Efficiency and Stability of a Financial Architecture with Too-Interconnected-to-Fail Institutions

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Abstract

The regulation of large interconnected financial institutions has become a key policy issue. To improve financial stability, regulators have proposed to limit banks’ size and interconnectedness. I estimate a network-based model of the over-the-counter interbank lending market in US and quantify the efficiency-stability implications of this policy. Trading efficiency decreases with limits on interconnectedness because the intermediation chains become longer. While restricting the interconnectedness of banks improves stability, the effect is non-monotonic. Stability also improves with higher liquidity requirements, when banks have access to liquidity during the crisis, and when failed banks’ depositors maintain confidence in the banking system.

Keywords: financial regulation, financial architecture, trading networks, trading efficiency, contagion risk, Federal funds market, simulated method of moments

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1 Introduction

The recent financial crisis left regulators more concerned than ever about the stability of the financial system. Too-interconnected-to-fail financial institutions are perceived to pose a substantial risk to financial stability. While testifying in front of the Financial Crisis Inquiry Commission of Congress, Ben Bernanke said: “If the crisis has a single lesson, it is that the too-big-to-fail problem must be solved” (Bernanke 2010). Paul Volcker, another former chairman of the Federal Reserve, argued that “the risk of failure of large, interconnected firms must be reduced, whether by reducing their size, curtailing their interconnections, or limiting their activities” (Volcker 2012).

In this paper, I develop a quantitative framework for computing the efficiency and stability of a financial architecture with and without too-interconnected-to-fail institutions. The analysis is based on a network-based model of trading in an over-the-counter (OTC) market, which is applied to the Fed funds market. In the model, banks trade only with partners with whom they have a long-term trading relationship. A financial architecture is a network of all trading relationships. I use the model to compute optimal trading decisions of banks and to compute efficiency of allocations in a financial architecture. Interbank trading generates exposures, which can result in a financial contagion if one of the banks fails. Regulators are particularly concerned that the failure of a very interconnected institution would result in a large cascade of failures, making these banks too-interconnected-to-fail. To measure stability of a financial architecture, I compute the effect of such failures on other banks and on market efficiency post-contagion. The counterfactual analysis compares efficiency and stability of a financial architecture, estimated using the network topology of the Fed funds market reported by Bech and Atalay (2010), with seven architectures that have less interconnected banks than in the estimated architecture.

In the estimated financial architecture, 0.56% of the potential gains from trade are lost due to the intermediation friction. The losses occur when banks with the highest need for liquidity cannot borrow funds. The losses are relatively small because banks use a price-setting mechanism that extracts a high share of surplus from the borrowers and thus provides incentives to banks with liquidity to lend. However, as long as intermediaries cannot extract the full surplus in each trade, we should expect positive welfare losses (Gofman 2014).¹ The estimated architecture has short intermediation chains, which further reduces the intermediation friction.

To quantify the stability of the estimated architecture, I compute interbank exposures based on the equilibrium trading decisions of banks. An exposure of bank $i$ to bank $j$ is equal to the loan amount from $i$ to $j$ divided by the total amount of the loans provided by $i$. Two contagion scenarios are considered. In the first scenario, a bank fails only if the losses from the failure of

¹If intermediaries had full bargaining power, then allocations would be (constrained) efficient in any financial architecture (Gale and Kariv 2007, Blume et al. 2009).
a counterparty are above a certain threshold, which is determined by a liquidity requirement. If banks are required to hold liquid assets equal to 15% of their interbank loans, a failure of the most interconnected bank triggers failure in 27% of banks and trading efficiency after contagion is 86% lower. The post-crisis efficiency level is still high in absolute terms because most of the failed banks are small periphery banks that do not play an important intermediation role. This result assumes that depositors of the failed banks reallocate their savings to the surviving banks. If they were to withdraw deposits from the banking system, a quarter of the potential surplus would be lost.

In the second scenario, a bank fails when its exposure to all failed counterparties exceeds its liquidity buffer. The second scenario is more severe than the first scenario because losses accumulate as contagion unravels, but banks cannot access additional liquidity to absorb these losses. With a liquidity requirement of 15%, almost all banks fail and no trading surplus can be created after contagion. This outcome highlights the importance of the unprecedented liquidity injection into the banking system by regulators during the recent financial crisis.

One of the benefits of the quantitative framework is that it allows me to generate several counterfactual architectures with different limits on the number of banks’ counterparties and to compute endogenous exposures between the banks in these architectures. In particular, I compare seven counterfactual architectures, where the maximum number of counterparties ranges from 150 to 24, to the estimated architecture that can have banks with more than 200 counterparties.

The counterfactual analysis shows that trading efficiency decreases with limits on interconnectedness because the intermediation chains become longer. In an architecture in which all banks have no more than 24 counterparties, the surplus losses are 137% higher than they are in the estimated architecture, even though both architectures have identical numbers of banks and of trading relationships. On the other side, failure of the most interconnected bank triggers more bank failures in the estimated architecture than in any other architecture. The number of bank failures declines monotonically as the limit on interconnectedness changes from 150 to 35, but it increases when the limit changes to 24. Efficiency measures post-contagion have a similar non-monotonic pattern. Combining the efficiency and stability results together, I find that the most homogeneous architecture is never optimal, but for all other architectures, there is a clear tradeoff between efficiency and stability. The optimal limit on interconnectedness depends on the probability of contagion and on how much efficiency we are willing to sacrifice in normal times to reduce the severity of a future crisis.

The efficiency and stability analyses rely on the parameters estimated to match four empirical moments of the Fed fund market. These moments capture the size of the daily network of

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2In section 5.2.1 I provide intuition for the non-monotonicity result using an analytical solution for interbank exposures in an architecture with six banks.
trades, its density, and the maximum number of lenders and borrowers from a single bank. The estimated model generates a daily network of trades with the low density and a small number of very interconnected banks. These characteristics observed not only in the Fed funds market, but also in many other OTC markets (Boss et al. 2004, Chang et al. 2008, Craig and Peter 2009). The model also generates a high persistence of trades, another robust feature of OTC markets (Afonso et al. 2013, Gabrieli and Georg 2014, Li and Schürhoff 2014). The persistence of trades is the highest between the most central banks in the architecture. Core banks are likely to borrow repeatedly from the same periphery banks, but they lend to different banks depending on who has the highest need for liquidity on a given day. More interconnected banks also intermediate a larger volume of trades. Consistent with the data, the model generates negative degree correlation, meaning that large interconnected banks are more likely to trade with small periphery banks. Overall, the model is able to match several important characteristics of the Fed funds market, even if they were not targeted in the estimation.

The paper is related to the theoretical and empirical studies of OTC markets. The theoretical modeling of OTC markets can be broadly divided into search-based and network-based models. The search-based approach pioneered by Duffie et al. (2005) has been used to study liquidity (Duffie et al. 2007, Vayanos and Weill 2008, Weill 2008, Feldhütter 2012, Praz 2014), and trading dynamics in the Fed funds market (Afonso and Lagos 2015). The original framework has also been extended to capture heterogeneous search intensities (Neklyudov 2013) and heterogeneous private values (Hugonnier et al. 2014, Shen et al. 2015). These extensions generate heterogeneity in the number of counterparties across traders and provide insights into which traders are more likely to become intermediaries. In these models, trading relationships are typically created at random, but the actual network of trading links is endogenous.

Although search-based models have been successful in contributing to our understanding of OTC markets, they cannot generate the persistent trading patterns observed in the data. When each trader searches randomly for counterparties, the probability of repeated trades is very low. It happens because this literature has focused on the search for spot trades, not for long-term relationships. In contrast, the network-based model used in this paper is designed to capture the presence of long-term trading relationships in OTC markets. A repeated pattern of trades is more likely to emerge in equilibrium when each trader has a limited number of trading partners. When banks trade persistently with a limited number of trading partners, it can increase the risk of contagion because some banks have some exposure to their counterparties. In general, the search literature does not focus on studying financial stability. A notable exception is Atkeson et al.

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3Density measures the percentage of links between banks that are observed in the data out of the maximum possible number of links. In the data, the density is only 0.7%, meaning that the network is very sparse, with a small average number of counterparties per bank.

4Several recent models combine elements of both approaches (e.g., Atkeson et al. 2015, Colliard and Demange 2014).
(2015), who address the issue of an endogenous exit after negative shocks.

Network-based models of the OTC markets have been used to understand the relationship between trading efficiency and market structure (Gale and Kariv 2007, Blume et al. 2009, Gofman 2011, Condorelli and Galeotti 2012), informational frictions (Babus and Kondor 2013, Glode and Opp 2014), and how networks form (Babus 2012, Farboodi 2014, Fainmesser 2014, Chang and Zhang 2015). Networks have also proved to be a useful analytical tool for studying financial contagion from a theoretical perspective (Allen and Gale 2000, Leitner 2005, Elliott et al. 2014, Cabrales et al. 2014, Acemoglu et al. 2015, Glasserman and Young 2015).

This paper is most closely related to empirical studies of contagion (Furfine 2003, Upper and Worms 2004, Gai and Kapadia 2010). The contribution of this paper is that it uses a theoretical model to compute exposures between banks. These exposures are rarely observable in the current architecture and are unobservable in counterfactual architectures. Although simulation-based approaches to study contagion risk help to compute the number of failures in a cascade, a model is needed to quantify the welfare implication of these failures.

This paper is among the first to structurally estimate a model of an OTC market. Blasques et al. (2015) employ an indirect inference approach to estimate a network formation model. Their paper relies on a Dutch interbank market, and its focus is on banks’ monitoring decisions and the monitory policy’s effect on interbank trading. Denbee et al. (2014) use a quasi-maximum likelihood approach to estimate a model of liquidity holding by banks in an interbank network. Their paper aims to identify which banks are most important for aggregate liquidity and for systemic risk. Stanton et al. (2014) use mortgage-origination and securitization network data to estimate a theoretical model of network formation to study contagion in the U.S. mortgage supply chain.

The structure of the paper is as follows. The next section presents a network-based model of the Fed funds market. In Section 3 I use a simulated method of moments to estimate the model. The analysis of the efficiency and stability of the estimated financial architectures appears in Section 4. Section 5 compares the estimated financial architecture to counterfactual financial architectures without too-interconnected-to-fail banks in terms of efficiency and stability. In Section 6 I summarize the main policy implications that arise from my analysis, while Section 7 discusses the limitations of my analysis and promising directions for future research. Section 8 presents my conclusions.


See Upper (2011) for a survey of this literature.

Early papers usually approximate the network of exposures using banks’ balance-sheet information. See Upper and Worms (2004) for German banks’ data and Wells (2004) for UK banks’ data. Regulators in the U.S. and Europe have only recently started to collect data that can reveal existing interbank exposures.
2 The Model

This section describes a network-based model of trading in an OTC market developed in Gofman (2014). In this section, the model is applied to the Fed funds market in which banks provide each other with short-term unsecured loans to satisfy reserve requirements. A single trade is a loan provided on one day and repaid with interest the next day. Trading in the Fed funds market is a mechanism that reallocates reserves from banks with excess reserves to those with shortages.

First, I describe how the model generates an endogenous network of trades for an exogenous given financial architecture. Then in a subsection 2.3 I describe a random network model that generates a financial architecture with large interconnected banks. The goal of the estimation in section 3 is to find parameters of the network formation model, such that trading in this architecture results in an endogenous network of trades with similar characteristics as the network of trades observed in the data.

There are \( n \) banks in the market, but not all of them trade every day. Banks belong to a financial architecture, which is unobservable. A financial architecture is represented by graph \( g \), which is a set of trading relationships between pairs of banks. If a trading relationship exists between bank \( i \) and bank \( j \), then \( \{i, j\} \in g \) (or \( ij \in g \)); otherwise, \( \{i, j\} \notin g \). Banks trade directly only if they have a trading relationship between them.

Some banks have excess liquidity and others need liquidity to satisfy their reserve requirements. A bank has excess liquidity when it receives a liquidity shock, such as a new deposit. If a bank lacks liquidity, it must pay a penalty on missing reserve requirements, or borrow at a higher rate from the discount window at the Federal Reserve and forgo profitable trading opportunities. Let vector \( E = \{E_1, ..., E_N\} \) describe the endowment of liquidity, so that \( E_i = 1 \) if bank \( i \) has excess liquidity. For simplicity, I assume that at any given time, one bank has one unit of excess liquidity. This assumption keeps the model both tractable and flexible enough to match empirical moments. After I characterize equilibrium trading for one endowment, I generalize the analysis to account for the multiple liquidity shocks that banks experience during a single trading day.

Each bank in the market has a private value for one unit of liquidity. The set of private values

\[ \text{The participants include commercial banks, savings and loan associations, credit unions, government-sponsored enterprises, branches of foreign banks, and others. For simplicity I will refer to all participants in the Fed funds market as “banks”.

I assume every bank can always use liquidity for its own needs (\( \{i, i\} \in g \) for all \( i \)), and that the trading network is undirected (if \( \{i, j\} \in g \), then \( \{j, i\} \in g \)).

The goal of the model is to capture the presence of trading relationships in the market, rather than to rationalize any particular reason for their presence. Consistent with further analysis, two banks may have a trading relationship if they extend each other a credit line to prior to the realization of shocks that determine the direction of trade, or if they know how to manage the counterparty risk better. The existence of persistent trading relationships between banks has been empirically documented in the United States (Afonso et al. 2013), Portugal (Cocco et al. 2009), Italy (Affinito 2012), and Germany (Bräuning and Fecht 2012).}
is captured by the vector \( V = \{ V_1, ..., V_N \} \in [0, 1]^n \), where \( V_i \in [0, 1] \) is the private value of bank \( i \). The interpretation of a bank’s private value is the highest gross interest rate a bank is willing to pay on a 24-hour loan without the possibility of reselling the loan. The bank with the greatest need for liquidity is willing to pay the highest interest rate and therefore has the highest private value. I normalize private values to be between 0 and 1. Heterogeneity in private values generates gains from trade in the market for liquidity. These private values change even over the course of a day. Later, I will generalize the model by introducing a distribution for shocks to private values, but prior to that I define equilibrium for a fixed set of private values.

**Definition (Equilibrium).** *Equilibrium trading decisions and valuations are defined as follows:

1. For all \( i \in N \), bank \( i \)'s equilibrium valuation is given by:

   \[
   P_i = \max \{ V_i, \delta \max_{j \in N(i, g)} V_i + B_i (P_j - V_i) \}.
   \]

2. For all \( i \in N \), bank \( i \)'s equilibrium trading decision is given by:

   \[
   \sigma_i = \arg \max_{\sigma_j \in N(i, g) \setminus \{i\}} P_j.
   \]

\( B_i \in (0, 1) \) is the share of surplus that bank \( i \) receives when provides a loan to another bank, \( N(i, g) \) is the set of direct trading partners of \( i \) in network \( g \), and \( \delta \) is the discount factor\footnote{In the empirical procedure, I assume that \( \delta \) is either 1 or \( 1 - 2^{-52} \), depending on the price-setting mechanism. The discount factor is assumed to be effectively 1 to reflect the fact that the time between intra-day trades is very short and to make sure that any welfare losses in trading are generated by the intermediation friction, not by discounting.}.

The endogenous valuation of bank \( i \), \( P_i \), is the maximum between bank \( i \)'s private value, \( V_i \), and a discounted continuation value from providing liquidity to one of the trading partners. In equilibrium, a lender never sells Fed funds for a price below his private value and a borrower never buys Fed funds at a price above his endogenous valuation. In equilibrium, bilateral prices and banks’ decisions to buy Fed funds, sell Fed funds, or act as intermediaries are jointly determined, although trading is sequential. Gofman (2014) showed that equilibrium valuations are unique because equation \( \text{(1)} \) is a contraction mapping.

Prices depend on the surplus from trade and on the split of this surplus. The surplus in trade between \( i \) and \( j \) is equal to the borrower’s endogenous valuation \( (P_j) \) minus the private valuation of the seller \( (V_i) \). If the surplus from trade between lender \( i \) and any of its potential borrowers is negative, then the lender does not lend the funds. In this case, the endogenous valuation of bank \( i \) is equal to its private value, \( P_i = V_i \). If lending to several borrowers generates positive surplus, who the equilibrium borrower will be depends on the share of surplus that \( i \) receives from trading
with each of these borrowers. When lender \( i \) trades with another bank, it receives a share of the surplus \( B_i \). \( B_i \) can either be fixed or depend on other parameters of the model.

To compute equilibrium prices and trading decisions, I start with an arbitrary vector of endogenous valuations and iterate the pricing equations until convergence. Then using equation (1) a new vector of endogenous valuations is computed. The same calculation is repeated, with the result of the previous calculation is used as an input for the new iteration. The solution is achieved when there is no change in the valuation vector between two subsequent iterations. After the vector of endogenous valuation is computed, I use equation (2) to compute each banks’ optimal trading decision. If a bank faces two counterparties with identical endogenous valuations, then I randomly choose one of them as a buyer. A detailed description of the process of computing the equilibrium is presented in section C of the appendix.

I proceed by describing four different price-setting mechanisms in the next subsection. Then in subsection 2.2 I allow for multiple endowment and private value shocks to generate an endogenous network of trades. I conclude this section by specifying a process for formation of trading relationships between banks.

### 2.1 Price-setting mechanisms

I consider four different specifications for \( B_i \) that I use in the estimation section of the paper. The first price-setting mechanism is a bilateral bargaining in which a borrower and a lender split the surplus equally. Formally, \( B(i) = 0.5 \) for all \( i \). The second mechanism assumes that a lender receives a higher share of the surplus when it has more borrowers. Formally, \( B(i) = 1 - \frac{0.5}{n(i,g)} \), where \( n(i,g) \) is the number of trading partners of bank \( i \) in network \( g \). In this case, when a bank has only one potential borrower, the surplus is divided equally, but as the number of borrowers increases, the share of the lender’s surplus converges to 100%. The third mechanism is when \( B_i = \frac{n(i,g)}{n(i,g)+n(j,g)} \). This mechanism assumes that the lender’s share of surplus depends not only on lender \( i \)’s number of trading partners, but also on the number of trading partners of borrower \( j \). It also ensures that bank \( i \) receives the same share of surplus when it trades with bank \( j \) regardless of whether \( i \) plays the role of lender or borrower in this transaction. The last price-setting mechanism resembles a second-price auction. According to this mechanism, a lender provides a loan to the bank with the highest endogenous valuation, and the price of the Fed funds traded is equal to the (discounted) second-highest endogenous valuation among the lender’s trading partners. This is the only price-setting mechanism of the four in which the share of the lender’s surplus is completely endogenous. If \( j \) has the highest endogenous valuation among \( i \)’s trading partners and \( k \) has the second highest valuation, then the share of \( i \)’s surplus when selling to \( j \) is \( B_i = \frac{P_k - V_i}{P_j - V_i} \). If we

\[ 12 \]The efficiency-stability analysis will be based on the price-setting mechanism that allows the model to fit the data.
substitute this share of surplus into equation (1), it simplifies to \( P_i = \max\{V_i, \delta P_k\} \). For this price-setting mechanism, \( \delta \) has to be smaller than 1 for contraction to work. For the estimation purposes, I use \( \delta = 1 - 2^{-52} \) for this mechanism and \( \delta = 1 \) for the other three mechanisms.

### 2.2 Endogenous Network of Trades

For a given set of private values, equilibrium trading decisions tell us what would be an equilibrium chain of intermediation that originates with a lender and ends with the final borrower. In the data we see multiple intermediation chains during the same day. To generate a network of trades, rather than a single intermediation chain, I assume that private values of banks change during the day, and that each bank has some endowment of liquidity. In particular, I assume that there are \( w \) iid draws of private values from a standard uniform distribution during a single trading day. This parameter needs to be estimated because intensity of shocks to private values is unobservable.

I further assume that the endowment of each bank is proportional to its interconnectedness. Formally, \( E_i = \frac{n(i,g)}{\sum_j n(j,g)} \cdot n \). The largest participants in the Fed fund market are big commercial banks, such as Bank of America and Wells Fargo. These banks have more deposits than small regional banks. Therefore, it is natural to assume that they have higher endowment. The aggregate endowment for each vector of private values is assumed to be \( n \). It means that banks’ private values are the same for up to \( n \) units of liquidity.

For each draw of private values and for each lender, the model generates a trading path with a volume equal to the endowment of the lender. Aggregating all trading paths across different lenders and for \( w \) vectors of private values generates an endogenous network of trades with heterogeneous volume of trade between any two banks.

Even if two banks have a positive trading volume after \( w \) shocks, it might not be sufficient for the link between them to be observable. If empirically only trades above some volume threshold are reported, then only links above this threshold will be observable. I introduce a parameter \( t \) to generate a truncated endogenous network of trades. This network consists only of links with volume above \( t \) units of liquidity. This parameter is estimated in section 3.

The topology of the equilibrium network of trades depends also on the underlying network of trading relationships, which is unobservable. The next section describes how the network of trading relationships is generated.

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13 For further analysis of this price-setting mechanism for a network setting, see Gofman (2014). Kotowski and Leister (2012) use this mechanism in a directed acyclic graph. Manea (2014) shows how a bilateral bargaining protocol converges to a second-price auction payoff when the number of potential buyers is sufficiently large.

14 The aggregate endowment can be set to any number because it is not affecting the network of trades, only the level of trading volume.
2.3 Process for Formation of Trading Relationships

To perform efficiency and stability analyses we need to know the network of trading relationships. It is different from the endogenous network of trades that was described in the previous section. The network of relationships describes the set of feasible trades in an OTC market. The network of trades includes the set of equilibrium trades. If a link between two banks is not part of the equilibrium network, it can be either because these two banks don’t have a trading relationship, or because it was not optimal to utilize this link. As a result, the network of trading relationships is not observable and it needs to be estimated. For this purpose, I specify a network formation process that is later used in the estimation.

The the network formation process is based on the preferential attachment model by Barabási and Albert (1999). This is a random network model that was designed to generate networks with a small number of very interconnected nodes and a large number of nodes with a small number of links. These are the features of OTC markets in general and the Fed funds market in particular.

The preferential attachment algorithm works as follows. I start with $s$ banks in the core of the financial architecture (e.g. JPMorgan Chase, Citibank, Bank of America, Wells Fargo) and assume that they are fully connected, meaning that each bank in the core can trade directly with any other bank in the core. Then I add more banks, one at a time. Each additional bank creates $s$ trading relationships with the existing banks. The process continues until all $n$ banks are added to the network. To generate a small number of very interconnected banks, new banks are assumed to be more likely to form a trading relationship with the most interconnected banks. In particular, if there are $k$ banks in the financial architecture and bank $k + 1$ needs to decide which banks it should connect to, the probability of an existing bank $i$ forming a trading relationship with bank $k + 1$ is $\frac{n(i)}{\sum_{j=1}^{k} n(j)}$, where $n(j)$ is the number of trading partners of bank $j$.

The network formation process is consistent with the assumption that more interconnected banks have higher endowment. On one side, banks are more likely to form relationships with banks that have more excess reserves because it ensures access to reserves when the need for funds is high. On the other side, it is unlikely that all banks would find it optimal to attach to a single bank, as they will need to compete for these funds. Even though the preferential attachment algorithm is a statistical model for generating networks with very interconnected banks, it could be interpreted as a reduced form of a game-theoretic model of network formation in which banks play a mixed strategy when they decide which banks to connect with.

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15 If there is a bank in the data that trades with hundreds of counterparties in equilibrium, the underlying network of trading relationships should have banks who can trade with at least as many counterparties.

16 I made two adjustments to the original algorithm by Barabási and Albert (1999): (1) I assume that all banks in the core are fully connected, and (2) I use the same parameter ($s$) to capture the number of banks initially in the core and the number of new trading relationships created by a new bank. A reduction of one parameters substantially reduces the computational needs for the estimation.
In the next section, I estimate the model using the Fed funds market characteristics.

3 Estimation

The goal of the estimation is to find three model parameters that match four empirical moments. These parameters are estimated using SMM. Next, I discuss the empirical moments used for the estimation, the estimation procedure and how these moments help to identify the parameters. I also present results of the estimation and derive empirical predictions from the estimated model.

3.1 Empirical Moments Used for Estimation

Usually, only regulators have access to data about interbank trades. Therefore for the estimation, I am restricted to using only those moments that have been reported in the literature. Specifically, my estimation relies on the results reported by Bech and Atalay (2010), who provide the most detailed description of the Fed funds network topology prior to the financial crisis. Although their paper covers a longer period, the estimation relies on network characteristics from 2006, for two reasons. First, 2006 is the only year with a detailed description of the Fed funds network topology in their paper, which is also the last year in their sample. Second, using data before the financial crisis and the consequent distortions of the market by the Federal Reserve’s policies allows me to estimate the model under normal market conditions and to perform an analysis from an ex-ante perspective.

Bech and Atalay (2010) report that during 2006, 986 banks traded in the market at least once. I take this number as the size of the financial architecture, so \( n = 986 \). For the estimation, I choose four empirical moments. Each moment is computed as an average of the network characteristics across 250 daily trading networks in 2006 (Table 5 in Bech and Atalay 2010). Section D in the appendix provides formulas to compute the moments. The empirical moments are: (1) the density of the network of trades, \( \alpha \), which is 0.7% in the data, (2) the maximum number of lenders to a single bank, \( k_{\text{max}}^{\text{in}} \), which is 127.6, (3) the maximum number of borrowers from a single bank, \( k_{\text{max}}^{\text{out}} \), which is 48.8, and (4) the average daily network size, \( \hat{n} \), measured at 470 banks\(^{17}\).

These moments are important because they capture the main characteristics of the observable network of trades. To study the efficiency and stability of a financial architecture with too-interconnected-to-fail banks, it is important to generate an architecture that has banks with many counterparties as manifested by moments 2 and 3. The density of the Fed funds market

\(^{17}\)The size of the daily network of trades can be smaller than the size of the financial architecture because the empirical network uses only loans above $1M. If all bilateral trades by a bank were below $1M, then it would appear in the data that this bank did not have any links during this day.
(moment 1) captures the sparsity of the network. Because of the low density, the average number of counterparties in the market is only 3.3. The first three moments together suggest that the market structure has a small number of large interconnected banks and a large number of small banks that trade with only a few counterparties. The fourth moment is important because it defines the size of the network for which other moments are computed. The density of 0.7% or the maximum in-degree of 127.6 have different implications if the network has 986 or 470 banks.

In the next section I describe the estimation procedure. Then in Section 3.3 I describe how these moments identify the parameters.

3.2 Simulated Method of Moments

I estimate the three model parameters in \( \Theta = [s \ w \ t] \) using SMM. The estimator is

\[
\hat{\Theta} \equiv \arg \min_{\Theta} (\hat{M} - \hat{m}(\Theta))^T W (\hat{M} - \hat{m}(\Theta)).
\]

(3)

\( \hat{M} \) is a vector of four empirical moments \( (\alpha, k_{\text{max}}^{\text{in}}, k_{\text{max}}^{\text{out}}, \hat{n}) \), and \( \hat{m}(\Theta) \) is a vector of moments generated by the model. \( W \) is a weighting matrix with the inverse variances of the empirical moments along the diagonal. This procedure minimizes the square of the difference between the model generated moments and the empirical moments, weighted by the variance of the empirical moment. This weighting assures that moments 1 and 4, which are less noisy, receive higher weight than moments 2 and 3, which have higher variation due to being maximum values of the distribution. To simulate model-generated models I follow the following procedure. For each value of the parameters, I compute an endogenous network of trades defined in Section 2.2. This is repeated 250 times, as the number of days in the empirical sample. Then for each daily network I compute the four network moments targeted in the estimation. The average of these moment estimates over the 250 days produces model-generated moments that correspond to the empirical moments. The optimal parameter estimates from the second stage are used in the efficiency and stability analyses. Section E in the appendix provides details for finding the optimal parameters.

3.3 Identification

In this section, I discuss which moments are most affected by which parameters. It helps to understand how the unique set of optimal parameters is selected in the estimation.

Both the total number of trading relationships and the maximum number of trading relationship-
ships for a single bank increase with \( s \). The first moment indicates that the density of the network of trades is very low, which is easier to match when the density of the financial architecture is also low. However, the second moment indicates that the maximum number of lenders to one bank is very high, which is easier to match when the financial architecture has banks with a high degree of interconnectedness. This tradeoff helps to identify the optimal value of \( s \). The third moment is also affected by \( s \), but to a smaller extend. A small value of \( s \) can be sufficient for banks to be able to lend to 48.8 counterparties (moment 3), but \( s \) needs to be substantially higher for banks to be able to borrow from 127.6 counterparties (moment 2).

The choice of \( w \) also involves a tradeoff. If we draw more vectors of private values, the third moment is easier to match. However, a higher \( w \) makes it harder to match the fourth moment. Holding \( t \) constant, as \( w \) increases, the number of banks in the network of trades increases until it reaches 986. The second moment is less important because a bank can have many lenders for a given vector of private values, but only one borrower. Therefore, \( w \) needs to be at least 48 for the model to be able to match the third moment.

The main role of the third parameter, \( t \), is to match the fourth moment. If \( t = 0 \), the number of banks in the network of trades is 986, which is two times larger than what the fourth moment indicates. Holding other parameters constant, as \( t \) increases, the size of the network of trades decreases. So the main moment that identifies \( t \) is the average number of banks in the network of trades.

### 3.4 Estimation Results

The estimation procedure described in the previous section results in a choice of three parameters \((s, w, \text{ and } t)\) and a price-setting model. To achieve the best fit of the data, the following parameters were chosen by the SMM procedure: \( s = 12 \), \( w = 78 \), \( t = 22 \). I also estimate the model for each of the 100 networks separately and compute the standard errors of the estimates. The mean and standard errors of the estimated parameters over the 100 networks are: \( \bar{s} = 12.4 \) (\( \sigma_s = 0.09 \)), \( \bar{w} = 75.7 \) (\( \sigma_w = 0.41 \)), and \( \bar{t} = 20.9 \) (\( \sigma_t = 0.16 \)). The standard errors are small, suggesting that the parameters are estimated with high precision. Only the first two parameters are used for the efficiency and stability analyses; the third one is only needed for the estimation of these two parameters.

The estimated financial architecture is two times larger and 3.5 times denser than the daily trading network. It has a small number of very interconnected banks that can have more than 200 trading partners. Figure 2 shows the distribution of the number of trading partners in a financial architecture generated using the estimated parameter. For comparison, the figure also shows the distribution of the number of trading partners for architectures without too-interconnected-to-fail
Out of the four possible price-setting mechanisms, specification 4 provided the best fit of the data. This mechanism resembles the second-price auction and generates an endogenous surplus sharing rule. This price-setting mechanism fits the data because it helps generate a high number of lenders to a single bank (moment 2). This moment emphasizes the role of large interconnected banks in the market. The other three price-setting mechanisms are not able to generate high enough number of lenders to a single bank. The first and the third mechanisms assume that banks in the core of the financial architecture extract a substantial share of the surplus when borrow from small periphery lenders, but in this case, periphery banks do not have incentives to lend to the core banks, reducing the maximum number of lenders to a single bank. The second mechanism also provides a higher share of surplus to a lender with more trading partners, but it does not account for the continuation values of these trading partners, only for their number. A more general insight from the estimation is that even if a bank has 234 trading partners to borrow from, not all of them will lend to it in equilibrium because they have tens or hundreds of alternative borrowers. To fit the data, a price-setting mechanism needs to provide incentives to the periphery banks to trade with the core banks in equilibrium.

Table 1 shows the comparison between the empirical moments and the simulated moments for the estimated parameters. In addition, it reports standard deviations of the simulated moments, which were not used in the estimation. There are 100 networks that are generated with the optimal s. The second column reports simulated moments for the network with the best fit. Column four shows the average fit when simulated moments are averaged not only across the 250 trading days, but also across the 100 networks. These moments were targeted in the estimation. Columns three and five report the 5th and the 95th percentiles of the simulated moments across the 100 networks. The last column reports t-statistics for testing a hypothesis that the empirical moment and the average simulated moment reported in the fourth column are equal.

The second and third moments are measured with more noise and consequently have less weight in the estimation. Not surprisingly, the fit of these two moments is not as good as the fit for the first and the fourth moments. The second moment is particularly difficult to fit, as is evident from the t-statistic. The first moment is very precisely measured in the data, but the model can fit this moment especially well. Moments 3 and 4 don’t have as good statistical fit as the first moment, but the percentage deviation of these two moments from the empirical moments is only 0.8% and 3.3% respectively. From the economic perspective, these small differences in the moments should not have any effect on the efficiency-stability analysis, especially because it is based on the financial architecture and not on the daily network of trades. All four moments are within the 5th and 95th percentiles of the simulated moment distribution across simulated networks. The standard deviations produced by the model are similar to the standard deviations.
in the data. The maximum number of borrowers has a substantially smaller standard deviation in
the model than in the data, while the density has a slightly higher standard deviation than in the
data. Overall, the model is able to generate a daily network of trades that has a core-periphery
equilibrium market structure, which is very similar to the one observed in the data.

In Table 2, twelve additional simulated moments are compared with the corresponding empir-
ical moments. These moments are not as important as the four used in the estimation, but they
are useful for understanding what additional network characteristics the model can and cannot
match. These additional moments are taken from Table 5 in Bech and Atalay (2010). Formal
definitions of these network measures appear in Section D of the appendix.

The model is able to generate a similar number of observable trading relationships and to
perfectly match the average number of counterparties. These two moments are not completely
independent from the first and the fourth moment used in the estimation, so it is not surprising
that the match is so good given that the model matches these two targeted moments quite well.
The remaining 10 moments are not functions of the four moments that were targeted. The next
five moments capture the amount of intermediation in the market. The model generates slightly
longer average distances between banks relative to the empirical distances, but it generates a
shorter maximum distance (diameter). Overall, the length of intermediation chains produced by
the model is very close to the amount of intermediation in the data. The next two moments
measure two clustering coefficients. The first one computes the probability that two lenders to a
bank also trade with each other. The model matches this moment perfectly. The second clustering
coefficient measures the probability that any two borrowers of a bank also trade with each other.
The empirical clustering coefficient is substantially higher than the one generated by the model.
The next moment measures the average reciprocity of the equilibrium network of trades, which
is the probability that we observe trades in both directions between a pair of banks during the
course of a single day. The model generates higher reciprocity than observed in the data. That can
suggest that some trading relationships in the data might be directional, limiting the possibility
of a trade happening in the opposite direction. However, the reciprocity weighted by volume is
43%, according to Bech and Atalay (2010). It suggests that most of the volume is traded using
undirected trading relationships. The last two moments measure the degree correlation between
banks. If the correlation is positive, it means that banks with a similar number of connections are
more likely to trade with each other. If it is negative, it indicates that dissimilar banks are more
likely to trade. For the set of observable trades, I compute the first measure as the correlation
between the number of banks that borrow from the lending bank and the number of lenders to
the borrowing bank. This correlation is -0.28 in the data and -0.35 in the model. This is a good
fit given that this moment was not targeted in the estimation. The second measure is computed
similarly, but the correlation is between the number of lenders to the lending bank and the number
of lenders to the borrowing banks. This correlation is -0.13 in the data and -0.26 in the model.
The quantitative fit of this moment is not as good as it is for the first degree correlation between borrowers and lenders. However, the model is able to generate an equilibrium network structure in which more interconnected banks are more likely to trade with less interconnected banks. Overall, the model provides a good fit for nine out of the twelve untargeted moments.

3.5 Empirical Predictions

In this section I provide two sets of empirical predictions that are derived from the estimated model. The first set of predictions is related to the persistence of trades, while the second set is related to the trading volume and degree of intermediation in the market. Some of this predictions can be verified based on the existing empirical evidence.

The first prediction is about the persistence of trades. Table 4 reports the persistence of the equilibrium network of trade for the estimated parameters. Persistence is a measure of how likely banks are to trade repeatedly with the same counterparties. The model predicts that if we observe a trade between two banks on one day, there is a 50% probability we will also observe a trade between these banks the next day. If we condition on the direction of the trade, the persistence is still very high. If we observe a loan from bank $i$ to bank $j$ on day $t$, there is a more than 40% probability of observing a loan from bank $i$ to bank $j$ at day $t + 1$. Not surprisingly, the persistence is even higher at weekly and monthly frequencies. In the network of trading relationships, banks are limited to trade with their trading partners making it very likely to observe persistent trades. A number of empirical studies have documented a strong persistence of trades in OTC markets in general (Li and Schürhoff 2014) and in interbank loan markets in particular (Afonso et al. 2013, Gabrieli and Georg 2014).

The second prediction is about the persistence of trades across banks at different positions in the financial architecture. Table 5 reports persistence across four groups of banks, sorted based on their betweenness centrality. Banks with a high betweenness centrality belong to many shortest paths between other banks in the financial architecture. The model predicts that banks with a high centrality position intermediate many trades. The daily persistence is almost 77% between the ten most central banks and only 1.7% between the least central banks in the fourth category. Moreover, the model also predicts that there is higher persistence in trades between lenders and the most central banks then in trades between borrowers and the most central banks.

The third empirical prediction is about the trading volume. The model predicts that a small number of banks trades disproportionately large volume of funds. Table 3 reports how the volume of trade is distributed across four groups of banks. The first group is composed of 10 banks with the largest total volume traded per day, averaged over one year. The second group includes banks ranked from 11 to 50. The third group consists of banks ranked from 51 to 100. The last
group includes all of the remaining banks with observable trades. Each cell in the table shows what percent of the average daily trading volume traded is traded between and within these four groups. We can see that although the top 10 banks constitute approximately 1% of all banks in the financial architecture, they borrow 22% and lend 20% of the total daily trading volume. For comparison, the lending and borrowing activities of banks ranked 51-100 in terms of trading volume account, respectively, for 10% and 9% of total volume. The first three groups with 100 banks trade more volume than the 886 remaining banks in the fourth group. Qualitatively, the first prediction of the model is confirmed by the empirical studies of the Fed funds market. Afonso and Lagos (2012) find that “trading activity across banks is very skewed: a few banks trade most loans, while most banks participate in few trades.”

The fourth prediction is related to the intermediation volume. Based on a simulated sample of 250 trading days, the model suggests that 56% of the total volume of trade is intermediated. Afonso and Lagos (2012) in Figure 10 report that in 2006 the proportion of intermediated funds in the Fed funds market was between 30% and 60%, with an average of 40%. The 56% estimate produced by the model is above the mean, but within the range of the daily estimates. The empirical estimates are computed based on trading in the last 2.5 hours of the trading session, which would be lower than the the full trading session estimate if intermediation activity is decreasing towards the end of the trading session.

The last prediction is about the degree of interconnectedness and trading. Figure 1 compare the total lending volume and the intermediation share of this volume across banks with different degrees of interconnectedness. More interconnected banks lend more and intermediate more than less interconnected banks. However, even stronger prediction is that the lending volume is a convex function of the number of counterparties, while the intermediation ratio is a concave function of the number of counterparties. If banks would only lend out their endowment, the relationship between the lending volume and interconnectedness would be linear. A convex relationship exist because more interconnected banks are also strong intermediaries, what attracts small periphery banks to lend them money. To understand the concavity of the intermediation ratio, we can define the intermediation ratio as one minus the volume borrowed and kept divided by the total lending volume. If each bank is equally likely to keep the borrowed funds, then the convex lending volume leads to the concave intermediation ratio prediction.

4 Efficiency and Stability Analyses

The first part of this section defines measures of the allocational efficiency of an OTC market. These measures are applied to quantify the efficiency of the estimated financial architecture with too-interconnected-to-fail banks. The second part of this section presents measures of financial
stability and applies them to the estimated architecture. These efficiency and stability measures are later used in the counterfactual analysis of financial architectures without too-interconnected-to-fail banks.

4.1 Efficiency Analysis

The goal of the interbank market is to allocate liquidity. The following example illustrates why equilibrium allocation can be inefficient and why the amount of intermediation and the bargaining power of the intermediaries matter for efficiency. The following example is taken from Gofman (2014). Imagine a simple financial architecture with seven banks. Bank A can provide a loan to banks B and C, bank B can provide a loan to banks B1 and B2. Bank C can provide a loan to banks C1 and C2. Assume $V_A = V_B = V_C = 0$, $V_{B1} = 1$, $V_{B2} = 0.5$, $V_{C1} = 0.9$ and $V_{C2} = 0.7$. Bank A has one unit of liquidity. Bank B1 has the highest private value for liquidity and is the most efficient borrower. Assume the discount factor is 1 and banks sell funds to the banks with the highest endogenous valuation for the price of the second-highest endogenous valuation. The endogenous valuation of bank B is 0.5 because this is the second-highest valuation among potential borrowers of bank B. Bank C’s endogenous valuation is 0.7, which is the endogenous valuation of bank C2. If bank A faces two borrowers with endogenous valuations 0.5 and 0.7, bank A will provide a loan to bank C and the cost of funds will be 0.5. Bank C will provide a loan to bank C1, and the cost of funds will be 0.7. The final allocation is inefficient because bank B1 does not receive the loan. The friction is present because bank B cannot extract enough surplus from bank B1. The share of the surplus received by B is determined by the endogenous valuation of bank B2. Bank C lends to a less efficient borrower but this intermediary can extract a higher share of the surplus, given that the valuation of C2 is higher than the valuation of B2. When each intermediary cannot extract the full surplus in each trade, a leakage of surplus happens each time an intermediary provides a loan, because the interest rate is always below the equilibrium valuation of the borrower. That is why in OTC markets the probability that the equilibrium allocation is inefficient can be substantial. That is also the reason why the financial regulation that changes a financial architecture affects trading efficiency.

4.2 Efficiency Measures

The challenge is to quantify the degree of inefficiency and to rank different financial architectures in terms of their efficiency. For a given realization of shocks, if the ultimate borrower in the chain of intermediated trades does not have the highest private value, then the equilibrium is inefficient. The role of a financial architecture is to allocate liquidity or risks in the economy for different

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19 This can be achieved by a sequence of second-price auctions.
realizations of shocks. Therefore, to rank architectures in terms of trading efficiency we need to define ex-ante measures that would measure expected surplus loss from the intermediation friction. The ex-ante measures allow us to answer the question of how efficient a particular financial architecture is before we observe the exact realization of shocks.

The main ex-ante measure of trading (in)efficiency used in this paper is expected surplus loss (ESL). This measure takes into account both the probability of inefficient allocation and how large the surplus loss is. Surplus loss is defined as: $SL = \frac{\max(V_i) - V_{\text{final buyer}}}{\max(V_i) - V_{\text{initial seller}}}$. For any initial allocation, the maximum surplus that can be created is the difference between the highest private value in the market and the private value of the initial seller. This maximum surplus appears in the denominator. Whenever the equilibrium allocation is inefficient, trading creates less surplus than the maximum possible surplus. This welfare loss appears in the numerator. Therefore, the surplus loss formula measures what percentage of the potential surplus is lost. The ESL measure computes the expected surplus loss from the ex-ante perspective by averaging the surplus loss for different endowment shocks, valuation shocks, and realizations of the network formation process. I also compute the probability of an inefficient allocation (PIA) as an additional measure of inefficiency. PIA measures the ex-ante probability that an equilibrium allocation is inefficient, but it does not account for the loss of surplus.

Table A1 in the appendix presents the steps to compute the efficiency measures. This calculation is a numerical integration to compute expectations for surplus loss by averaging surplus losses over endowment shocks, private value shocks and network draws. In the numerical procedure I draw 1000 networks using the estimated network formation parameter $s$ to ensure that results are not driven by some outlier realization of the network formation process.

### 4.3 Efficiency Results

The efficiency results for the estimated architecture are reported in Table 6. The ESL is only 56 basis points, even though the probability that the equilibrium allocation is inefficient is 66%. This result suggests that even when the final allocation of liquidity is inefficient, the difference between the private value of the final borrower and the highest private value is not large. Overall, the price-setting mechanism is very efficient because it allows intermediaries to extract a high share of the surplus, thanks to the competition for funds between trading partners of each lender. The presence of large interconnected institutions improves efficiency because such institutions have connections with many potential borrowers and reduce the lengths of intermediation chains. The fact that endowment is perfectly correlated with interconnectedness also helps to achieve high market efficiency, as it puts large amounts of liquidity in the center of the network, making the

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20 The surplus loss is zero when the initial allocation is first-best.
21 See the formal definition of these two measures in Gofman (2014).
distance to the final borrowers shorter relative to the counterfactual case in which small periphery banks would have most of the endowment.

To convert the ESL estimate into dollar terms, one first needs to determine the total dollar value of the surplus that is created each day in the market and then multiply it by 0.56%. Because of the size of the Fed funds market and OTC markets more generally, even a small surplus loss could be meaningful in dollar terms. However, in percentage terms we should not expect a very high ESL. A too-high ESL would mean that mergers between some banks could allow them to profit from the inefficiency because mergers reduce the length of intermediation chains.

In addition to the ESL and PIA, the table also reports daily trading volume measured at 173,333. Without intermediation, the volume of trade would be approximately 2.25 times smaller (78 draws of private values, 986 units of liquidity for each draw). The trading volume serves as a benchmark for the amount of trading in normal periods. I will later study how the trading volume changes after the failure of the most interconnected bank.

How trading efficiency is related to the structure of the financial network is an important policy question. The answer to this question appears in Section 5, but first I compute the stability of the estimated architecture with too-interconnected-to-fail banks.

4.4 Stability Measures

The analysis of contagion requires us to address three questions: What triggers contagion? How does it spread from one bank to another? What are the measures of contagion outcome? The contagion analysis in this paper focuses on contagion triggered by the failure of the most interconnected bank. This is interpreted as a stress-test scenario that attempts to understand the cost of having large interconnected banks in the financial architecture. During the recent financial crisis the risk of contagion from the failure of large interconnected banks was one of the major arguments for a bailout.

The spread of contagion from one bank to another depends on the bilateral exposures between them. Usually this network of exposures is exogenous or reconstructed using balance sheet information. In this paper the network of exposures is generated endogenously by solving for

\[ \text{The average daily volume of trade in the Fed funds market in 2006 was $338 billion. The same friction is likely to be present in other OTC markets because they also require intermediation.} \]

\[ \text{If two or more banks are tied in having the most counterparties, one of them is chosen randomly.} \]

\[ \text{See Upper (2011) for an excellent survey of 15 studies of contagion in different countries. Twelve such studies use the exogenous failure of a single bank as a trigger for contagion. Eight papers use the maximum entropy method to compute bilateral exposures. This statistical method overstates the density of the network relative to the empirical density, and it does not take into account trading relationships between banks. Thirteen papers use the sequential contagion approach, which is similar to that used in this paper. All the papers focus on computing the number of bank failures, and none compute the welfare cost of contagion.} \]
equilibrium trading paths. The result of this computation is a volume of trade matrix $W$ with element $w_{ij}$ representing the amount of loans that bank $i$ provides to bank $j$ during a single trading day. If we normalize each row of $W$ to add up to 1 by dividing each element by the sum of the row, then we get a matrix of exposures $X$. Element $x_{ij}$ in this matrix represents the share of loans that $j$ owes to $i$ out of all loans $i$ provided.

I study two contagion scenarios. In the first, a bank fails in contagion when it has exposure above some threshold to a failed counterparty. This scenario assumes that banks have access to interim liquidity. In the second scenario, a bank fails in contagion when it has exposure above some threshold to all failed counterparties. This is when banks do not have access to interim liquidity. The default threshold depends on the ratio of a bank’s capital invested in liquid assets relative to the size of loans provided to other banks. Section F in the appendix provides formulas to compute contagion dynamics under the two scenarios.

There is an important difference between the two contagion scenarios. The difference emerges because not all banks fail simultaneously in financial contagion. If a bank suffers losses due to the failure of one of its counterparties, it will try to prepare itself for further defaults by raising equity, selling assets, borrowing at the discount window, etc. Whether banks can accomplish it depends on the market conditions and on the availability of funding from the discount window. The first scenario assumes that banks have access to the discount window or other sources of liquidity during the interim period of contagion. In this case, banks default only if a single loss is above their current liquidity buffer. The second scenario assumes that banks cannot do anything as the contagion unravels. Their liquidity pre-crisis is the only resource they can count on to absorb losses during the cascade. The second scenario better captures a severe financial crisis in which markets for new equity are frozen, there are large fire sale discounts on asset sales, and banks lack collateral of sufficient quality to allow them to borrow at the discount window.

Three parameter values for the default threshold are analyzed: 15%, 20%, and 25%. The threshold represents the amount of liquidity available relative to the aggregate or single exposure of a bank to its counterparties. This threshold can be a result of a regulatory requirement that sets the same threshold for all banks. If banks hold only as much liquidity as required by the

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25 Failures happen based on the gross exposure between banks. One reason is that interbank loans are unsecured and should have lower seniority in case of default. Another reason for using gross exposures is that Fed funds transactions represent bought and sold reserves and are not necessarily treated as loans that can be netted out. According to Upper (2011), ten out of fifteen studies of contagion in the interbank markets (including the Fed funds market) assumed there is no netting.

26 Given the 24-hour maturity of the Fed funds loans used in the estimation, I assume that even if there is some positive recovery rate on the defaulted loans, any recovered funds would be collected after a substantial delay and cannot be used to absorb other losses.

27 It can be also interpreted as a capital requirement. However, not only the amount of capital held against the outstanding loans determines whether a bank fails, but also whether this capital is held in liquid assets that can be used to repay overnight loans when a bank faces losses on the loans it provided.
regulation, then using the same threshold is particularly appealing.

Next, I introduce measures that capture the severity of the contagion risk. The first measure is the number of banks that fail in contagion. This is a standard measure in the literature. The second measure is more novel and requires a trading model. This measure attempts to assess the consequences of bank failures on welfare. We can expect efficiency to decline after contagion for three reasons. First, the length of intermediation paths can increase when some banks fail. Longer intermediation chains can result in higher trading inefficiency. Second, banking relationships with firms and retail investors are destroyed when banks fail. To capture this type of welfare loss, I assume that the distribution of private value shocks is the same as it was before contagion, but if a failed bank receives the highest private value, the surplus loss will be measured relative to this private value. A high private value could represent an investment opportunity, such as a loan to a business, but such a loan cannot be provided given that the bank that could generate this private value failed in contagion. The third reason for increased inefficiency is that depositors of the failed banks withdraw liquidity from the banking system, reducing the total endowment of liquidity. So instead of depositing money in one of the surviving banks, they put the money "under the mattress". The welfare implication is that a surplus that could be created by this withdrawn liquidity endowment is lost. The main post-crisis measure assumes that depositors are not withdrawn from the banking system by adjusting the aggregate level of endowment post-crisis to the pre-crisis level. However, I also compute the ESL2 post-crisis measure that assumes that endowment of the failed banks is not reallocated to the surviving banks. In this case, there is a 100% surplus loss associated with the endowment of the failed banks. The ESL2 measure assumes that private values are drawn only for surviving banks, meaning that the second reason for welfare reduction is not part of this measure. To better understand the sources of the reduction in trading efficiency, I also compute the probability that all banks fail in a cascade. In this case, 100% of surplus is lost.

After contagion, banks trade optimally on the remaining financial architecture. The endogenous adjustment of the trading paths is an important mechanism that mitigates the severity of the crisis. To measure the benefit of the endogenous adjustment of the trading network, I compute a counterfactual measure of post-crisis inefficiency. This measure, ESL3, computes what the expected surplus loss would be if banks were to continue trading along the same trading paths used prior to the contagion. If all intermediaries on the pre-crisis trading path survived, then the surplus loss would be unchanged. However, if one of the banks along the pre-crisis trading path failed, then the last surviving intermediary keeps the liquidity. For example, assume that in the pre-crisis architecture bank A sells Fed funds to bank B, bank B resells them to bank C, and bank C provides a loan to bank D. If, during contagion, bank C fails, then without adjustment to trading, bank B would be the final surviving borrower in this pre-crisis chain. The surplus loss will be the difference between the highest private value and the private value of bank B divided by the
difference between the highest private value and the private value of bank A. Trading around the "holes" in the financial architecture created by bank failures, might make it possible to allocation liquidity from bank A to bank D, but this measure deliberately forces banks to continue to trade as before. The difference between this measure and the post-crisis ESL measure represents the benefit of an endogenous readjustment of trading paths post contagion.

Table A2 in the appendix summarizes the procedure to compute stability measures. In the next section, I report the stability results for the estimated architecture.

4.5 Stability Results

The results for the contagion risk analysis are reported in Panel B of Table 6. The number of surviving banks increases monotonically as the default threshold increases. When the threshold is 15%, contagion without interim liquidity almost always destroys the entire financial architecture. If banks hold liquid capital equal to 25% of their interbank loans, slightly more than 60% of banks survive the crisis. There is a 33% probability that the entire financial architecture is destroyed. This statistic is very important because when all banks fail, the welfare losses are 100% as no trading surplus can be created. That is why the post-crisis ESL measures are particularly high when banks cannot get access to liquidity during contagion. If all banks fail in the architecture with high probability, all measures of trading inefficiency produce similar results. When the threshold is 25%, more than 60% of banks survive the contagion, even if they don’t have access to interim liquidity. A third of the trading surplus is lost in this scenario, but without endogenous trading readjustments the expected surplus loss would be more than 50%. If depositors of the failed banks would withdraw liquidity from the banking system then the expected surplus loss would be 39%.

Access to liquidity during contagion reduces the contagion risk substantially, but it can be still high. With a 15% threshold, more than 25% of banks fail. Although we did not see this high a percentage of bank failures during the recent financial crisis, perhaps because of the bailout policy, it is not an unrealistic level of bank failure compared to the Great Depression. Bernanke (1983) reports that 50% of banks failed between 1929 and 1933, albeit for different reasons. In this scenario, most of the banks failing are small. This was also true during the Great Depression.

An important implication of the stability analysis is that the number of bank failures is not a good measure of post-crisis efficiency. This can be seen by comparing the ESL and the number of bank failures in the case with a default threshold of 25% but no interim liquidity, and a case with interim liquidity and a default threshold of 15%. The ESL is 32 times higher in the first case than in the second case, while the number of bank failures is (only) 45% higher. The ESL increase is so much higher than the the increase in the defaults is because banks redirect trades
endogenously in the surviving network. In the second case, most of the failed banks are small banks, which does not disrupt intermediation services provided by large interconnected banks in the post-crisis architecture. Small periphery banks are more likely to have exposures to a single counterparty above 15% than are large core banks with hundreds of counterparties. So contagion that originates from failure of the most interconnected bank is likely to spread to periphery banks, but it is unlikely that it will affect other core banks, as they are well diversified. In the case without interim liquidity, many large interconnected banks fail, causing a sharp increase in the ESL. These banks fail when many small banks fail because each bank represents a small share of the core bank’s loans portfolio, but the aggregate loss goes above the threshold. I conclude that what matters for efficiency is not only the number of bank failures, but also the type of the failed banks.

On the other side, the probability of a complete failure of a financial architecture provides a good estimate for post-crisis efficiency. This is particularly relevant to cases when banks don’t have access to interim liquidity. The probability of complete failure is highly correlated with the post-crisis ESL because when all banks fail, all potential trading surplus is lost.

A complete failure of a financial architecture is unlikely when banks have access to interim liquidity. The main welfare loss in this case comes from the possibility that the depositors of failed banks will withdraw liquidity from the banking system and not deposit their funds with the surviving banks. If it happens, the expected surplus loss would increase to 7.27% even if the threshold is 25%. If the default threshold is 15%, the ESL after the withdrawal of deposits would be as high as 25%.

The ESL3 measure reported in the last column of Table 6 highlights the importance of endogenous trading. It is especially pronounced in the case with interim liquidity. If banks would continue to trade along pre-crisis trading paths, the expected surplus loss would be as high as 81% when the default threshold is 15%. That is 80% more than the benchmark case with an endogenous choice of trading paths.

The third column of Table 6 reports the volume of trade in the market after a crisis. The trading volume is smaller after contagion, but this is not a mechanical result driven by a smaller number of endowment shocks, because I keep the aggregate post-crisis endowment at the pre-crisis level. The volume drop occurs for two reasons. First, although the pre-crisis network has one component, making all final allocations feasible, the post-crisis network can have several components. The infeasibility of trades between network components increases surplus losses and triggers a decline in trading volume. The second reason for the decline in trading volume is related to intermediation friction. Post-crisis trading endogenously reroutes trades around failed banks and toward the surviving part of the network. New intermediation chains required for efficient allocation become longer and are less likely to be part of the equilibrium trading network because
of the intermediation friction. When the default threshold is 20% or 25% and banks have access to interim liquidity, there is no substantial drop in the volume of trade. However, if banks don’t have access to interim liquidity, even with a default threshold of 25%, the drop in trading volume is almost 40%. This result again emphasizes the significance of the interim liquidity for mitigating severity of a financial contagion.

So far the analysis has been only applied to the estimated architecture with too-interconnected-to-fail banks. I next examine study how efficiency and stability change as the network structure changes.

5 The Effect of Financial Architecture on Efficiency and Stability

To understand how efficiency and stability depend on the presence of too-interconnected-to-fail institutions, we need to study counterfactual financial architectures in which the maximum number of counterparties to a single bank is smaller. One approach to reducing banks’ interconnectedness is to study financial architectures of the same size but with fewer trading relationships. However, if we did, we would not know whether the reduction in efficiency stems from the reduction in the number of trading relationships in the network or from a reduction in the interconnectedness of the most interconnected banks. To avoid this identification problem, I use a novel comparative statics approach that allows me to change the interconnectedness of banks, while keeping the number of trading relationships constant. In particular, I generate seven financial architectures with the same number of banks and links as in the estimated architecture, but with a different allocation of links across banks and with a smaller maximum number of connections per bank.

To generate counterfactual architectures, I use the estimated preferential attachment process with $s = 12$ but put a constraint ($c$) on the maximum number of counterparties each bank can have. As $c$ decreases, the financial architecture changes. The smallest $c$ possible, holding the number of trading relationships constant, is $c = 24$. When $c = 24$, most of the banks have exactly 24 trading partners. In this homogeneous architecture, no bank is too interconnected relative to other banks.

Figure 2 illustrates the effect of the cap on the formed networks by comparing three architectures: (1) estimated (no cap), (2) $c = 50$, and (3) $c = 24$. For each of these architectures, the figure reports an adjacency matrix that presents whether banks $i$ and $j$ are connected (cell $ij$ is

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28 See Gofman (2014) for an analysis of the relationship between the ESL and network density, which is non-monotonic for some networks.

29 I am grateful to Matt Jackson for suggesting this approach.

30 The smallest $c$ is computed by dividing the total number of directed links between banks by the number of banks and rounding up: $c_{\text{min}} = \left\lceil \frac{n(n-1)}{s} \right\rceil$. 
colored), and also a histogram for the number of trading partners per bank. The seven counterfactual architectures have a cap \( c \) equal to 150, 125, 100, 70, 50, 35, and 24. The maximum number of trading partners of a single bank is 234 in the unconstrained financial architecture across 1000 simulated networks, so even a cap of 150 is binding.

5.1 Efficiency Analysis of Counterfactual Financial Architectures

Figure 3 shows the relationship between trading efficiency and the maximum number of counterparties in the network. The results suggest that the estimated financial architecture is more efficient than any of the counterfactual financial architectures. There is a monotonic decrease in trading efficiency as the cap on the maximum number of trading partners tightens. This result suggests that the presence of large interconnected banks in a financial architecture improves welfare. Trading efficiency declines as \( c \) decreases because the intermediation chains are longer in the counterfactual architectures. The correlation between the ESL and the average shortest distance between any pair of banks in each architecture is 97%.

Quantitatively, the increase in the ESL is gradual with a noticeable increase for the most homogenous architecture. The ESL increases from 0.56% to 0.74% when we compare the estimated architecture to the architecture in which banks have no more than 35 trading partners. This is a 30% increase in trading inefficiency. However, the increase from \( c = 35 \) to \( c = 24 \) is 80%. Overall, moving from the estimated core-periphery architecture to the most homogenous architecture reduces trading efficiency by 137%. The reason for this increase is that with \( cap = 24 \) the intermediation chains become very long.

The efficiency losses from restricting the number of counterparties of large banks are consistent with the argument by Saunders and Walter (2012, p. 48) that “systemically important financial institutions (SIFIs) are at least in part the product of market forces whose benefits would have to be sacrificed in any institutional restructuring that breaks them up.” However, while this is true from the qualitative perspective, quantifying the effect on welfare is ultimately what matters for the policy to reduce the interconnectedness of large banks. Another crucial input for that policy discussion is the policy’s effect on stability. That is the subject of the next section.

5.2 Stability Analysis of Counterfactual Financial Architectures

Figure 4 plots the expected surplus loss from trading in eight financial architectures after contagion with interim liquidity. This figure assumes that banks fail if their exposure to a failed

\footnote{Distance is a measure of the shortest number of links between banks. If two banks can trade directly, then the distance is 1. If they need no more than one intermediary, then the distance is 2.}
counterparty is above 15%. I compute two measures of post-crisis efficiency. The first measure assumes that depositors of the failed banks do not withdraw money from the banking system, but there are welfare losses from the inability to trade with failed banks that have access to high investment opportunities. This measure of the expected surplus loss declines from 1.05% for the estimated architecture to 0.8% in the architecture with $c = 35$. However, when the cap becomes 24 counterparties, it increases to 4.58%. This architecture is never optimal because it is less efficient and less stable. Optimality of the other six architectures depends on the preferences for stability versus efficiency. Architectures with a cap between 150 and 35 are more stable, but less efficient. The second measure of post-crisis efficiency, confirms this conclusion. This measure assumes that depositors of failed banks withdraw money from the banking system, but that surviving banks have access to investment opportunities previously available to all banks. The level of inefficiency is much higher in this scenario, as it is very costly to be unable to allocate the failed banks’ endowment to the banks that need liquidity. This post-crisis efficiency measure declines from 25% to less than 5%, but then increases to more than 15% when the cap is 24 counterparties. Both measures suggest that limiting banks to have no more than 35 counterparties results in the most stable architecture of the eight.

Figure 5 plots the same two measures for a scenario in which banks lack access to liquidity during the crisis. In this case, banks fail if their exposure to all failed counterparties is above 15%. As we saw in Table 6, the post-crisis ESL in the estimated architecture is almost 100% in this scenario. The ESL is substantially smaller in the regulated architectures, with $c = 35$ again being the most stable architecture among the eight alternatives. When banks can trade with no more than 35 counterparties, the post-crisis ESL can be reduced to less than 20%, even when banks cannot replenish their liquid capital during contagion. In this scenario, there is no difference between the two measures because most of the losses are caused by a complete failure of the financial architecture. If that is the case, the post-crisis reallocation of deposits or investment opportunities across banks is not relevant. When all banks fail, all potential surplus is lost.

The number of bank failures is another measure of financial stability. Figure 6 shows that overall, the number of bank failures declines as the cap decreases. The only exception is the most homogenous architecture, which experiences more bank failures than an architecture with a cap of 35 counterparties. This stability result is consistent with stability results based on the post-crisis welfare measures. When banks have access to interim liquidity, the number of bank failures can be reduced from 267 to 26 by limiting banks to have no more than 35 counterparties. The reduction will be from 985 to 169 defaults, if banks lack access to interim liquidity.

It should not be surprising that the failure of the most interconnected bank in the estimated architecture triggers a larger number of failures among its direct counterparties than does the

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32 I draw private values from the standard uniform distribution only for the surviving banks.
failure of the most interconnected banks in the counterfactual architectures. However, the total number of failures also depends on the number of default waves in the cascade. In the first wave, only some of the direct counterparties of the failed bank also fail. As the contagion spreads, more and more banks are affected. Figure 7 shows the average number of the default waves for each architecture and each type of contagion. The average number of default waves increases as the cap reaches 100, decreases until the cap is 35, then increases again. The most homogeneous architecture experiences a large total number of defaults because it experiences very long cascades. Even if the original failure that triggers the cascade in this architecture results in a small number of failures, almost half of the banks will eventually eventually because the banks are not diversified enough and have high exposure to each other.

Overall, by combining stability results with the efficiency results in the previous section we learn that there is a strong efficiency-stability tradeoff. The only architecture that is not optimal is the most homogeneous one. The optimality of other architectures depends on the probability of contagion and on a social preference for stability over efficiency.

In all of the figures for contagion we find a surprising non-monotonicity of the stability measures with respect to the cap on interconnectedness. It suggests that more strict regulation of interconnectedness is not necessarily better. This non-monotonicity is not specific to the estimated parameters, distributional assumptions, or to the price-setting mechanism. The next section provides intuition for this result using a financial architecture with only six banks.

5.2.1 The Non-Monotonic Relationship Between the Cap on Interconnectedness and Contagion Risk

Figure 8 shows three architectures with six banks and presents endogenous exposures between them. The exposures are computed assuming that all banks are equally likely to receive an endowment and to have the highest private value. The non-monotonicity does not depend on the price-setting mechanism. This example assumes that sellers extract the full surplus in each trade. The financial architectures are constructed using a preferential attachment. I start with two connected banks (banks 1 and 2), then I add new banks until the architecture has six banks. Each new bank adds one trading relationship to the bank with the highest number of trading relationships, unless this bank has reached the cap on the maximum number of counterparties.

The top architecture has no cap, so the resulting network structure of trading relationships is shaped like a star with bank 1 in the center and banks 2-6 on the periphery. The middle architecture is computed so that each bank can have no more than three counterparties; its structure has two banks in the core and four banks on the periphery. Each core bank trades

\[33\] If two banks have the same number of links, then a new bank forms a link to the bank with the lowest index.
with two peripheral banks. The bottom architecture is shaped as a line. In this architecture, every bank trades with no more than two counterparties. The interbank exposures in this figure are computed analytically in section G of the appendix. Assume that exposures above 50% trigger contagion because banks are required to hold liquid assets equal to 50% of their loan portfolio. In the star-shaped architecture, when bank in the center fails, all other banks also fail because they have 100% exposure to the failed bank. In the architecture with two core banks, when one of the banks fails, then only two peripheral banks fail. The second core bank survives because its exposure to the failed bank is below its liquidity buffer. In the third architecture, when bank 1 or 2 fails, all other banks fail as well. As in the case with 986 banks, there is a non-monotonic relationship between the number of bank failures and the degree of concentration in the architectures with six banks.

The number of bank failures in the architecture with \( \text{cap} = 2 \) is higher than the number of bank failures with \( \text{cap} = 3 \) because when the architecture becomes more homogeneous, the exposure between banks 1 and 2 increases from 47% to 53%. The exposure in the architecture with \( \text{cap} = 3 \) is lower because each of the core banks intermediates between two peripheral banks. This type of intermediation is absent in the third architecture: banks 1 and 2 can trade only with one other counterparty in addition to trading between themselves. When the core banks are less interconnected and cannot intermediate between peripheral banks, their loan portfolios are smaller and so their liquidity buffer for absorbing losses is smaller. With a smaller liquidity buffer, bank 1 is more likely to fail when bank 2 fails and vice versa. Liquidity held by a core bank against loans it intermediated between two peripheral banks helps to absorb losses from the failure of another core bank. This liquidity externality highlights the non-trivial effects that caps have on contagion. The same effects are present in financial architectures with thousands of banks.

6 Policy Implications

The Dodd-Frank Act directs the chairperson of the Financial Stability Oversight Council (FSOC), a new entity established by this act, to recommend limitations on the activities or structure of large financial institutions that will help to mitigate systemic risk in the economy (Section 123). The recommendation should also estimate the benefits and costs of these limitations on the efficiency of capital markets, on the financial sector, and on national economic growth. One possible limitation can be on the size or number of banks’ counterparties. A number of regulators have suggested

\(^{34}\) An arrow from bank \( i \) to bank \( j \) and a number next to it represent bank’s \( i \) exposure to bank \( j \).

\(^{35}\) The non-monotonic relationship also exists when I compute the average cascade size by failing each one of the six banks and averaging the size of the cascades triggered by these failures. The same non-monotonicity for an average cascade size is present in the model with 986 banks as can be seen from Figure A7 in the appendix.
this approach as a solution to the too-big-to-fail problem. A number of regulatory steps have been already taken to address the too-interconnected-to-fail problem. Section 622 in Dodd-Frank prohibits banks from having more than 10% of all liabilities in the financial system. The Basel III regulation limits the maximum exposure to a single counterparty, which can force peripheral banks to direct trades away from core banks to which they currently have high exposures (BCBS 2014). Sections 115 and 165 of the Dodd-Frank Act authorize regulators to impose capital requirements, liquidity requirements and concentration requirements on very interconnected institutions. Consequently, systemically important financial institutions (SIFIs) are required to perform stress tests and hold more capital. As a result of such regulation, SIFIs voluntarily decide to downsize.

The analysis of the counterfactual financial architectures presented in Section 5 is a first attempt to quantify the implications of regulatory actions aimed at reducing the interconnectedness of large banks. The policy implications of this analysis are as follows. First, the efficiency of a financial architecture will be reduced as a result of regulation because the intermediation chains will be longer. Second, the stability results show that such restrictions will improve financial stability. Both the number of surviving bank and post-crisis trading efficiency increase when there is a cap on interconnectedness. However, the most homogenous financial architecture is not the most stable one, despite it being the least efficient.

The results also suggest that providing banks with access to liquidity as the contagion unravels has a significant effect on the number of bank failures and post-crisis efficiency. With a 15% liquidity requirement, 985 banks fail in the estimated architecture if the pre-crisis liquidity buffers cannot be replenished, but (only) 267 banks fail if banks can raise equity, sell assets, or borrow at a discount window as the crisis unravels. Higher pre-crisis buffers of liquidity also reduce the severity of the crisis. If the pre-crisis liquidity buffer is increased from 15% to 25%, instead of the failure of all the banks in the architecture, the model predicts a failure of less than 40% of the banks, assuming that banks have no access to additional liquidity during the crisis. Another way to reduce the severity of the crisis is to maintain confidence in the banking system. The stability analysis shows that if depositors of the failed banks pull liquidity from the banking

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36Richard Fisher, president and CEO of the Federal Reserve Bank of Dallas said, “I favor an international accord that would break up these institutions into more manageable size.” (Fisher 2011). Size and interconnectedness are highly correlated, so decreasing a bank’s size also implies that bank’s interconnectedness will decrease. In his speech, Fisher quoted Mervyn King, former governor of the Bank of England, who said that “If some banks are thought to be too big to fail, then . . . they are too big” (King 2009). A similar view has been voiced by the former president and CEO of the Kansas Fed, Thomas Hoenig, and by the president and CEO of the St. Louis Fed, James Bullard (Hoenig 2010, Bullard 2012).

37For example, General Electric was designated as a SIFI, but to avoid this designation it decided, in April 2015, to sell GE Capital to other financial institutions.

38Section A in the appendix verifies that the policy implications of limiting interconnectedness are robust to changes in the model parameters and distributional assumptions.
system instead of depositing funds at the surviving banks, then the efficiency of the resource allocation process after the crisis is significantly reduced. Quantitatively, the post-crisis ESL in the estimated architecture is 1% when banks have access to liquidity during the crisis and the liquidity buffer is 15%. The ESL increases to 25% if depositors of the failed banks put their money “under the mattress”.

To conclude, the optimality of caps on interconnectedness depends on a social preference for stability over efficiency and on the probability of failure of the most interconnected bank. I also find that providing banks with liquidity during a crisis, requiring banks to hold high liquidity buffers pre-crisis, and maintaining the failed banks’ depositors’ confidence in the banking system all reduce the severity of financial contagion.

7 Limitations and Future Directions

In this section, I discuss the limitations of the results as well as interesting possibilities for future research. First, it is important to emphasize that the stability measures do not take into account the probability of failure of the most interconnected bank fails. These measures should be viewed as stress tests that address the question of what would happen should such a failure occur. In addition, the threat of contagion triggered by the first failure could trigger government bailouts, and these scenarios might therefore never be observed. A bailout of a very interconnected bank is especially likely in the estimated architecture, given the severity of the contagion that such a failure can trigger. That is why the most interconnected bank is considered too-interconnected-to-fail. The contagion results attempt to assess what would happen without governmental intervention.

The stability results presented in the paper assume that the failure of the most interconnected banks is the trigger for contagion. Regulators could also be concerned about contagion under alternative scenarios. A discussion of two alternative measures of stability appears in Section B of the appendix. The first measure ranks architectures based on the largest possible contagion triggered by a single bank. The second measure assesses the average resilience of an architecture. The ranking of the architecture depends on the measure. While regulators have focused on discussing policies towards too-interconnected-to-fail banks, other stability measures can also be important to the regulation of contagion risk.

Another potential limitation of the model is that pricing equations do not account for the counterparty risk. This specification would be easily justified if the bank failures that trigger contagion were unanticipated. It is also possible that banks do not charge any extra interest because they anticipate to be bailed out. However, even without these two assumptions, counterparty risk might not need to be priced on a trade by trade basis. Instead of charging counterparties a credit
risk premium in each trade, two banks could form a long-term relationship that allows them to trade without credit risk mark-ups. This type of arrangement will work if the direction of trade between the two banks changes frequently. Bech and Atalay (2010) report that weighted reciprocity at a daily frequency is 43% in the Fed funds market, suggesting that some pairs of bank trade in opposite directions during a single day. Even accounting for these three explanations of why the model matches the data well without explicitly pricing counterparty risk, it would still be interesting to extend the model and to allow banks to form beliefs about the probability of different triggers of contagion and of the government’s bailout decisions. In equilibrium, banks’ beliefs should be consistent with the government’s equilibrium bailout strategy. The bailout strategy depends on endogenous interbank exposures that in turn depend on banks’ beliefs. This would entail a significant extension of the current model, one that I am leaving to future research.

Another interesting direction would be to reestimate the model by using the topological characteristics of other OTC markets. Recently, a number of empirical papers have documented the differences and similarities of OTC markets across assets and maturities (Hollifield et al. 2012, Roukny et al. 2014, Langfield et al. 2014, Aldasoro and Alves 2015). The presence of a significant overlap between different networks suggests that the efficiency and stability results reported in this paper could be relevant beyond the Fed funds market. However, because the topological structures are not identical, it would be interesting to use the model to identify structural differences across different OTC markets.

8 Conclusion

The analysis presented in this paper relies on four components. The first is a model of the Fed funds market in which banks trade and allocate liquidity. The model is needed to study welfare and to compute endogenous exposures between banks. The second component is a estimation of the model using an observed network of trades in the interbank market for short-term unsecured loans in the U.S. The third involves computing the efficiency and stability of the estimated financial architecture with large interconnected banks. The last component studies the costs and benefits of large interconnected financial institutions by comparing the efficiency and stability of the estimated financial architecture to alternative financial architectures with more equal distribution of trading relationships across banks. This comparison is used to draw policy implications for regulating the current market structure.

The results suggest that large interconnected banks improve efficiency mainly by decreasing the length of intermediation chains in the market. Therefore, alternative architectures without very interconnected banks will result in lower allocational efficiency.
The stability analysis generates a number of novel results. First, the welfare implications of contagion depend not only on the number of banks that fail, but also on the intermediation role of the failed banks. If banks have access to liquidity during a crisis, the probability that large banks fail is small because they do not have high exposure to any given counterparty. The failure of small periphery banks does not impose high welfare losses. However, if banks have to solely rely on pre-crisis liquidity buffers to absorb losses from all failed counterparties, then large core banks can fail. Their failure causes significant welfare losses.

Second, an important measure of contagion risk is the probability that all banks fail in contagion. This measure has a direct implication on welfare because if all banks fail, 100% of the potential trading surplus is lost and the resource allocation function of an OTC market ceases to exist. Even if banks have access to interim liquidity and the complete collapse of a financial architecture is not a realistic concern, it is important that depositors of the failed banks do not lose confidence in the banking system. The post-crisis efficiency is reduced substantially if these depositors choose not to move their deposits to surviving banks.

Another novel result presented in the paper is that the number of bank failures and the post-crisis trading efficiency are non-monotonically related to the cap on the maximum number of counterparties that a bank can have. This non-monotonicity suggests that more strict regulation does not necessarily improve financial stability. In particular, the most homogenous financial architecture is never optimal.

Overall, my results suggest that two ex-ante policies can improve stability: restrictions on the interconnectedness of large banks and an increase in liquidity requirements. These policies would need to be implemented prior to a crisis. However, if the crisis has already begun and some banks suffer losses from failed counterparties, then the provision of interim liquidity and maintaining the failed banks' depositors' confidence in the banking system are the most important tools for reducing the severity of the crisis.
### Table 1: Moments from SMM Estimation

<table>
<thead>
<tr>
<th>Moment</th>
<th>Empirical Value</th>
<th>Best Fit</th>
<th>Simulated Value</th>
<th>t-stat</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Value</td>
<td></td>
<td>5th Mean 95th</td>
<td></td>
</tr>
<tr>
<td><strong>Means:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Network density (%) ((\alpha))</td>
<td>0.70</td>
<td>0.70</td>
<td>0.63 0.70 0.77</td>
<td>0.38</td>
</tr>
<tr>
<td>Max. number of lenders ((k_{\text{max}}^{\text{in}}))</td>
<td>127.6</td>
<td>122.6</td>
<td>89.5 111.8 141.6</td>
<td>15.33</td>
</tr>
<tr>
<td>Max. number of borrowers ((k_{\text{max}}^{\text{out}}))</td>
<td>48.8</td>
<td>49.0</td>
<td>42.8 47.2 52.4</td>
<td>3.98</td>
</tr>
<tr>
<td>Number of active banks ((\hat{n}))</td>
<td>470.2</td>
<td>470.0</td>
<td>446.8 473.9 501.6</td>
<td>-3.79</td>
</tr>
<tr>
<td><strong>Standard Deviations (not used in the estimation):</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Network density (%)</td>
<td>0.03</td>
<td>0.05</td>
<td>0.04 0.05 0.05</td>
<td></td>
</tr>
<tr>
<td>Maximum number of lenders</td>
<td>16.3</td>
<td>17.98</td>
<td>11.0 15.1 20.9</td>
<td></td>
</tr>
<tr>
<td>Maximum number of borrowers</td>
<td>6.4</td>
<td>3.34</td>
<td>2.5 3.0 3.8</td>
<td></td>
</tr>
<tr>
<td>Number of active banks</td>
<td>15.3</td>
<td>17.92</td>
<td>15.9 17.8 20.1</td>
<td></td>
</tr>
</tbody>
</table>

Empirical values are taken from Table 5 in Bech and Atalay (2010). Each empirical value represents a time-series mean or standard deviation of the corresponding Fed funds network characteristic taken over 250 trading days in 2006. To compute the simulated moments, I draw 100 architectures according to the estimated preferential attachment process and for each architecture I solve for 250 endogenous networks of equilibrium trades. Each endogenous network represents one day of trading according to the estimated shock intensity and truncation parameters. For each simulated architecture, I compute the mean and standard deviation of each moment over 250 days to compute the simulated moments that correspond to the empirical moments. The first column (“Best Fit”) presents simulated moments for a financial architecture with the four moments that are closest to the empirical moments. The second column ("Best Fit") presents simulated moments for a financial architecture with the four moments that are closest to the empirical moments. The next three columns report the 5th percentile, the mean and the 95th percentile across 100 architectures given the optimal parameters (\(s = 12, w = 78, t = 22\)). The last columns reports t-statistics for a hypothesis that the empirical moment and the mean simulated moment across the 100 networks are equal. Only the time-series means of the four moments were used in the estimation. Empirical standard deviations were used to compute the t-stats assuming that the data is iid. Formal definitions of these network moments appear in Section D of the appendix.
Table 2: Model Fit for Moments Not Used in the Estimation

<table>
<thead>
<tr>
<th>Moments (Time-Series Means)</th>
<th>Empirical Value</th>
<th>Simulated Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of links ($m$)</td>
<td>1543</td>
<td>1523 1557 1590</td>
</tr>
<tr>
<td>Average number of counterparties ($k$)</td>
<td>3.3 3.2 3.3 3.4</td>
<td></td>
</tr>
<tr>
<td>Average path length-in ($t^\text{in}$)</td>
<td>2.4 2.8 2.8 2.9</td>
<td></td>
</tr>
<tr>
<td>Average path length-out ($t^\text{out}$)</td>
<td>2.7 2.7 2.8 2.8</td>
<td></td>
</tr>
<tr>
<td>Average maximum path length-in ($\overline{t}^\text{in}$)</td>
<td>4.1 4.5 4.6 4.8</td>
<td></td>
</tr>
<tr>
<td>Average maximum path length-out ($\overline{t}^\text{out}$)</td>
<td>4.5 4.5 4.6 4.8</td>
<td></td>
</tr>
<tr>
<td>Diameter ($D$)</td>
<td>7.3</td>
<td>6.5 6.7 7.0</td>
</tr>
<tr>
<td>Clustering by lenders ($C^\text{in}$)</td>
<td>0.10 0.09 0.10 0.11</td>
<td></td>
</tr>
<tr>
<td>Clustering by borrowers ($C^\text{out}$)</td>
<td>0.28 0.10 0.12 0.14</td>
<td></td>
</tr>
<tr>
<td>Reciprocity (%) ($\rho$)</td>
<td>6.5</td>
<td>25.27 26.11 26.92</td>
</tr>
<tr>
<td>Degree Correlation (borrowers,lenders)</td>
<td>-0.28 -0.40 -0.35 -0.31</td>
<td></td>
</tr>
<tr>
<td>Degree Correlation (lenders,lenders)</td>
<td>-0.13 -0.29 -0.26 -0.22</td>
<td></td>
</tr>
</tbody>
</table>

Means for the empirical moments not used in the estimation are taken from Table 5 in Bech and Atalay (2010). Each mean of the network characteristics is computed over 250 trading days in the Fed funds market in 2006. To compute similar moments for the model, I draw 100 architectures according to the estimated preferential attachment process and for each architecture I solve for 250 endogenous networks of equilibrium trades. Each endogenous network represents one day of trading according to the estimated shock intensity and truncation parameters. First, I compute the time-series means and standard deviations of the moments and then I take a mean over different architectures. I also report 5th and 95th percentiles of the distribution for these statistics generated by the random draws of architectures. Parameters that generate the above moments are: $s = 12$, $w = 78$, $t = 22$. Formal definitions of these network measures appear in Section D of the appendix.

Table 3: Distribution of Trading Volume Across Groups of Banks

<table>
<thead>
<tr>
<th>Borrowers</th>
<th>1-10</th>
<th>11-50</th>
<th>51-100</th>
<th>Other</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-10</td>
<td>2.0</td>
<td>3.5</td>
<td>2.0</td>
<td>14.1</td>
<td>22%</td>
</tr>
<tr>
<td>Lenders</td>
<td>11-50</td>
<td>4.4</td>
<td>5.2</td>
<td>2.4</td>
<td>15.9</td>
</tr>
<tr>
<td></td>
<td>51-100</td>
<td>2.3</td>
<td>2.6</td>
<td>0.7</td>
<td>4.6</td>
</tr>
<tr>
<td>Other</td>
<td>10.8</td>
<td>13.1</td>
<td>4.0</td>
<td>12.4</td>
<td>40%</td>
</tr>
<tr>
<td>Total</td>
<td>20%</td>
<td>24%</td>
<td>9%</td>
<td>47%</td>
<td>100%</td>
</tr>
</tbody>
</table>

This table presents the distribution of trading volume generated by the model across four groups of banks. The 1-10 group includes the top 10 banks in terms of average trading volume during a year, Group 2 has the next 40 banks, Group 3 the next 50 banks, and the remaining banks are in Group 4. The table reports what percent of the total average daily volume was lent by the group of banks in the row to the group of banks in the column. The volume was averaged over 250 trading days and 100 draws of a financial architecture.
### Table 4: Persistence of Trades

<table>
<thead>
<tr>
<th>Measures</th>
<th>daily</th>
<th>weekly</th>
<th>monthly</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prob. of a trade between bank $i$ and bank $j$ at time $t + 1$ conditional on a trade at time $t$</td>
<td>49.5%</td>
<td>59.6%</td>
<td>74.6%</td>
</tr>
<tr>
<td>Prob. of absence of trading between bank $i$ and bank $j$ at time $t + 1$ conditional on absence of trading at time $t$</td>
<td>99.7%</td>
<td>99.5%</td>
<td>99.0%</td>
</tr>
<tr>
<td>Prob. of a loan from bank $i$ to bank $j$ at time $t + 1$ conditional on a loan from bank $i$ to bank $j$ at time $t$</td>
<td>40.7%</td>
<td>57.7%</td>
<td>72.5%</td>
</tr>
<tr>
<td>Prob. of absence of lending by bank $i$ to bank $j$ at time $t + 1$ conditional on absence of lending by bank $i$ to bank $j$ at time $t$</td>
<td>99.9%</td>
<td>99.8%</td>
<td>99.8%</td>
</tr>
<tr>
<td>Number of periods</td>
<td>2500</td>
<td>500</td>
<td>125</td>
</tr>
</tbody>
</table>

The table reports the persistence of the equilibrium network of trades for the estimated set of parameters. The measures were computed at three frequencies: daily, weekly, and monthly. The difference between the first two and the next two measures is that the former do not account for the direction of trade. The parameters used to compute persistence measures are: $s = 12$, $w = 78$, $t = 22$.

### Table 5: Persistence of Trade between Groups of Banks

<table>
<thead>
<tr>
<th>Borrowers</th>
<th>1-10</th>
<th>11-50</th>
<th>51-100</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lenders</td>
<td>1-10</td>
<td>76.8%</td>
<td>45.3%</td>
<td>25.6%</td>
</tr>
<tr>
<td></td>
<td>11-50</td>
<td>77.4%</td>
<td>37.8%</td>
<td>16.5%</td>
</tr>
<tr>
<td></td>
<td>51-100</td>
<td>79.7%</td>
<td>27.4%</td>
<td>4.7%</td>
</tr>
<tr>
<td></td>
<td>Other</td>
<td>69.3%</td>
<td>28.3%</td>
<td>1.2%</td>
</tr>
</tbody>
</table>

This table presents the persistence of trade volume generated by the model across four groups of banks. The 1-10 group includes the top 10 banks in terms of average betweenness centrality. Group 2 has the next 40 banks, Group 3 the next 50 banks, and the remaining banks are in Group 4. The table reports what the probability is that a bank from the group in the row provides a loan to the bank in the group in the column at day $t + 1$, conditional that the first bank provided a loan to the second bank at day $t$. Both the centrality measures and the persistence measures were computed using 2500 endogenous trading networks representing 10 years of trading.
Table 6: Results of Efficiency and Stability Analyses

<table>
<thead>
<tr>
<th></th>
<th>ESL(%)</th>
<th>PIA(%)</th>
<th>Volume</th>
<th>% of banks</th>
<th>Prob. all banks</th>
<th>ESL2(%)</th>
<th>ESL3(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Efficiency Results</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pre-crisis</td>
<td>0.56</td>
<td>66.19</td>
<td>173,333</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Panel B: Stability Results</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Contagion with Interim Liquidity</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.25</td>
<td>0.67</td>
<td>68.83</td>
<td>161,442</td>
<td>92.01</td>
<td>0</td>
<td>7.27</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>0.73</td>
<td>70.05</td>
<td>154,816</td>
<td>88.27</td>
<td>0</td>
<td>10.89</td>
</tr>
<tr>
<td></td>
<td>0.15</td>
<td>1.05</td>
<td>74.58</td>
<td>128,366</td>
<td>72.86</td>
<td>0</td>
<td>25.10</td>
</tr>
<tr>
<td><strong>Contagion without Interim Liquidity</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.25</td>
<td>33.76</td>
<td>79.43</td>
<td>106,420</td>
<td>60.75</td>
<td>33.3</td>
<td>38.84</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>87.69</td>
<td>96.22</td>
<td>19,452</td>
<td>11.09</td>
<td>87.6</td>
<td>88.79</td>
</tr>
<tr>
<td></td>
<td>0.15</td>
<td>99.90</td>
<td>99.97</td>
<td>161</td>
<td>0.09</td>
<td>99.9</td>
<td>99.91</td>
</tr>
</tbody>
</table>

This table reports the average results for 1000 different financial architectures produced by using the estimated preferential attachment model. The columns report the expected surplus loss (ESL), the probability of inefficient allocation, trading volume, the percent of surviving banks, the probability that all banks fail in contagion, the expected surplus loss when endowments are not reallocated to surviving banks (ESL2), and the expected surplus loss when banks don’t adjust trading post-crisis (ESL3). Panel A reports results for the estimated parameters in the pre-crisis period. Panel B reports results after the most interconnected bank fails and triggers cascades of failures whenever each bank’s exposure to any of the failed banks is above 15%, 20%, or 25% respectively. The results at the bottom correspond to contagion in which a bank fails when its aggregate exposure to all failed banks is above 15%, 20%, or 25%.
The two plots represent model-generated total lending volume and the ratio of the intermediated volume to the total lending volume as a function of the number of counterparties each bank could potentially trade with. The results are computed based on 250 trading days.
The figure presents an adjacency matrix (blue dot if two banks are connected) and the distribution of the number of counterparties for three architectures: (1) the estimated financial architecture (left); (2) the counterfactual financial architecture with a cap of 50 (center); and (3) the counterfactual financial architecture with a cap of 24. All three financial architectures were generated using a version of a preferential attachment model in which the maximum number of trading relationships is capped. The preferential attachment model in the estimated financial architecture does not put any restriction on the maximum number of counterparties, so the cap is equal to the maximum number of counterparties that each bank can have. The bottom plots report the distribution each bank’s number of trading partners.
The figure presents the expected surplus loss (ESL) in the estimated financial architecture and in seven counterfactual financial architectures. The ESL for each architecture is an average of surplus losses in 1000 networks generated by the preferential attachment process. The financial architecture with a maximum of 234 trading partners is the estimated architecture with too-interconnected-to-fail banks that was generated without a cap on the maximum number of counterparties. Each of the other seven architectures is simulated 1000 times with a cap on the maximum number of trading partners that corresponds to the value on the x-axis. Two standard error bounds are reported as bars around the point estimates.
The figure presents two measures of the expected surplus loss (ESL) after contagion in the estimated financial architecture and in seven counterfactual financial architectures. The contagion is triggered by the failure of one of the most interconnected banks. Banks fail if they have exposure above 15% to a failed counterparty. The efficiency measures are computed for the remaining financial architecture after the cascade of failures stops. The first ESL measure (dotted line) is computed assuming the pre-crisis procedure for drawing private values. It captures the case of entrepreneurs who used to borrow from the failed banks, and who now cannot borrow from the surviving banks due to the lack of a long-term relationship. For this measure, the total post-crisis endowment is reset to the pre-crisis level, which is equivalent to assuming that post-crisis depositors deposit their savings with the remaining banks. The second measure (solid line) assumes that depositors withdraw their savings from the banking sector, such that the aggregate endowment is not readjusted to the pre-crisis level. This measure assumes that entrepreneurs are able to borrow from the surviving banks, meaning that the vector of private values is drawn for the surviving banks only. Each endogenous contagion scenario was computed for 1000 network draws using the estimated parameters for the trading model. Two standard error bounds are reported as bars around the point estimates.
Figure 5: Post-Crisis Efficiency Without Interim Liquidity

The figure presents two measures of the expected surplus loss (ESL) after contagion in the estimated financial architecture and in seven counterfactual financial architectures. The contagion is triggered by the failure of one of the most interconnected banks. Banks fail if they have aggregate exposure above 15% to all failed counterparties. The efficiency measures are computed for the remaining financial architecture after the cascade of failures stops. The first ESL measure (dotted line) is computed assuming the pre-crisis procedure for drawing private values. It captures the case of entrepreneurs who used to borrow from the failed banks, and who now cannot borrow from the surviving banks due to the lack of a long-term relationship. For this measure, the total post-crisis endowment is reset to the pre-crisis level, which is equivalent to assuming that post-crisis depositors deposit their savings with the remaining banks. The second measure (solid line) assumes that depositors withdraw their savings from the banking sector, such that the aggregate endowment is not readjusted to the pre-crisis level. This measure assumes that entrepreneurs are able to borrow from the surviving banks, meaning that the vector of private values is drawn for the surviving banks only. Each endogenous contagion scenario was computed for 1000 network draws using the estimated parameters for the trading model. Two standard error bounds are reported as bars around the point estimates.
Figure 6: Consequences of Failure of the Most Interconnected Bank(s)

The figure presents the number of bank failures after contagion with a 15% threshold. The contagion is triggered by the failure of one of the most interconnected banks. Banks fail either if they have exposure above the threshold to a failed counterparty (with interim liquidity case) or to all failed counterparties (without interim liquidity). For each bank with the largest number of counterparties, the size of the cascade is computed and then averaged across all banks that are the most interconnected. The calculation is repeated for 1000 network draws. Two standard error bounds are reported as bars around the point estimates for each graph. The number of banks before contagion is 986.
The figure presents the average number of default waves when the most interconnected bank fails and triggers contagion. The contagion happens when a bank’s exposure to its failed counterparty is above 15% (with interim liquidity case) or when an aggregate exposure to all failed counterparties (without interim liquidity). For each bank that has the greatest number of counterparties, I compute the length of the cascade of failures. If there are several most interconnected banks then the average length of the cascades that they trigger is computed. The cascade lengths are also averaged over 1000 network draws. Two standard error bounds are reported as bars around the point estimates.
Figure 8: Endogenous Exposures in Three Financial Architectures with Six Banks

This figure shows the expected endogenous exposures for three financial architectures with different caps on the maximum number of trading partners. An arrow from bank $i$ to bank $j$ represents bank $i$’s exposure to bank $j$. The exposures are computed analytically assuming that a seller extracts the full surplus in each trade, that each bank has the same endowment, and that each bank is equally likely to have the highest private value for liquidity. A bank is assumed to fail if its exposure is above 50% to a counterparty that failed. Exposures above 50% are in bold to represent links that cause contagion to spread.
References


BCBS (2014): “Supervisory framework for measuring and controlling large exposures (Standards),” BIS publication.


Appendix

A Robustness of the Efficiency-Stability Tradeoff

There are three estimated parameters: \( s \), \( w \), and \( t \), but only the first two are used in the efficiency and stability analyses. In this section, I report results of robustness tests that assess how sensitive the policy conclusions are with respect to these two parameters. I also compute how the distributional assumption about the private shocks affects the results.

First, I study how the results change when \( s \) changes. This parameter controls the network formation process. The estimated value of \( s \) is 12. The resulting network of relationships has a density of 2.4% (986 banks and 23,520 links). Figure A1 reports the trading efficiency for \( s = 11 \), \( s = 12 \), and \( s = 13 \). The number of trading relationships is 8% less (more) in the case of \( s = 11 \) (\( s = 13 \)) relative to the estimated network of relationships. The maximum number of trading partners in the unrestricted architecture when \( s = 11 \) (\( s = 13 \)) is 225 (245). This is computed as an average over 1000 network draws for each \( s \). Given that the number of trading relationships is different across the three values of \( s \), the smallest cap is also different. With \( s = 11 \), each bank can have no more than 22 counterparties in the most homogenous architecture with the same number of links. With \( s = 13 \), the minimum cap is 26.

An unrestricted architecture without a cap is more efficient when \( s \) is higher. Higher \( s \) shortens the intermediation chains makes core banks more interconnected, helping them to reduce the intermediation friction. Relative to \( s = 12 \), the ESL for \( s = 11 \) is 7.8% larger and is 7% smaller for \( s = 13 \), with an absolute difference being of less than 0.05%. The ESL is increasing when the cap gets tighter for all three values of \( s \). For all the three values of \( s \), the ESL in the most homogeneous architecture is about 140% higher than in the respective unrestricted architecture. The earlier finding that restricting the interconnectedness of banks reduces efficiency is robust to different values of \( s \). One additional interesting result is worth mentioning. In figure A1 the ESL of the architecture with \( s = 11 \) and a cap of 24 is 33.6% smaller than the ESL of the architecture with \( s = 12 \) and the same cap. This is a clear example that network’s density is not the only factor that determines trading efficiency; how trading relationships are distributed across banks also matters.

Figure A2 presents stability measures for the two alternative values of \( s \). The post-crisis ESL with interim liquidity does not change much with \( s \), part from when the most homogeneous architectures are compared. The ESL measures without interim liquidity differ mostly for architectures with intermediate caps. The number of bank failures (two bottom plots) suggest that a higher \( s \) results in a more stable financial architecture, but the non-monotonic relationship between the cap and the number of bank failures is robust to alternative values of \( s \). The policy conclusions stay
the same. Restricting interconnectedness improves financial stability, but the most homogeneous network is network optimal. We can also learn from this figure that an architecture with \( s = 11 \) and a cap of 24 is more stable than an architecture with \( s = 12 \) and the same cap. That means that whether a limit of 24 counterparties per bank achieves the most stable architecture depends crucially on \( s \). For \( s = 11 \), this restriction would lead to the most stable architecture. But for \( s = 12 \), that policy would result in a substantially worse architecture in terms of efficiency and stability. Overall, all the main conclusions in the paper are robust to alternative values of \( s \).

The next robustness test is with respect to the number of draws of private values per day (\( w \)). I study the implications of changing this number to 56 or 100 instead of the estimated 78. The pre-crisis ESL is not significantly different when the number of shocks changes. The stability results without interim liquidity also do not change. The figures for the stability measures without interim liquidity are almost identical across the three values of \( w \), and therefore they are not plotted. The main difference is in the results for contagion with interim liquidity. Figure ?? reports the number of bank failures and the post-crisis trading efficiency for \( w = \{56, 78, 100\} \). The affect of the cap on post-crisis ESL is qualitatively similar across the three values of \( w \). The post-crisis ESL is higher when the number of shocks is smaller. Banks trade with more counterparties when the number of shocks increases. A more diversified loan portfolio reduces the risk of contagion. The effect of \( w \) on the number of bank failures is similar. The number of bank failures is higher when banks face fewer shocks during the day. The architecture with the smallest number of bank failures is when each bank has no more than 35 counterparties, and the architecture does not depend on \( w \). I conclude that the main results in the paper are robust to different values of \( w \).

The quantitative results for pre-crisis efficiency and post-crisis stability without interim liquidity are the same. The main quantitative differences are in stability measures with interim liquidity.

Lastly, I check how robust the results are with respect to the distribution of private values. A uniform distribution is a natural choice when there is no prior knowledge about the distribution of shocks to private values. To test how much the results depend on this distributional choice, I redo the analysis assuming that private values are distributed according to a beta distribution with \( \alpha = \beta = 2 \) or \( \alpha = \beta = 0.5 \). The first distribution, \( Beta(2, 2) \), is a hump-shaped distribution, with the highest density at 0.5. It assumes that most banks have similar private values for liquidity, but a small number of banks that have a high need for liquidity and a small number of banks who have a low need for it. The second distribution, \( Beta(0.5, 0.5) \), is an inverse hump-shape with the lowest density at 0.5. This distribution assumes that most of the banks have either a high or low private value for liquidity.

The effect of the distribution on trading efficiency appears in Figure [A4]. When the density of banks with high private values is low, trading efficiency is lower. When there are many banks with high private values, trading efficiency improves relative to the uniform distribution case.
The quantitative effect of the distributional assumption is substantial. The ESL triples in the unrestricted architecture when $Beta(2, 2)$ and is halved when the distribution is $Beta(0.5, 0.5)$. The difference in trading efficiency does not affect the relationship between the ESL and the cap on the maximum number of counterparties, indicating that the policy implications are robust. A cap on interconnectedness reduces trading efficiency. Figure A5 reports four stability measures and how they change when the distribution for private values changes. Out of the four measures, only the ESL measure with interim liquidity differs across the three distributions. The ESL is higher after contagion, but there is little difference in the ESL across architectures, conditional on the distribution. The only exception is the architecture with a cap of 24, which is significantly less efficient than the other architectures. Interestingly, the number of bank failures (with and without interim liquidity) and the ESL without interim liquidity do not differ significantly across the three distributions. The result that the number of bank failures does not depend on the distribution of private values is important to policy that attempts to improve stability via restrictions on interconnectedness. I conclude that while the distributional assumption affects the level of the inefficiency, the efficiency-stability results hold qualitatively. Moreover, three out of four stability measures do not change quantitatively, when the distribution for liquidity shocks changes.

B Additional Measures of Financial Stability

This section reports the results for two alternative measures of financial stability. The first measure assumes that the systemically important bank fails, and computes the size of the cascade. A bank is *systemically important* if its failure results in the largest number of bank failures. By definition, the failure of a systemically important bank is more severe than the failure of the most interconnected bank. Ranking financial architectures based on the worst case scenario is useful if policy makers seek to minimize the maximum number of defaults triggered by a single bank’s failure.

Figure A6 reports the maximum number of bank failures triggered by the failure of a single bank with a 15% default threshold. When banks have no access to liquidity, all banks fail in the worst case scenario in all architectures. As expected, if banks have access to interim liquidity, the number of defaults after a systemically important bank fails is substantially higher than when the most interconnected bank fails. This is true for all eight architectures. There is a non-monotonic relationship between the cap on the maximum number of counterparties and the number of failures. When banks have access to interim liquidity, the estimated architecture is more resilient to the failure of systemically important banks than all other architectures.

The second measure of stability focuses on less extreme scenarios. It does not assume that any particular bank triggers a contagion, but rather allows for any of the banks to be the cause
of contagion. To compute this measure, first I compute the number of bank failures triggered by
the failure of each of the 986 banks, and then compute the size of the average cascade. If each
bank is equally likely to trigger a cascade, this measure of stability is particularly useful.\footnote{For example, if operational risk is the cause of the original failure, it is reasonable to assume that banks are equally exposed to this type of risk.}

Figure [A7] reports the fragility of each architecture based on average cascade size. The aver-
age cascade size is smaller than the size of the cascade triggered by the failure of the most
interconnected bank in all architectures but one. In the most homogeneous architecture, there
is no substantial difference in the number of bank failures under the two scenarios because most
of the banks in this architecture have the same number of counterparties. A tighter cap on the
maximum number of counterparties results in a more fragile financial architecture when there
is no interim liquidity. With interim liquidity, there is a small non-monotonicity. The average
fragility increases until it reaches a cap of 70, slightly decreases until a cap of 35, then increases
again for a cap of 24. In both cases, the estimated architecture is the most stable according to
this measure.

These results suggest that if policy makers’ main concern is the worst case scenario or if the
objective is to reduce the expected number of defaults when each bank has the same probability
of triggering contagion, then putting a cap on the most interconnected bank is not optimal.

C Solution algorithm

The trading mechanism in equation (1) is a contraction mapping (Gofman 2014). Therefore,
according to the contraction mapping theorem (see Stokey et al. (1989), Theorem 3.2), the vector
of equilibrium valuation is unique. The benefit of the contraction mapping theorem is that it
allows me to solve for equilibrium valuations and trading decisions in large trading networks by
using an iterative approach. This approach is described below.

\textit{Step 1:} Let \( k = 0 \) and \( P(0) = V \) is the initial vector of endogenous valuations.

\textit{Step 2:} Let \( k = k + 1 \); compute each bank’s new valuation according to Equation (1) and
using \( P(k - 1) \) as an endogenous vector of valuations on the right-hand side of the equation. The
pricing equation for the fourth price-setting mechanism can be simplified to:

\[ P_i = \max \{ V_i, \delta \max_{P_j \in N(i,g)} P_j \} \]

(4)

where the second max operator picks the second-highest value in the set.

After we compute each bank’s new valuation, we get a new vector of valuations \( P(k) \).
Step 3: If \( P(k) = P(k - 1) \), then \( P(k) \) is the equilibrium vector of valuations. Otherwise we need to make another iteration by returning to Step 2 and computing \( P(k + 1) \) until we find a fixed point at which an additional iteration does not change the vector of valuations. The contraction mapping theorem ensures that this fixed point is unique and can be reached using a sequence of iterations. After we solve for the equilibrium valuations, equilibrium trading decisions are computed using Equation (2).

D Network Measures

In this section, I formally define network measures used for computing targeted and untargeted moments in the paper. I use the same definitions as in Bech and Atalay (2010) to make sure that the simulated moments and the empirical moments are defined in the same way.

A network of trades has \( \hat{n} \) nodes and \( m \) links. Nodes are banks that are observed to be trading in the market on a particular day, and links are trades between these banks. The number of links relative to the maximum possible number of links defines the \textit{density} of a network. The density of a directed network is given by:

\[
\alpha = \frac{m}{\hat{n}(\hat{n} - 1)}.
\] (5)

Reciprocity of a network measures what percent of links in a directed network also have a link in the opposite direction. In a daily network of trades, a link in both directions occurs if bank \( i \) provides a loan to bank \( j \) and later in the day, bank \( j \) provides a loan to bank \( i \). Let \( a_{ij} = 1 \) if bank \( i \) provides a loan to bank \( j \), otherwise \( a_{ij} = 0 \). Reciprocity is defined as:

\[
\rho = \frac{\sum_i \sum_j a_{ij} a_{ji}}{m}.
\] (6)

Degree distribution in a network captures how many counterparties each bank has. Formally, \( k_{in}^i = \sum_j a_{ji} \) is the number of lenders to bank \( i \), and \( k_{out}^i = \sum_j a_{ij} \) is the number of borrowers from bank \( i \). Then the maximum number of lenders to a single bank is \( k_{in}^{maxi} = \max_i k_{in}^i \) and the maximum number of borrowers is \( k_{out}^{max} = \max_i k_{out}^i \). For any network, the average number of lenders per bank is equal to the average number of borrowers. So the average number of counterparties of a bank can be computed as \( \bar{k} = \frac{1}{\hat{n}} \sum_i k_{in}^i = \frac{1}{\hat{n}} \sum_i k_{out}^i \).

Degree correlation measures how much the likelihood of a trade between two banks depends on the number of counterparties they have. The \textit{degree correlation} \((\text{borrowers, lenders})\) is computed as \( \text{corr}(k_{out}^i, k_{in}^j) \) for all banks \( i \) and \( j \), such that \( a_{ij} = 1 \). If this correlation is negative (positive) it means that banks with many borrowers are less (more) likely to lend to banks with many lenders. Similarly, the \textit{degree correlation} \((\text{lenders, lenders})\) is \( \text{corr}(k_{in}^i, k_{in}^j) \). If this correlation is negative

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(positive) it means that banks with many lenders are less (more) likely to lend to banks with many lenders.

Next, I define the distance measures between the banks in a network of trades. A distance from \( i \) to \( j \), \( d_{ij} \), equals to the length of the shortest directed path from \( i \) to \( j \). If there is no directed path between these two banks, then \( d_{ij} = \infty \). The average in-path length of node \( i \), \( l_{i}^{in} \), is equal to the mean of \( \{d_{ji} : d_{ji} < \infty \} \). The average out-path length of node \( i \), \( l_{i}^{out} \), is equal to the mean of \( \{d_{ij} : d_{ij} < \infty \} \). The maximum in-path length of node \( i \), \( e_{i}^{in} \), is equal to the maximum of \( \{d_{ji} : d_{ji} < \infty \} \). The maximum out-path length of node \( i \), \( e_{i}^{out} \), is equal to the maximum of \( \{d_{ij} : d_{ij} < \infty \} \). The average path-in and path-out for the network are \( \bar{l}^{in} = \frac{1}{n} l_{i}^{in} \) and \( \bar{l}^{out} = \frac{1}{n} l_{i}^{out} \) respectively. The maximum path-in and path-out of the network are \( \bar{e}^{in} = \frac{1}{n} e_{i}^{in} \) and \( \bar{e}^{out} = \frac{1}{n} e_{i}^{out} \) respectively. The diameter of a network is equal to the maximum finite distance between any two nodes \( D = \max_i e_{i}^{in} = \max_i e_{i}^{out} \).

Clustering coefficients compute the probability that two banks connected to a third bank are also connected to each other. Clustering can be computed between either a bank’s lenders or its borrowers. A clustering by borrowers coefficient is defined as

\[
C_{i}^{out} = \frac{1}{k_{i}^{out}(k_{i}^{out} - 1)} \sum_{j,h} a_{ij} + a_{ih} - \frac{1}{2} a_{ij}a_{ih}a_{jh}.
\]

A clustering by lenders coefficient is defined as

\[
C_{i}^{in} = \frac{1}{k_{i}^{in}(k_{i}^{in} - 1)} \sum_{j,h} a_{ji} + a_{hi} - \frac{1}{2} a_{ji}a_{hi}a_{jh}.
\]

The clustering coefficients for the network are computed as an average of the individual clustering coefficients \( C^{out} = \frac{1}{n} C_{i}^{out} \) and \( C^{in} = \frac{1}{n} C_{i}^{in} \).

E Estimation Procedure

This section provides details about the estimation procedure.

To find the optimal parameters, I follow a two-stage procedure. In the first stage, I compute moments for one network draw and 250 trading days of trading. This stage narrows down the set of pleasurable parameters. This stage also allows me to narrow down which price-setting mechanism provides the best fit to the data. In the second stage, I compute simulated moments as an average of moments in 100 networks and 250 days of trading in each network. The model is reestimated based on these moments. This calculation achieves low standard errors of the moment estimates and ensures that the result are not driven by some extreme realization of the process for formation of long-term trading relationships. Overall, this two-stage procedure generates an
efficient way to search for optimal parameters in the wide grid of possible parameter values.

In the first stage, I consider 17 values for \( s \), ranging between 4 and 17. It means that the core of the financial architecture can have as little as four banks or as much as 20 banks. Also, it means that each bank can form between 4 and 20 trading relationships with existing banks. The number of shocks to the private values, \( w \), is allowed to be between 1 and 300. The threshold on the minimum volume of trade for links to be observable, \( t \), is assumed to be between 2 and 200, but I compute moments only for even values of \( t \) in this range. I evaluate the loss function in 3 for every possible set of parameter values. The optimal parameters that minimize the distance between the empirical moments and the simulated moments are: \( \hat{s}_1 = 13, \hat{w}_1 = 82, \) and \( \hat{t}_1 = 22 \).

In the second stage, I increase the number of network draws to 100, and consider 6 values for \( s \), ranging between 10 and 15; 100 values for \( w \), ranging between 1 and 100; and 31 values for \( t \), ranging between 15 and 30. These parameter values are selected based on the results of the estimation in the first stage. The optimal parameters from the second-stage are very similar to the first-stage estimates: \( \hat{s}_2 = 12, \hat{w}_2 = 78, \) and \( \hat{t}_2 = 22 \). The parameters from the second stage are used for the stability and efficiency analysis.

The simulated moments in the second stage are computed as follows:

*Step 1:* Draw a network of 986 banks for each value of \( s \).

*Step 2:* Draw a vector of private values.

*Step 3:* Compute optimal trading decisions and construct a network of realized trades according to the distribution of endowment shocks.

*Step 4:* Compute moments for the truncated daily equilibrium network of trades for each value of \( t \).

*Step 5:* Repeat steps 2 to 4 100 times (maximum number of liquidity shocks), each time adding the new links uncovered in Step 3.

*Step 6:* Repeat steps 1 to 5 250 times, as the number of trading days in 2006.

*Step 7:* Repeat steps 1 to 6 100 times to average out realizations of the random network formation process.

Formally, the construction of the moments can be described as follows. Let \( M^i_t \) be an empirical moment \( i \) measured for a network of trades observed on day \( t \), where \( t = 1, \ldots, T \). Let \( m^i_{tk} (\Theta) \) be a simulated moment \( i \) measured for the equilibrium network of trades in architecture \( k \) on day \( t \), \( k = 1, \ldots, K \), and \( t = 1, \ldots, T \). It means that the model is simulated \( K \) times and each time has \( T \) trading days. The simulated moments depend on the vector of structural parameters \( \Theta \). The empirical moment \( i \) is defined as \( \hat{M}^i = \frac{1}{T} \sum_{t=1}^{T} M^i_t \) and the simulated moment is computed as \( \hat{m}^i (\Theta) = \frac{1}{K} \sum_{k=1}^{K} \left( \frac{1}{T} \sum_{t=1}^{T} m^i_{tk} (\Theta) \right) \). Then, \( \hat{M} = [\hat{M}^1 \ldots \hat{M}^I]' \) is a column vector of \( I \) empirical moments, and \( \hat{m} (\Theta) = [\hat{m}^1 (\Theta) \ldots \hat{m}^I (\Theta)]' \) is a column vector of the simulated moments.
F Contagion Dynamics

In this section, I provide formulas for the contagion dynamics triggered by the failure of a bank. Let \( X \) be an \( n \times n \) matrix of endogenous exposures between banks, where \( x_{ij} \) is an exposure of bank \( i \) to bank \( j \). Let \( F(t) \) be a set of banks that failed at round \( t \) of the cascade. Then we can define \( B(t) = \sum_{l=0}^{t} F(l) \) as the set of banks that failed by the end of round \( t \). Without loss of generality, assume that bank \( j \) fails and triggers a cascade. Formally, \( F(0) = j \) and \( B(0) = j \). In the first round of defaults, the counterparties of bank \( j \) fail if they have exposure above \( r \) to bank \( j \). The interpretation of \( r \) is a liquidity buffer that banks are required to hold against their loan portfolio to other banks. Formally, \( F(1) = \{ i | x_{ij} > r \} \). Even within the same round of contagion, bank failures happen at different times because the 24-hours loans that bank provided are not all due at the same time. Therefore, for the second round of contagion I consider two scenarios. In the first scenario, banks have access to interim liquidity that allows them to replenish their liquidity buffer as they suffer losses from the failure of their counterparties. In this case, the contagion propagates according to the following recursive formula

\[
F(t) = \{ i | x_{ij} > r, j \in F(t-1) \}. \tag{9}
\]

Contagion ends at time \( T \) if \( F(T) = \emptyset \) or if \( |B(T)| = n \), meaning that the size of the set of all failed banks is equal to the total number of banks in the financial architecture.

The second scenario assumes that banks do not have access to interim liquidity and can only rely on their pre-crisis liquidity buffer to absorb all the losses from contagion. In this case the set of failed banks changes as follows

\[
F(t) = \{ i | \sum_{j \in B(t-1)} x_{ij} > r \}. \tag{10}
\]

Contagion ends when either no further failures happen or after all banks have failed.

G Analytical Solution for Exposures in the Six Banks Example

In this section, I solve for exposures between the six banks in Figure 8. An exposure of bank \( i \) to bank \( j \) is defined as the ratio of loans provided by \( i \) to \( j \) divided by all loans provided by \( i \). I make three assumptions: (i) each seller receives the full surplus in each trade; (ii) each bank is equally likely to have the highest private value; and (iii) all banks have the same probability of receiving an endowment of one unit of liquidity. The first assumption ensures that the equilibrium trading path will be from the bank with the endowment to the bank with the highest private value. The second assumption ensures that each bank is equally likely to be the destination of this trading
path. The third assumption ensures that each bank is equally likely to be the origin of this trading path and that the volume of trade is the same across all trading paths. With 6 banks, 30 trading paths (6 possible sellers and 5 different buyers for each seller) are observed in equilibrium. To compute the exposure of bank $i$ to bank $j$, we need to count the number of equilibrium trading paths that include a link from $i$ to $j$ and divide it by the number of trading paths that include a link from $i$ to some other bank.

In the top figure with a cap of 5, each periphery bank has an exposure of 100% to the bank in the core because this is the only counterparty with which they trade. Bank in the center is equally likely to sell its endowment to each of the periphery banks because of the second assumption. The optimal trading decision of this bank does not depend on whether it sells its own endowment or acts as an intermediary. Therefore, the expected exposure to the periphery banks is 20%.

In the second architecture, each bank can trade with no more than three counterparties. There are four periphery banks connected to two core banks in this architecture. Each periphery bank trades only with banks 1 or 2, meaning that its exposure to one of the core banks is 100%. The architecture is symmetric (relabeling the names of banks 1 and 2 would not change the architecture). Combining this fact with assumptions (ii) and (iii) ensures that the inter-core exposure between banks 1 and 2 is the same. The symmetry produced by the assumptions also means that the exposure of core banks to each of the periphery banks connected to them should be the same. Without loss of generality, I solve for the exposure of bank 1 to banks 2 and 3. Bank 1 provides a loan to bank 2 when banks 1, 2, and 3 have an endowment and banks 2, 5, and 6 have the highest private value. So a link from bank 1 to bank 2 is utilized in 9 out of the 30 trading paths. A link from bank 1 to bank 3 is utilized when bank 3 has the highest private value and other banks have an endowment, which happens in 5 trading paths. Let exposure from bank 1 to bank 3 be equal to $x$, then we have the following equation to solve: $x + x + \frac{9}{5}x = 1$. This equation says that exposures from bank 1 to banks 3, 4, and 2 add up to 100%. Solving this equation gets us $x = \frac{5}{19}$ or 26.3%. If the exposure between banks 1 and 3 is 26.3%, then the exposure between banks 1 and 2 is 47.4%. The rest of the exposures in this architecture follow because of the symmetry.

In the third architecture, each bank can trade with no more than two counterparties (bottom of figure 8). There are two periphery banks with an exposure of 100% to their only counterparty. I solve for the exposure of bank 1 (bank 2 is symmetric). Bank 1 provides a loan to bank 2 when bank 2, 4, or 6 has the highest private value and bank 5, 3, or 1 has an endowment. That means in 9 out of 30 trading paths. Bank 1 provides a loan to bank 3 when bank 3 or 5 has the highest private value, and bank 1, 2, 4, or 6 has an endowment. In these cases, this link is utilized in 8 trading paths. Let $x$ be the exposure between bank 1 and 3, then we need to solve the following equation: $x + \frac{9}{5}x = 1$ or $x = \frac{8}{17}$. If the exposure between banks 1 and 3 is 47% then the exposure
between banks 1 and 2 is 53%. Finally, I solve for bank 3’s exposure to banks 1 and 5. Bank 3 provides a loan to bank 1 when bank 1, 2, 4 or 6 has the highest private value and bank 5 or 3 has an endowment, which is the case in 8 out of the 30 possible trading paths. Bank 3 provides a loan to bank 5 when this bank has the highest private value and five other banks have an endowment, which is 5 out of the 30 trading paths. Let exposure of bank 3 to bank 5 be \( y \), then we need to solve the following equation: \( y + \frac{8}{5}y = 1 \). The solution is \( y = \frac{5}{13} \), which means that 38% of bank’s 3 portfolio are loans to bank 5 and 62% are loans to bank 1. The rest of the exposures in this architecture follow because of the symmetry.
H Appendix: Tables

Table A1: Steps to Compute Welfare Measures

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 1</td>
<td>Draw a network of 986 banks with 12 core banks</td>
</tr>
<tr>
<td>Step 2</td>
<td>Draw a vector of private values</td>
</tr>
<tr>
<td>Step 3</td>
<td>Compute optimal trading decisions and equilibrium allocation for each endowment</td>
</tr>
<tr>
<td>Step 4</td>
<td>Compute welfare measures for every initial allocation</td>
</tr>
<tr>
<td>Step 5</td>
<td>Average welfare measures across different initial allocations</td>
</tr>
<tr>
<td>Step 6</td>
<td>Repeat steps 2-5 78 times and average welfare measures across draws of private values</td>
</tr>
<tr>
<td>Step 7</td>
<td>Repeat steps 1-6 1000 times and average welfare measures across different realizations of network draws</td>
</tr>
</tbody>
</table>

Table A2: Steps to Compute Stability Measures

<table>
<thead>
<tr>
<th>Step 1</th>
<th>Draw a network of 986 banks with 12 core banks.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 2</td>
<td>Draw a vector of private values.</td>
</tr>
<tr>
<td>Step 3</td>
<td>Compute optimal trading decisions and equilibrium allocation for each endowment.</td>
</tr>
<tr>
<td>Step 4</td>
<td>Compute the network of endogenous exposures for a single trading day.</td>
</tr>
<tr>
<td>Step 5</td>
<td>Assume the most interconnected bank fails.</td>
</tr>
<tr>
<td>Step 6</td>
<td>Fail all banks that have exposure above the default threshold to this bank.</td>
</tr>
<tr>
<td>Step 7a</td>
<td>Fail all banks that have exposure above the default threshold to a failed counterparty.</td>
</tr>
<tr>
<td>Step 7b</td>
<td>Fail all banks that have exposure above the default threshold to all failed counterparties.</td>
</tr>
<tr>
<td>Step 8</td>
<td>Repeat steps 7a and 7b until there are no bank failures.</td>
</tr>
<tr>
<td>Step 9</td>
<td>Compute how many banks fail and welfare measures in the survived network.</td>
</tr>
<tr>
<td>Step 10</td>
<td>Repeat steps 1-9 1000 times and average stability measures across different realizations of network draws.</td>
</tr>
</tbody>
</table>
Figure A1: Robustness of the Pre-crisis ESL with Respect to $s$

The figure presents the expected surplus loss (ESL) in eight different architectures for $s = \{11, 12, 13\}$. The ESL for each architecture is an average of surplus losses in 1000 networks generated by the preferential attachment process with a parameter $s$. Two standard error bounds are reported as bars around the point estimates.
Figure A2: Robustness of Stability Measures with Respect to $s$

The figure presents four post crisis measures of stability for $s = \{11, 12, 13\}$. The left column assumes that a bank fails when its exposure to a failed counterparty is above 15%. The right column assumes that a bank fails when its exposure to all failed counterparties is above 15%. Each measure is an average over 1000 network draws with a parameter $s$. Two standard error bounds are reported as bars around the point estimates.
The figure presents how the post-crisis ESL and the number of bank failures change when the number of draws of private values per day, estimated at 78, increases to 100 or decreases to 56. The contagion is triggered by the failure of one of the most interconnected banks. Banks fail if they have exposure above 15% to a failed counterparty. The efficiency measures are computed for the remaining financial architecture after the cascade of failures stops. Each endogenous contagion scenario was computed for 1000 network draws. Two standard error bounds are reported as bars around the point estimates.
Figure A4: Robustness of the Pre-crisis ESL with Respect to the Distribution of Private Values

The figure presents how the pre-crisis ESL changes when, instead of the uniform distribution, private values are drawn from a beta distribution with $\alpha = \beta = 2$ or $\alpha = \beta = 0.5$. The ESL for each architecture is an average of surplus losses in 1000 networks generated by the estimated preferential attachment process. Two standard error bounds are reported as bars around the point estimates.
The figure presents how the post-crisis ESL and the number of bank failures change when, instead of the uniform distribution, private values are drawn from a beta distribution with $\alpha = \beta = 2$ or $\alpha = \beta = 0.5$. The contagion is triggered by the failure of one of the most interconnected banks. Banks fail if they have exposure above 15% to a failed counterparty. The efficiency measures are computed for the remaining financial architecture after the cascade of failures stops. Each endogenous contagion scenario was computed for 1000 network draws. Two standard error bounds are reported as bars around the point estimates.
The figure presents the maximum number of bank failures triggered by the failure of a single bank. The calculation is repeated 1000 times. The mean and two standard error bounds are reported. If banks cannot receive interim liquidity as they face a cascade, then all banks fail in the worst case scenario regardless of the architecture. A failure is triggered when bank's aggregate losses to all failed counterparties are above 15% of its total loan portfolio. If banks can receive interim liquidity as they face the failures of counterparties, then a bank fails when its exposure to a single failed counterparty is above 15%. Two standard error bounds are reported as bars around the point estimates.
The figure presents the average cascade size computed by sequentially failing each bank and averaging the size of the cascade that this failure triggers across banks. The average cascade size represents the expected number of bank failures when each bank has the same probability of failure. This calculation is repeated 1000 times, and means are reported for each architecture. With interim liquidity, a bank fails when its exposure to a failed counterparty is above 15%. Without interim liquidity, a bank fails when its exposure to all failed counterparties is above 15%. Two standard error bounds are reported as bars around the point estimates.