BELIEF HETEROGENEITY WITH UNINSURABLE RISKS

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INTRODUCTION

Household survey data on macroeconomic forecasts contain large systematic biases

- substantial belief dispersion in the cross section
- large variation in the time series

Introduce a model of subjective beliefs

- links subjective beliefs to risk exposure of the consumption process

Estimate risk exposures from consumption micro data

- study whether the model can rationalize observed belief dispersion
Interested in studying departures from rational expectations

- **parsimony**: model of subjective beliefs does not depend on details of the environment
- **discipline**: model-implied subjective forecasts can be compared to survey data

**Hypothesis**: agents’ subjective beliefs have a quantitatively large impact on aggregate dynamics, wealth distribution and welfare

- Bhandari, Borovička and Ho (2018) show that common fluctuations in subjective beliefs play an important role in unemployment dynamics
- here we focus on cross-sectional evidence on belief heterogeneity
• Facts from expectational survey data
  • document cross-sectional and time series patterns using FRBNY Survey of Consumer Expectations and Michigan Survey of Consumers

• A framework for subjective beliefs
  • theoretically link agents’ subjective beliefs to consumption risk exposures

• Empirical implementation using micro consumption data
  • use Consumer Expenditure Surveys (CEX) data to measure consumption risk exposures across demographic groups
Study household survey evidence on forecasts of macroeconomic variables

- **Belief wedge**: difference between subjective and rational forecast

Document

- Large heterogeneity in forecasts across demographic groups
- Large and time-varying common biases in the time series
SURVEY DATA

Data source

- New York Fed Survey of Consumer of Expectations
  - cross-sectional information (2013–2018)
- University of Michigan Survey of Consumers

Macroeconomic variables

- Unemployment rate
- Inflation rate
- Stock market

Demographic groups

- Age
- Income
- Education
Survey question: “What do you think is the percent chance that 12 months from now the unemployment rate in the U.S. will be higher than it is now?”

**Bold line**: average response for the group; **dashed lines**: 95% confidence interval.
Survey question: "In your view, what would you say is the percent chance that, over the next 12 months, the rate of inflation will be [within 10 listed bins]." → answer computed as average over the histogram
Survey question: “What do you think is the percent chance that 12 months from now, on average, stock prices in the U.S. stock market will be higher than they are now?”
UNEMPLOYMENT TWO-WAY Sorts

- **Income / Education**
  - Below 40k
  - 40k-75k
  - Above 75k

- **Income / Age**
  - Under 40
  - 40-59
  - Above 60

- **Age / Education**
  - Under 40
  - 40-59
  - Above 60
INFLATION TWO-WAY SORTS

income / education

income / age

age / education

(%)

below 40k
40k-75k
above 75k

under 40
40-59
above 60

below 40k
40k-75k
above 75k

under 40
40-59
above 60

HS or less
(some) college
graduate
SUMMARY OF FINDINGS

Unemployment (probability of unemployment increase)

- decreasing with age
- decreasing in income

Inflation

- decreasing with income

Stock market (probability of stock market increase)

- increasing with income
- increasing with education
BELIEF WEDGES: MICHIGAN SURVEY MINUS VAR FORECAST

See Bhandari, Borovička and Ho (2018) for details on construction.
Substantial dispersion in forecasts across demographic groups.

Systematic relationship across questions, also holds for other variables

- Forecasts of earnings growth, job separation rates, and job finding rates in FRBNY Survey of Consumer Expectations
- Consistent patterns in the Michigan Survey of Consumers

**Interpretation:** Household-level heterogeneity in magnitude of belief biases but these biases share a common origin.

Next introduce a model of subjective beliefs to rationalize these findings.
**Assumption:** Survey responses are consistent with agents’ decisions under a subjective belief model.
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Asset pricing

- Greenwood and Shleifer (2014): return expectations and subsequent realized returns
- Nagel and Xu (2018): stock market dynamics

Corporate finance

- Gennaioli, Ma, Shleifer (2015): managers’ surveys and firm investment

Macroeconomics

- Malmendier and Nagel (2016): Michigan survey responses and borrowing and lending decisions
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MODEL OF SUBJECTIVE BELIEFS

• preferences of an agent with concern for model misspecification

\[ V_t = \log C_t + \beta E_t [ V_{t+1} ] \]
• preferences of an agent with concern for model misspecification

\[ V_t = \log C_t + \beta E_t [m_{t+1} V_{t+1}] \]
MODEL OF SUBJECTIVE BELIEFS

• preferences of an agent with concern for model misspecification

\[ V_t = \min_{m_{t+1}} \log C_t + \beta E_t [m_{t+1} V_{t+1}] \]
MODEL OF SUBJECTIVE BELIEFS

• preferences of an agent with concern for model misspecification

\[ V_t = \min_{m_{t+1}^{E[m_{t+1}]=1}} \log C_t + \beta E_t [m_{t+1} V_{t+1}] + \beta^1 \theta E_t [m_{t+1} \log m_{t+1}] \]

• penalty parameter \( \theta \) (rational expectations \( \theta = 0 \))
preferences of an agent with concern for model misspecification

\[ V_t = \min_{m_{t+1}} \log C_t + \beta E_t [m_{t+1} V_{t+1}] + \beta \frac{1}{\theta} E_t [m_{t+1} \log m_{t+1}] \]

penalty parameter \( \theta \) (rational expectations \( \theta = 0 \))

implied belief distortion \( m_{t+1} \) defines a probability measure \( \tilde{P} \)

\[ m_{t+1} = \frac{\exp(-\theta V_{t+1})}{E_t[\exp(-\theta V_{t+1})]} \]

\( \theta \) controls the magnitude of the belief distortion

\( \theta \) sign determines whether beliefs are pessimistic or optimistic
Belief wedges

\[ \Delta_t^{(1)} = \tilde{E}_t [x_{t+1}] - E_t [x_{t+1}] = \text{Cov}_t (m_{t+1}, x_{t+1}) \approx -\theta \text{Cov}_t (V_{t+1}, x_{t+1}) \]

• \( \text{Cov}_t (V_{t+1}, x_{t+1}) \) endogenously determines which states are ‘bad’
  • \( \tilde{P} \) overweighs states with low continuation utility \( V_{t+1} \) (when \( \theta \) positive)

Theory of subjective beliefs

• testable using consumption exposures that determine \( V_t \)

\[ V_t = \log C_t - \frac{\beta}{\theta} \log E_t [\exp (-\theta V_{t+1})] \]

• belief heterogeneity emerges from heterogeneity in exposures or \( \theta \)
  • can be embedded in GE framework with endogenous \( x_{t+1} \) and \( V_{t+1} \)
• Facts from expectational survey data
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Specify dynamics for consumption growth for demographic group $i$
  • implies heterogeneous subjective beliefs across demographics

Use Consumer Expenditure Surveys (CEX) data
  • construct average consumption time series by demographic groups
  • estimate exposures of group-specific consumption to aggregate shocks

Assess model-implied heterogeneity in belief wedges
State dynamics

\[ X_{t+1} = \phi_x X_t + \phi_w W_{t+1} \]

Consumption growth for demographic group \( i \)

\[ \log C_{i,t+1} - \log C_{i,t} = \nu_{i,c}' X_t \]

Scaled continuation value

\[ v_{i,t} = V_{i,t} - \log C_{i,t} = \nu_{i,v}' X_t \]

- exposure vector \( \nu_{i,v} \) is a solution to the continuation value recursion

Belief distortion

\[ m_{i,t+1} = \frac{\exp \left( -\theta_i (\nu_{i,v} + \nu_{i,c})' X_{t+1} \right)}{E_t \left[ \exp \left( -\theta_i (\nu_{i,v} + \nu_{i,c})' X_{t+1} \right) \right]} \]
Belief Wedges

Forecasted variable (measured in survey data)

\[ Z_t = \nu_z' X_t \]

One-period belief distortions

\[ \Delta_{i,t}^{(1)} = \tilde{E}_t [Z_{t+1}] - E_t [Z_{t+1}] = -\theta_i \nu_z' \phi_w \phi'_w \nu_i, \nu \]

Multi-period belief distortions

\[ \Delta_{i,t}^{(j)} = \tilde{E}_t [Z_{t+j}] - E_t [Z_{t+1}] = -\theta_i \nu_z' \left( \sum_{k=0}^{j-1} (\phi_x)^k \right) \phi_w \phi'_w \nu_i, \nu \]
State dynamics

• $X_t$ — aggr. consumption growth, labor income growth, real S&P500 return, log price-dividend ratio, realized variance of industrial production growth, inflation, unemployment rate
• quarterly data

Consumption data

• CEX interview micro data (total expenditure, food expenditure)
• Construct quarterly average consumption time series by demographic group (1985–2017)

Specification of $\theta_i$ and $\beta_i$

• Common across groups: $\theta_i = \theta$, $\beta_i = \beta$
Model-implied belief wedges: Unemployment

Food consumption used. Wedges relative to belief of a representative agent.
MODEL-IMPLIED BELIEF WEDGES: INFLATION

- Age:
  - Under 40
  - 40-59
  - 60 and above

- Income:
  - Below 40k
  - 40k-75k
  - Above 75k

- Education:
  - HS or less
  - (Some) college
  - Graduate

The graphs illustrate the model-implied belief wedges for inflation across different age groups, income levels, and education categories.
A fully specified model of consumption exposures

- incomplete markets
- life-cycle dynamics

Relaxing restrictions on $\theta$

- $\theta$ can be heterogeneous across households / groups
- exploit cross-question restrictions

Explore implications for forecasts of household-specific variables

- relevant for household decision-making
TBW if needed.