Dynamic Valuation in Macroeconomic Models with Financing Restrictions

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Impulse Problem

Ragnar Frisch (1933):

There are several alternative ways in which one may approach the impulse problem .... One way which I believe is particularly fruitful and promising is to study what would become of the solution of a determinate dynamic system if it were exposed to a stream of erratic shocks that constantly upsets the continuous evolution ...

Irving Fisher (1930):

The manner in which risk operates upon time preference will differ, among other things, according to the particular periods in the future to which the risk applies.
Risk-Return Tradeoffs

Dynamic asset pricing through altering cash flow exposure to shocks.

- Study implication on the price today of changing the exposure tomorrow on a cash flow at some future date.
- Represent shock price elasticities by normalizing the exposure and studying the impact on the logarithms of the expected returns.
- Construct pricing counterpart to impulse response functions.
Outline

- VAR Analysis and Campbell-Shiller Decompostion
- Stochastic Discount Factors
- Dynamic Value Decomposition
- VAR’s reconsidered
- Market Segmentation and Financial Frictions
- Long-term Valuation and Permanent Shocks
Log-linear Decomposition

Let $R_{t+1}$ be a date $t + 1$ gross return on an asset with dividend process $G$ and date $t$ price $P_t$.

$$\log R_{t+1} = \log \left( \frac{P_{t+1} + G_{t+1}}{P_t} \right)$$

$$= \log \left( \frac{P_{t+1}}{G_{t+1}} + 1 \right) + (\log G_{t+1} - \log G_t) - (\log P_t - \log G_t)$$

Approximate: $\log \left( \frac{P_{t+1}}{G_{t+1}} + 1 \right)$ around a constant value for $\log \frac{P}{G} = \log \bar{p}$:

$$\log \left( \frac{P_{t+1}}{G_{t+1}} + 1 \right) = \text{constant} + \lambda (\log P_{t+1} - \log G_{t+1})$$

where $\lambda = \frac{1}{\bar{p}+1}$
Log-linear Decomposition

*Ex post relation:*

\[ \log P_t - \log G_t = \text{constant} + \sum_{j=0}^{\infty} \lambda^j \left( \log G_{t+1+j} - \log G_{t+j} - \log R_{t+1+j} \right) \]

- Future shocks to dividends must be offset by future shocks to returns
- Discounted (by \( \lambda \)) impulse responses to cash flow growth offset by the discounted impulse responses to returns
- Requires the identification of shocks - typically use VAR methods
Log-linear Decomposition

Ex ante relation:

\[ \log P_t - \log G_t = \text{constant} \]

\[ + \sum_{j=0}^{\infty} \lambda^j E \left( \log G_{t+1+j} - \log G_{t+j} - \log R_{t+1+j} \mid \mathcal{F}_t \right) \]

- Express logarithm of the dividend-price ratio as the sum of two correlated components
  - Expected discounted dividend growth
  - Expected discounted returns
- Primitive shocks may effect both components
- Expected discounted returns depend on exposure to shocks and price of that exposure
Models of Asset Valuation

Two channels:

- **Stochastic growth** modeled as a process $G = \{G_t\}$ where $G_t$ captures growth between dates zero and $t$.

- **Stochastic discounting** modeled as a process $S = \{S_t\}$ where $S_t$ assigns risk-adjusted prices to cash flows at date $t$.

Date zero prices of a payoff $G_t$ are

$$\pi = E (S_t G_t | \mathcal{F}_0)$$

where $\mathcal{F}_0$ captures current period information.

Stochastic discounting reflects investor preferences through the intertemporal marginal rate of substitution for marginal investors.
Risk-Return Tradeoffs

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Elasticities

Counterparts to impulse response functions pertinent to valuation:

- shock-exposure elasticities
- shock-price elasticities

These are the ingredients to risk premia, and they have a term structure induced by the changes in the investment horizons.

Hansen-Borovička (forthcoming in Handbook of Macroeconomics) with many references
Proportional Risk Premia

Recall: Date zero price of a payoff $G_t$ is

$$\pi = E(S_t G_t|\mathcal{F}_0)$$

$$r_{\text{prem}} = \underbrace{\log E(G_t|\mathcal{F}_0)}_{\text{log expected return}} - \underbrace{\log E(S_t G_t|\mathcal{F}_0)}_{\text{log expected return}} + \underbrace{\log E(S_t|\mathcal{F}_0)}_{\text{minus log riskfree return}}$$

Trace out a risk-return frontier by changing payoff $G_t$ by altering the risk exposure. Characterize market compensations for risk exposure.
Discrete-time model

Ingredients:

- $W$ is a vector of shocks
- State evolution:
  \[ X_{t+1} = \psi(X_t, W_{t+1}) \]
- Multiplicative process $M$ evolution:
  \[ \log M_{t+1} - \log M_t = \phi(X_t, W_{t+1}) \]

Comments:

- $\log M$ has stationary increments
- Use $M$ to model stochastic discount factor $S$ and macroeconomic growth $G$. 
Special structure

- Shock vector $W_{t+1}$ is a multivariate standard normal
- $M$ evolution:

$$\log M_{t+1} - \log M_t = \beta_m(X_t) + \alpha_m(X_t) \cdot W_{t+1}$$
One-period shock elasticities

Parameterize a family of random variables $H_1 (r)$

$$\log H_1 (r) = r \eta (X_0) \cdot W_1 - \frac{r^2}{2} |\eta (X_0)|^2$$

where $r$ is an auxiliary scalar parameter. The vector of exposures $\eta (X_0)$ is normalized to

$$E \left[ |\eta (X_0)|^2 \right] = 1.$$ 

By construction:

$$E [H_1 (r) |X_0] = 1.$$
Stochastic perturbations continued

- Form a parameterized family of payoffs $G_1H_1(r)$:

$$\log M_1 - \log M_0 + \log H_1(r) = [\alpha_m(X_0) + r\eta(X_0)] \cdot W_1 + \beta_m(X_0) - \frac{r^2}{2} |\eta(X_0)|^2.$$

- Differentiate with respect to $r$ and evaluate at $r = 0$ to obtain the one-period shock elasticity

$$\varepsilon_m(x,1) = \frac{d}{dr} \log E \left[ \left( \frac{M_1}{M_0} \right) H_1(r) \mid X_0 = x \right] \bigg|_{r=0} = \alpha_m(x) \cdot \eta(x)$$
Shock elasticities

- The one-period shock elasticity

\[ \varepsilon_m (x, 1) = \frac{d}{dr} \log E \left[ \left( \frac{M_1}{M_0} \right) H_1 (r) \mid X_0 = x \right] \bigg|_{r=0} = \alpha_m (x) \cdot \eta (x) \]

- The \( t \) period elasticity replaces \( M_1 \) with \( M_t \) but continues to use \( H_1 (r) \):

\[ \varepsilon_m (x, t) = \frac{d}{dr} \log E \left[ \left( \frac{M_1}{M_0} \right) H_1 (r) \mid X_0 = x \right] \bigg|_{r=0} = \eta (x) \cdot \frac{E \left[ \left( \frac{M_t}{M_0} \right) W_1 \mid X_0 = x \right]}{E \left[ \left( \frac{M_t}{M_0} \right) \mid X_0 = x \right]} \]
Shock elasticity types

○ Shock exposure

\[ \varepsilon_g (x, t) = \eta (x) \cdot \frac{E \left[ \left( \frac{G_t}{G_0} \right) W_1 \mid X_0 = x \right]}{E \left[ \left( \frac{G_t}{G_0} \right) \mid X_0 = x \right]} \]

○ Shock price

\[ \varepsilon_p (x, t) = \eta (x) \cdot \frac{E \left[ \left( \frac{G_t}{G_0} \right) W_1 \mid X_0 = x \right]}{E \left[ \left( \frac{G_t}{G_0} \right) \mid X_0 = x \right]} - \eta (x) \cdot \frac{E \left[ \left( \frac{S_t G_t}{S_0 G_0} \right) W_1 \mid X_0 = x \right]}{E \left[ \left( \frac{S_t G_t}{S_0 G_0} \right) \mid X_0 = x \right]} \]
Observations

- Continuous-time **diffusion** limits are well behaved
- Allow alternative shock distributions. An analogous approach applies but need to construct a **meaningful** way to **denominate** risk compensation
- Incorporate **jump** processes (continuous time) or **Markov regime shift** models (discrete time) with an interpretable perturbation and implied compensation
- **Global** compensations require **weighted** integrals
VAR example

- **State evolution:**
  \[ X_{t+1} = \bar{\mu}X_t + \bar{\sigma}W_{t+1}. \]
  where the absolute values of eigenvalues of the matrix \( \bar{\mu} \) are strictly less than one.

- **Multiplicative process evolution:**
  \[
  \log M_{t+1} - \log M_t = \bar{\nu} + \bar{\beta} \cdot X_t + \bar{\alpha} \cdot W_{t+1}. \tag{1}
  \]
  The shock \( W_{t+1} \) is distributed as a multivariate standard normal.

- **Impulse response recursions:**
  \[
  \bar{\varrho}_{t+1} - \bar{\varrho}_t = (\bar{\zeta}_t)' \bar{\beta}
  \]
  with initial condition \( \bar{\varrho}_1 = \bar{\alpha} \),
  \[
  \bar{\zeta}_{t+1} = \bar{\mu} \bar{\zeta}_t
  \]
  with initial condition \( \bar{\zeta}_1 = \bar{\sigma} \).
Moving-average representation

Write $\log M_t$ as:

$$\log M_t - \log M_0 = \sum_{j=1}^{t} \bar{\rho}_j \cdot W_{t-j+1} + E (\log M_t - \log M_0 \mid X_0)$$

$$= \sum_{j=1}^{t-1} \bar{\rho}_j \cdot W_{t-j+1} + \bar{\rho}_t \cdot W_1$$

$$+ E (\log M_t - \log M_0 \mid X_0).$$

where we have purposely separated out the date 1 term.
Shock elasticity computation

\[ E \left[ \left( \frac{M_t}{M_0} \right) \mid X_0 = x, W_1 = w \right] = \exp \left( \frac{1}{2} \sum_{j=1}^{t-1} \bar{\varrho}_j \cdot \bar{\varrho}_j \right) \]
\[ \times \exp (\bar{\varrho}_t \cdot W_1) \]
\[ \times \exp (E \left[ \log M_t - \log M_0 \mid X_0 \right]) \]

\[ \varepsilon (x, t) = \frac{E \left[ \left( \frac{M_t}{M_0} \right) W_1 \mid X_0 = x \right]}{E \left[ \left( \frac{M_t}{M_0} \right) \mid X_0 = x \right]} = \frac{E \left[ \exp (\bar{\varrho}_t \cdot W_1) W_1 \right]}{E \left[ \exp (\bar{\varrho}_t \cdot W_1) \right]} = \bar{\varrho}_t. \]

Note that following random variable is positive and has a unit expectation.

\[ \frac{\exp (\bar{\varrho}_t \cdot W_1)}{E \left[ \exp (\bar{\varrho}_t \cdot W_1) \right]} \]
Shock elasticities (lognormal)

- Shock elasticities coincide with the impulses responses measured by $\bar{\eta} \cdot \bar{\varrho}_t$ for $t = 1, 2, \ldots$ where $\bar{\eta}$ selects the shock under consideration.
- Shock exposure elasticities are $\bar{\eta} \cdot \bar{\varrho}_{g,t}$ computed with $M = G$.
- Shock price elasticity consists of the difference of shock elasticities for $G$ and $SG$. The additivity of the construction implies that the shock-price elasticities are $-\bar{\eta} \cdot \bar{\varrho}_{s,t}$. 
Stationary Density for $X_t$

$X$ is the relative wealth of specialists for He-Krishnamurthy (2013)
Shock-Price Elasticity for $C_t$

He-Krishnamurthy
Stationary Density for $X_t$

$X$ is the wealth share of the experts in Brunnermeier-Sannikov (2014)
Shock-Price Elasticity for $C^a_t$
Two martingale constructions

- **Multiplicative factorization**

  \[
  \frac{M_t}{M_0} = \exp(\nu t) \frac{H_t}{H_0} \left[ \frac{f(X_t)}{f(X_0)} \right]
  \]

  where \( H \) is a positive martingale and \( \log H \) has stationary increments. Valuable for characterizing long-term risk return tradeoffs. Alvarez-Jermann (2005) and Hansen-Scheinkman (2009)

- **Additive decomposition**

  \[
  \log M_t - \log M_0 = \eta^* t + (\log H^*_t - \log H^*_0) + [g(X_t) - g(X_0)]
  \]

  where \( \log H^* \) is a martingale with stationary increments. Valuable for identifying permanent shocks.

  For a discussion of the comparison see Hansen (2012).