# Learning from Others' Outcomes

Alexander Wolitzky

MIT

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# Scarcity of Cost-Saving Innovation

Development and diffusion of new technologies is a fundamental driver of economic growth.

Some kinds of technologies are introduced and adopted at higher rates than others.

Often, innovations that (improve outcomes, increase costs) seem to be introduced and adopted more than innovations that (hurt outcomes, reduce costs).

# Overuse of Fertilizer in Kenya

Duflo, Kremer, and Robinson (2008):

| | Median % Increase in Yield | Median % Annualized Return | Obs. |
|---|---|---|---|
| 1/4 tsp. | 9 | −43 | 112 |
| 1/2 tsp. | 24 | 44 | 200 |
| 1 tsp. | 31 | −27 | 273 |
| 1 tsp.+hybrid seed (recommended) | 49 | −60 | 82 |

Why don't farmers learn to use less fertilizer?

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  - Good outcome is bad news about adoption.
  - As #(observations)$\rightarrow \infty$, effects exactly cancel out.
- No such problem with outcome-improving innovation.
- General rationale for scarcity of cost-saving innovations.

# Action-Based vs. Outcome-Based Learning

Most social learning models assume **action-based learning** (actions observable, outcomes unobservable).

- Natural assumption if actions are consumption choices, outcomes are subjective utility realizations.
  - Choosing between two restaurants.
  - Choosing among different brands of a good.
  - Investing in privately held companies.
  - Fads and fashions.

I assume **outcome-based learning** (outcomes observable, actions unobservable).

- Natural assumption if actions are input choices, outcomes are objective outputs.
  - Fertilizer usage/choice of seed variety.
  - Choice of medicine or surgical technique.
  - Choice of manufacturing process.

# Literature

**Herding:** Banerjee 1992, Bikhchandani-Hirshleifer-Welch 1992, Smith-Sørensen 2000

**Word-of-Mouth Learning:** Banerjee-Fudenberg 2000, Smith-Sørensen 2014

**Outcome-Based Learning:** McLennan 1984, Piketty 1995, Smith-Sørensen 2000

**Limited Memory:** Banerjee 1993, Acemoglu-Wolitzky 2014

**Herding and Collective Experimentation:** Murto-Välimäki 2011, Frick-Ishii 2016

# Outline

1. Model + Regular Equilibrium Existence
2. Main Results: Efficiency with Large Samples
3. Efficiency with Small Samples
4. Dynamics
5. Extensions/Conclusion

# Environment

2 states: $\Theta = \{0, 1\}$ ("innovation is cost-ineffective or cost-effective")
Prior $\Pr(\theta = 1) = p$.

2 actions: $X = \{0, 1\}$ ("use status quo or innovation")

2 outcomes: $Y = \{0, 1\}$ ("failure or success")

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Each player chooses action once.

Choose $x = 1$ in state $\theta \implies$ get $y = 1$ wp $\pi_\theta$, get $y = 0$ wp $1 - \pi_\theta$.
Assume $\pi_0 < \pi_1$.

Choose $x = 0 \implies$ get $y = 1$ wp $\chi$, get $y = 0$ wp $1 - \chi$.

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Choose $x = 0 \implies$ get $y = 1$ wp $\chi$, get $y = 0$ wp $1 - \chi$.

Payoff $y - cx$, where $c \in \mathbb{R}$ is cost of using innovation.
($c$ can be positive or negative.)

# Information

Before choosing her action, each players observes **outcomes** (not actions) of random sample of $K$ other players.

(Formalized below.)

# Assumptions

Always choosing status quo is not an eqm:

$$p\pi_1 + (1 - p)\,\pi_0 - c > \chi$$

(Equivalent to assuming $p > p^* :=$ cutoff belief where indifferent between actions.)

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Always choosing innovation is not an eqm:

$$\left(\frac{1-\pi_0}{1-\pi_1}\right)^K > \frac{p}{1-p}\frac{1-p^*}{p^*}$$

(If think everyone else experiments and see $K$ failures, prefer not to experiment.)

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Assumptions imply innovation optimal in good state, status quo optimal in bad state:

$$\pi_1 - c > \chi > \pi_0 - c$$

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$\chi < \pi_1$ (allows $c < 0$ or $c \geq 0$): **Outcome-improving innovation.**

- New crop variety that could give higher or lower yield.
- Expensive new medical technology that always gives better outcomes, but may or may not justify cost.

$\chi = 0$: **Pure** outcome-improving innovation.

- Planting pineapple rather than maize.

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$\chi > \pi_1$ (implies $c < 0$): **Cost-saving innovation.**

- Using less than standard amount of fertilizer.
- Just-in-time input delivery.
- Forgoing standard vaccine.

$\chi = 1$: **Pure** cost-saving innovation.

- Standard vaccine is perfectly effective.

# Notation/Terminology

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$$
\begin{aligned}
& p(k; x_0, x_1) \\
= & \left[ 1 + \frac{1-p}{p} \frac{[x_0 \pi_0 + (1 - x_0)\chi]^k [1 - x_0 \pi_0 - (1 - x_0)\chi]^{K-k}}{[x_1 \pi_1 + (1 - x_1)\chi]^k [1 - x_1 \pi_1 - (1 - x_1)\chi]^{K-k}} \right]^{-1}.
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$p(k; x_0, x_1)$ increasing in $k \iff \Pr(\mathit{success})$ higher in state 1:

$$x_1 (\pi_1 - \chi) \geq x_0 (\pi_0 - \chi).$$

Say a pair $(x_0, x_1)$ is **regular** if this holds; **irregular** otherwise.

- Regular $\iff$ success is good news.

# Equilibrium Paths: Informal Description

Equilibrium path models dynamic process where new players enter population at rate 1 and best-respond to random samples of outcomes generated by existing players.

Consists of, at each point in time:

1. Fraction of existing players experimenting in each state.
2. Cutoff number of successes above or below which new players experiment.
3. New players' mixing probability if see exactly cutoff number of successes.

# One More Piece of Notation

Probability that observe $k$ successes when state is $\theta$ and fraction $x$ experiment equals

$$\phi_\theta(k;x) = \binom{K}{k}\left[x\pi_\theta + (1-x)\chi\right]^k \left[1 - x\pi_\theta - (1-x)\chi\right]^{K-k}.$$

# Equilibrium Paths: Definition

### Definition

An *equilibrium path* is a list of measurable functions of time $(x_0, x_1, k^*, s)$ such that

1. Trajectories respect individual optimization: for $\theta = 0, 1$, $x_\theta$ is absolutely continuous, with derivative a.e given by

$$\dot{x}_\theta(t) = \begin{cases} \phi_\theta(k^*(t); x_\theta(t))\, s(t) + \sum_{k=k^*(t)+1}^{K} \phi_\theta(k; x_\theta(t)) - x_\theta(t) \\ \quad \text{if } (x_0(t), x_1(t)) \text{ regular} \\ \phi_\theta(k^*(t); x_\theta(t))\, s(t) + \sum_{k=0}^{k^*(t)-1} \phi_\theta(k; x_\theta(t)) - x_\theta(t) \\ \quad \text{if } (x_0(t), x_1(t)) \text{ irregular} \end{cases}$$

...

# Equilibrium Paths: Definition (cntd.)

2. Cutoffs are consistent with Bayes' rule.

$$p\left(k^*\left(t\right)-1; x_0\left(t\right), x_1\left(t\right)\right) \leq p^* \leq p\left(k^*\left(t\right)+1; x_0\left(t\right), x_1\left(t\right)\right)$$
$$\text{if } \left(x_0\left(t\right), x_1\left(t\right)\right) \text{ regular}$$
$$p\left(k^*\left(t\right)-1; x_0\left(t\right), x_1\left(t\right)\right) \geq p^* \geq p\left(k^*\left(t\right)+1; x_0\left(t\right), x_1\left(t\right)\right)$$
$$\text{if } \left(x_0\left(t\right), x_1\left(t\right)\right) \text{ irregular}$$

3. Decisions are optimal at the cutoff:

$$s\left(t\right)\left\{\begin{array}{cc} =1 & \text{if } p\left(k^*\left(t\right); x_0\left(t\right), x_1\left(t\right)\right) > p^* \\ =0 & \text{if } p\left(k^*\left(t\right); x_0\left(t\right), x_1\left(t\right)\right) < p^* \\ \in\left[0,1\right] & \text{if } p\left(k^*\left(t\right); x_0\left(t\right), x_1\left(t\right)\right) = p^* \end{array}\right\}.$$

# Existence of Equilibrium Paths

## Proposition

*For any point* $(\hat{x}_0, \hat{x}_1) \in [0,1]^2$*, there exists an equilibrium path* $(x_0, x_1, k^*, s)$ *with* $(x_0(0), x_1(0)) = (\hat{x}_0, \hat{x}_1)$.

## Proof.

Equilibrium paths correspond to solution to differential inclusion of form

$$(\dot{x}_0(t), \dot{x}_1(t)) \in F(x_0(t), x_1(t)),$$

where $F$ is non-empty, convex- and compact-valued, and upper hemi-continuous. Standard existence result applies. □

# Steady States

**Definition**

An *equilibrium* (or *steady state*) $(x_0, x_1, k^*, s)$ is a constant equilibrium path.

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**Definition**

An equilibrium $(x_0, x_1, k^*, s)$ is *regular* if $(x_0, x_1)$ is regular and *irregular* if $(x_0, x_1)$ is irregular.

# Main Result 1: Existence/Robustness of Regular Equilibrium

**Theorem**

*A regular equilibrium exists.*
*In addition, if* $(x_0, x_1)$ *is an equilibrium path and* $(x_0(0), x_1(0))$ *is regular, then* $(x_0(t), x_1(t))$ *is regular for all* $t \in \mathbb{R}_+$.

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Regularity of initial point is mild requirement.

- Holds if initial generation chooses based on prior.
- If give initial generation some exogenous information, holds as long as probability of very informative signal is low.

Justifies focusing on regular points when analyzing efficiency.

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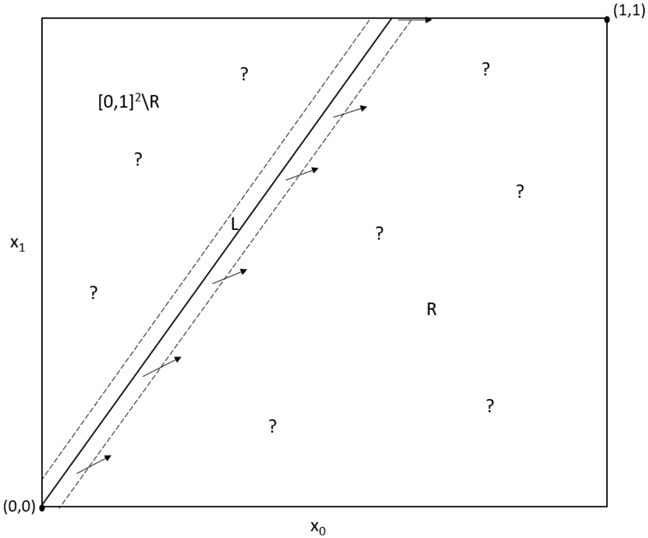
**Intuition:**

- Let $L$ be be boundary of regular region.
- Close to $L$, $\Pr(\textit{success})$ hardly depends on state, so observations uninformative.
- Observations uninformative $\implies$ follow prior $\implies$ experiment.
- Point where everyone experiments is regular.
- $\implies$ if population dynamic gets close to boundary of regular region, heads back toward interior.

# Proof by Picture



# Outline

1. Model + Regular Equilibrium Existence
2. **Main Results: Efficiency with Large Samples**
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# Efficiency

**Expected welfare** at point $(x_0, x_1)$ equals

$$p\left[x_1\left(\pi_1 - c\right) + \left(1 - x_1\right)\chi\right] + \left(1 - p\right)\left[x_0\left(\pi_0 - c\right) + \left(1 - x_0\right)\chi\right].$$

**First-best expected welfare** equals

$$p\left(\pi_1 - c\right) + \left(1 - p\right)\chi.$$

# Main Result 2: Inefficiency with Cost-Saving Innovation

## Theorem

*Assume $\chi > \pi_1$. For every $K$, expected welfare at every regular point is uniformly bounded away from efficiency.*
*Specifically, expected welfare falls short of the first-best by at least*

$$(1-p)\left(\frac{\chi-\pi_1}{\chi-\pi_0}\right)(\chi-\pi_0-c) > 0.$$

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- Efficient point $(0,1)$ is irregular: success rates are $\chi$ vs. $\pi_1$.
- No regular point can be close to $(0,1)$.

Combining Theorems 1+2, can't get close to efficiency starting from $(1,1)$ or other regular point.

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**Quantifying loss:**

- Proof shows most efficient regular point is $\left(\frac{\chi-\pi_1}{\chi-\pi_0},1\right)$.
- Completely uninformed player plays $(1,1)$.
- With cost-saving innovation, best-case scenario is don't mess up performance in good state, improve performance in bad state from wrong wp 1 to wrong wp $\frac{\chi-\pi_1}{\chi-\pi_0}$.
- Best-case scenario better when status quo worse, innovation less noisy.

# Main Result 3: Efficiency with Outcome-Improving Innovation

**Theorem**

*Assume* $\chi < \pi_1$. *Fix any sequence of equilibria* $(x_{0,K}, x_{1,K}, k_K^*, s_K)$ *indexed by* $K$. *Then* $\lim_{K\to\infty} x_{0,K} = 0$ *and* $\lim_{K\to\infty} x_{1,K} = 1$.

# Main Result 3: Efficiency with Outcome-Improving Innovation

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*Assume $\chi < \pi_1$. Fix any sequence of equilibria $(x_{0,K}, x_{1,K}, k^*_K, s_K)$ indexed by $K$. Then $\lim_{K\to\infty} x_{0,K} = 0$ and $\lim_{K\to\infty} x_{1,K} = 1$.*

- Rationality implies $x_1 \geq x_0$ and $x_1 \gg 0$.
- $\implies$ success rates $x_1(\pi_1 - \chi) + \chi$ and $x_0(\pi_0 - \chi) + \chi$ bounded away from each other when $\chi < \pi_1$.
- $\implies$ efficiency as $K \to \infty$ follows from LLN.

# Confounded Learning with Cost-Saving Innovation

How can players fail to learn state as $K \to \infty$?

# Confounded Learning with Cost-Saving Innovation

How can players fail to learn state as $K \to \infty$?

- Success rate must be same in both states.
- Steady state must lie on line $L$.
- Players can't distinguish between cases where
  - innovation reduces success rate a little and is widely adopted vs.
  - innovation reduces success rate a lot but is narrowly adopted.

**Proposition**

*Assume $\chi > \pi_1$. Fix any sequence of regular equilibria $(x_{0,K}, x_{1,K}, k_K^*, s_K)$ indexed by $K$. Then*

$$\lim_{K\to\infty} \frac{x_{0,K}(\chi - \pi_0)}{x_{1,K}(\chi - \pi_1)} = 1.$$

# Summary of Main Results

1. As long as initial adoption rates regular, population dynamic visits only regular points.
2. With cost-saving innovation, welfare at regular points uniformly bounded away from efficiency.
3. With outcome-improving innovation, equilibrium welfare $\longrightarrow$ first-best as $K \longrightarrow \infty$.
4. With cost-saving innovation, confounded learning as $K \longrightarrow \infty$.

"Hard to learn about cost-saving innovations, as not clear if success is good news or bad news."

- Success good news for some adoption rates; bad news for others.
- Sucess uninformative in regular eqm as $K \longrightarrow \infty$.

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# Additional Results: Small Samples

Is adoption of outcome-improving innovations also more efficient when $K$ is small?

No hope for efficient adoption unless $\pi_0 \to 0$ and $\pi_1 \to 1$
$\implies$ focus on this case.
(And hence also on pure outcome-improving/pure cost-saving cases.)

$K = 1$:

- Inefficiency in both cases.
- Under-adoption in pure outcome-improving case; over-adoption in all other cases.

Fixed $K > 1$:

- Efficiency in pure outcome-improving case.
- Persistent over-adoption in pure cost-saving case.

# Single Observation

- Adoption inefficient in both cases.
- Under-adoption in pure outcome-improving case; over-adoption in all other cases.

### Proposition

*Assume $K = 1$. Then,*

1. *Outside of the pure outcome-improving case, if $\pi_1 \longrightarrow 1$ then $x_1 \longrightarrow 1$ but $x_0 \not\rightarrow 0$. That is, the innovation is over-adopted.*
2. *In the pure outcome-improving case, if $\pi_1 \longrightarrow 1$ then $x_0 \longrightarrow 0$ but $x_1 \not\rightarrow 1$. That is, the innovation is under-adopted.*

# Intuition: Maize vs. Pineapple

- Suppose community plants traditional maize, faces introduction of either high-yield maize or pineapple.
- Innovation succeeds wp $\approx 1$ if soil condition is favorable.
- High-yield maize = impure outcome-improving innovation. (Traditional maize can yield large harvest.)
- Pineapple = pure outcome-improving innovation. (Traditional maize can't yield pineapple.)
- Result says high-yield maize over-adopted, pineapple under-adopted.

# Intuition: High-Yield Maize Case

- Farmers sometimes observe large harvest even if soil bad.
- Large harvest good news
  $\implies$ plant high-yield maize if see large harvest
  $\implies$ positive adoption rate even if soil bad.

- If soil good, high-yield maize almost always yields large harvest, traditional maize sometimes yields large harvest.
- Strategy of planting high-yield maize iff see large harvest
  $\implies$ in eqm plant high-yield maize wp $\approx 1$.

# Intuition: Pineapple Case

- If farmers never plant pineapple after observing no-pineapple, then rate of planting pineapple would fall to 0 in both states. (As maize can't yield pineapple.)
- $\implies$ farmers must mix after observing no-pineapple.
- For farmers to be indifferent after seeing no-pineapple, pineapple planting rate must be low enough that seeing no-pineapple isn't terrible news.
- $\implies$ pineapple under-adopted when soil good.
- But pineapple almost always succeeds when soil good. So probability of planting pineapple after seeing no-pineapple must be $\approx 0$.
- $\implies$ pineapple not planted when soil bad.

# Multiple Observations

- Efficient adoption in pure outcome-improving case.
- Persistent over-adoption in pure cost-saving case.

### Proposition

*Assume $K > 1$.*

1. *In the pure outcome-improving case, if $\pi_0 \longrightarrow 0$ and $\pi_1 \longrightarrow 1$ then along any sequence of equilibria $x_0 \longrightarrow 0$ and $x_1 \longrightarrow 1$.*
2. *In the pure cost-saving case, if $\pi_0 \longrightarrow 0$ and $\pi_1 \longrightarrow 1$ then along any sequence of regular equilibria $x_1 \longrightarrow 1$ but $x_0 \not\longrightarrow 0$.*

# Intuition

Why efficiency in pure outcome-improving case for any $K > 1$?

Suppose players experiment iff see at least one success.

Then eqm $x_1$ given by

$$x_1 = 1 - (1 - x_1 \pi_1)^K .$$

$K = 1$: for all $\pi_1 < 1$, only solution is $x_1 = 0$.

$K > 1$: for $\pi_1 \to 1$, admits positive solution converging to 1.

No mixing/indifference after failure required when $K > 1$.

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# Convergence to Equilibrium

Lack general proof that population dynamic converges to equilibrium.

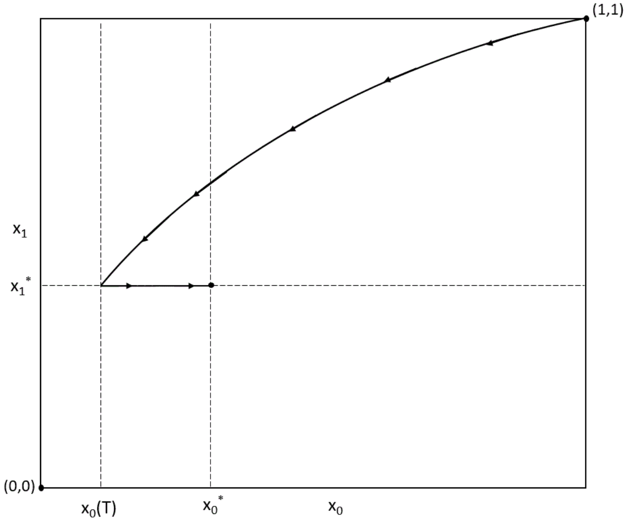
Can show this when $K = 1$.

**Proposition**

*When $K = 1$, there is a unique equilibrium, and it is globally asymptotically stable.*

# Pure Outcome-Improving Case

$\chi = \pi_0 = 0$: solve dynamics from initial condition $(1,1)$ in closed form.



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- Failure of *improvement principle:*
  earlier generations can be better-off than later ones.
- In discrete-time version of model where entire population turns over each period, can get cycles with deterministic period as in Acemoglu-Wolitzky 2014.

# Outline

# Extensions/Robustness Checks

- More general physical environments
  - Continuous actions/convex costs
  - Heterogeneous players
  - Multiple states of the world
- More general information structures
  - Additional signals of the state
  - Additional signals of others' actions
  - Unknown calendar time.

# Continuous Actions/Convex Costs

Replace binary adoption choice with continuous adoption intensity choice:

- Action $x \in [0, 1]$.
- Payoff
$$x\pi_\theta + (1-x)\chi - c(x)$$
for continuously differentiable, convex cost function $c$.
- Let $x^*(\tilde{p})$ be optimal action at belief $\tilde{p}$.
- Observe $K$ independent Bernoulli trials with parameter
$$x_\theta(t)\pi_\theta + (1 - x_\theta(t))\chi$$
where $x_\theta(t)$ is average action in population at time $t$.

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where $x_\theta\,(t)$ is average action in population at time $t$.

When $(x_0\,(t)\,, x_1\,(t))$ is close to $L$, population dynamic drifts toward regular point $(x^*\,(p)\,, x^*\,(p))$
$\implies$ main results continue to hold.

# Heterogeneous Players

Assume $|Q|$ types of players, with population shares $\sigma_q$ and success rates $\pi_{\theta,q}$.

Two models:

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   - Sample informativeness depends only on average success rate $\implies$ main results continue to hold.
2. Player observes random samples of $K$ other players' outcomes **and** their types.
   - If learning from one type becomes confounded, can still learn from other types $\implies$ generically, efficient adoption even in cost-saving case.

# Multiple States

Example where results overturned:

- Add "very good" state to cost-saving case: $\Theta = \{0, 1, 2\}$, $\pi_0 < \pi_1 < \pi_2 < \chi$.
- Suppose innovation is optimal under prior but status quo is optimal conditional on $\theta \in \{0, 1\}$ and conditional on $\theta \in \{0, 2\}$.

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Can have path with following properties:

- Early on, $SR_2 > SR_1 > SR_0$. Success good news.
- Then $SR_1$ and $SR_0$ cross. Success *mixed* news.
  (Players experiment iff observe *extreme* number of successes.)
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2 states: success is always good news or bad news, can't switch from one to the other along an equilibrium path.

Multiple states: success can go from good news to bad news by passing through region where it's neither good news nor bad news.

# Multiple States (cntd.)

Sufficient conditions for main results to continue to hold:

Let $\pi_0 < \pi_1 < \ldots \pi_n$.
Let $\theta^*$ be best state at which innovation is still inefficient.

## Proposition

*Let $a = \sum_{\theta=0}^{\theta^*} p_\theta$. Suppose there exists a set of efficient states $\Theta^*$ with $\sum_{\theta \in \Theta^*} p_\theta = b$ such that*

1. $\chi > \pi_{\max \Theta^*}$
2. *the innovation is optimal when $\theta = 0$ wp $\frac{a}{a+b}$ and $\theta = \min \Theta^*$ wp $\frac{b}{a+b}$:*

$$\frac{a}{a+b}\pi_0 + \frac{b}{a+b}\pi_{\min \Theta^*} - c > \chi$$

*Then there does not exist a sequence of equilibrium paths converging to efficiency as $t \to \infty$ and $K \to \infty$.*

# Multiple States (cntd.)

Alternatively, say an equilibrium path is **simple** if success is always good or bad news.

Proposition

*If $\chi > \pi_\theta$ for some efficient state $\theta$, then there does not exist a sequence of simple equilibrium paths converging to efficiency as $t \to \infty$ and $K \to \infty$.*

# Additional Signals of the State

If players get private signals of state in addition to observing others' outcomes, main results continue to hold as long as point that results from private signals alone is regular.

- Same result holds a fortiori for signals of variables measurable with respect to the state, like the adoption rate.

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**Intuition:**

- When others' outcomes uninformative, population dynamic drifts toward point that results from private signals alone.
- If this point is regular, then population dynamic is trapped in regular region.

# Additional Signals of Others' Actions

If perfectly observe one other player's action and outcome, population dynamic is still trapped in regular region.
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(Follows from Banerjee-Fudenberg 2004.)

If $K \to \infty$ and get iid signals of actions, can back out adoption rate $\implies$ population dynamic converges to efficiency.

# Unknown Calendar Time

Alternative model: all players draw inferences as if samples drawn from steady state
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Fix steady state $(x_0^*, x_1^*, k^*, s^*)$. An *unknown calendar time path relative to* $(x_0^*, x_1^*, k^*, s^*)$ is defined by

$$\dot{x}_\theta(t) = \left\{ \begin{array}{c} \phi_\theta(k^*; x_\theta(t)) s^* + \sum_{k=k^*+1}^{K} \phi_\theta(k; x_\theta(t)) - x_\theta(t) \\ \text{if } (x_0^*, x_1^*) \text{ regular} \\ \phi_\theta(k^*; x_\theta(t)) s^* + \sum_{k=0}^{k^*-1} \phi_\theta(k; x_\theta(t)) - x_\theta(t) \\ \text{if } (x_0^*, x_1^*) \text{ irregular} \end{array} \right\}.$$

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## Proposition

*If $(x_0^*, x_1^*, k^*, s^*)$ is an irregular steady state, $(x_0, x_1)$ is an unknown calendar time path relative to $(x_0^*, x_1^*, k^*, s^*)$, and $(x_0(0), x_1(1))$ is regular, then $(x_0(t), x_1(t))$ is regular for all $t \in \mathbb{R}_+$.*

# Summary

- Developed simple of model of *outcome-based* social learning.
- Outcome-improving innovations adopted efficiently when samples are large.
- Cost-saving innovations adopted inefficiently in any regular eqm, regardless of sample size.
- "Hard to learn about cost-saving innovations, as not clear if success is good news or bad news."

# A Positive Implication

Empirical prediction: correlation between underlying suitability to innovation and realized adoption rate is greater for outcome-improving innovation than for cost-saving innovation.

Ex. Correlation between soil suitability and adoption is greater for high-yield crop varieties than for labor-saving (ex. herbicide-resistant) varieties.

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Interventions that could help:

1. Gathering + releasing info on adoption rate:
   equilibrium is efficient when adoption rate observable.
2. Providing technology that lets individuals run their own tests:
   if tests sufficiently informative,
   $\Longrightarrow$ initial adoption rates irregular
   $\Longrightarrow$ adoption rates converge to efficiency for large $K$.

Efficient adoption of cost-saving innovation requires overcoming ambiguous info content of good outcomes.