambda > \Lambda^*$), where the constraint binds globally and, thus, the loan rate stays strictly above the unconditional default rate ($R_{min}^\Lambda > p$).

Finally, note that for $\Lambda \rightarrow 0$, the equilibrium loan rate at the recapitalization boundary R_{max}^Λ (indicated by the solid line in the right panel of Figure 6) does not converge to its unregulated counterpart R_{max} . The reason is that the loan rate has to ensure market clearing and, thus, a capital constraint that binds over a nonempty (albeit small) region drives up the market clearing loan rate by discrete amount. As a result, even a small minimum capital ratio leads to significant increase in R_{max}^Λ .

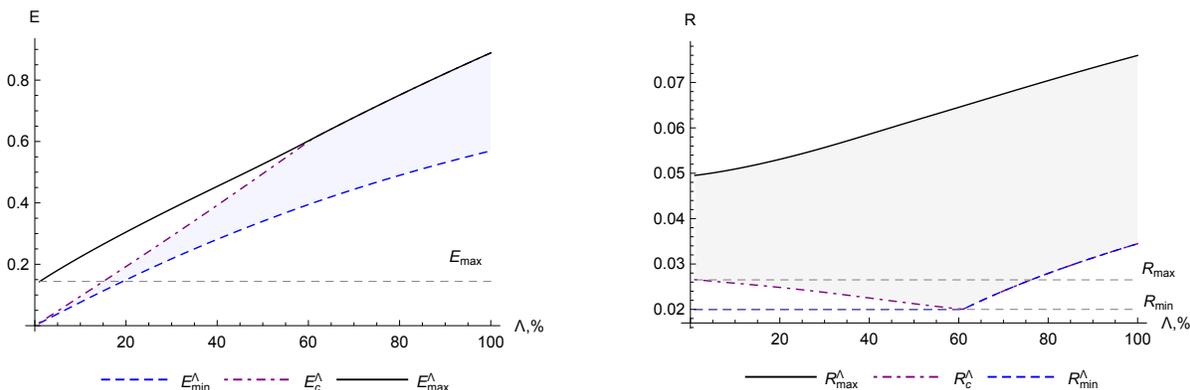
5.3 Lending

The direct impact of the minimum capital ratio on lending is illustrated in the left-hand side panel of Figure 7 where, for sake of comparability, the respective recapitalization boundaries E_{min}^Λ are normalized to zero.³³ The dashed and dash-dotted lines in this graph indicate the second best and unregulated levels of lending, respectively.

Consider first a relatively mild minimum capital ratio of, say 5%, which in fact leads to a significant reduction in exposure when the constrained is binding (i.e., when the system is poorly capitalized). Interestingly, the effect of the minimum capital requirements propagates over the unconstrained region. This result is driven by the banks' precautionary motive: anticipating a potentially binding constraint in the future, banks also reduce lending when the constraint is slack,

³³That is lending is plotted as a function of the actual capital buffer $E - E_{min}^\Lambda$. Note however, that the absolute level of capital indeed matters only in so far as it determines the maximum feasible level of lending.

Figure 6: Minimum capital requirements and bank policies



Notes: this figure illustrate the impact of minimum capital ratio on the optimal dividend and recapitalization policies. The dashed parallel lines indicates the values of the target equity buffer (E_{max}), the minimum and maximum loan rates (R_{min} and R_{max}) in the unregulated set-up. The shaded areas on both panels marks the regions (for aggregate capital E or the loan rate R) in which the regulatory constraint is binding. For $\Lambda > \Lambda^*$, a critical level E_c^Λ does not exist, i.e., the regulatory constraint is binding for any $E \in [E_{min}^\Lambda, E_{max}^\Lambda]$. Parameter values: $\rho = 0.02$, $\sigma_0 = 0.05$, $p = 0.02$, $\gamma = 0.2$, $\bar{R} = 0.15$.

albeit less severely. Thus, while mitigating inefficiencies (excessive lending) in poorly capitalized states, the constant minimum capital ratio exacerbates inefficiencies (unsufficient lending) in good states.

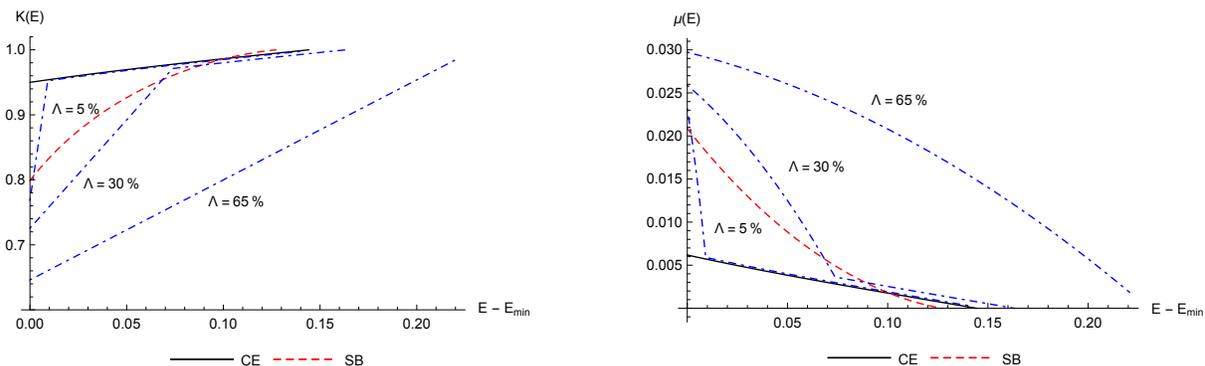
Upon raising the minimum capital ratio to 30%, the region in which the constraint binds expands (cf. the shaded area in the left-hand side panel of Figure 6) and the constrained level of lending declines further, falling far below the second best level.³⁴ With a stricter minimum capital ratio also the precautionary motive and, thus, the reduction of lending in the unconstrained region becomes more pronounced. Finally, increasing the minimum capital ratio further to, say, 65% leads to inefficiently low lending for all feasible levels of aggregate capitalization.

By inducing banks to lend less, minimum capital requirements reduce the exposure of the banking system to aggregate shocks and, thus, mitigate the endogenous volatility of aggregate capital. At the same time, restricting lending boosts banks' profit flows, and, thus, improves the banking system's ability to accumulate capital. This effect of minimum capital requirements is illustrated in the right-hand side panel of Figure 7 that plots the endogenous drift of aggregate capital, $\mu(E) \equiv K(E)(R(E) - p)$, for different values of Λ .

The fact that minimum capital requirements simultaneously reduce the endogenous volatility and increase the profit margins in the banking system may suggest that capital regulation goes hand in hand with enhanced stability. To explore this conjecture, we now turn to the analysis of the probabilistic behavior of the regulated equilibrium and show that it is not necessarily true.

³⁴Note that banks react to the stricter regulation by increasing E_{min}^Λ , yet, not sufficiently to maintain the same level of lending. Therefore, R_{max}^Λ , which is represented by the solid line in the right panel of Figure 6, is strictly increasing in Λ .

Figure 7: Impact of capital regulation on aggregate lending and expected profit margins



Notes: this figure illustrates the impact of minimum capital requirements on aggregate lending and on the drift of aggregate equity. Dashed lines refer to second best and dash-dotted lines to the unregulated equilibrium. In the scenarios with $\Lambda = 5\%$ and $\Lambda = 30\%$, the regulatory constraint is binding when E is low and is slack when E is high. In the scenario with $\Lambda = 65\%$, the regulatory constraint is always binding. Parameter values: $\rho = 0.02$, $\sigma_0 = 0.05$, $p = 0.02$, $\gamma = 0.2$, $\bar{R} = 0.15$.

5.4 Stability

To discuss financial stability issues in the equilibrium with minimum capital regulation, we numerically compute the ergodic density function of E :

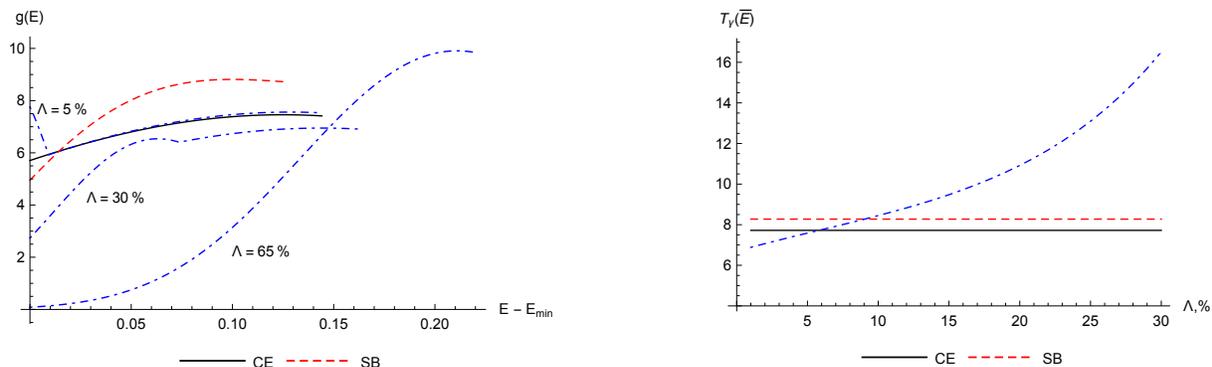
$$g_\Lambda(E) = \frac{C_\Lambda}{\sigma_0^2 K^2(E)} \exp\left(\int_{E_{min}^\Lambda}^{E_{max}^\Lambda} \frac{2(R(E) - p)}{\sigma_0^2 K(E)} dE\right),$$

where $K(E)$ and $R(E)$ are defined in Proposition 5, $\mu(E) \equiv K(E)(R(E) - p)$, $\sigma(E) \equiv -\sigma_0 K(E)$ and the constant C_Λ is such that the mass of densities over the set of feasible values of aggregate capital is normalized to one.

The left hand panel of Figure 8 illustrates the patterns of the ergodic density function $g_\Lambda(\cdot)$ for different levels of regulation (again, for sake of comparability, $g_\Lambda(\cdot)$ is plotted as a function of the actual capital buffer $E - E_{min}^\Lambda$). An interesting phenomenon revealed by this graph is the density concentration in the states with poor capitalization that emerges for mild minimum capital requirements. The reason is that, for low capital ratios, banks un-lever only when the constraint is binding, which occurs in the states where the banking system is poorly capitalized. For most states, however, the constraint is not binding and exposure remains high, almost as high as in the unregulated set-up. The system, therefore, enters the poorly capitalized region almost as often as in the unregulated equilibrium. Yet, as banks have to severely un-lever when the system enters the poorly capitalized region, it can get trapped for a long time in the states where lending is substantially lower than in the second best level. For sufficiently high minimum capital ratios, exposure to aggregate shocks (lending) is also substantially reduced when the system is well capitalized and, as a result, the "trap" disappears.

The above observations show that lax regulation can destabilize the system, inducing protracted credit crunches. This feature is also illustrated in the right-hand side panel of Figure 8, which reports the banking system's stability, measured by the average time to recapitalization, as a function of the minimum capital ratio. This graph shows that the average time to recapitalization under the mild minimum capital ratios is lower than in the unregulated set-up, despite the fact that banks operate with lower exposure to aggregate shocks and higher expected profits. This instability paradox, which is entirely driven by the density concentration in poorly-capitalized states, disappears for relatively high levels of Λ . As stability is strictly increasing in the minimum capital ratio Λ , for sufficiently high capital ratios the banking system becomes even more stable than in the second best allocation, which comes at the cost of substantial reductions in lending and, thus, output.

Figure 8: Impact of capital regulation on financial stability



Notes: this figure illustrates the impact of a constant minimum capital ratio on financial stability. Dashed lines refer to second best and dash-dotted lines to the unregulated equilibrium. Parameter values: $\rho = 0.02$, $\sigma_0 = 0.05$, $p = 0.02$, $\gamma = 0.2$, $\bar{R} = 0.15$.

Overall, our numerical analysis conducted above demonstrates that the minimum capital requirement is a natural tool to control the endogenous volatility in the banking system. However, the *constant* minimum capital ratio turns out to be highly imperfect in terms of replicating the second-best allocation.³⁵ Ideally, the socially optimal level of lending (as well as the socially optimal probabilistic behaviors of the system) can be implemented by inducing banks to maintain the state-contingent capital ratio conditioned on the level of aggregate bank capital. Compared with the unregulated case, such state-dependent capital regulation would imply higher capital ratios (lower leverage) in the states with lower aggregate capital and lower capital ratios (higher leverage) in the states with abundant capital. This observation suggests that the "countercyclical" capital requirement proposals (widely discussed in the aftermath of the financial crisis) would be inadequate in our set-up.³⁶

³⁵In fact, by using the constant minimum capital ratio, one can only replicate the first moment (the average) of the probability distribution of aggregate lending across states.

³⁶It must be acknowledged, however, that our framework is not perfectly suitable for analyzing the proposals of the

6 Conclusion

This paper develops a general equilibrium model of commercial banking, in which banks satisfy households' needs for safe deposits and channel funds to the productive sector. Bank capital plays the role of a loss-absorbing buffer that insulates banks from the need to undertake costly recapitalizations too often. In our model, the aggregate level of bank capitalization drives the cost and the volume of lending. Specifically, we establish a negative relation between the equilibrium loan rate and the level of aggregate bank capital that is supported by data. The competitive equilibrium is not constrained efficient because of a pecuniary externality which leads to excessive lending in the states with poor bank capitalization and insufficient lending in well-capitalized states. Moderate minimum capital requirements can mitigate inefficiencies in the states with poor capitalization. However, too lax capital regulation can destabilize the system.

It should be acknowledged that our model suffers from several limitations. First, it only considers commercial banking activities (deposit taking and lending), while neglecting market activities such as investment in securities and derivatives trading. Second, it only considers diffusion risks that do not lead to actual bank defaults, but merely fluctuations in the size of the banking system. A consequence of these limitations is that we do not address the important questions of banks' excessive risk-taking and the role of capital regulation in the mitigation of this behavior, which, however, have already been the subject of a large academic literature. Finally, in this paper we have focused on the scenario in which private bank recapitalizations prevent systemic crises from happening. A potential direction of further investigations would be to explore the alternative scenario, where shareholders do not spontaneously inject new capital, and public authorities are forced to intervene.

"countercyclical" capital requirements as it does not accommodate business cycles. We, therefore, leave a rigorous investigation of this issue for further research.

Appendix

I. Proofs

Proof of Proposition 1. Omitted.

Proof of Proposition 2. By the standard dynamic programming arguments, shareholder value $v(e, E)$ must satisfy the Bellman equation:³⁷

$$\begin{aligned} \rho v = & \max_{k \geq 0, d\delta \geq 0, di \geq 0} \left\{ d\delta(1 - v_e) - di(1 + \gamma - v_e) \right. \\ & + \left[re + k(R(E) - p - r) \right] v_e + kK(E)\sigma_0^2 v_{eE} + \frac{k^2\sigma_0^2}{2} v_{ee} \\ & \left. + \left[rE + K(E)(R(E) - p - r) \right] v_E + \frac{\sigma_0^2 K^2(E)}{2} v_{EE} \right\}. \end{aligned} \quad (\text{I.1})$$

Using the fact that $v(e, E) = eu(E)$, one can rewrite the Bellman equation (I.1) as follows:

$$\begin{aligned} (\rho - r)u(E) = & \max_{d\delta \geq 0} \left\{ \frac{d\delta}{e} [1 - u(E)] \right\} - \max_{di \geq 0} \left\{ \frac{di}{e} [1 + \gamma - u(E)] \right\} \\ & + \max_{k \geq 0} \left\{ \frac{k}{e} \left[(R(E) - p - r)u(E) + \sigma_0^2 K(E)u'(E) \right] \right\} \\ & + \left(rE + K(E)[R(E) - p - r] \right) u'(E) + \frac{\sigma_0^2 K^2(E)}{2} u''(E). \end{aligned} \quad (\text{I.2})$$

A solution to the maximization problem in k only exists when

$$\frac{u'(E)}{u(E)} \leq -\frac{R(E) - p - r}{\sigma_0^2 K(E)}, \quad (\text{I.3})$$

with equality when $k > 0$.

Under conjecture that $R(E) \geq p + r$ (which is verified ex-post), it follows from the above expression that $u(E)$ is a decreasing function of E . Then, the optimal payout policy maximizing the right-hand side of (I.2) is characterized by a critical barrier E_{max} satisfying

$$u(E_{max}) = 1, \quad (\text{I.4})$$

and the optimal recapitalization policy is characterized by a barrier E_{min} such that

$$u(E_{min}) = 1 + \gamma. \quad (\text{I.5})$$

In other words, dividends are only distributed when E_t reaches E_{max} , whereas recapitalization occurs only when E_t reaches E_{min} . Given (I.3), (I.4), (I.5) and $k > 0$, it is easy to see that, in the region $E \in (E_{min}, E_{max})$, market-to-book value $u(E)$ satisfies:

$$(\rho - r)u(E) = \left[rE + K(E)(R(E) - p - r) \right] u'(E) + \frac{\sigma_0^2 K^2(E)}{2} u''(E). \quad (\text{I.6})$$

³⁷For the sake of space, we omit the arguments of function $v(e, E)$.

Note that, at equilibrium, $K(E) = L(R(E))$. Taking the first derivative of (I.3), we can compute $u''(E)$. Inserting $u''(E)$ and $u'(E)$ into (I.6) and rearranging terms yields:

$$R'(E) = -\frac{1}{\sigma_0^2} \frac{2(\rho - r)\sigma_0^2 + (R(E) - p - r)^2 + 2(R(E) - p - r)rE/L(R(E))}{L(R(E)) - (R(E) - p - r)L'(R(E))}. \quad (\text{I.7})$$

Since $L'(R(E)) < 0$, it is clear that $R'(E) < 0$ if $R(E) > p + r$. To verify that $R(E) > p + r$ for any $E \in [E_{min}, E_{max}]$, it is sufficient to show that $R_{min} \equiv R(E_{max}) \geq p + r$.

To obtain R_{min} , let

$$V(E) \equiv Eu(E)$$

denote the market value of the entire banking system. At equilibrium, dividends are distributed when the marginal value of bank capital equals the marginal value of dividends, which implies

$$V'(E_{max}) = u(E_{max}) + E_{max}u'(E_{max}) = 1.$$

Similarly, recapitalizations take place when the marginal value of bank capital equals the marginal costs of recapitalizing the banks, which implies

$$V'(E_{min}) = u(E_{min}) + E_{min}u'(E_{min}) = 1 + \gamma.$$

Given (I.4) and (I.5), it must hold that³⁸

$$u'(E_{max}) = 0,$$

and

$$E_{min} = 0.$$

Inserting $u'(E_{max}) = 0$ into the binding condition (I.3) immediately shows that $R_{min} = p + r$, so that $R(E) > p + r$ for any $E \in [E_{min}, E_{max}]$.

Hence, for any given E_{max} , the loan rate $R(E)$ can be computed as a solution to the differential equation (I.7) with the boundary condition $R(E_{max}; E_{max}) = p + r$.

To obtain E_{max} , we use the fact that individual banks' optimization with respect to the recapitalization policy implies $u(0) = 1 + \gamma$. Integrating equation (I.3) in between $E_{min} = 0$ and E_{max} , while taking into account the condition $u(E_{max}) = 1$, yields an equation that implicitly determines E_{max} :

$$\underbrace{u(E_{max}) \exp\left(\int_0^{E_{max}} \frac{R(E; E_{max}) - p - r}{\sigma_0^2 L(R(E; E_{max}))} dE\right)}_{u(0)} = 1 + \gamma. \quad (\text{I.8})$$

Note that, since $R(E; E_{max})$ is increasing with E_{max} , the right-hand side of Equation (I.8) increases with E_{max} . When the left-hand side of equation (I.8) is larger than $1 + \gamma$ when $E_{max} \rightarrow \infty$, it has the (unique) solution.³⁹ **Q.E.D.**

Proof of Proposition 3. For any given barriers E_{min}^{sb} and E_{max}^{sb} , solving for the second best

³⁸As we show in Appendix B, these properties can be alternatively established by looking at the limit of the discrete-time equilibrium characterization for $h \rightarrow 0$.

³⁹Throughout the paper we focus on this case. In the alternative case all banks default simultaneously when E_t falls to zero.

allocation requires solving for the functions $R^{sb}(E)$ and $W(E)$ that simultaneously satisfy equations (45) and (40), subject to the boundary conditions $W'(E_{min}^{sb}) = W'(E_{max}^{sb}) - \gamma = 1$. From the concavity of the welfare function it immediately follows that $E_{min}^{sb} = 0$, whereas the optimal target level of equity E_{max}^{sb} must satisfy the super-contact condition $W''(E_{max}^{sb}) = 0$. **Q.E.D.**

Proof of Proposition 4. Consider the case where the firms' demand for loans is constant and equal to 1 as long as the loan rate does not exceed some maximum rate \bar{R} .⁴⁰

$$L(R) = \begin{cases} 1 & \text{for } R \leq \bar{R}, \\ 0 & \text{for } R > \bar{R}. \end{cases} \quad (\text{I.9})$$

The single state variable in the social planner's problem is aggregate bank equity E and the market for bank credit must clear, i.e., $K_t = L(R_t)$. Over $[0, E_{max}^{sb}]$, the social welfare function satisfies the following ODE:

$$\rho W(E) = \max_{R \leq \bar{R}, K \leq 1} \underbrace{K(\bar{R} - R)}_{\text{firms' profits}} + \underbrace{\lambda(D)}_{\text{utility from deposits}} + \underbrace{K(R - p)}_{\text{expected banks' profits}} W'(E) + \frac{\sigma_0^2 K^2}{2} W''(E), \quad (\text{I.10})$$

subject to the boundary conditions

$$W'(0) = 1 + \gamma, \quad (\text{I.11})$$

$$W'(E_{max}^{sb}) = 1. \quad (\text{I.12})$$

The optimal dividend barrier E_{max}^{sb} should satisfy the super-contact condition

$$W''(E_{max}^{sb}) = 0. \quad (\text{I.13})$$

As a consequence, it holds that $W'(E) > 1$ for $E \in [0, E_{max}^{sb})$, implying that social welfare is maximized at the highest possible ("reservation") loan rate $R^{sb}(E) \equiv \bar{R}$.⁴¹ Therefore, with inelastic demand, firm profits are equal to zero and the social welfare function coincides with the value function of a monopolistic bank, satisfying ODE

$$\rho W(E) = \max_{K \leq 1} K(\bar{R} - p) W'(E) + \frac{\sigma_0^2 K^2}{2} W''(E). \quad (\text{I.14})$$

Maximizing the right-hand side of (I.14) with respect to aggregate lending yields:

$$K^{sb}(E) = \min \left\{ 1, -\frac{(\bar{R} - p) W'(E)}{\sigma_0^2 W''(E)} \right\}. \quad (\text{I.15})$$

Substituting K^{sb} back into HJB (I.14) implies that for interior loan volumes ($K^{sb} < 1$), the

⁴⁰This loan demand specification can be obtained from the demand specification (38) by taking the limit case $\beta \equiv 0$.

⁴¹To see this, note that the partial derivative of (I.10) with respect to R is equal to $K[W'(E) - 1] > 0$.

social welfare function has an explicit solution given by

$$W_1(E) = c_2 \left(\frac{E}{1-\eta} - c_1 \right)^{1-\eta}, \quad \text{with } \eta \equiv \frac{(\bar{R}-p)^2}{(\bar{R}-p)^2 + 2\rho\sigma_0^2}, \quad (\text{I.16})$$

where $\{c_1, c_2\}$ are constants to be determined.⁴²

Substituting the general solution of the welfare function (I.16) into Equation (I.15) shows that the optimal volume of lending, if interior, linearly increases with aggregate capital E :

$$K^{sb}(E) = \min \left\{ 1, \frac{(\bar{R}-p)}{\eta\sigma_0^2} (E - (1-\eta)c_1) \right\}. \quad (\text{I.17})$$

Next, let \hat{E}_K denote the critical level of aggregate capital for which

$$K^{sb}(\hat{E}_K) = 1, \quad (\text{I.18})$$

such that the welfare maximizing loan volume satisfies $K^{sb}(E) < 1$ for $E < \hat{E}_K$ and $K^{sb}(E) = 1$ for $E \geq \hat{E}_K$. That is, for low levels of aggregate bank capital, the social planner reduces aggregate lending below the level that is attained in a competitive equilibrium.

From (I.17), we have⁴³

$$\hat{E}_K = \max \left\{ 0, \frac{\eta\sigma_0^2}{\bar{R}-p} + c_1(1-\eta) \right\}.$$

Combining (I.15) and (I.11), it is easy to see that $\hat{E}_K > 0$ when the following condition is satisfied:

$$W''(0) < -\frac{(\bar{R}-p)}{\sigma_0^2}(1+\gamma).$$

For completeness we show how the constants c_1 and c_2 can be computed. To this end, consider the region $(\hat{E}_K, E_{max}^{sb}]$, in which the social planner would allow the banking system to operate at the maximum feasible scale, i.e., $K^{sb}(E) = 1$. Substituting $K^{sb}(E) = 1$ into (I.14) and solving the obtained ODE under the boundary conditions (I.12) and (I.13), yields the explicit solution of the welfare function for $E \in (\hat{E}_K, E_{max}^{sb}]$,

$$W_2(E) = \frac{1}{\beta_1 - \beta_2} \left[\frac{\beta_2}{\beta_1} e^{\beta_1(E-E_{max}^{sb})} + \frac{\beta_1}{\beta_2} e^{\beta_2(E-E_{max}^{sb})} \right], \quad (\text{I.19})$$

where $\beta_1 > 0$ and $\beta_2 < 0$ are the roots of the characteristic equation $\rho = (\bar{R}-p)\beta + \frac{\sigma_0^2}{2}\beta^2$.⁴⁴

For any fixed E_{max}^{sb} , the values of c_1 and c_2 are implicitly given by the system of the value-

⁴²We indicate the solution by W_1 in order to differentiate it from its counterpart W_2 in the region where the corner solution $R = 1$ applies.

⁴³Note that if γ and σ_0 are sufficiently low, \hat{E}_K might not be interior, in which case $K^{sb}(E) = 1$ for all $E \in [E_{min}^{sb}, E_{max}^{sb}]$.

⁴⁴In particular, we have $\beta_{1,2} = -\frac{\bar{R}-p \pm \sqrt{(\bar{R}-p)^2 + 2\rho\sigma_0^2}}{\sigma_0^2}$.

matching and smooth-pasting conditions at \hat{E}_K :

$$W_1(\hat{E}_K) = W_2(\hat{E}_K), \quad (\text{I.20})$$

$$W'_1(\hat{E}_K) = W'_2(\hat{E}_K). \quad (\text{I.21})$$

Finally, the optimal target level of equity E_{max}^{sb} can be pinned down from the boundary condition (I.11). **Q.E.D.**

Proof of Proposition 5. Consider the shareholders' maximization problem stated in (47). By the standard dynamic programming arguments and the fact that $v_\Lambda(e, E) = eu_\Lambda(E)$, where $u_\Lambda(E)$ satisfies the following Bellman equation:

$$\begin{aligned} \rho u_\Lambda(E) = & \max_{d \geq 0, di \geq 0} \left\{ \frac{d\delta}{e} [1 - u_\Lambda(E)] - \frac{di}{e} [1 + \gamma - u_\Lambda(E)] \right\} + \\ & + \max_{0 < k \leq e/\Lambda} \left\{ \frac{k}{e} [(R(E) - p)u_\Lambda(E) + \sigma_0^2 K(E)u'_\Lambda(E)] \right\} + \\ & + K(E)[R(E) - p]u'_\Lambda(E) + \frac{\sigma_0^2 K^2(E)}{2} u''_\Lambda(E). \end{aligned} \quad (\text{I.22})$$

A solution to (I.22) exists only if $K(E) \leq E/\Lambda$, and

$$B(E) := \frac{R(E) - p}{\sigma_0^2 K(E)} \geq -\frac{u'_\Lambda(E)}{u_\Lambda(E)} \equiv y(E), \quad (\text{I.23})$$

with equality when $K(E) < E/\Lambda$.

The optimal dividend and recapitalization policies are characterized by barriers E_{max}^Λ and E_{min}^Λ such that $u_\Lambda(E_{max}^\Lambda) = 1$ and $u_\Lambda(E_{min}^\Lambda) = 1 + \gamma$. Moreover, by the same reason that in the unregulated equilibrium, it must hold that

$$E_{max}^\Lambda u'_\Lambda(E_{max}^\Lambda) = 0,$$

and

$$E_{min}^\Lambda u'_\Lambda(E_{min}^\Lambda) = 0.$$

From the first equation we immediately have $u'_\Lambda(E_{max}^\Lambda) = 0$. The implication of the second equation is less trivial. Recall that in the unregulated equilibrium, we had $E_{min} = 0$. However, under capital regulation, this is no longer possible, as $K(E) = E/\Lambda$ on the constrained region and thus equation (I.22) could not hold when $E \rightarrow 0$. Therefore, it must be that $u'_\Lambda(E_{min}^\Lambda) = 0$.

With the conjecture that there exists some critical threshold E_c^Λ such that the regulatory constraint is binding for $E \in [E_{min}^\Lambda, E_c^\Lambda]$ and is slack for $E \in [E_c^\Lambda, E_{max}^\Lambda]$, equation (I.22) can be rewritten as the first-order differential equation:

$$\rho = -\pi_B(E)y(E) + \frac{\sigma_0^2 K^2(E)}{2} [y^2(E) - y'(E)] + \mathbb{1}_{\{E \in [E_{min}^\Lambda, E_c^\Lambda]\}} \frac{[R(E) - p - \sigma_0^2 K(E)y(E)]}{\Lambda} \quad (\text{I.24})$$

where $\mathbb{1}_{\{\cdot\}}$ is the indicator function and $\pi_B(E)$ denotes the aggregate expected profit of banks:

$$\pi_B(E) = K(E)[R(E) - p],$$

the volume of lending $K(E)$ satisfies

$$K(E) = \begin{cases} E/\Lambda, & E \in [E_{min}^\Lambda, E_c^\Lambda] \\ L(R(E)), & E \in (E_c^\Lambda, E_{max}^\Lambda], \end{cases} \quad (\text{I.25})$$

and the loan rate $R(E)$ is given by

$$R(E) = \begin{cases} L^{-1}(E/\Lambda), & E \in [E_{min}^\Lambda, E_c^\Lambda] \\ R'(E) = -\frac{2\rho\sigma_0^2 + (R-p)^2}{\sigma_0^2[L(R) - (R-p)L'(R)]}, & R(E_c^\Lambda) = L^{-1}(E_c^\Lambda/\Lambda), \quad E \in (E_c^\Lambda, E_{max}^\Lambda], \end{cases} \quad (\text{I.26})$$

where L^{-1} is the inverse function of the demand for loans.

The critical threshold E_c^Λ must satisfy equation

$$y(E_c^\Lambda) = B(E_c^\Lambda).$$

If $y(E) < B(E)$ for any $E \in [E_{min}^\Lambda, E_{max}^\Lambda]$, then the regulatory constraint is always binding and $y(E)$ satisfies equation (I.24) with $E_c^\Lambda = E_{max}^\Lambda$. The condition $u'_\Lambda(E_{min}^\Lambda) = 0$ yields the boundary condition $y(E_{min}) = 0$. Similarly, the condition $u'_\Lambda(E_{max}^\Lambda) = 0$ translates into the boundary condition $y(E_{max}) = 0$. **Q.E.D.**

II. Solving for the regulated equilibrium

a) Numerical procedure

This numerical algorithm solving for the competitive equilibrium with minimum capital regulation can be implemented with the *Mathematica* software.⁴⁵

- Pick a candidate value \hat{E}_{min}^Λ .
- Assume that the regulatory constraint always binds. Solve ODE (I.24) for $y(E)$ under the boundary condition $y(\hat{E}_{min}^\Lambda) = 0$.
- Compute a candidate value \hat{E}_{max}^Λ such that satisfies equation $y(\hat{E}_{max}^\Lambda) = 0$.
- Check whether $y(\hat{E}_{max}^\Lambda) \leq B(\hat{E}_{max}^\Lambda)$.
- Conditional on the results of the previous step, one of the two scenarios is possible:
 - a) if $y(\hat{E}_{max}^\Lambda) \leq B(\hat{E}_{max}^\Lambda)$, then the regulatory constraint is always binding for a given Λ , i.e., there is a single “constrained” region. In this case market-to-book value $u_\Lambda(\hat{E}_{min}^\Lambda)$ can be computed according to

$$u_\Lambda(\hat{E}_{min}^\Lambda) = u_\Lambda(\hat{E}_{max}^\Lambda) \exp\left(\int_{\hat{E}_{min}^\Lambda}^{\hat{E}_{max}^\Lambda} y(E) dE\right) = \exp\left(\int_{\hat{E}_{min}^\Lambda}^{\hat{E}_{max}^\Lambda} y(E) dE\right).$$

⁴⁵In all computations Λ is taken as a parameter.

- b) observing $y(\hat{E}_{max}^\Lambda) > B(\hat{E}_{max}^\Lambda)$ means that, for given \hat{E}_{min}^Λ , the constrained and unconstrained regions coexist. To find the critical level of aggregate equity \hat{E}_c^Λ above which the regulatory constraint is slack, one needs to solve the following equation:

$$y(\hat{E}_c^\Lambda) = B(\hat{E}_c^\Lambda).$$

- using \hat{E}_c^Λ and (I.26), define the function $R(E)$ for $E > \hat{E}_c^\Lambda$;
- compute a new candidate for the dividend boundary, \check{E}_{max}^Λ , such that $B(\check{E}_{max}^\Lambda) = 0$ (note that this is equivalent to solving equation $R(\check{E}_{max}^\Lambda) = p$);
- compute the market-to-book value $u_\Lambda(\hat{E}_{min}^\Lambda)$ according to:

$$u_\Lambda(\hat{E}_{min}^\Lambda) = \exp\left(\int_{\hat{E}_{min}^\Lambda}^{\hat{E}_c^\Lambda} y(E)dE\right)\exp\left(\int_{\hat{E}_c^\Lambda}^{\check{E}_{max}^\Lambda} B(E)dE\right).$$

- If $u_\Lambda(\hat{E}_{min}^\Lambda) = 1 + \gamma$, then $E_{min}^\Lambda = \hat{E}_{min}^\Lambda$, $E_{max}^\Lambda = \hat{E}_{max}^\Lambda$ (or \check{E}_{max}^Λ if 2 regions) and $E_c^\Lambda = \hat{E}_c^\Lambda$ (if 2 regions). Otherwise, pick a different \hat{E}_{min}^Λ , repeat the procedure from the beginning.

b) Expected time to reach the recapitalization barrier

Let $\bar{T}_\gamma(E)$ denote the expected time it takes to reach the recapitalization boundary E_{min}^Λ starting from any $E \geq E_{min}^\Lambda$. Since $t + \bar{T}_\gamma(E_t)$ is a martingale, function $\bar{T}_\gamma(E)$ satisfies the following differential equation:

$$\frac{[K(E)\sigma_0]^2}{2}\bar{T}_\gamma''(E) + K(E)[R(E) - p]\bar{T}_\gamma'(E) + 1 = 0, \quad (E_{min}^\Lambda, E_{max}^\Lambda),$$

where functions $K(E)$ and $R(E)$ are defined in (I.25) and (I.26) respectively.

The above equation is subject to the following two boundary conditions: $\bar{T}_\gamma(E_{min}^\Lambda) = 0$ (i.e., it takes no time to reach E_{min}^Λ from E_{min}^Λ), and $\bar{T}_\gamma'(E_{max}^\Lambda) = 0$, which emerges due to the reflection property of aggregate equity. To measure the impact of minimum capital requirements on financial stability in Section 5, for each level of Λ , we compute the expected time to reach the recapitalization boundary E_{min}^Λ starting from the long-run average level of aggregate equity, \bar{E}^Λ .

Online Appendix

A. Discrete-time dynamic model

A.1. Competitive equilibrium in the dynamic discrete-time model

In this Appendix we develop a discrete-time dynamic model and show that, when the length of time periods tends to zero, the outcomes of this model converges to the outcomes of the continuous-time model.

Let h denote the length of each time period. We work with the particular specification of the firms' default probability which allows the convergence to the diffusion process in the continuous

time limit:⁴⁶

$$\tilde{p}_t(h) = \begin{cases} p - \frac{\sigma_0}{\sqrt{h}}, & \text{with probability } 1/2 \text{ (positive shock),} \\ p + \frac{\sigma_0}{\sqrt{h}}, & \text{with probability } 1/2 \text{ (negative shock).} \end{cases}$$

To simplify the presentation of further results, let

$$\tilde{A}_{t+h}(E_t) = \begin{cases} (R^h(E_t) - p - r)h + \sigma_0\sqrt{h} \equiv A^+(E_t, h), \\ (R^h(E_t) - p - r)h - \sigma_0\sqrt{h} \equiv A^-(E_t, h), \end{cases}$$

denote the marginal loan return realized in period $(t, t+h]$.

Then, the dynamics of individual equity follows:

$$\tilde{e}_{t+h} = (1 + rh)(e_t + i_t + \delta_t) + k_t \tilde{A}_{t+h}(E_t). \quad (\text{A1})$$

Similarly, aggregate equity evolves according to

$$\tilde{E}_{t+h} = (1 + rh)(E_t + I_t + \Delta_t) + L(R^h(E_t))\tilde{A}_{t+h}(E_t), \quad (\text{A2})$$

where we have used the equilibrium condition $K_t = L(R^h(E_t))$.

The non-default constraint imposed by the interbank borrowing market implies

$$e_{t+h}^- := (1 + rh)(e_t + i_t + \delta_t) + k_t A^-(E_t, h) \geq 0. \quad (\text{A3})$$

Let $\theta^h(E_t)$ denote the Lagrangian multiplier associated with the non-default constraint (A3). Given the dynamics of individual (equation (A1)) and aggregate (equation (A2)) equity, the maximization problem of an individual bank can be stated as follows:

$$v(e_t, E_t) \equiv e_t u^h(E_t) = \max_{\delta_t \geq 0, i_t \geq 0, k_t \geq 0} \delta_t - (1 + \gamma)i_t + \mathbb{E} \left[\frac{\tilde{e}_{t+h} u^h(\tilde{E}_{t+h}) + \theta^h(E_t) e_{t+h}^-}{1 + \rho h} \right], \quad t = 0, \dots, +\infty. \quad (\text{A4})$$

Then, the maximization problem (A4) can be rewritten in the following way:

$$\begin{aligned} e_t u_t^h(E_t) = e_t & \frac{(1 + rh)(\mathbb{E}[u(\tilde{E}_{t+h})] + \theta^h(E_t))}{1 + \rho h} + \max_{\delta_t \geq 0} \delta_t \left[1 - \frac{(1 + rh)(\mathbb{E}[u^h(\tilde{E}_{t+h})] + \theta^h(E_t))}{1 + \rho h} \right] \\ & + \max_{i_t \geq 0} i_t \left[-(1 + \gamma) + \frac{(1 + rh)(\mathbb{E}[u^h(\tilde{E}_{t+h})] + \theta^h(E_t))}{1 + \rho h} \right] \\ & + \max_{k_t > 0} k_t \left[\frac{\mathbb{E}[\tilde{A}_{t+h} u^h(\tilde{E}_{t+h})] + \theta^h(E_t) A_{t+h}^-(E_t)}{1 + \rho h} \right]. \end{aligned} \quad (\text{A5})$$

⁴⁶ Another interesting example could be constructed for the specification of the default probability generating a jump-process in the continuous-time limit:

$$\tilde{p}_t(h) = \begin{cases} p, & \text{with probability } 1 - \phi h \text{ (positive shock),} \\ p - \frac{l}{h}, & \text{with probability } \phi h \text{ (negative shock),} \end{cases}$$

where ϕ is the intensity of large losses and l is the size of a (proportional) large loss.

Since the value function is linear in all policy variables, optimizing with respect to the bank's policies yields:

$$1 - \frac{(1 + rh)(\mathbb{E}[u^h(\tilde{E}_{t+h})] + \theta^h(E_t))}{1 + \rho h} \leq 0 \quad (= \text{ if } \delta_t > 0), \quad (\text{A6})$$

$$-(1 + \gamma) + \frac{(1 + rh)(\mathbb{E}[u^h(\tilde{E}_{t+h})] + \theta^h(E_t))}{1 + \rho h} \leq 0 \quad (= \text{ if } i_t > 0), \quad (\text{A7})$$

$$\mathbb{E}[\tilde{A}_{t+h}(E_t)u^h(\tilde{E}_{t+h})] + \theta^h(E_t)A^-(E_t, h) = 0, \quad (\text{A8})$$

and thus

$$u(E_t) = \frac{(1 + rh)(\mathbb{E}[u^h(\tilde{E}_{t+h})] + \theta^h(E_t))}{1 + \rho h}. \quad (\text{A9})$$

Under (A9), condition (A6) transforms to

$$u(E_t) \geq 1, \quad (= \text{ if } \delta_t > 0), \quad (\text{A10})$$

and condition (A7) can be rewritten as

$$u(E_t) \leq 1 + \gamma, \quad (= \text{ if } i_t > 0). \quad (\text{A11})$$

Thus, provided that $u^h(\cdot)$ is a decreasing function of its argument (this has to be verified ex-post), the optimal dividend and recapitalization policies are of the barrier type. Let E_{min}^h and E_{max}^h denote, respectively, the levels of aggregate capital at which (A10) and (A11) holds with equality. Banks distribute any excess profits as dividends so as to maintain aggregate equity at or below E_{max}^h . Similarly, recapitalizations are undertaken so as to offset losses and to maintain aggregate equity at or above E_{min}^h .

To complete the characterization of the equilibrium, one has to solve for the functions $\theta^h(E)$, $u^h(E)$ and $R^h(E)$. Rewriting (A8) and (A9), we obtain the system of equations:

$$\frac{1}{2} \left[A^+(E, h)u^h(E^+) + A^-(E, h)u^h(E^-) \right] = -\theta^h(E)A^-(E, h), \quad (\text{A12})$$

$$\frac{1}{2} \left[u^h(E^+) + u^h(E^-) \right] = \frac{(1 + \rho h)}{(1 + rh)} u^h(E) - \theta^h(E), \quad (\text{A13})$$

Moreover, the fact that the non-default constraint (A3) is binding at the individual level if and only if it is binding at the aggregate level implies an additional equation:

$$\theta(E) \left\{ (1 + rh)E + L(R^h(E))A^-(E, h) \right\} = 0 \quad (\text{A14})$$

We proceed by conjecturing and verifying the existence of two non-empty regions: $[E_{min}^h, \hat{E}^h)$, where the non-default constraint is binding and $\theta^h(E) > 0$, and $[\hat{E}^h, E_{max}^h]$ where the non-default constraint is slack and $\theta^h(E) = 0$. The critical barrier \hat{E}^h is implicitly given by $\theta^h(\hat{E}^h) = 0$.

First, consider the constrained region $[E_{min}^h, \hat{E}^h)$. The equilibrium interest rate, $R^h(E)$, is implicitly given by the binding non-default constraint:

$$L(R^h(E))[\sigma_0\sqrt{h} - (R^h(E) - p - r)h] = (1 + rh)E.$$

The above equation implies that $E^- \equiv 0$ in the region $[E_{min}^h, \hat{E}^h]$. Furthermore, we have $E^+ = 2\sigma_0\sqrt{h}L(R^h(E))$ and $u(E^-) = u(E_{min}^h) = 1 + \gamma$. Solving the system of equations (A13) and (A12) with respect to $\theta^h(E)$ and $u^h(E)$ yields:

$$\theta^h(E) = \frac{1}{2} \left(\frac{1 + \beta^h[R^h(E)]}{1 - \beta^h[R^h(E)]} u(2\sigma_0\sqrt{h}L(R^h(E))) - 1 - \gamma \right), \quad (\text{A15})$$

$$u^h(E) = \left(\frac{1 + rh}{1 + \rho h} \right) \frac{u(2\sigma_0\sqrt{h}L(R^h(E)))}{1 - \beta^h[R^h(E)]}, \quad (\text{A16})$$

where

$$\beta^h[R^h(E)] = \frac{R^h(E) - p - r}{\sigma_0} \sqrt{h}.$$

In the unconstrained region, $[\hat{E}^h, E_{max}^h]$, the equilibrium loan rate $R^h(E)$ and the market-to-book ratio $u^h(E)$ satisfy the system of equations:

$$\left(1 + \beta^h[R^h(E)]\right) u^h(E^+) = \left(1 - \beta^h[R^h(E)]\right) u^h(E^-), \quad (\text{A17})$$

$$2(1 + \rho h)u^h(E) = (1 + rh) \left[u^h(E^+) + u^h(E^-) \right]. \quad (\text{A18})$$

Rewriting Equation (A17) after taking expectation yields:

$$R^h(E) = p + r + \underbrace{\frac{\sigma_0}{\sqrt{h}} \left[\frac{u^h(E^-) - u^h(E^+)}{u^h(E^+) + u^h(E^-)} \right]}_{\text{lending premium, } > 0} > p + r.$$

It remains to check that $E_{max}^h > \hat{E}^h > E_{min}^h > 0$.

First, let us show that $E_{min}^h > 0$. Suppose by way of contradiction that $E_{min}^h = 0$. Then, from the binding leverage constraint at the aggregate level it follows that $R^h(0) = p + r + \frac{\sigma_0}{\sqrt{h}}$ and thus $\beta^h[R^h(0)] = 1$. The optimal recapitalization policy implies that at E_{min}^h it must hold that $u^h(E_{min}^h) = 1 + \gamma$. However, evaluating (A16) at $E_{min}^h = 0$ yields $u(0) \equiv \infty$, which is incompatible with the previous statement. Therefore, $E_{min}^h > 0$, as claimed.

Next, we show that $\hat{E}^h > E_{min}^h$. Combining (A13) and $u^h(E_{min}^h) = 1 + \gamma$, it is immediate to see that $\theta^h(E_{min}^h) > 0$. Then, by continuity, it must be that $\theta^h(E) > 0$ in the vicinity of E_{min}^h .

Finally, let us show that $E_{max}^h > \hat{E}^h$. Assume by way of contradiction that $E_{max}^h = \hat{E}^h$. Then it must hold that $\theta^h(E_{max}^h) \geq 0$. The optimal dividend policy implies that $u(E^+) = 1$ for $E = E_{max}^h$. Note that $\theta^h(E_{max}^h) \geq 0$ if and only if

$$\xi(h) := 1 + \beta^h[R^h(E_{max}^h)] - (1 + \gamma)(1 - \beta^h[R(E_{max}^h)]) > 0.$$

Yet, when $h \rightarrow 0$, we have $\beta^h[R(E_{max}^h)] \rightarrow 0$ and thus $\xi(h) < 0$, a contradiction. Hence, $E_{max}^h > \hat{E}^h$.

A.2. Convergence to the continuous-time results

We now establish convergence of the properties of the discrete-time version of the competitive equilibrium studied above to the properties of its continuous-time counterpart studied in Section 3. Namely, we are going to show that, for $h \rightarrow 0$,

- $\hat{E}^h \rightarrow E_{min}^h$ and $E_{min}^h \rightarrow 0$;
- $R(E_{max}^h) \rightarrow p + r$;
- Equations (A17) and (A18) converge to Equations (22) and (26).

a) First, let us show that $\hat{E}^h \rightarrow E_{min}^h$ when $h \rightarrow 0$. Recall that \hat{E}^h is implicitly defined by Equation $\theta(\hat{E}^h) = 0$, which holds if and only if

$$(1 + \beta^h[R^h(\hat{E}^h)])u(2\sigma_0\sqrt{h}L(R^h(\hat{E}^h))) = (1 + \gamma)(1 - \beta^h[R^h(\hat{E}^h)]).$$

When $h \rightarrow 0$, we have $\lim_{h \rightarrow 0} \beta^h[R^h(\hat{E}^h)] \rightarrow 0$, so that the above equality transforms to:

$$u(2\sigma_0\sqrt{h}L(R^h(\hat{E}^h))) = 1 + \gamma. \quad (\text{A19})$$

At the same time, E_{min}^h satisfies $u^h(E_{min}^h) = 1 + \gamma$, which is equivalent to

$$\left(\frac{1 + rh}{1 + \rho h}\right) \frac{u(2\sigma_0\sqrt{h}L(R^h(E_{min}^h)))}{1 - \beta^h[R^h(E_{min}^h)]} = 1 + \gamma.$$

When $h \rightarrow 0$, the above equation transforms to

$$u(2\sigma_0\sqrt{h}L(R^h(E_{min}^h))) = 1 + \gamma. \quad (\text{A20})$$

Given that $u^h(\cdot)$ is a monotonically-decreasing function of its argument, equations (A19) and (A20) simultaneously hold only when $\hat{E}^h = E_{min}^h$.

Second, the property $E_{min}^h \rightarrow 0$ for $h \rightarrow 0$ immediately follows from the aggregate market leverage constraint that holds with equality at $E = E_{min}^h$.

b) Next, we demonstrate that $R(E_{max}^h) \rightarrow p + r$ when $h \rightarrow 0$. To this end, consider the system of Equations (A17) and (A18) evaluated at E_{max}^h :

$$\left(1 + \beta^h[R^h(E_{max}^h)]\right)u^h(E^+) = \left(1 - \beta^h[R^h(E_{max}^h)]\right)u^h(E^-), \quad (\text{A21})$$

$$2(1 + \rho h)u^h(E_{max}^h) = (1 + rh)\left[u^h(E^+) + u^h(E^-)\right]. \quad (\text{A22})$$

The optimal dividend policy implies that $E^+ \equiv E_{max}^h$ and $u^h(E_{max}^h) = 1$. Then, solving the above system one obtains:

$$u^h(E^-) = \frac{1 - \beta^h[R^h(E_{max}^h)]}{1 + \beta^h[R^h(E_{max}^h)]},$$

and

$$\beta^h[R^h(E_{max}^h)] = \frac{(\rho - r)h}{1 + \rho h}.$$

Using the definition of $\beta^h(\cdot)$, one can show that:

$$R^h(E_{max}^h) = p + r + \sigma_0 \frac{(\rho - r)\sqrt{h}}{1 + \rho h}.$$

It is easy to see from the above equation that $R^h(E_{max}^h) \rightarrow p + r$ when $h \rightarrow 0$.

c) Finally, let us show that Equations (A17) and (A18) converge to Equations (22) and (26) when $h \rightarrow 0$. To see this, consider first a first-order Taylor expansion of Equation (A17). Neglecting the terms of order higher than \sqrt{h} , we get:

$$\left(1 + \beta^h [R^h(E)]\right) \left[u^h(E) + [u^h(E)]' L(R^h(E)) \sigma_0 \sqrt{h} \right] = \left(1 - \beta^h [R^h(E)]\right) \left[u(E) - [u^h(E)]' L(R^h(E)) \sigma_0 \sqrt{h} \right], \quad (\text{A23})$$

which, after simplification, yields

$$u^h(E)(R^h(E) - p - r)h + \sigma_0^2 [u^h(E)]' L(R^h(E))h = 0, \quad (\text{A24})$$

or,

$$\frac{[u^h(E)]'}{u^h(E)} = -\frac{R^h(E) - p - r}{\sigma_0^2 L(R^h(E))},$$

which corresponds to (22).

Similarly, applying a second-order Taylor expansion to the right-hand side of Equation (A18) and neglecting terms of order higher than h yields:

$$(\rho - r)u^h(E) = [u^h(E)]' \left(rE + (R^h(E) - p - r)L(R^h(E)) \right) + \frac{\sigma_0^2 L(R^h(E))^2}{2} [u^h(E)]''(E), \quad (\text{A25})$$

which corresponds to (26).

Appendix B. Time-varying financing conditions

In the core of the paper we have considered the setting in which the refinancing cost γ was constant over time. In this section we extend the basic model to time-varying financing conditions. Assume that the cost of issuing new equity depends on a macroeconomic state that can be Good or Bad, with the respective costs of recapitalization γ_G and γ_B , such that $\gamma_B > \gamma_G$. Let ψ_B denote the intensity of transition from the Good to the Bad state and ψ_G denote the intensity of transition from the Bad to the Good state.

Note that the homogeneity property of the individual decision problem is still preserved, so we can again work directly with the market-to-book value of banks. To ensure that the maximization problem of bank shareholders has a non-degenerate solution, the market-to-book value must satisfy the system of simultaneous equations:

$$\rho u_G(E) = \frac{[K_G(E)\sigma_0]^2}{2} u_G''(E) + K_G(E) [R_G(E) - p] u_G'(E) - \psi_B [u_G(E) - u_B(E)], \quad E \in (E_{min}^G, E_{max}^G) \quad (\text{B1})$$

$$\rho u_B(E) = \frac{[K_B(E)\sigma_0]^2}{2} u_B''(E) + K_B(E) [R_B(E) - p] u_B'(E) - \psi_G [u_B(E) - u_G(E)], \quad E \in (E_{min}^B, E_{max}^B) \quad (\text{B2})$$

along the system of the FOCs for the individual choices of lending in each state:

$$u_G(E)[R_G(E) - p] = -\sigma_0^2 K_G(E) u'_G(E), \quad (\text{B3})$$

$$u_B(E)[R_B(E) - p] = -\sigma_0^2 K_B(E) u'_B(E). \quad (\text{B4})$$

Thus, the market-to-book value, the loan rate and aggregate lending functions will have different expressions conditional on financing conditions. The differential equations characterizing them take into account the possibility of transitions between the states. Note that equations (B1)-(B2) are similar to equation (22) obtained in the setting with the time-invariant financing cost. However, compared to (22), each equation carries the additional term $-\psi_{\bar{j}} [u_j(E) - u_{\bar{j}}(E)]$ reflecting the possibility that the cost of raising new equity can suddenly change from γ_j to $\gamma_{\bar{j}}$ (here \bar{j} denotes the state complementary to the state j).

For given recapitalization (E_{min}^G, E_{min}^B) and dividend (E_{max}^G, E_{max}^B) boundaries, the boundary conditions can be established by solving for the optimal recapitalization and payout policies, which yields:

$$u_G(E_{max}^G) = u_B(E_{max}^B) = 1, \quad (\text{B5})$$

$$u_G(E_{min}^G) = 1 + \gamma_G, \quad u_B(E_{min}^B) = 1 + \gamma_B. \quad (\text{B6})$$

To define the boundaries $E_{min}^G, E_{max}^G, E_{min}^B, E_{max}^B$, we follow the same logic that was used in the setting with the time-invariant cost and consider the marginal value of the entire banking system at the boundaries. The absence of arbitrage opportunities implies that the following condition must hold at $E_{max}^j, j \in \{G, B\}$:

$$V'_j(E_{max}^j) = u_j(E_{max}^j) + E_{max}^j u'_j(E_{max}^j) = 1.$$

Combining the above condition with (B5) and taking into account the fact that $E_{max}^j > 0$ yields us a couple of equations needed to compute the values of $E_{max}^j, j \in \{G, B\}$:

$$u'_j(E_{max}^j) = 0, \quad j \in \{G, B\}. \quad (\text{B7})$$

Similarly, at refinancing boundaries $E_{min}^j, j \in \{G, B\}$, it must hold that

$$V'_j(E_{min}^j) = u_j(E_{min}^j) + E_{min}^j u'_j(E_{min}^j) = 1 + \gamma_j,$$

which implies

$$E_{min}^j u'_j(E_{min}^j) = 0, \quad j \in \{G, B\}. \quad (\text{B8})$$

Note that, compared with the setting involving the time-invariant refinancing cost, two polar cases are possible now: a) $u'_j(E_{min}^j) > 0$ with $E_{min}^j = 0$; or b) $u'_j(E_{min}^j) = 0$ with $E_{min}^j > 0$. As we discuss below, which of these cases ultimately materializes depends on the magnitude of the refinancing cost in the Bad state, γ_B .

Finally, before we turn to the numerical illustration of the equilibrium properties, note that the

loan rate in state $j \in \{G, B\}$ satisfies the following differential equation:⁴⁷

$$R'_j(E) = -\frac{1}{\sigma_0^2} \frac{2\rho\sigma_0^2 + (R_j(E) - p)^2 + \psi_j \left(1 - \frac{u_j(E)}{u_j(E)}\right) \sigma_0^2}{L(R_j(E)) - (R_j(E) - p)L'(R_j(E))}.$$

Compared with equation (28) in Section 3, the right-hand side of the above equation contains an additional term in the numerator, $\psi_j \left(1 - \frac{u_j(E)}{u_j(E)}\right) \sigma_0^2$. This suggests that in the current set-up the loan rate will carry an extra premium/discount.

Numerical example. To illustrate the numerical solution and properties of the competitive equilibrium in the setting with time-varying refinancing costs, we resort to the simple linear specification of the demand for loans:

$$K_j(E) = \bar{R} - R_j(E), \quad j \in \{G, B\}.$$

We first work with the systems of equations (B1)- (B2) and (B3)- (B4). Replacing $K_j(E)$ in (B3)- (B4), one can express $R_j(E)$ as a function of $u_j(E)$ and $u'_j(E)$. In our simple linear case, this yields:

$$R_j(E) = \frac{pu_j(E) - \bar{R}\sigma_0^2 u'_j(E)}{u_j(E) - \sigma_0^2 u'_j(E)}, \quad j \in \{G, B\}. \quad (\text{B9})$$

Substituting the above expression(s) in the system (B1)- (B2) leaves us with a system of two simultaneous second-order differential equations that can be solved numerically by using the four boundary conditions stated in (B5)-(B6). To obtain a solution to this system, we conjecture that $E_{max}^G < E_{max}^B$ (this is verified ex-post) and use the fact that $u_G(E) \equiv 1$ when $E \in [E_{max}^G, E_{max}^B]$.⁴⁸ To solve numerically for the equilibrium, we proceed in three steps:

- i) first, we take the boundaries $\{E_{max}^G; E_{max}^B\}$ as given and solve for the optimal E_{min}^G and E_{min}^B that satisfy conditions (B8);
- ii) second, we search for the couple $\{E_{max}^G; E_{max}^B\}$ that satisfies conditions (B7);
- iii) finally, we uncover the equilibrium loan rates by substituting the functions $u_G(E)$ and $u_B(E)$ into equations (B9).

We now turn to the discussion of two possible cases that may arise depending on the magnitude of the refinancing cost γ_B . For this discussion, it is helpful to introduce two *benchmarks*: in the first benchmark, the economy never leaves the Good state, i.e., $\psi_B \equiv 0$; in the second benchmark, the economy always remains in the Bad state, i.e., $\psi_G \equiv 0$. The first benchmark gives us the lower bound for E_{max}^G that we will further label E_{max}^* , whereas the second benchmark gives the upper bound for E_{max}^B that we will label E_{max}^{**} . We also denote $u^{\psi_B=0}(E)$ ($R^{\psi_B=0}(E)$) the market-to-book value (loan rate) in the set-up with $\psi_B = 0$ and $u^{\psi_G=0}(E)$ ($R^{\psi_G=0}(E)$) the market-to-book value (loan rate) in the set-up with $\psi_G = 0$.

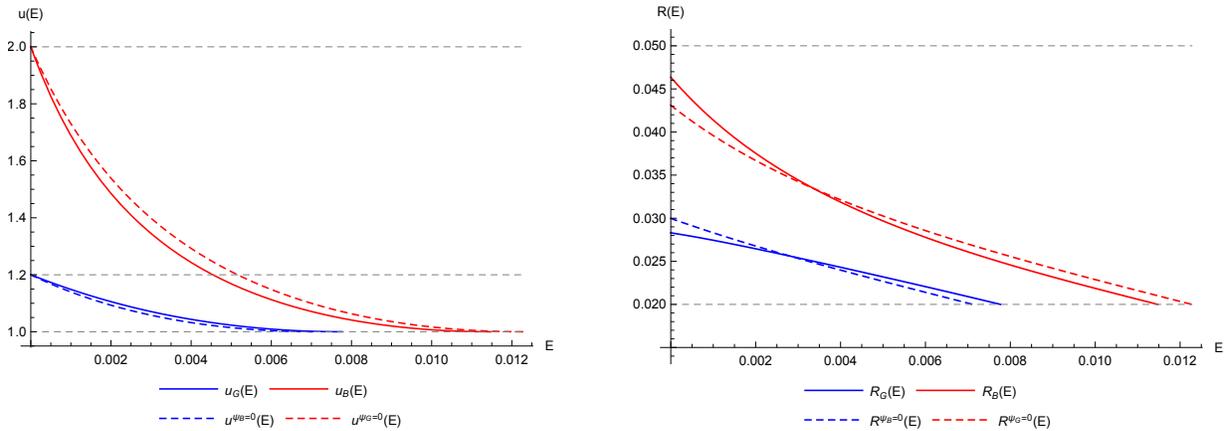
⁴⁷This equation can be obtained by combining equations (B1) and (B3) (or, equivalently, (B2) and (B4)).

⁴⁸To find a numerical solution, one can start with a guess for $u_B(E)$ and proceed iteratively until the difference between the solutions on the consequent iterations vanishes. It turns out to be convenient to take as the initial guess for $u_B(E)$ the solution to the ODE (B2) with $\psi_G \equiv 0$.

Case 1: γ_B low. First we consider the case in which the refinancing cost in the Bad state is relatively low. The solid lines in Figure 9 depict the typical patterns of the market-to-book ratios (left panel) and the corresponding loan rates (right panel) computed in each state. These outcomes are contrasted with the patterns emerging in the two benchmarks (dashed lines).

Several features of the equilibrium are worth mentioning at this stage. First, one can note that $E_{min}^G = E_{min}^B = 0$, so that banks do not change their recapitalization policies as compared to those implemented in the benchmark settings. By contrast, $E_{max}^G > E_{max}^*$ and $E_{max}^B < E_{max}^{**}$, which means that banks will delay the distribution of dividends as compared to the setting in which the economy permanently remains in the Good state and will accelerate dividend payments as compared to the setting in which the economy is permanently locked in the Bad state. Second, in the Good state, the loan rate carries a “discount” (as compared to its benchmark value obtained for $\psi_B \equiv 0$) when E is low and an extra “premium” when E is high enough. By contrast, the loan rate in the Bad state carries an extra “premium” (as compared to its benchmark value obtained for $\psi_G \equiv 0$) for lower level of E and a “discount” when E is high. Finally, the fact that $R_B(E) > R_G(E)$ implies $L(R_B(E)) < L(R_G(E))$, i.e., lending is procyclical.

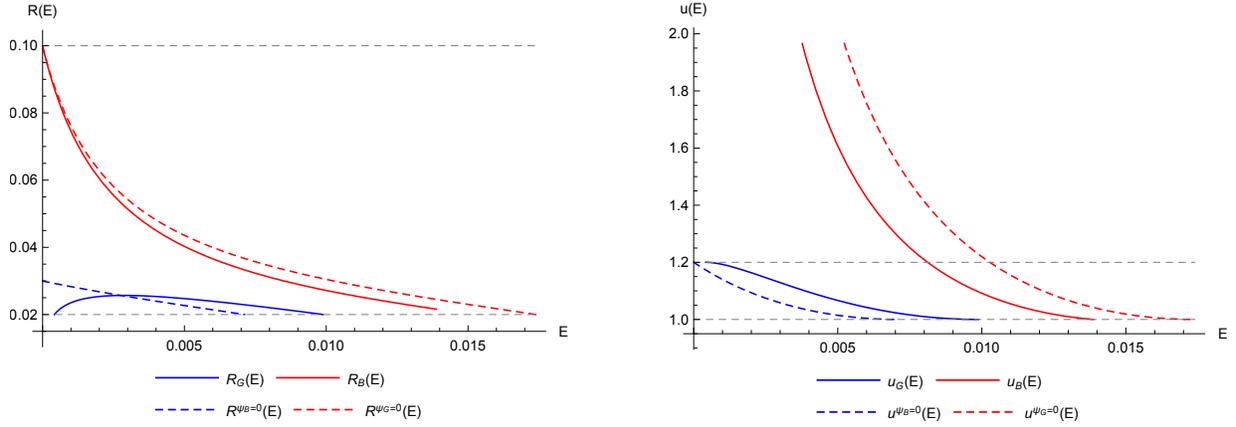
Figure 9: Competitive equilibrium with time-varying financing costs: γ_B low



Parameter values: $\rho = 0.05$, $\sigma_0 = 0.05$, $p = 0.02$, $\gamma_G = 0.2$, $\gamma_B = 1$, $\psi_G = 0.1$, $\psi_B = 0.05$; $L(R) = \bar{R} - R$ with $\bar{R} = 0.1$. The benchmark payout barriers are: $E_{max}^* \approx 0.007$ and $E_{max}^{**} \approx 0.012$. The payout barriers in the Good and Bad regimes are: $E_{max}^G \approx 0.008$ and $E_{max}^B \approx 0.011$.

Case 2: γ_B high. We now turn to the setting involving very high refinancing costs in the Bad state. The properties of the competitive equilibrium emerging in this case are illustrated in Figure 10. The key distinction from the previous case is that, for fear of incurring substantial recapitalization costs if the economy slides in the Bad state, in the Good state banks will raise new equity capital at a *strictly positive* level of aggregate capitalization, i.e., $E_{min}^G > 0$. In a dynamic setting, this phenomenon of “market timing” was first identified by Bolton et al. (2013) within a partial-equilibrium liquidity-management model with the stochastically changing fixed cost of equity issuance. The main general-equilibrium implication of this feature in our setting is that the loan rate (and, therefore, the volume of lending) converges to its First-Best level when $E \rightarrow E_{min}^G$ and financing conditions are good (i.e., $j = G$).

Figure 10: Competitive equilibrium with time-varying financing costs: γ_B high



Parameter values: $\rho = 0.05$, $\sigma_0 = 0.05$, $p = 0.02$, $\gamma_G = 0.2$, $\gamma_B = +\infty$, $\psi_G = 0.1$, $\psi_B = 0.05$; $L(R) = \bar{R} - R$ with $\bar{R} = 0.1$. The benchmark payout barriers are: $E_{max}^* \approx 0.007$ and $E_{max}^{**} \approx 0.017$. The payout barriers in the Good and Bad regimes are: $E_{max}^G \approx 0.01$ and $E_{max}^B \approx 0.014$. The recapitalization barriers are: $E_{min}^G \approx 0.0005$ and $E_{min}^B = 0$. Note that functions $u_G(E)$ and $u^{\psi_G=0}(E)$ tend to +

Appendix D. Empirical analysis

This Appendix presents a simple assessment of the consistency of the key predictions of our model with the data. As stated in Section 3, our model delivers two key predictions: bank loan rates and the ratio of market-to-book equity are (weakly) decreasing functions of aggregate bank equity. We assess these predictions by estimating conditional correlations via regression analysis between measures of bank gross returns on earning assets, the ratio of market-to-book equity, and aggregate bank equity. We use a large bank-level panel dataset. Its use, and the attendant heterogeneity of the data for banks belonging to a specific country group, places a strong consistency requirement on the predictions of our model, which is constructed under the simplifying assumption of homogeneous banks.

Our results indicate that the key predictions of our model are not rejected by simple statistical tests, are consistent with a large variety of country circumstances, and are robust to data heterogeneity owing to our use of a firm-level panel dataset. This suggests that our model is useful and relevant.

G.1. Data and statistics

The data consists of consolidated accounts and market data for a panel of publicly traded banks in 43 advanced and emerging market economies for the period 1982-2013 taken from the Wordscope database retrieved from Datastream.⁴⁹ The sample is split into three sub-samples: U.S. banks (753

⁴⁹The Standard Industrial Classification (SIC) System is used to identify the types of financial institutions included in the sample, which are: National Commercial Banks (6021), State Commercial Banks (6022), Commercial Banks Not Elsewhere Classified (6029), Savings Institution Federally Chartered (6035), Savings Institutions Not Federally Chartered (6036). In essence, the sample includes all publicly quoted depository institutions in the database. This panel dataset is unbalanced due to mergers and acquisitions, but all banks active in each period are included in the

banks with about 10,500 bank-year observations); banks in advanced economies excluding the U.S. (500 banks with about 8,000 bank-year observations); banks in emerging market economies (185 banks with about 2,000 bank-year observations). Table 1 summarizes the definitions of the variables considered.

Table 1: Definition of variables

Identifier	Variable	Measurement
<i>ret</i>	bank gross return on assets	total interest income/earning assets
<i>mtb</i>	market-to-book equity ratio	market equity/book equity
<i>logta</i>	bank size	Log(assets)
<i>loanasset</i>	% of loans to assets	total loans/total assets
<i>bequity</i>	bank book equity	bank book equity
<i>mmr</i>	short-term rate	short-term rate
<i>TBE</i>	total bank equity	sum of bequity

Note that bank gross return on assets include revenues accruing from investments other than loans; however, in the analysis below we will condition our estimates on asset composition using the % of loans to assets as a bank control. Furthermore, the variable total bank equity is the sum of the equity of banks belonging to a particular country: this amounts to assuming that the relevant banking market is the country. All other variables are exact empirical counterparts of the variables defined in the model.

Table 2 reports sample statistics (Panel A) and some (unconditional) correlations (Panel B). Note that the correlation between bank returns, the market-to-book equity ratio, and total bank equity is negative and significant. However, we wish to gauge conditional correlations, to which we now turn.

G.2. Conditional correlations via panel regressions

We test whether there exist a negative conditional correlation between bank returns, market-to-book equity and total bank equity by estimating panel regressions with $Y_{it} \in (ret, mtb)$ as the dependent variable of the form:

$$Y_{it} = \alpha + \beta E_t + \gamma_1 bequity_{it} + \gamma_2 Logta_{it} + \gamma_3 loanasset_{it} + \gamma_4 mmr_t + \gamma_5 rgdpg_t + \gamma_6 infl_t + \epsilon_{it}. \quad (D1)$$

Model (D1) is used for the US sample, while we add to Model (D1) country specific effects in the estimation for the advanced economies and emerging market samples. Our focus in on the coefficient β . Bank specific effects are controlled for by the triplet (*bequity*, *logta*, *loanasset*), while country specific time-varying effects are controlled for by the short-term rate, real GDP growth and inflation (*mmr*, *rgdpg*, and *infl* respectively).

Table 3 reports the results. The coefficient β is negative and (strongly) statistically significant in all regressions. The quantitative impact of changes in total bank equity on both bank returns and the market-to-book ratio is substantial as well. Thus, we conclude that the two key predictions of our model are consistent with the data.

sample to avoid survivorship biases.

Table 2: Sample statistics and unconditional correlations

Panel A: Sample Statistics

Variable	US			Advanced (ex. US)			Emerging		
	Obs.	Mean	Std.Dev.	Obs.	Mean	Std.Dev.	Obs.	Mean	Std.Dev.
ret (%)	10712	6.68	1.78	8606	6.68	4.3	3074	10.08	5.07
mtb	10094	1.41	0.71	8645	1.42	0.87	3028	1.62	0.98
logta	11630	13.58	1.65	9671	16.49	2.05	3644	15.62	1.93
loanasset	11442	65.6	13.37	8816	68.25	15.35	3240	66.28	15.91
bequity (US\$ bln)	11555	943.52	8942.38	9632	3748.55	11154.98	3591	2832.71	11411.89
mmr	25874	5.33	3.92	14977	5.52	5.14	4987	9.78	5.89
TBE (US\$ bln)	25113	330209.8	363168.8	15164	98893.61	126907.5	6416	32144.24	102500.9

Panel B: Correlations

	ret	mtb	logta	ret	mtb	logta	ret	mtb	logta
mtb	0.1653*	1		0.0471*	1		0.0584*	1	
logta	-0.1163*	0.2124*	1	-0.2418*	0.1637*	1	-0.3680*	0.0818*	1
TBE	-0.7860*	-0.3016*	0.1747*	-0.5266*	-0.1340*	0.2914*	-0.1991*	-0.0539*	0.3755*

Notes: * indicates significance at 5% level.

Table 3: Panel regressions

Explanatory variables	US		Advanced (ex. US)		Emerging	
	(1) ret	(2) mtb	(3) ret	(4) mtb	(5) ret	(6) mtb
<i>E</i>	$-2.17e-06^{***}$ [0.00]	$-6.75e-07^{***}$ [0.00]	$-8.65e-07^{**}$ [0.0112]	$-2.52e-06^{***}$ [0.00]	$-1.67e-05^{**}$ [0.01]	$-8.33e-06^{***}$ [0.00]
<i>bequity</i>	$-1.33e-06$ [0.12]	$-7.05e-06^{***}$ [0.00]	$-9.42e-06^{***}$ [0.00]	$-1.73e-06^{**}$ [0.03]	$-2.12e-06$ [0.92]	$-5.23e-06$ [0.26]
<i>logta</i>	0.00419 [0.58]	0.139*** [0.00]	-0.276*** [0.00]	0.0762*** [0.00]	-0.590*** [0.00]	0.107*** [0.00]
<i>loanasset</i>	0.0183*** [0.00]	0.00291*** [0.00]	-0.0238*** [0.00]	-0.00302*** [0.00]	0.0368*** [0.00]	-0.00202 [0.24]
<i>mmr</i>	0.413*** [0.00]	-0.0375*** [0.00]	0.589*** [0.00]	-0.00545 [0.210]	0.410*** [0.00]	-0.0222*** [0.00]
<i>rgdpg</i>	-0.110*** [0.00]	0.116*** [0.00]	-0.0636*** [0.00]	0.100*** [0.00]	0.0179 [0.347]	0.0781*** [0.00]
<i>infl</i>	-0.0350*** [0.00]	-0.0509*** [0.00]	0.0657*** [0.00]	-0.00966 [0.303]	0.148*** [0.00]	-0.00444 [0.567]
<i>Constant</i>	5.689*** [0.00]	-0.349*** [0.00]	11.71*** [0.00]	0.660*** [0.00]	18.47*** [0.00]	-0.111 [0.72]
<i>Observations</i>	10,572	9,937	7,450	7,386	1,818	1,799
<i>R-squared</i>	0.773	0.233	0.736	0.282	0.643	0.150

Notes: robust p-values in brackets (***) $-p < 0.01$, (**) $-p < 0.05$, (*) $-p < 0.1$.

References

- Admati, A., DeMarzo, P., Hellwig, M., and P. Pfleiderer, 2011. Fallacies, Irrelevant Facts, and Myths in the Discussion of Capital Regulation: Why Bank Equity is Not Expensive. Mimeo.
- Admati, A., and M. Hellwig, 2013. The Bankers' New Clothes: What's Wrong with Banking and What to Do about It. Princeton University Press.
- Adrian, T., and H. Shin, 2010. Liquidity and leverage. *Journal of Financial Intermediation* 19, 418 - 437.
- Aikman, D., Haldane, A. G. and B. D. Nelson, 2014. Curbing the Credit Cycle. *The Economic Journal*. doi: 10.1111/eoj.12113.
- Baron, M., 2015. Countercyclical Bank Equity Issuance. Princeton University, Working Paper.
- Becker, B., and V. Ivashina, 2014. Cyclicity of credit supply: Firm level evidence. *Journal of Monetary Economics* 62, 76-93.
- Begenau, J., 2015. Capital Requirements, Risk Choice, and Liquidity Provision in a Business Cycle Model. Working Paper.
- Berrospide, J. M. and R. M. Edge, 2010. The Effects of Bank Capital on Lending: What Do We Know, and What Does it Mean? *International Journal of Central Banking* 6, 5-54.
- Bernanke, B., Gertler, M., and S. Gilchrist, 1996. The Financial accelerator and the flight to quality. *The Review of Economics and Statistics* 78, 1 - 15.
- Biais, B., Mariotti, T., Plantin, G. and J.-Ch. Rochet, 2007. Dynamic security design: convergence to continuous time and asset pricing implications. *The Review of Economic Studies* 74, 345 -390.
- Bolton, P., H. Chen, and N. Wang, 2011. A Unified Theory of Tobin's q, Corporate Investment, Financing, and Risk Management. *Journal of Finance*, 66, 1545 - 1578.
- Bolton, P., H. Chen, and N. Wang, 2013. Market Timing, Investment, and Risk Management. *Journal of Financial Economics* 109, 40 - 62.
- Bridges, J., Gregory, D., Nielsen, M., Pezzini, S., Radia, A. and M. Spaltro, 2014. The impact of capital requirements on bank lending. Bank of England Working Paper n°486.
- Brunnermeier, M. K. and Y. Sannikov, 2014. A Macroeconomic Model with a Financial Sector. *American Economic Review* 104, 379 - 421.
- Brunnermeier, M. K. and Y. Sannikov, 2015. International Credit Flows and Pecuniary Externalities. *American Economic Journal: Macroeconomics* 7, 297 - 338.
- DeAngelo, H., and R. M. Stulz, 2014. Why High Leverage is Optimal for Banks. Fisher College of Business WP 2013-03-08.
- Décamps, J.P., T. Mariotti, J.C. Rochet, and S. Villeneuve, 2011. Free Cash Flow, Issuance Costs, and Stock Prices. *Journal of Finance*, 66, 1501-1544.

- Dell’Ariccia, G., and R. Marquez, 2006. Lending Booms and Lending Standards. *Journal of Finance* 61, 2511 - 2546.
- De Nicolò, G., A. Gamba, and M. Lucchetta, 2014. Microprudential Regulation in a Dynamic Model of Banking. *Review of Financial Studies* 27, 2097 - 2138.
- Diamond, D., and P. Dybvig, 1983. Bank runs, deposit insurance, and liquidity. *Journal of Political Economy* 99, 689-721.
- Diamond, D., and R. Rajan, 2000. A theory of bank capital. *Journal of Finance* 55, 2431-2465.
- Di Tella, S., 2014. Uncertainty Shocks and Balance Sheet Recessions. Working Paper, Stanford GSB.
- Freixas, X. and J.-Ch. Rochet, 2008. *Microeconomics of Banking*, MIT Press.
- Gennaioli, N., Schleifer, A., and R.W. Vishny, 2013. A Model of Shadow Banking. *Journal of Finance* 68, 1331 - 1363.
- Gornall, W., and I. A., Strebulaev, 2015. Financing as a Supply Chain: The Capital Structure of Banks and Borrowers. Working Paper.
- Gorton, G., 2010. *Slapped by the Invisible Hand: The Panic of 2007*. Oxford University Press, Oxford, UK.
- Gorton, G., Pennacchi, G., 1990. Financial intermediaries and liquidity creation. *Journal of Finance* 45, 49-72.
- He, Z. and A. Krishnamurthy, 2012. A Model of Capital and Crises. *Review of Economic Studies* 79, 735-77.
- He, Z. and A. Krishnamurthy, 2013. Intermediary Asset Pricing. *American Economic Review* 103, 732 - 770.
- Hellwig, M., 2015. Liquidity provision and Equity Funding of Banks. Discussion Paper, Max Planck Institute.
- Hennessy, C. A. and T. M. Whited, 2007. How costly is external financing: Evidence from a structural estimation. *Journal of Finance* 62, 1705 - 1745.
- Holmström, B., Tirole, J., 1997. Financial intermediation, loanable funds, and the real sector. *Quarterly Journal of Economics* 112, 663-691.
- Holmström, B., Tirole, J., 1998. Private and public supply of liquidity. *Journal of Political Economy* 196, 1-40.
- Hugonnier, J. N., and E. Morellec, 2015. Bank Capital, Liquid Reserves, and Insolvency Risk. Working Paper, Swiss Finance Institute and EPFL.
- Institute for International Finance, 2010. Interim Report on the Cumulative Impact on the Global Economy of Proposed Changes in Banking Regulatory Framework, Washington, DC: Institute for International Finance.

- Isohätälä, J., Milne A., and D. Robertson, 2014. The net worth trap: investment and output dynamics in the presence of financing constraints. Bank of Finland Research Discussion Paper No. 26/2014.
- Isohätälä, J., Klimenko, N., and A. Milne, 2015. Post-crisis macro financial modelling: continuous time approaches. Forthcoming in "Handbook of post crisis nancial modelling", Palgrave MacMillan.
- Jeanblanc, M., and A. N. Shiryaev, 1995. Optimization of the flow of dividends. *Russian Mathematical Surveys* 50, 257 - 277.
- Jiménez, G., and J. Saurina, 2006. Credit Cycles, Credit Risk, and Prudential Regulation. *International Journal of Central Banking* 2, 65-98.
- Klimenko, N., Pfeil, S., and J.-Ch. Rochet, 2015. The Apples of Wrath: A Lucas Model with Extreme Financial Frictions. Working Paper.
- Kiyotaki, N. and J. Moore, 1997. Credit Cycles. *Journal of Political Economy* 105, 211 - 48.
- Lee, I., S. Lochhead, J. Ritter, and Q. Zhao, 1996. The Costs of Raising Capital. *Journal of Financial Research* 19 , 59-74.
- Lucas, R., 1978. Asset Prices in an Exchange Economy. *Econometrica* 46, 1429 - 1445.
- Malherbe, F., 2015. Optimal capital requirements over the business and financial cycles. ECB Working Paper 1830.
- Martinez-Miera, D., and R. Repullo, 2010. Does Competition Reduce the Risk of Bank Failure? *Review of Financial Studies* 23, 3638 -3664.
- Martinez-Miera, D., and J. Suarez, 2014. A Macroeconomic Model of Endogenous Systemic Risk Taking. Mimeo, CEMFI.
- Meh, C., and K. Moran, 2010. The role of bank capital in the propagation of shocks. *Journal of Economic Dynamics and Control* 34, 555 - 576.
- Nguyên, T., 2014. Bank Capital Requirements: A quantitative Analysis. Working Paper.
- Pandit, V., 2010. We must rethink Basel, or growth will suffer. *Financial Times*, 10 November.
- Phelan, G., 2015. Financial Intermediation, Leverage, and Macroeconomic Instability. Forthcoming in *American Economic Journal: Macroeconomics*.
- Stein, J., 2012. Monetary Policy as Financial Stability Regulation. *Quarterly Journal of Economics* 127, 57-95.
- TheCityUK report, October 2013. Alternative finance for SMEs and mid-market companies.
- Van den Heuvel, S. J., 2008. The welfare cost of bank capital requirements. *Journal of Monetary Economics* 55, 298 - 320.