Fiscal Rules and Discretion under Self-Enforcement

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Motivation

- Countries impose rules to constrain governments' policy decisions
  - Fiscal rules in place in 92 countries in 2015, up from 7 in 1990
- Credible enforcement mechanisms are critical for institution of rules
  - E.g., Chile feared breaking fiscal rule in 2007 would set bad precedent
  - Rule broken in 2009 due to extraordinary circumstances
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  - Rule broken in 2009 due to extraordinary circumstances
  - Lax policy persisted, including under next govt; rule reinstated in 2011
- What is an optimal fiscal rule when enforcement is limited?
  - How restrictive should a fiscal rule be?
  - Should we expect governments to occasionally violate their rules?
  - What is the optimal structure of penalties for violating rules?

Tradeoffs Behind Rules

- Benefit: Rules can fix commitment problems
  - Governments are present-biased $\implies$ Excessive deficits or spending

- Cost: Rules reduce flexibility. Some discretion can be desirable
  - Not all contingencies/shocks are contractible or observable

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- Cost: Rules reduce flexibility. Some discretion can be desirable
  - Not all contingencies/shocks are contractible or observable
- Model tradeoff in self-control framework using mechanism design
  - Government privately observes shock. Must truthfully report it
  - Literature assumes perfect enforcement of rules
    - E.g., Amador-Werning-Angeletos 2006 (AWA), Halac-Yared 2014
  - **This paper**: Fiscal rules must be self-enforcing

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  - Marginal value of public spending is stochastic
- Frictions
  - Government is present-biased towards public spending
  - Marginal value of spending privately observed by government
  - **New**: Government has full policy discretion
- Rule must satisfy private information and self-enforcement constraints
  - Perfect enforcement: Optimal rule is a deficit limit (AWA)
  - Self-enforcement: Observable deviations punished off path
    - Worst continuation equilibrium sustains optimum (Abreu 1988)

Results

- Optimal fiscal rule is a maximally enforced deficit limit
  - Any on- or off-path violation leads to the worst punishment
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  - Key technical result: Bang-bang dynamic incentives
- Necessary and sufficient conditions for violation of deficit limit
  - Severe deficit bias, rare extreme shocks
- Worst punishment takes form of temporary overspending
  - Maximally enforced surplus limit. Deficit limit eventually reinstated
  - Periods of fiscal rectitude and fiscal profligacy sustain each other

Related Literature

- Commitment versus flexibility
  - Athey-Atkeson-Kehoe 2005, Amador-Werning-Angeletos 2006, Halac-Yared 2014, 2017, Amador-Bagwell 2016
- Political economy of fiscal policy
  - Yared 2010, Aguiar-Amador 2011, Song-Storesletten-Zilibotti 2012, Azzimonti-Battaglini-Coate 2015, Dovis-Kirpalani 2017
- Hyperbolic discounting and commitment devices
  - Phelps-Pollack 1968, Laibson 1997, Bernheim-Ray-Yeltekin 2015, Bisin-Lizzeri-Yariv 2015
- Price wars and bang-bang equilibria
  - Green-Porter 1984, Abreu-Pearce-Stacchetti 1990, Athey-Bagwell-Sanchirico 2004

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- Welfare at time t:
  - Society:

$$\sum_{k=0}^{\infty} \delta^k \mathbb{E}[\theta_{t+k} U(G_{t+k})]$$

  - Government after θ_t's realization, when choosing policy:

$$\theta_t U(G_t) + \beta \sum_{k=1}^{\infty} \delta^k \mathbb{E}[\theta_{t+k} U(G_{t+k})], \text{ where } \beta \in (0, 1)$$

Interpretation

- Suppose two-period economy
  - First best: $\theta_0 U'(G_0) = \delta(1+r)\mathbb{E}[\theta_1]U'(G_1)$
  - Full flexibility: $\theta_0 U'(G_0) = \beta\delta(1+r)\mathbb{E}[\theta_1]U'(G_1)$
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- Politicians temporarily in charge of budget overweigh spending
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  - Arises under aggregation of heterogeneous preferences (Jackson-Yariv)
- Realization of θ_t is private information
  - Rules cannot depend on θ_t explicitly
  - Heterogeneous citizen preferences; government sees aggregate
  - Exact cost of public goods not perfectly observed

Assumption on Preferences

- **Assumption**: $U(G_t) = \log(G_t)$
  - Spending rate $g_t = G_t/((1+r)\tau/r - B_t)$, savings rate $x_t = 1 - g_t$
  - Welfare from savings $W(x_t) = \delta\mathbb{E}[\theta_t]U(x_t)/(1-\delta)$

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- Implies welfare is separable with respect to debt

Society: $$\sum_{k=0}^{\infty} \delta^k \mathbb{E}\left[\theta_{t+k} U(g_{t+k}) + W(x_{t+k})\right] + \chi(B_t)$$

Govt: $$\theta_t U(g_t) + \beta W(x_t) + \beta \sum_{k=1}^{\infty} \delta^k \mathbb{E}\left[\theta_{t+k} U(g_{t+k}) + W(x_{t+k})\right] + \varphi(B_t)$$

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- Two benchmarks
  - First best: $\theta_t U'(g^{fb}(\theta_t)) = W'(x^{fb}(\theta_t))$
  - Full flexibility: $\theta_t U'(g^{f}(\theta_t)) = \beta W'(x^{f}(\theta_t))$

Self-Enforcing Rules

- Perfect public equilibria
  - Government chooses g_t given $\{g_0, g_1, \ldots, g_{t-1}\}$ and private info θ_t

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  - Continuation value at t (normalized by debt) is

$$V_t(\theta^{t-1}) = \sum_{k=0}^{\infty} \delta^k \mathbb{E}\left[\theta_{t+k} U(g_{t+k}(\theta^{t+k-1}, \theta_{t+k})) + W(x_{t+k}(\theta^{t+k-1}, \theta_{t+k}))\right]$$

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- Equilibrium iff govt at θ^{t-1} prefers $\{g_t(\theta^{t-1}, \theta_t), V_{t+1}(\theta^{t-1}, \theta_t)\}$ to:
  - Unobservable deviation: $\{g_t(\theta^{t-1}, \theta'), V_{t+1}(\theta^{t-1}, \theta')\}$ for $\theta' \neq \theta_t$
  - Observable deviation: $\{g^f(\theta_t), \underline{V}\}$
    - Where $\underline{V}$ is lowest value supported by equilibrium strategies

Optimal Self-Enforcing Rule

$$\overline{V} = \max_{\{g(\theta),x(\theta),V(\theta)\}_{\theta\in\Theta}} \mathbb{E}\left[\theta U(g(\theta)) + W(x(\theta)) + \delta V(\theta)\right]$$

subject to

$$\theta U(g(\theta)) + \beta W(x(\theta)) + \beta\delta V(\theta) \geq \theta U(g(\theta')) + \beta W(x(\theta')) + \beta\delta V(\theta')$$

(private information constraint)

$$\theta U(g(\theta)) + \beta W(x(\theta)) + \beta\delta V(\theta) \geq \theta U(g^f(\theta)) + \beta W(x^f(\theta)) + \beta\delta \underline{V}$$

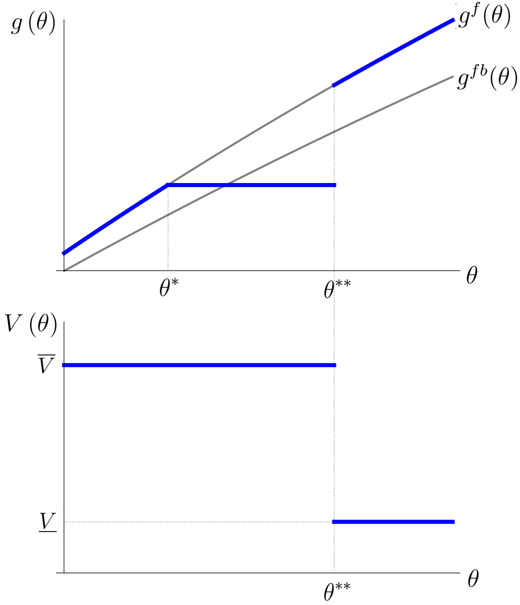
(self-enforcement constraint)

$$g(\theta) + x(\theta) = 1 \text{ and } V(\theta) \in \left[\underline{V}, \overline{V}\right]$$

(feasibility)

Definition: Maximally Enforced Deficit Limit

$\theta^* \in [0, \overline{\theta})$ and $\theta^{**} > \max\{\theta^*, \underline{\theta}\}$



Preliminaries

- Envelope condition: Government welfare given θ equals

$$\underline{\theta} U(g(\underline{\theta})) + \beta W(x(\underline{\theta})) + \beta \delta V(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} U(g(\widetilde{\theta})) d\widetilde{\theta} \tag{1}$$

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- Social welfare (normalized by debt) equal to

$$\frac{1}{\beta}\underline{\theta}U(g(\underline{\theta})) + W(x(\underline{\theta})) + \delta V(\underline{\theta}) + \frac{1}{\beta}\int_{\underline{\theta}}^{\overline{\theta}} U(g(\theta))Q(\theta)d\theta$$

where $Q(\theta) \equiv 1 - F(\theta) - \theta f(\theta)(1-\beta)$

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- $Q(\theta)$: Weight on allowing spending distortions by type θ
  - Higher $Q(\theta)$ $\implies$ Lower social welfare cost of distorting θ's spending

Bang-Bang Incentives

- **Proposition**: *Suppose $Q(\theta)$ satisfies these generic properties:*
  1. $Q'(\theta) \neq 0$ *almost everywhere*
  2. *If* $Q(\theta^L) = Q(\theta^H) = \widehat{Q}$, *then* $\int_{\theta^L}^{\theta^H} Q(\theta) d\theta \neq \int_{\theta^L}^{\theta^H} \widehat{Q} d\theta$

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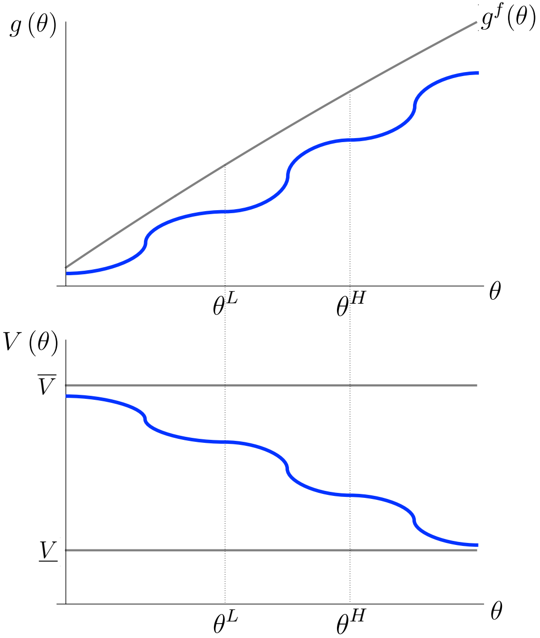
- Relationship to Abreu-Pearce-Stacchetti 1990
  - Instead of moral hazard, we study adverse selection with self-control:
    - Continuation value set is one-dimensional
    - Locally spreading out continuation values may not be IC or beneficial

Sketch of Proof: Three Steps

- Step 1: $V(\theta)$ is a step function (no local dynamic incentives)
- Step 2: $V(\theta) \in \{\underline{V}, \overline{V}\}$ whenever $g(\theta)$ is strictly increasing
- Step 3: $V(\theta) \in \{\underline{V}, \overline{V}\}$ whenever $g(\theta)$ is constant

Step 1: Rule Out Local Dynamic Incentives

- Suppose $V'(\theta) < 0$ with $g'(\theta) > 0$
- If $Q'(\theta) < 0$, flattening perturbation increases welfare
- If $Q'(\theta) > 0$, steepening perturbation increases welfare



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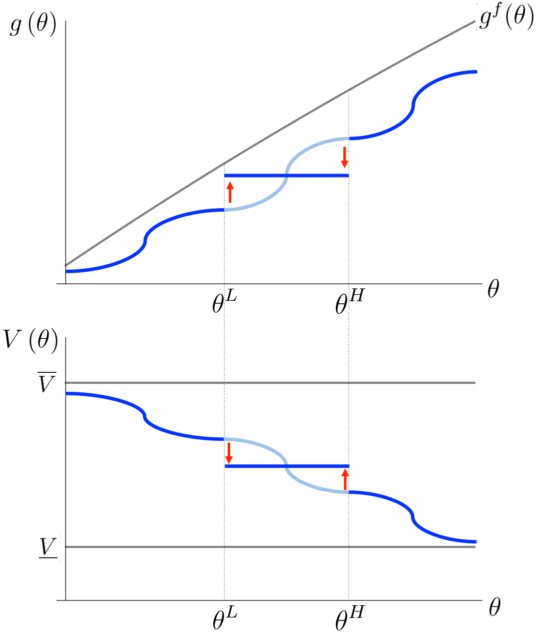

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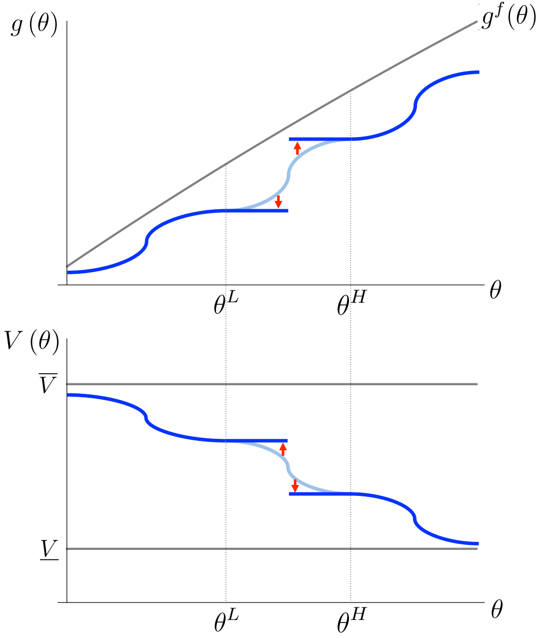

Step 2: Rule Out Interior Values under Rising Spending

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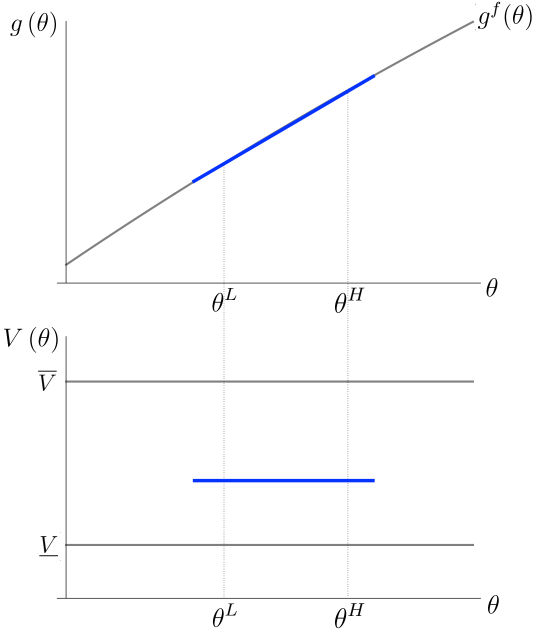
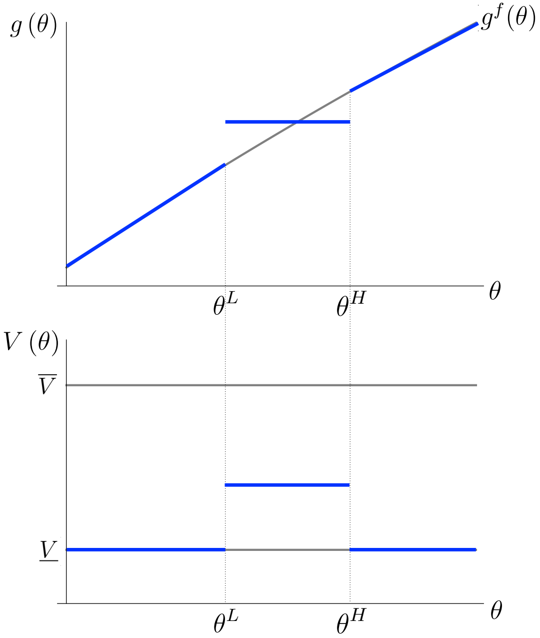


Step 3: Rule Out Interior Values under Constant Spending

- Suppose $V(\theta) \in (\underline{V}, \overline{V})$ with $g'(\theta) = 0$
- If $Q(\theta^L) > \int_{\theta^L}^{\theta^H} Q(\theta) d\theta$, segment shifting perturbation increases welfare
- Analogous perturbations under different conditions



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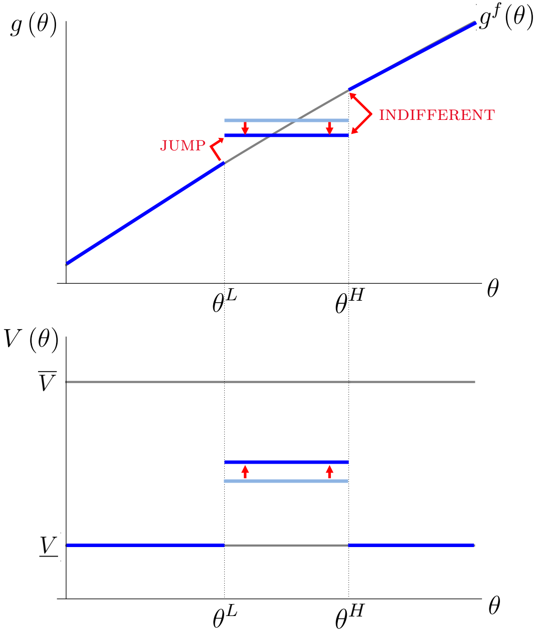

Monotonic Incentives

- **Assumption**: *There exists* $\widehat{\theta} \in \Theta$ *such that*

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- **Lemma**: *In any optimal rule,* $V(\theta)$ *is weakly decreasing,* $V(\underline{\theta}) = \overline{V}$

  - Implies either $V(\theta) = \overline{V}$ $\forall\theta$ or $V(\theta)$ jumps down to $\underline{V}$ at interior θ^{**}
  - $\theta > \widehat{\theta} \implies$ Load spending distortions at top, high-powered incentives
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- Proof makes use of perturbations like previous ones

Sketch of Proof: Three Steps

- Step 1: If $V(\theta) = \underline{V}$, then $\theta \geq \widehat{\theta}$
  - Otherwise, $Q'(\theta) < 0$, and $g(\theta) = g^f(\theta)$ by self-enforcement
  - Improve w/flattening perturbation

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- Step 2: If $V(\theta') = \underline{V}$, then $V(\theta) = \underline{V}$ for all $\theta \geq \theta'$
  - Otherwise, $V(\theta) = \overline{V}$ and $Q'(\theta) > 0$ over $[\theta^L, \theta^H]$, $\theta^L > \theta'$
  - Improve w/steepening ($g' > 0$) or segment-shifting ($g' = 0$) perturbation

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- Step 3: $V(\underline{\theta}) = \overline{V}$
  - Otherwise, $V(\theta) = \underline{V}$ for all $\theta \in \Theta$
  - Improve w/global perturbation that increases $V(\theta)$ for all $\theta \in \Theta$

Optimal Self-Enforcing Fiscal Rule

- **Proposition**: *Any optimal rule is a maximally enforced deficit limit*
  - Follows from previous results, after proving $g(\theta)$ must be continuous
  - If $g(\theta)$ is discontinuous, improve with perturbation that closes "hole"

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- **Corollary**: *Suppose θ_e is self-enforcing, that is:*

$$\overline{\theta}U(g^f(\theta_e)) + \beta W(x^f(\theta_e)) + \beta\delta\overline{V} \geq \overline{\theta}U(g^f(\overline{\theta})) + \beta W(x^f(\overline{\theta})) + \beta\delta\underline{V}$$

Then $\theta^* = \theta_e$ and $\theta^{**} \geq \overline{\theta}$. No dynamic incentives

Use of Punishment

- If θ_e is not self-enforcing, define θ_b:

$$\overline{\theta}U(g^f(\theta_b)) + \beta W(x^f(\theta_b)) + \beta\delta\overline{V} = \overline{\theta}U(g^f(\overline{\theta})) + \beta W(x^f(\overline{\theta})) + \beta\delta\underline{V}$$

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  1. *If $\int_{\theta_b}^{\overline{\theta}}(Q(\theta) - Q(\overline{\theta}))d\theta \geq 0$, then $\theta^* = \theta_b$ and $\theta^{**} = \overline{\theta}$*
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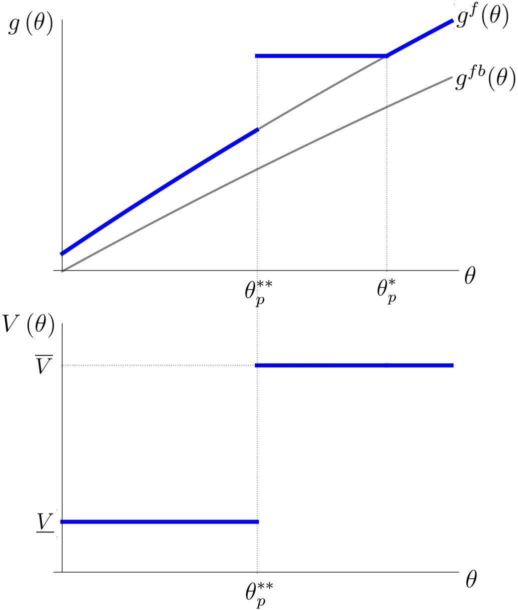
- Condition reflects benefits and costs of dynamic incentives
  - Discipline lower types; no discipline and dynamic costs for higher types
  - For any $\overline{V} - \underline{V}$, no punishment if $Q'(\theta) < 0\ \forall\theta$. True if $f'(\theta) \geq 0\ \forall\theta$
  - For any $\overline{V} - \underline{V}$, punishment if $f(\theta) \to 0$ as $\theta \to \overline{\theta}$

Definition: Maximally Enforced Surplus Limit

$\theta_p^* > \underline{\theta}$ and $\theta_p^{**} \in [\underline{\theta}, \min\{\theta_p^*, \overline{\theta}\})$



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- Intuition: Government cares more about present spending than society
  - Inducing overspending relaxes self-enforcement and minimizes welfare
  - Best incentives achieved with maximal reward and punishment
- Proof: Characterizing $\underline{V}$ analogous to characterizing $\overline{V}$ (but reverse)
  - Key step establishes that punishment is not absorbing
  - Rewarding overspending by high types reduces welfare

Bang-Bang Dynamics

- Optimal fiscal rule is solution to two problems:
  - $\{\theta^*, \theta^{**}\}$ which yield maximum social welfare given $\overline{V} - \underline{V}$
  - $\{\theta_p^*, \theta_p^{**}\}$ which yield minimum social welfare given $\overline{V} - \underline{V}$
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- Phases of fiscal rectitude and fiscal profligacy sustain each other

Conclusion

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- Other applications
  - Self-control and going off the wagon
  - Regulation with socially costly penalties

Thank you!

Discussion of Distributional Assumptions

- Recall distributional assumption: There exists $\widehat{\theta} \in \Theta$ such that

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- Implies our distributional assumption is necessary for characterization
  - Weaker distributional assumption needed under perfect enforcement