Discussion of: Merging Simulation and Projection Approaches to Solve High-Dimension Problems by K. Judd, L. Maliar, & S. Maliar

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Overall Approach

• Problem: Solve for \( y(s,z), s'(s,z), (z',y') \):
  \[ E_t[G(s, z, y, s', z', y')] = 0, \quad z' = Z(Z, \epsilon). \]

• Method:
  – Start with a guess
  – Simulate the system
  – Construct an \( \epsilon \)-distinguishable set (EDS)
  – Solve over the EDS using a projection method
  – Iterate until the grid converges
Innovation and Significance

• EDS method

• General ideas:
  – Restrict the grid region to high-probability events by deleting points below threshold on kernel-based density approximation
  – Obtain uniform collection of points by eliminating ε-neighbors iteratively (and using PCA)

• Results: Effective and efficient solutions even in high dimensions
Potential Problems in Paper

- Lack of convergence proof
- Unstable simulation results (use restricted rules with growth limits)
- Failed fixed-point iterations (use Newton or global search)
- Equilibrium not attained (check on second order conditions)
Other Issues and Extensions

• Alternative grids/bounds, e.g.: sparse grid (Smolyak)
  For Sobolev spaces of functions $f$ on $R^d$ with smoothness $r$, error:

$$O\left(\frac{(\log n)^{(d-1)(r+1)}n^r}{n^r}\right).$$

• Issue: may not have smoothness (although bounds maybe still be possible)

• Can adapt to different regions

• For smooth functions, irregular points can help.
Simulation Setup and Convexity

• Use of low-discrepancy sequences for simulation
  Pseudo-random draws may help for building confidence intervals but here?
  Low-discrepancy sequences would appear to offer better initial coverage

• Use of structural properties (e.g., convexity) in the approximations
  Possible in neoclassical growth example
  Can use global information
Monte Carlo Methods

- MCMC and particle-type methods with re-sampling for reducing variance and importance sampling

\[ \int G(s'(s, z'), y'(s, z'), s, y, z') \phi(z'|z)dz' = \int G(s', y', s, y, z') \frac{\phi(z'|z)}{g(z')} g(z')dz' \]

- Latent variables for extended Gaussian mixtures

\[ \int G(s', z', y', s, z, y) \phi(z'|z)dz' = \int \int G(s', z', y', s, z, y) g(w, s', z', y') \phi(z'|z)dw dz' \]
Analytical Results

• Convergence
  – Characterize conditions?
  – Guidance on implementation?

• Error bounds
  – Asymptotic
  – Explicit
  – Estimated