Discussion of: *Merging Simulation and Projection Approaches to Solve High-Dimension Problems* by K. Judd, L. Maliar, & S. Maliar

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Overall Approach

- Problem: Solve for *y(s,z),s'(s,z),(z',y')* :

$$E_t[G(s, z, y, s', z', y')] = 0, z' = Z(Z, \epsilon).$$

- Method:
  - – Start with a guess
  - – Simulate the system
  - – Construct an ϵ-distinguishable set (EDS)
  - – Solve over the EDS using a projection method
  - – Iterate until the grid converges

Innovation and Significance

- EDS method
- General ideas:
  - Restrict the grid region to high-probability events by deleting points below threshold on kernel-based density approximation
  - Obtain *uniform* collection of points by eliminating ϵ-neighbors iteratively (and using PCA)
- Results: Effective and efficient solutions even in high dimensions

Potential Problems in Paper

- Lack of convergence proof
- Unstable simulation results (use restricted rules with growth limits)
- Failed fixed-point iterations (use Newton or global search)
- Equilibrium not attained (check on second order conditions)

Other Issues and Extensions

- Alternative grids/bounds, e.g.: sparse grid (Smolyak)

For Sobolev spaces of functions f on R^d with smoothness r, error:

$$O(\frac{(\log n)^{(d-1)(r+1)}}{n^r}).$$

- Issue: may not have smoothness (although bounds maybe still be possible)
- Can adapt to different regions
- For smooth functions, irregular points can help.

Simulation Setup and Convexity

- Use of low-discrepancy sequences for simulation
  - Pseudo-random draws may help for building confidence intervals but here?
  - Low-discrepancy sequences would appear to offer better initial coverage
- Use of structural properties (e.g., convexity) in the approximations
  - Possible in neoclassical growth example
  - Can use global information

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Monte Carlo Methods

- MCMC and particle-type methods with re-sampling for reducing variance and importance sampling

$$\int G(s'(s,z'),y'(s,z'),s,y,z')\phi(z'|z)dz' = \int G(s',y',s,y,z')\frac{\phi(z'|z)}{g(z')}g(z')dz'$$

- Latent variables for extended Gaussian mixtures

$$\int G(s',z',y',s,z,y)\phi(z'|z)dz' = \int\int G(s',z',y',s,z,y)g(w,s',z',y')\phi(z'|z)dwdz'$$

Analytical Results

- Convergence
  - Characterize conditions?
  - Guidance on implementation?
- Error bounds
  - Asymptotic
  - Explicit
  - Estimated