Dynamic Valuation in

in Macroeconomic Models

with Financing Restrictions

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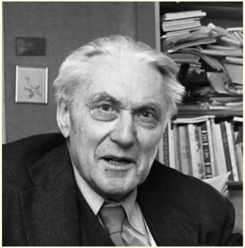
Impulse Problem



Ragnar Frisch (1933):

There are several alternative ways in which one may approach the impulse problem .... One way which I believe is particularly fruitful and promising is to study what would become of the solution of a determinate dynamic system if it were exposed to a stream of erratic shocks that constantly upsets the continuous evolution ... .

Irving Fisher (1930):

The manner in which risk operates upon time preference will differ, among other things, according to the particular periods in the future to which the risk applies.

Risk-Return Tradeoffs

Dynamic asset pricing through altering cash flow exposure to shocks.

- Study implication on the price today of changing the exposure tomorrow on a cash flow at some future date.
- Represent shock price elasticities by normalizing the exposure and studying the impact on the logarithms of the expected returns.
- Construct pricing counterpart to impulse response functions.

Outline

- VAR Analysis and Campbell-Shiller Decompostion
- Stochastic Discount Factors
- Dynamic Value Decomposition
- VAR’s reconsidered
- Market Segmentation and Financial Frictions
- Long-term Valuation and Permanent Shocks

Log-linear Decomposition

Let R_{t+1} be a date $t+1$ gross return on an asset with dividend process G and date t price P_t.

$$\log R_{t+1} = \log\left(\frac{P_{t+1}+G_{t+1}}{P_t}\right)$$
$$= \log\left(\frac{P_{t+1}}{G_{t+1}}+1\right) + (\log G_{t+1} - \log G_t) - (\log P_t - \log G_t)$$

Approximate: $\log\left(\frac{P_{t+1}}{G_{t+1}}+1\right)$ around a constant value for $\log\frac{P}{G} = \log\bar{p}$:

$$\log\left(\frac{P_{t+1}}{G_{t+1}}+1\right) = \text{constant} + \lambda\left(\log P_{t+1} - \log G_{t+1}\right)$$

where $\lambda = \frac{1}{\bar{p}+1}$

Log-linear Decomposition

Ex post relation:

$$\log P_t - \log G_t = \text{constant} + \sum_{j=0}^{\infty} \lambda^j \left(\log G_{t+1+j} - \log G_{t+j} - \log R_{t+1+j} \right)$$

- Future shocks to dividends must be offset by future shocks to returns
- Discounted (by λ) impulse responses to cash flow growth offset by the discounted impulse responses to returns
- Requires the identification of shocks - typically use VAR methods

Log-linear Decomposition

Ex ante relation:

$$\log P_t - \log G_t = \text{constant} + \sum_{j=0}^{\infty} \lambda^j E\left(\log G_{t+1+j} - \log G_{t+j} - \log R_{t+1+j} | \mathcal{F}_t\right)$$

- Express logarithm of the dividend-price ratio as the sum of two correlated components
  - Expected discounted dividend growth
  - Expected discounted returns
- Primitive shocks may effect both components
- Expected discounted returns depend on exposure to shocks and price of that exposure

Models of Asset Valuation

Two channels:

- Stochastic growth modeled as a process $G = \{G_t\}$ where G_t captures growth between dates zero and t.
- Stochastic discounting modeled as a process $S = \{S_t\}$ where S_t assigns risk-adjusted prices to cash flows at date t.

Date zero prices of a payoff G_t are

$$\pi = E\left(S_t G_t | \mathcal{F}_0\right)$$

where $\mathcal{F}_0$ captures current period information.

Stochastic discounting reflects investor preferences through the intertemporal marginal rate of substitution for marginal investors.

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Elasticities

Counterparts to impulse response functions pertinent to valuation:

- shock-exposure elasticities
- shock-price elasticities

These are the ingredients to risk premia, and they have a term structure induced by the changes in the investment horizons.

Hansen-Borovička (forthcoming in Handbook of Macroeconomics) with many references

Proportional Risk Premia

Recall: Date zero price of a payoff G_t is

$$\pi = E\left(S_t G_t | \mathcal{F}_0\right)$$

$$\text{rprem} = \underbrace{\log E\left(G_t | \mathcal{F}_0\right) - \log E\left(S_t G_t | \mathcal{F}_0\right)}_{\text{log expected return}} + \underbrace{\log E\left(S_t | \mathcal{F}_0\right)}_{\text{minus log riskfree return}}$$

Trace out a risk-return frontier by changing payoff G_t by altering the risk exposure. Characterize market compensations for risk exposure.

Discrete-time model

Ingredients:

- W is a vector of shocks
- State evolution:

$$X_{t+1} = \psi(X_t, W_{t+1})$$

- Multiplicative process M evolution:

$$\log M_{t+1} - \log M_t = \phi(X_t, W_{t+1})$$

Comments:

$\log M$ has stationary increments

Use M to a model stochastic discount factor S and macroeconomic growth G.

Special structure

- Shock vector W_{t+1} is a multivariate standard normal
- M evolution:

$$\log M_{t+1} - \log M_t = \beta_m(X_t) + \alpha_m(X_t) \cdot W_{t+1}$$

One-period shock elasticities

Parameterize a family of random variables $H_1(\mathsf{r})$

$$\log H_1(\mathsf{r}) = \mathsf{r}\eta(X_0) \cdot W_1 - \frac{\mathsf{r}^2}{2} |\eta(X_0)|^2$$

where r is an auxiliary scalar parameter. The vector of exposures $\eta(X_0)$ is normalized to

$$E\left[|\eta(X_0)|^2\right] = 1.$$

By construction:

$$E[H_1(\mathsf{r}) | X_0] = 1.$$

Stochastic perturbations continued

- Form a parameterized family of payoffs $G_1 H_1(\mathsf{r})$:

$$\log M_1 - \log M_0 + \log H_1(\mathsf{r}) = [\alpha_m(X_0) + \mathsf{r}\eta(X_0)] \cdot W_1 + \beta_m(X_0) - \frac{\mathsf{r}^2}{2} |\eta(X_0)|^2 .$$

- Differentiate with respect to r and evaluate at $\mathsf{r} = 0.$ to obtain the one-period shock elasticity

$$\varepsilon_m(x,1) = \frac{d}{d\mathsf{r}} \log E\left[\left(\frac{M_1}{M_0}\right) H_1(\mathsf{r}) \mid X_0 = x\right]\Bigg|_{\mathsf{r}=0} = \alpha_m(x) \cdot \eta(x)$$

Shock elasticities

- The one-period shock elasticity

$$\varepsilon_m(x,1) = \frac{d}{d\mathsf{r}} \log E\left[\left(\frac{M_1}{M_0}\right) H_1(\mathsf{r}) \mid X_0 = x\right]\Bigg|_{\mathsf{r}=0}$$
$$= \alpha_m(x)\cdot\eta(x)$$

- The t period elasticity replaces M_1 with M_t but continues to use $H_1(\mathsf{r})$:

$$\varepsilon_m(x,t) = \frac{d}{d\mathsf{r}} \log E\left[\left(\frac{M_1}{M_0}\right) H_1(\mathsf{r}) \mid X_0 = x\right]\Bigg|_{\mathsf{r}=0}$$
$$= \eta(x)\cdot\frac{E\left[\left(\frac{M_t}{M_0}\right) W_1 \mid X_0 = x\right]}{E\left[\left(\frac{M_t}{M_0}\right) \mid X_0 = x\right]}$$

Shock elasticity types

- Shock exposure

$$\varepsilon_g(x,t) = \eta(x) \cdot \frac{E\left[\left(\frac{G_t}{G_0}\right) W_1 \mid X_0 = x\right]}{E\left[\left(\frac{G_t}{G_0}\right) \mid X_0 = x\right]}$$

- Shock price

$$\begin{aligned}\varepsilon_p(x,t) = &\eta(x) \cdot \frac{E\left[\left(\frac{G_t}{G_0}\right) W_1 \mid X_0 = x\right]}{E\left[\left(\frac{G_t}{G_0}\right) \mid X_0 = x\right]} \\ &- \eta(x) \cdot \frac{E\left[\left(\frac{S_t G_t}{S_0 G_0}\right) W_1 \mid X_0 = x\right]}{E\left[\left(\frac{S_t G_t}{S_0 G_0}\right) \mid X_0 = x\right]}\end{aligned}$$

Observations

- Continuous-time diffusion limits are well behaved
- Allow alternative shock distributions. An analogous approach applies but need to construct a meaningful way to denominate risk compensation
- Incorporate jump processes (continuous time) or Markov regime shift models (discrete time) with an interpretable perturbation and implied compensation
- Global compensations require weighted integrals

VAR example

- State evolution:

$$X_{t+1} = \bar{\mu} X_t + \bar{\sigma} W_{t+1}.$$

  where the absolute values of eigenvalues of the matrix $\bar{\mu}$ are strictly less than one.

- Multiplicative process evolution:

$$\log M_{t+1} - \log M_t = \bar{\nu} + \bar{\beta} \cdot X_t + \bar{\alpha} \cdot W_{t+1}. \tag{1}$$

  The shock W_{t+1} is distributed as a multivariate standard normal.

- Impulse response recursions:

$$\bar{\varrho}_{t+1} - \bar{\varrho}_t = \left(\bar{\zeta}_t\right)' \bar{\beta}$$

  with initial condition $\bar{\varrho}_1 = \bar{\alpha}$,

$$\bar{\zeta}_{t+1} = \bar{\mu} \bar{\zeta}_t$$

  with initial condition $\bar{\zeta}_1 = \bar{\sigma}$.

Moving-average representation

Write $\log M_t$ as:

$$\begin{aligned}\log M_t - \log M_0 &= \sum_{j=1}^{t} \bar{\varrho}_j \cdot W_{t-j+1} + E\left(\log M_t - \log M_0 \mid X_0\right) \\ &= \sum_{j=1}^{t-1} \bar{\varrho}_j \cdot W_{t-j+1} + \bar{\varrho}_t \cdot W_1 \\ &\quad + E\left(\log M_t - \log M_0 \mid X_0\right).\end{aligned}$$

where we have purposely separated out the date 1 term.

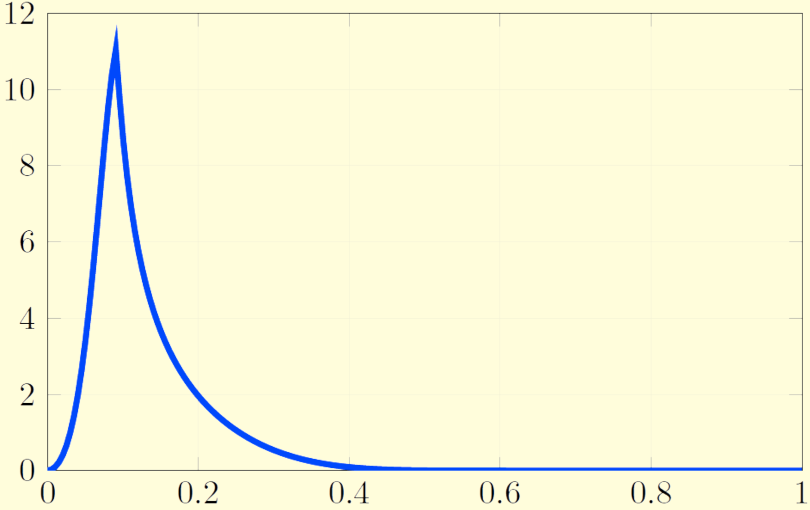
Shock elasticity computation

- $$E\left[\left(\frac{M_t}{M_0}\right) \mid X_0 = x, W_1 = w\right] = \exp\left(\frac{1}{2}\sum_{j=1}^{t-1} \bar{\varrho}_j \cdot \bar{\varrho}_j\right) \times \exp\left(\bar{\varrho}_t \cdot W_1\right) \times \exp\left(E\left[\log M_t - \log M_0 \mid X_0\right]\right)$$
- $$\varepsilon(x,t) = \frac{E\left[\left(\frac{M_t}{M_0}\right) W_1 \mid X_0 = x\right]}{E\left[\left(\frac{M_t}{M_0}\right) \mid X_0 = x\right]} = \frac{E\left[\exp\left(\bar{\varrho}_t \cdot W_1\right) W_1\right]}{E\left[\exp\left(\bar{\varrho}_t \cdot W_1\right)\right]} = \bar{\varrho}_t.$$
- Note that following random variable is positive and has a unit expectation.
$$\frac{\exp\left(\bar{\varrho}_t \cdot W_1\right)}{E\left[\exp\left(\bar{\varrho}_t \cdot W_1\right)\right]}$$

Shock elasticities (lognormal)

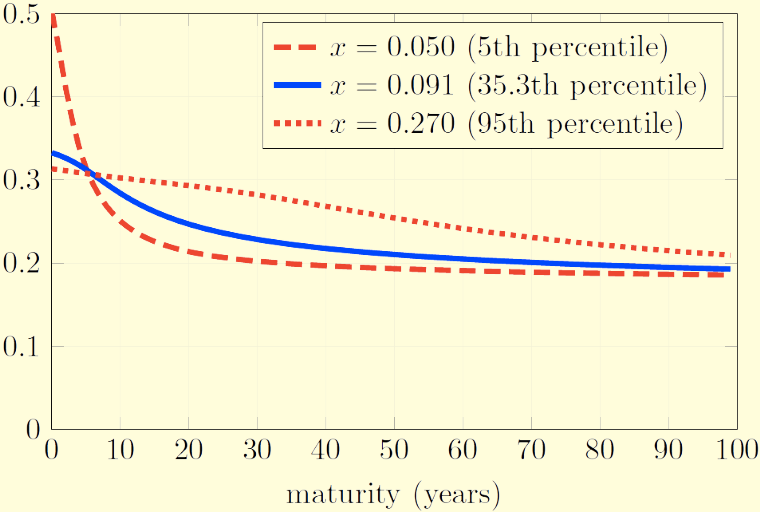
- Shock elasticities coincide with the impulses responses measured by $\bar{\eta} \cdot \bar{\varrho}_t$ for $t = 1, 2, \ldots$ where $\bar{\eta}$ selects the shock under consideration.
- Shock exposure elasticities are $\bar{\eta} \cdot \bar{\varrho}_{g,t}$ computed with $M = G$.
- Shock price elasticity consists of the difference of shock elasticities for G and SG. The additivity of the construction implies that the shock-price elasticities are $-\bar{\eta} \cdot \bar{\varrho}_{s,t}$.

Stationary Density for X_t



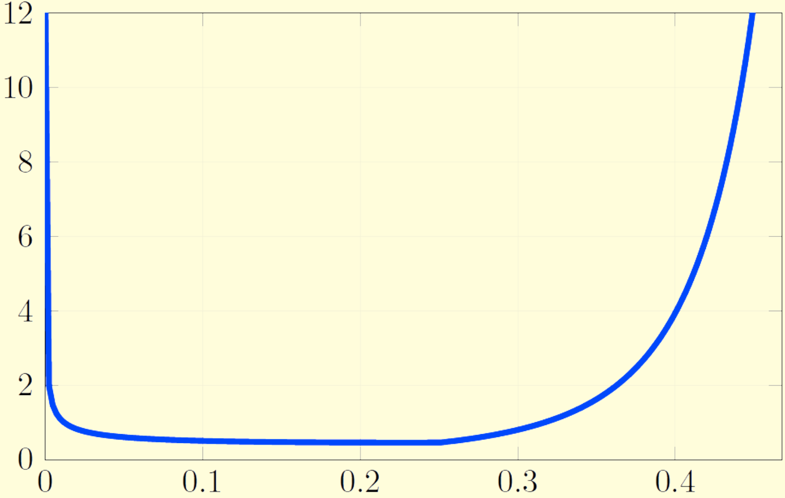
X is the relative wealth of specialists for He-Krishnamurthy (2013)

Shock-Price Elasticity for C_t



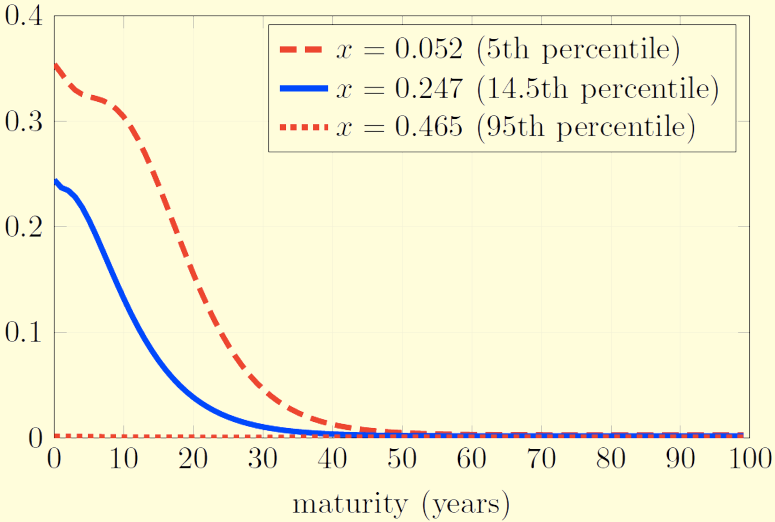
He-Krishnamurthy

Stationary Density for X_t



X is the wealth share of the experts in Brunnermeier-Sannikov (2014)

Shock-Price Elasticity for C_t^a



Brunnermeier-Sannikov

Two martingale constructions

- Multiplicative factorization

$$\frac{M_t}{M_0} = \exp(\nu t)\frac{H_t}{H_0}\left[\frac{f(X_t)}{f(X_0)}\right]$$

where H is a positive martingale and $\log H$ has stationary increments. Valuable for characterizing long-term risk return tradeoffs. Alvarez-Jermann (2005) and Hansen-Scheinkman (2009)

- Additive decomposition

$$\log M_t - \log M_0 = \eta^* t + (\log H_t^* - \log H_0^*) + [g(X_t) - g(X_0)]$$

where $\log H^*$ is a martingale with stationary increments. Valuable for identifying permanent shocks.

For a discussion of the comparison see Hansen (2012).