Efficient Collective Decision-Making, Marginal Cost Pricing, and Quadratic Voting

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Abstract

We trace the developments that led to quadratic voting, from Vickrey's counterspeculation mechanism and his second-price auction through the family of Groves mechanisms, the Clarke mechanism, the expected externality mechanism, and the Hylland-Zeckhauser mechanism. We show that these mechanisms are all applications of the fundamental insight that for a process to be efficient, *all parties involved* must bear the marginal costs of their actions.

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1. Introduction

Quadratic voting is such a simple and powerful idea that it is remarkable that it took economists so long to understand it. At one level, its discovery was a flash of insight. At another level, the discovery is the latest step in a series of incremental understandings over more than half a century.

Our account of the history of insights that led up to quadratic voting begins with a detour. In his 1954 paper *The pure theory of public expenditures*, Paul Samuelson set out the conditions for efficient provision of public goods and told us that we should not expect to ever achieve those conditions. Samuelson (1954, p. 389) wrote:

> One could imagine every person in the community being indoctrinated to behave like a "parametric decentralized bureaucrat" who *reveals* his preferences by signalling in response to price parameters or Lagrangean multipliers, to questionnaires, or to other devices. But there is still this fundamental technical difference going to the heart of the whole problem of *social* economy: by departing from his indoctrinated rules, any one person can hope to snatch some selfish benefit in a way not possible under the self-policing competitive pricing of private goods; and the "external economies" or "jointness in demand" intrinsic to the very concept of collective goods and governmental activities makes it impossible for the grand ensemble of optimizing equations to have that special pattern of zeros which makes *laissez-faire* competition even *theoretically* possible as an analogue computer.

This discouraging pronouncement led many economists to believe that there was no point in looking for ways to achieve efficiency in the provision of public goods. But partly because the solution came from an unexpected direction and partly because not everyone was discouraged by Samuelson's pronouncement, economists eventually realized that the solution to the problem of motivating people to report their preferences for public goods truthfully was to employ novel applications of the principle of marginal cost pricing.

2. Efficiency requires that all parties bear the marginal costs of their actions

2.1 *Valuing a single good*

The path to understanding the role of marginal cost pricing in motivating truthful revelation of preferences for public goods began with William Vickrey's 1961 paper *Counterspeculation, Auctions, and Competitive Sealed Tenders*. In this paper, Vickrey introduced the idea of a single-item second-price sealed-bid auction in which the item goes to the bidder who makes the highest bid, who then pays the amount of the second-highest bid.[1] Vickrey showed that such an auction provides each bidder with an incentive to reveal his genuine valuation of the item, as long as bidders do not collude. To understand why, consider bidder *i* who values the item at auction at V_i and must determine the amount X_i to bid. Let *B* be the bid that will be the highest bid if bidder i does not bid, let $f(B)$ be the probability density function of *B*, and assume that this distribution is independent of X_i.[2] Bidder *i* maximizes his expected net benefit function[3]

$$E[\pi(X_i)] = \int_0^{X_i}(V_i - B)f(B)dB \tag{2.1}$$

by announcing the value X_i^* that solves the first order condition

$$\frac{dE[\pi(X_i)]}{dX_i} = (V_i - X_i)f(X_i) = 0, \tag{2.2}$$

that is, by announcing $X_i^* = V_i$.

The second-price auction achieves truthful revelation of V_i by severing the direct connection between the bidder's announcement and his expected payoff. The bidder cannot increase his expected payoff by announcing $X_i \neq V_i$, but he will pass up the opportunity to profit from the auction if $X_i < B < V_i$, and he will win the auction at a loss to himself if $X_i > B > V_i$. In both cases, his loss can be described as the absolute value of the difference between his own valuation and the valuation of the bidder whose valuation is closest to his, that is, the loss in social (consumer) surplus that arises if the item is not acquired by the person who values it most highly. Hence bidder i bears the entire social cost $|V_i - B|$ of announcing $X_i \neq V_i$. If bidder i announces $X_i = V_i$, then he pays the entire loss B of the person who would have won the auction in bidder i's absence.

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[1] Vickrey (1961, p.23) introduced the second-price auction as a variation on the traditional Dutch auction, in which the auctioneer lowers the asking price successively and the winner is the bidder who makes the first bid, which he is required to pay. If the first bid is recorded but kept a secret until the second bid has been made and the first bidder then pays the second bid, then such a modified Dutch auction has the same properties as the single-item second-price sealed-bid auction. However, as Gordon Tullock remarked in conversation, a second-price auction can be described more simply as a one-shot version of an English auction with continuous bids.

[2] It is irrelevant whether $f(B)$ describes the actual distribution of *B* or whether it describes bidder *i*'s beliefs about this distribution. What matters is that bidder i does not believe that the value X_i that he announces affects the distribution of *B*.

[3] If $X_i > B$, then *B* is the second highest bid and bidder *i* pays B for an item that he values at V_i. If $X_i < B$, then bidder i does not win the auction and keeps X_i. Since bidder *i*'s wealth does not change if $X_i < B$, the corresponding integral for values of *B* in $[X_i, \infty]$ is zero.

Without any apparent awareness of Vickrey's work, Becker, DeGroot, and Marschak proposed a corresponding mechanism in Becker et al. (1964) that induces a person to reveal his valuation of an item that he owns rather than that of an item that he wishes to acquire. This mechanism is often called the BDM mechanism. Let X_i be the amount that owner *i* announces as his own valuation of his property, and let B be an offer for this property that is drawn from a distribution with density function *f*(B). If $X_i > B$, then the owner keeps his property that he values at V_i, and he must sell his property at B if $B > X_i$. As long as B is independent of the amount X_i that owner *i* announces, owner *i* maximizes his expected net benefit function

$$E[\pi(X_i)] = \int_0^{X_i} V_i f(B)dB + \int_{X_i}^{\infty} Bf(B)dB \quad (2.3)$$

by announcing the value X_i^* that solves a first order condition identical to equation 2.2. Any loss that arises from announcing $X_i \neq V_i$ equals $|V_i - B|$, which is the loss in social surplus that arises if the property is not owned by the person who values it highest. Thus the owner bears the full marginal social cost of not revealing V_i.

In contrast to Vickrey's second-price auction where the highest bidder's net benefit is $V_i - B$, the BDM mechanism leaves owners who announce $X_i = V_i$ with the entire social benefit, $max\{V_i, B\}$.[4] Thus even though both mechanisms are applications of marginal cost pricing, they apply this principle in different ways. The connection between the ideas went long unnoticed, and the first paper to cite both Vickrey (1961) and Becker *et al*. (1964) was published only in 1985.[5]

Karni and Safra (1985, 1987) showed that neither the second-price auction nor the BDM mechanism necessarily elicit truthful valuations if the item in question has the character of a lottery with known probabilities and owners maximize expected utility with rank-dependent probabilities, rather than simply expected utility. Horowitz (2006) showed that, for owners who do not maximize expected utility because they have, for example, disappointment aversion, the same result holds even if there is no uncertainty associated with the item in question.

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[4] For example, B might be the net social benefit of a public redevelopment project that the government plans to implement on owner *i*'s property.

[5] Harrison and McKee (1985) undertook an evaluation of alternative approaches to the efficiency problem in single-product monopolies; they referenced Vickrey's second-price auction as a method of extracting rent from the owner of a monopoly and referred the reader to Becker *et al.* (1964) for their research design. To our knowledge, Karni and Safra were the first to reference both mechanisms in one paper—Karni and Safra (1987) examined the BDM mechanism, and they referred the reader to Karni and Safra (1985), which does not mention BDM, for an implication of their results to Vickrey's mechanism.

2.2 *Counterspeculation*

Vickrey's 1961 paper is generally cited for its discussion of the second-price auction. However, in Section 1 of that paper, Vickrey developed an idea that might be described as a generalization of the BDM mechanism. Vickrey offered this ingenious idea as an answer to the question of how one might implement the concept of "counterspeculation" that Abba P. Lerner mentioned in his 1944 book *The Economics of Control*. Lerner considered situations of imperfect competition in which prices differ from competitive equilibrium prices. To obtain efficient allocations nevertheless, Lerner suggested that, for any such imperfectly competitive market, the government estimate, through a *Board of Counterspeculation*, the competitive equilibrium price and guarantee that anyone be able purchase or sell the good at this estimated price. The government would fulfill this guarantee by buying, in the free market, the amounts that buyers wish to acquire and by selling all units that sellers wish to sell. Lerner expressed the expectation that, over time, the profits and losses from these trades would roughly cancel each other out as the government became more proficient in estimating equilibrium prices. If (1) the government's estimate of the competitive equilibrium price is correct, (2) buyers and sellers expect that nothing they do will change this estimate, and (3) buyers and sellers are unable to collude, then buyers and sellers have an incentive to behave as if they were part of a competitive market.

To estimate the equilibrium price, the *Board of Counterspeculation* would need to learn the marginal benefit and cost schedules without giving buyers and sellers the opportunity to affect this estimate by misrepresenting these schedules. Lerner did not suggest how the *Board of Counterspeculation* might do this. Vickrey proposed the following mechanism to learn these schedules: (1) require that all buyers and sellers report their respective marginal benefit and marginal costs schedules, (2) aggregate these schedules to obtain an estimate of the equilibrium price, (3) require each seller willing to sell units at this price to accept, for each unit, a price that is calculated as the marginal social value of that unit, considering the buyers' marginal willingnesses to pay for units and the marginal cost schedules of the other sellers, and (4) require each buyer willing to acquire units at the equilibrium price to pay, for each unit, a price that is calculated as the marginal social cost of that unit, considering the marginal cost schedules of sellers and the other buyers' marginal willingnesses to pay for units.

Here is how Vickrey's counterspeculation mechanism can be interpreted as a continuous multi-person version of the second-price auction and the BDM mechanism. As with the second-price auction, every buyer states an amount X_i that he is willing to pay for every unit i that he wants to purchase (or a continuous marginal benefit function to accommodate a continuum of units). The amount B_i that the buyer must pay if a sale occurs is the larger of (the marginal benefit if unit i was consumed by one of the other buyers) and (the marginal cost of producing unit i), that is, the value that unit i has for the person who values it second-highest. Because the buyer cannot change B_i, he does not benefit from announcing an amount X_i that differs from his marginal benefit MB_i: he might have to acquire unit i at a price above his marginal benefit if $X_i > B_i > MB_i$, and he might forego the acquisition of unit i at a price below his marginal benefit if $MB_i > B_i > X_i$. Because the buyer bears the full cost of announcing $X_i \neq MB_i$, he has an incentive to announce $X_i = MB_i$.

Similarly, the BDM mechanism requires that one person state the amount X at which he values a particular good, and that a second person announce an amount B that is independent of X at which he is willing to purchase the good. The exchange takes place if $B > X$, in which case the first person receives B. If $B \leq X$, then the first person keeps the item that he values at X. In Vickrey's mechanism, every seller states, for every unit i, an amount X_i at which he is willing to sell that unit.[6] The amount B_i that a seller receives if a sale occurs is the smaller of (the buyers' marginal benefit of consuming unit i) and (the marginal cost of having unit i be produced by one of the other sellers). Because the seller cannot change B_i, he does not benefit from announcing an amount X_i that differs from his marginal cost MC_i: the seller might lose a profitable sale if he announces $X_i > MC_i$ and it happens that $X_i > B_i > MC_i$, and he has to sell unit i below his marginal cost if he announces $X_i < MC_i$ and it happens that $MC_i > B_i > X_i$. Because the seller bears the full cost of announcing $X_i \neq MC_i$, he has an incentive to announce $X_i = MC_i$.

The main drawback of Vickrey's counterspeculation mechanism is that, because sellers receive more than the equilibrium price for all units other than the marginal unit and buyers pay less than the equilibrium price for all units other than the marginal unit, the mechanism leads to a budget deficit equal to the sum of the amounts that sellers receive in excess of, and that buyers'

[6] To accommodate a continuum of units, every seller announces a continuous marginal cost function rather than a discrete set of amounts.

payments fall short of, the equilibrium price of the marginal unit. Since Vickrey did not see a solution to this problem, he turned to analyses of Dutch auctions and second-price auctions.

Vickrey's counterspeculation mechanism is an early application of the fundamental insight that for a process to be efficient, *all parties involved* must bear the marginal costs of their actions and be rewarded with the marginal social values of their actions. Coase applied the same insight in his famous 1960 paper *The problem of social cost*, in his discussion of a smoke-emitting factory that causes damages to near-by residents (Coase, 1960, p. 41):[7]

> If the factory owner is to be made to pay a tax equal to the damage caused, it would clearly be desirable to institute a double tax and to make residents of the district pay an amount equal to the additional cost incurred by the factory owner (or the consumers of his products) in order to avoid the damage.

However, the situation is more complicated than Coase indicates. Each individual should be charged the amount that his presence adds to the factory's cost, which generally becomes very small as the number of persons increases. But if there is a non-convexity in the cost structure that creates a possible efficiency from having all residents move away simultaneously, then the community should be sent a bill for the cost of its presence, and the community should make a collective decision as to whether to all move away or to pay the bill and remain.

In 1968, Vickrey applied the insight about charging everyone their marginal costs to the theory of accidents (Vickrey, 1968, pp.466 – 467):

> Indeed, if it were not for the catastrophic nature of accident losses in relation to the resources of the individuals involved, economic efficiency would be best served by exacting from every individual who takes action as a result of which damage is inflicted on others that would have not have occurred in the absence of this action a payment equal to the full amount of the damage so inflicted. Moreover, what will sound strange to the juristic mind, the assessment should be independent of any criterion of fault or relative contributory negligence and furthermore should not be abated or offset in any way by compensation for injuries suffered by the actor himself! … Economically speaking, it is just as important to

[7] Even though the date on Coase's publication precedes that of Vickrey's paper, Vickrey might deserve credit for an earlier publication because the 1960 issue of the *Journal of Law and Economics* has a 1961 copyright date.

provide an adequate incentive for driving defensively rather than merely nonnegligently as it is to provide an incentive for driving nonnegligently rather than recklessly. Systems which require payments by the actors only in case of fault and only to the extent of the compensation received by others (even with expenses of adjudication and administration added) fail to give an adequate incentive for seeking out alternatives not involving the increased risk of vehicular accident.

Richard Zeckhauser provided a formalization of this same application of marginal cost pricing to the theory of accidents in his 1968 dissertation. Writing at the same time, Vickrey and Zeckhauser were not aware of each other's work. Theodore Groves, in his 1970 dissertation, applied a reverse version of the same principle to the design of efficient incentives in teams—teams have an incentive to behave efficiently if every team member is promised to receive the entire benefit of the team's joint output.[8]

3. Public goods: Solutions to Samuelson's discouraging pronouncement

Edward Clarke (1971) had the idea for a mechanism that would overturn Samuelson's 1954 pronouncement regarding the apparent impossibility of efficient provision of public goods. Clarke's mechanism can be characterized as an application of Vickrey's second-price auction to the problem of decisions about public goods. It achieves efficiency by assigning to all persons involved the marginal costs that their choices impose on all other persons. Groves and Loeb (1975), unaware of Clarke's work, proposed the same mechanism. Groves (1976) characterized a family of mechanisms of which the Clarke mechanism is a member. These mechanisms are generally called *Groves mechanisms* in view of Groves' generalization. Green and Laffont (1977) established that a mechanism motivates truthful revelation of preferences as dominant strategies if and only if it is a Groves mechanism.

We restrict our discussion of the family of Groves mechanisms to the binary case of the choice between two options. Although analyses of choices between two options obscure limitations that arise in cases of more than two options,[9] they permit straightforward illustration

[8] This approach might be feasible if team members must pay fixed fees to be on a team.

[9] For further discussion, see Tideman (2006, pp. 298 - 319).

of the different ways in which marginal cost pricing works in different mechanisms and allow us to show the relationship between Groves mechanisms and quadratic voting.

Consider a society with N citizens. Assume that there are two mutually exclusive policies between which the government will choose. Without loss of generality, assume that the first n citizens favor policy 1 while the remaining $N - n$ citizens favor policy 2. Every citizen $j, j = 1, \dots, n$ is willing to pay V_j to see policy 1 adopted, while every citizen $k, k = n + 1, \dots, N$ is willing to pay V_k to see policy 2 adopted. The government asks each citizen $i, i = 1, \dots, N$ to provide an announcement of the amount X_i he is willing to pay to have his preferred policy adopted. Let $P(N) = 1, 2$ be the policy that the government adopts when it considers the announcements made by all N citizens; the government adopts the policy $P(N)$ that maximizes

$$W = \begin{cases} \sum_{j=1}^{n} X_j & \text{if } P(N) = 1 \\ \sum_{k=n+1}^{N} X_k & \text{if } P(N) = 2 \end{cases} . \tag{3.1}$$

Thus the government adopts policy 1 if $\sum_{j=1}^{n} X_j \geq \sum_{k=n+1}^{N} X_k$ and policy 2 otherwise.[10]

The government uses a transfer function $T_i(X_i)$ to provide citizen i with an incentive to announce his true willingness to pay X_i, where $T_i(X_i) > 0$ describes a subsidy and $T_i(X_i) < 0$ describes a tax. The mechanisms that we discuss below differ mainly in the type of transfer function that they use.

3.1. *Groves mechanisms*

Groves (1976) proposed a transfer function of the form

$$T_i(X_i) = \begin{cases} \sum_{j, j \neq i} X_j - A_i & \text{if } P(N) = 1 \\ \sum_{k, k \neq i} X_k - A_i & \text{if } P(N) = 2 \end{cases} , \tag{3.2}$$

where A_i is an adjustment function that is independent of X_i. With Groves' transfer function, each citizen i receives the total willingness to pay of all citizens who support the policy that the government adopts, excluding his own willingness to pay if he also supports the chosen policy, minus an amount A_i that does not depend on X_i and that therefore does not affect citizen i's choice of X_i. Thus the net benefit function of any citizen $i \leq n$, is

[10] To keep the notation simple, we assume that the government adopts policy 1 if $\sum_{j=1}^{n} X_j = \sum_{k=n+1}^{N} X_k$, implying that both policies have the same support. For example, policy 1 might represent the status quo and citizens agree to change the status quo only if their support of the alternative policy exceeds their support of the status quo.

$$\pi_i(X_i) = \begin{cases} V_i + T_i(X_i) = V_i + \sum_{j,j\neq i} X_j - A_i & \text{if } P(N) = 1 \\ T_i(X_i) = \sum_k X_k - A_i & \text{if } P(N) = 2 \end{cases}, \qquad (3.3a)$$

while any citizen $i > n$ has the net benefit function

$$\pi_i(X_i) = \begin{cases} T_i(X_i) = \sum_j X_j - A_i & \text{if } P(N) = 1 \\ V_i + T_i(X_i) = V_i + \sum_{k,k\neq i} X_k - A_j & \text{if } P(N) = 2 \end{cases}. \qquad (3.3b)$$

Because citizen i ignores A_i when deciding which X_i to announce, any citizen $i \leq n$ maximizes his net benefit by maximizing

$$\pi_i'(X_i) = \begin{cases} V_i + \sum_{j,j\neq i} X_j & \text{if } P(N) = 1 \\ \sum_k X_k & \text{if } P(N) = 2 \end{cases}, \qquad (3.4a)$$

while any citizen $i > n$ maximizes his net benefit by maximizing

$$\pi_i''(X_i) = \begin{cases} \sum_j X_j & \text{if } P(N) = 1 \\ V_i + \sum_{k,k\neq i} X_k & \text{if } P(N) = 2 \end{cases}. \qquad (3.4b)$$

If citizen i announces $X_i = V_i$, then his respective equation 3.4 equals equation 3.1, which is the function that the government maximizes. Hence a citizen can do no better than announce his true willingness to pay.[11]

Transfer rule 3.2 describes an entire family of mechanisms because its incentive property holds for all A_i that are independent of X_i. The Groves mechanism with $A_i = 0$ uses the same principle as the BDM mechanism that we discuss in Section 2: both mechanisms align a citizen's interest with society's interest by assigning to each citizen the entire net social benefit that others receive, so that the sum for all citizens—including the focal citizen—is maximized when the focal citizen announces $X_i = V_i$. Hence a citizen who changes the social outcome by announcing $X_i \neq V_i$ bears the entire decrease in net social benefit to others, if any, that such a change causes.

3.2. *The Clarke mechanism*

The Groves mechanism with $A_i = 0$ leads to severe budget deficits. Assigning $A_i < 0$ mitigates the deficit, but the requirement that A_i be independent of X_i might result in net payments from some citizens to the government that violate citizens' budget constraints. Thus Groves

[11] The requirement that all of $\sum_{j,j\neq i} X_j$, $\sum_{k,k\neq i} X_k$, and A_i be independent of X_i for all i entails ruling out all coalitions of citizens that seek to increase their members' payoffs by coordinating the amounts that each group member announces. If the requirement is not satisfied, then members of coalitions may increase their expected gains by coordinated untruthful statements, though not without risk.

mechanisms offer a challenge to find a specification of A_i that is independent of X_i and neither leads to budget deficits nor imposes infeasible taxes.

The Clarke mechanism for a choice between two policies achieves this. It can be viewed as a member of the family of Groves mechanisms with the specific adjustment function

$$A_i(X_i) = \begin{cases} \sum_{j,j\neq i} X_j & \text{if } P(N\backslash i) = 1 \\ \sum_{k,k\neq i} X_k & \text{if } P(N\backslash i) = 2 \end{cases}, \tag{3.5}$$

where $P(N\backslash i)$ denotes the policy that the government adopts when it considers the announcements of all citizens except citizen i, leading to the transfer rule

$$T_i^C(X_i) = \begin{cases} \sum_{j,j\neq i} X_j - \sum_{j,j\neq i} X_j = 0 & \text{if } P(N) = 1 \text{ and } P(N\backslash i) = 1 \\ \sum_{j,j\neq i} X_j - \sum_{k,k\neq i} X_k & \text{if } P(N) = 1 \text{ and } P(N\backslash i) = 2 \\ \sum_{k,k\neq i} X_k - \sum_{j,j\neq i} X_j & \text{if } P(N) = 2 \text{ and } P(N\backslash i) = 1 \\ \sum_{k,k\neq i} X_k - \sum_{k,k\neq i} X_k = 0 & \text{if } P(N) = 2 \text{ and } P(N\backslash i) = 2 \end{cases}, \tag{3.6}$$

which is a special case of equation 3.2.

Transfer rule 3.6 is known as the Clarke tax, so named by Tideman and Tullock (1976). Citizen i's Clarke tax payment is zero, that is, $T_i^C(X_i) = 0$, if the government chooses the same policy when citizen i announces X_i as it would have chosen if citizen i had made no announcement, that is, if $P(N) = P(N\backslash i)$. Thus citizens pay no tax if their announcements do not alter the socially chosen policy. However, citizen i pays a tax $T_i^C(X_i) < 0$ if his announcement changes the socially chosen policy, that is, if $P(N) \neq P(N\backslash i)$. It is straightforward to verify this claim: if the government were to adopt policy 2 in citizen i's absence, then it must be the case that $\sum_{j,j\neq i} X_j < \sum_{k,k\neq i} X_k$. Similarly, if the government were to adopt policy 1 in citizen i's absence, then it must be the case that $\sum_{k,k\neq i} X_k < \sum_{j,j\neq i} X_j$. In both cases, the difference between the two sums—the amount that citizen i is required to pay—is the margin by which the policy that loses when citizen i is present would win in citizen i's absence. Thus every citizen whose announcement alters the government's choice—every Pivotal citizen—pays a Clarke tax that equals the net social loss that arises because of the citizen's presence. Citizen i's Clarke tax payment will be within the interval $[0, X_i]$, because citizen i can be Pivotal only as long as X_i is at least as large as the winning margin in his absence. Thus the Clarke mechanism ensures that no citizen is assigned a positive transfer and that no citizen who announces his true willingness to pay must pay a tax that exceeds his budget constraint.

Clarke taxes create a budget surplus. It is not possible to redistribute the actual surplus among the citizens while maintaining the incentives of the Clarke mechanism because the question of whether or not a citizen is pivotal—and hence pays a tax—depends on the announcements of the other citizens. Thus if the actual surplus were distributed, citizen i could increase the amount to be distributed, and hence increase his share, by altering X_i so as to increase the set of pivotal voters. This would disturb his incentive to announce $X_i = V_i$. Bailey (1997) suggested that the government return to each citizen i a fraction of the hypothetical budget surplus that the government *would* collect if citizen i were absent, or $\frac{N-2}{(N-1)^2}\sum_{l=1,l\neq i}^{N} T_l^{Clarke}(X_l)\big|_{i \text{ is absent}}$.[12] Because the hypothetical budget surplus generally differs from the actual budget surplus, Bailey's refund mechanism will generally either not fully exhaust the actual budget surplus or lead to a (probably exceedingly small) budget deficit.[13]

If the government's task is to allocate a single item to the higher of two bidders, then the Clarke mechanism and Vickrey's second-price auction are identical. To allocate the item, the government adopts the policy $P(N)$ that maximizes

$$W = \begin{cases} X_1 & \text{if } P(N) = 1 \\ X_2 & \text{if } P(N) = 2 \end{cases}, \tag{3.7}$$

setting $P(N) = i$, where i is the identity of the bidder for whom $X_i = max\{X_1, X_2\}$.

Bidder i's Clarke tax is

$$T_i(X_i) = \begin{cases} -X_{k,k\neq i} & \text{if } P(N) = i \text{ and } P(N\backslash i) \neq i \\ X_{k,k\neq i} - X_{k,k\neq i} & \text{if } P(N) \neq i \text{ and } P(N\backslash i) \neq i \end{cases}, \tag{3.8}$$

where bidder $k, k \neq i$ would have been the high bidder in bidder i's absence. Thus bidder i pays the second highest bid $X_{k,k\neq i}$ if his bid is highest, and nothing otherwise.

Vickrey's second-price auction and the Clarke mechanism align each citizen's interest with society's interest by charging each citizen i the aggregate social cost to all citizens other than citizen i of announcing X_i, which is positive if the announcement of X_i changes the social choice. Thus, although the Clarke mechanism is a member of the family of Groves mechanisms, it works differently than the Groves mechanism with $A_i = 0$, which aligns the interests of

[12] See Bailey (2001, pp. 216 – 217).

[13] Plassmann and Tideman (2011, pp. 71 – 72) provide a numerical example.

citizens and society by assigning to each citizen the entire net social benefit of the eventual social choice for other citizens, so that every citizen bears the cost of lowering this net social benefit.

Until 1975, the researchers who were aware of Clarke's work were not aware of the work of Groves and Loeb, and vice versa. But in that year the connections were made between these two strands of research and Vickrey's earlier work, and Gordon Tullock commissioned a special issue of *Public Choice* (published as Volume 29:2 in 1977, edited by Tideman) to bring together the various perspectives.

4. The expected externality mechanism

Green and Laffont (1977) showed that the family of Groves mechanisms includes all mechanisms that are efficient and have the revelation of V_i as a dominant strategy for every citizen, that is, all mechanisms that lead to an efficient outcome. Under Groves mechanisms, the revelation of V_i is a dominant strategy for each citizen *i* because transfer function 3.2 is designed in such a way that citizen *i* can do no better than announce $X_i = V_i$, regardless of what he expects other citizens to do. Walker (1980) showed that no mechanism that has truth-telling as a dominant strategy can guarantee a balanced budget. Arrow (1979) and separately d'Aspremont and Gérard-Varet (1979) showed that one can achieve budget balance with a revelation mechanism that is defined in terms of the citizens' expectations, achieving truth-telling as a Bayesian Nash equilibrium rather than as a dominant strategy.

As before, we consider the special case of a social choice between two policies. Consider citizen $j, j = 1, \dots, n$ who is willing to pay X_j for the implementation of policy 1. Let $B_j = \sum_{m,m\neq j} X_m - \sum_k X_k$ be society's net willingness to pay for policy 1 when citizen j's contribution is ignored, and let $F(B_j)$ be (citizen j's belief of) the cumulative density function of B_j, with corresponding probability density function $f(B_j)$. Citizen j's expected net benefit can then be written as

$$E[\pi(X_j)] = 0 \int_{-\infty}^{-X_j} f(B_j) dB_j + V_i \int_{-X_j}^{\infty} f(B_j) dB_j + T_j(X_j)$$
$$= V_j[1 - F(-X_j)] + T_j(X_j), \tag{4.1}$$

where $T_j(X_j)$ is the transfer function and $1 - F(-X_j)$ denotes the probability that $B_j + X_j > 0$, so that the government will adopt policy 1. Citizen j maximizes his expected payoff function by identifying the value of X_j that solves his first-order condition

$$\frac{dE[\pi(X_j)]}{dX_j} = V_j f(-X_j) + \frac{dT_j(X_j)}{dX_j} = 0. \quad (4.2)$$

Any citizen $k, k = n+1, \dots, N$ who is willing to pay X_k for the implementation of policy 2 solves a corresponding problem, where $B_k = \sum_{m,m\neq k} X_m - \sum_j X_j$ defines society's net willingness to pay for the status quo rather than policy 2 when citizen k's contribution is ignored.

The term $f(-X_j)$ denotes the density of B_j at the smallest value of X_j at which citizen j's announcement leads to the adoption of policy 1, because $B_j = -X_j$ implies $\sum_{m=1}^{n} X_m = \sum_{k=n+1}^{N} X_k$.[14] Thus $V_j f(-X_j)$ is citizen j's expected benefit per dollar of additional announced value X_j at the smallest X_j for which citizen j's preferred policy 1 will win. If the government knew citizen j's beliefs regarding $f(-X_j)$, then it could set $\frac{dT_j(X_j)}{dX_j} = -X_j f(-X_j)$ to provide each citizen j with an incentive to announce $X_j = V_j$. Thus at the margin, citizen j's expected benefit of increasing X_j would equal his expected cost, $-X_j f(-X_j)$. Because $-X_j f(-X_j) = B_j f(B_j)$ at the smallest X_j for which policy 1 wins, the mechanism charges citizen j the expected social cost of changing the social choice, which explains why the mechanism is called the expected externality mechanism.[15]

If the government charges every citizen $i, i = 1, \dots, N$ an announcement tax $h_i(X_i)$ that depends only on X_i but not on the announcement of any other citizen $m, m \neq i$, then the government can distribute the entire tax revenue among the citizens with the transfer function

$$T_i(X_i) = -h_i(X_i) + \frac{1}{N-1} \sum_{m=1,m\neq i}^{N} h_m(X_m) \quad (4.3)$$

so that the budget is balanced. Balancing the budget in this way does not affect citizen i's choice of X_i because adjustment function $A_i = \frac{1}{N-1} \sum_{m=1,m\neq i}^{N} h_m(X_m)$ is independent of X_i. This is in contrast to the Clarke mechanism where every citizen's pivotalness—and hence the citizen's Clarke tax—depends on the announcements of the other citizens.

To understand the relationship between the expected externality mechanism and the Clarke mechanism, consider citizen j's transfer function (his Clarke tax):

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[14] The expected net benefit of any citizen $k, k = n+1, \dots, N$ who favors policy 2 is $E[\pi(X_k)] = V_k F(-X_k) + T_k(X_k)$, where $f(-X_k)$ denotes the probability that citizen k will be pivotal if he increases X_k by a marginal amount.

[15] This assumes that all citizens other than j reveal their true valuations so that $-X_j = B_j = \sum_{m,m\neq j} X_m - \sum_k X_k$ is a correct measure of the net social benefit of adopting policy 2 when citizen j's contribution is ignored.

$$T_j^{Clarke}(X_j) = \begin{cases} B_j & \text{if } B_j < 0 \text{ and } X_j \geq |B_j| \\ 0 & \text{if } B_j < 0 \text{ and } X_j < |B_j| \\ 0 & \text{if } B_j > 0 \end{cases} \quad . \tag{4.4}$$

First-order condition 4.2 is not directly applicable to the Clarke mechanism because transfer function 4.4 is discontinuous at the only point at which the transfer value changes, but the first-order condition is nevertheless helpful in interpreting the Clarke mechanism. The term $-X_j f(-X_j)$ is the subjective expected marginal cost of increasing X_j at the smallest X_j that makes citizen j pivotal, because the Clarke tax equals $B_j = X_j$ at this value of X_j. Thus if citizen j and the government have the same beliefs about $f(-X_j)$, then one way of interpreting first-order condition 4.2 is that citizen j has an incentive to announce $X_j = V_j$ if he must pay a tax that equals, at the smallest value of X_j that makes the citizen pivotal, the expected value of his Clarke tax.

The expected externality mechanism ensures that the marginal change in the tax at the optimal value of X_i equals the expected marginal net social cost of increasing X_i, while the actual tax $h_i(X_i)$ might exceed the net social cost. In contrast, Groves mechanisms assign to each citizen i the full net social cost of announcing X_i, which ensures that citizen i bears the marginal cost of announcing $X_i \neq V_i$. With both types of mechanism, charging citizen i the net social cost of announcing X_i *at the margin* provides the incentive to announce $X_i = V_i$.

5. Quadratic Voting

The major short-coming of the expected externality mechanism is its requirement that citizen j and the government have the same beliefs about $f(-X_j)$. The severity of this short-coming depends on the distribution of $f(-X_j)$. Consider the case of a large electorate in which every citizen has only a very small chance of being pivotal. If all citizens take the possibility of being pivotal as being beyond their control, then, subjectively, $f(-X_j)$ is a constant function for every citizen, and it is reasonable to treat $p = f(-X_j)$ as being the same for all citizens. Thus citizen j's belief about the probability of being pivotal by virtue of the last cent that he announces is assumed to be, for all practical purposes, independent of the value of X_j that he announces, so that citizen j's expected announcement cost (a triangle of lost surplus for others) is proportional to X_j^2. For a large electorate, first-order condition 4.2 suggests that a transfer function of the form

$$T_j(X_j) = -aX_j^2 + A_j, \tag{5.1}$$

that is quadratic in X_i will provide the appropriate incentive to reveal V_j.

This is indeed the case for transfer functions with $a = -1$, as conjectured by Weyl (2012) and shown by Lalley and Weyl (2014), and with $a = \frac{1}{4\sigma\sqrt{n\pi}}$, as shown by Goeree and Zhang (2013). Consider first Goeree and Zhang's contribution: the Central Limit Theorem says that as *n* approaches infinity, $F(-X_j)$ converges to the cdf of the normal distribution

$$\Phi\left(\frac{(-X_j)}{(\sigma\sqrt{2n})}\right) \approx \frac{1}{2}\left(1 + \frac{(-X_j)}{(\sigma\sqrt{n\pi})}\right), \tag{5.2}$$

where the right-hand side is the first-order Taylor series approximation of the normal cdf around zero. The derivative of the Taylor series approximation of $F(-X_j)$ is $\frac{1}{2}/(\sigma\sqrt{n\pi})$, which is an approximation of $f(-X_j)$. Thus for an electorate of infinite size, the transfer function

$$T_j(X_j) = -\frac{1}{4\sigma\sqrt{n\pi}}X_j^2 + A_j^{GZ}, \tag{5.3}$$

which, using equation 4.2, leads to the first-order condition

$$\frac{dE[\pi(X_j)]}{dX_j} = \frac{1}{2}/(\sigma\sqrt{n\pi})V_j - \frac{1}{2}/(\sigma\sqrt{n\pi})X_j = \frac{1}{2}/(\sigma\sqrt{n\pi})(V_j - X_j) = 0, \tag{5.4}$$

provides citizen *j* with an incentive to announce $X_j = V_j$.

To distribute the tax revenue $\sum_{i=1}^{N}\frac{1}{4\sigma\sqrt{n\pi}}X_i^2$ among the citizens, Goeree and Zhang proposed the adjustment function[16]

$$A_j^{GZ} = \frac{1}{N-1}\sum_{m=1,m\neq j}^{N}\frac{1}{4\sigma\sqrt{n\pi}}X_m^2, \tag{5.5}$$

so that whether a citizen pays a tax or receives a subsidy depends on whether his announcement is greater than or less than the average announcement of all his peers. Because citizen *m*'s gross tax $h(X_m) = \frac{1}{4\sigma\sqrt{n\pi}}X_m^2, m \neq j$, does not depend on citizen *j*'s announcement X_j, adjustment function 5.5 preserves citizen *j*'s incentive to announce $X_j = V_j$. The sum of all taxes and

[16] Our notation is slightly imprecise because *n* voters are in favor of policy 1 while *N* – *n* voters are in favor of policy 2. Because the difference between the magnitudes of the two parts of the electorate becomes inconsequential as these magnitudes become infinitely large, we use *n* to denote the magnitude of either part of the electorate.

subsidies equals zero because $\sum_{i=1}^{N} T_i(X_i) = -\sum_{i=1}^{N} \frac{1}{4\sigma\sqrt{n\pi}} X_i^2 + \frac{1}{N-1}\sum_{i=1}^{N}\left(\sum_{m,m\neq i}^{N} \frac{1}{4\sigma\sqrt{n\pi}} X_m^2\right) = \frac{1}{4\sigma\sqrt{n\pi}}\sum_{i=1}^{N} X_i^2 - \frac{N-1}{N-1}\frac{1}{4\sigma\sqrt{n\pi}}\sum_{m=1}^{N} X_m^2 = 0$.

One shortcoming of Goeree and Zhang's mechanism is that, to be able to specify transfer function 5.3, the government needs to know the number of citizens and the standard deviation of $F(-X_j)$. This shortcoming does not apply to the transfer function proposed by Lalley and Weyl (2014), who showed that the simple transfer function

$$T_j(X_j) = -X_j^2 + A_j^{QV}, \tag{5.6}$$

which does not require the government to know anything about $F(-X_j)$, permits the government to make an efficient decision on the basis of the values that citizens announce, even if the mean of $F(-X_j)$ differs from zero. If $A_j^{QV} = 0$, then citizen *j* pays an amount equal to the square of the number of dollars that he is willing to spend on (= votes that he wants to cast for) his preferred policy, which explains why their mechanism is called "quadratic voting."

Transfer function 5.6 leads to the first-order condition

$$X_j = \frac{p}{2} V_j, \tag{5.7}$$

and summing both sides of equation 5.7 over all *n* citizens who prefer policy 1 and all *N* – *n* citizens who prefer policy 2 implies

$$\sum_{j=1}^{n} X_j - \sum_{k=n+1}^{N} X_k = \frac{p}{2}\left(\sum_{j=1}^{n} V_j - \sum_{k=n+1}^{N} V_k\right). \tag{5.8}$$

In contrast to Groves mechanisms and the expected externality mechanism, quadratic voting does not provide citizen *i* with an incentive to announce $X_i = V_i$ but rather to announce a value that is proportional to V_i, with the same constant of proportionality for all citizens, provided that they have the same beliefs.[17] Thus the attractiveness of quadratic voting lies in the fact that it enables the government to make a socially optimal choice on the basis of the sign of $\sum_{j=1}^{n} X_j - \sum_{k=n+1}^{N} X_k$, which is the same as the sign of $\sum_{j=1}^{n} V_j - \sum_{k=n+1}^{N} V_k$ as long as all

[17] The Thompson insurance mechanism (see Thompson, 1966) has the same property. Like the expected externality mechanism, the Thompson insurance mechanism requires that citizens and the government have the same beliefs regarding the probability that policy 1 is implemented. Unlike the expected externality mechanism, the Thompson insurance mechanism requires that citizens be risk-averse and is applicable only to the choice between two policies. In addition, the Thompson insurance mechanism fails to provide appropriate incentives in cases in which citizens do not wish to buy insurance, for example, when the choice of one policy over the other leads to non-pecuniary gains or losses that affect total but not marginal utility (see Calfee and Rubin, 1992, and Bailey, 2001, p. 182 – 188).

citizens report the valuations that maximize their respective expected utilities under the same beliefs.[18]

Lalley and Weyl propose the adjustment function

$$A_j^{QV} = \frac{1}{N-1}\left(\sum_{m,m\neq j}^{n} X_m^2 + \sum_k^N X_k^2\right) = \frac{1}{N-1}\sum_{l,l\neq j}^{N} X_l^2, \tag{5.9}$$

so that whether a citizen pays a net tax or receives a net subsidy depends on whether his number of votes is greater than or less than the average number of votes of all his peers. The sum of all taxes and subsidies is zero because $\sum_{i=1}^{N} T_i(X_i) = \sum_{i=1}^{N} X_i^2 - \sum_{i=1}^{N} \frac{1}{N-1}\left(\sum_{l,l\neq i}^{N} X_l^2\right) = \sum_{i=1}^{N} X_i^2 - \frac{N-1}{N-1}\sum_{l=1}^{N} X_l^2 = 0$.

6. Simultaneous choices of continuous options using points

A payment that is a quadratic function of the intensity of a preference arises also in the point voting mechanism developed by Hylland and Zeckhauser. If strengths of preferences are expressed in terms of points rather than money, then for the points to have an opportunity cost, several decisions must be made simultaneously. If in addition the decisions under consideration are the levels of continuous variables, then quadratic voting morphs into the mechanism proposed by Hylland and Zeckhauser (1979), hereafter HZ. Unlike standard point-voting mechanisms, which almost always motivate voters to allocate all of their points to a single issue for reasons explained by Mueller (1973, pp 67-68; 1977), the HZ mechanism motivates voters to allocate their points, at the margin, in proportion to their marginal utilities for changes in the parameters to be determined. It achieves this goal by specifying that a voter's influence on a decision will be proportional to the square root of the number of points that the voter allocates to the issue. This rule is equivalent to specifying that the cost in points of a given amount of influence is proportional to the square of the amount of influence sought, as in quadratic voting.

The HZ mechanism invites each citizen to allocate a fixed number of points among decisions about the quantities of a variety of public goods, with the *change* in the quantity of a public good that is induced by the citizen's allocation of points to that good being the *signed* square root of the number of points allocated to the decision. Thus equilibrium announcements

[18] The version of quadratic voting that we describe here assumes an infinite number of citizens. Lalley and Weyl (2014) show that the assumptions for the efficiency of quadratic voting are approximately satisfied when the number of citizens is large but finite.

can be either positive or negative, which is in contrast to mechanisms where citizens announce the quantity of the public good that they desire, as they do with Groves mechanisms. As with the mechanisms for which citizens announce how much they would like to add to the quantity that would be chosen if they abstained, equilibrium must be approached by successive approximations, because the changes that a citizen wants depend on the quantities that would be chosen without his participation.

The HZ mechanism works as follows: Consider the vector $Q = (Q_1, \dots, Q_K)$, where Q_k denotes the quantity of public good *k*. The government begins by proposing quantities of the *K* public goods, $Q' = (Q'_1, \dots, Q'_K)$. Every citizen *i* is asked to announce, for each public good *k*, the number of units X_{ik} by which he would like to change Q'_k, subject to the constraint that the sum of the squares of the requested changes may not exceed the number of points with which each citizen is endowed. The government then determines revised quantities $Q'' = (Q''_1, \dots, Q''_K)$, with $Q''_k = Q'_k + \sum_{i=1}^{N} X_{ik}$. If the suggested changes cancel each other out so that

$$\sum_{i=1}^{N} X_{ik} = 0 \;\forall k, \tag{6.1}$$

when every citizen *i* announces the units X_{ik} that maximize his utility, given the constraint on points used and the announcements of the other citizens, then $Q' = Q''$ describes a Nash equilibrium.[19]

Let *A* be the number of "influence points" that each citizen can use to express his preferences for either increasing or decreasing the proposed quantity of each of *K* public goods. Let $a_i = (a_{i1}, \dots, a_{iK})$ denote the vector of influence points that citizen *i* spends on the public goods, with $\sum_{k=1}^{K} |a_{ik}| \leq A$. Spending a_{ik} influence points leads to a suggested change X_{ik} in the quantity of public good *k* according to

$$X_{ik} = \mathbb{I}_{ik}\sqrt{\mathbb{I}_{ik} a_{ik}}, \tag{6.2}$$

where $\mathbb{I}_{ik}$ is an indicator function with $\mathbb{I}_{ik} = 1$ if $a_{ik} = 1$ and $\mathbb{I}_{ik} = -1$ if $a_{ik} = -1$. Given the government's initial quantity Q'_k of public good k and the announcements $X_{lk} = \mathbb{I}_{lk}\sqrt{\mathbb{I}_{lk} a_{lk}}$ of all citizens $l \neq i$, citizen *i*'s expenditure of a_{ik} influence points leads to the revised quantity

$$Q''_k = Q'_k + \sum_{l=1}^{N} X_{lk} = Q'_k + \sum_{l,l \neq i}^{N} X_{lk} + \mathbb{I}_{ik}\sqrt{\mathbb{I}_{ik} a_{ik}}. \tag{6.3}$$

[19] Hylland and Zeckhauser (1979) also describe a procedure by which society might reach such an equilibrium.

Citizen i maximizes his utility $U_i(Q)$ by identifying the vector a_i^* that satisfies his budget constraint $A - \sum_{k=1}^{K} |a_{ik}^*| \geq 0$ and solves his K first-order conditions [20]

$$\frac{\partial U_i(Q)}{\partial X_{ik}} \frac{\partial X_{ik}}{\partial a_{ik}} - \mathbb{I}_{ik}\mu_i = 0$$

$$\Leftrightarrow \frac{\partial U_i(Q)}{\partial X_{ik}} \frac{1}{2\sqrt{\mathbb{I}_{ik}a_{ik}}} - \mathbb{I}_{ik}\mu_i = 0$$

$$\Leftrightarrow \frac{1}{2\mu_i} \frac{\partial U_i(Q)}{\partial X_k} = X_{ik}, \qquad (6.4)$$

where μ_i is the Lagrange multiplier for citizen i's point budget constraint. Thus a citizen who uses his points optimally allocates them in such a way that the ratio of the marginal utility of a good to the square root of the number of points allocated on it is constant across goods.

At the socially efficient allocation, the ratio of the signed square roots of citizen i's announcements $|a_{ik}|$ and $|a_{ig}|$ equals the sum of the ratio of weighted marginal utilities that all other citizens obtain from any two public goods k and g, or

$$\frac{\mathbb{I}_{ik}\sqrt{\mathbb{I}_{ik}a_{ik}}}{\mathbb{I}_{ig}\sqrt{\mathbb{I}_{ig}a_{ig}}} = \frac{\sum_{l,l\neq i}^{N} \lambda_l \frac{\partial U_l(Q)}{\partial Q_k}}{\sum_{l,l\neq i}^{N} \lambda_l \frac{\partial U_l(Q)}{\partial Q_g}} = \frac{\frac{\partial U_i(Q)}{\partial Q_k}}{\frac{\partial U_i(Q)}{\partial Q_g}}. \qquad (6.5)$$

The way that the HZ mechanism relates to marginal cost pricing is that under the HZ mechanism, the marginal cost to an individual of changing the social decision about one dimension of the multi-dimensional continuum of possibilities, when measured in influence points, is equal to the amount of change that the individual has already caused. Thus when an individual has spread his influence points efficiently, the relative marginal utilities to an individual of changes in different dimensions are proportional to the square roots of the numbers of points spent on the different dimensions.

The Nash equilibrium under the HZ mechanism does not necessarily achieve Samuelson's condition for the optimal provision of a public good, $\sum_{i=1}^{N} \frac{\partial U_{ik}(Q_k^*)}{\partial Q_k} = q_k$, where q_k is the marginal cost per unit of public good k. This failure to achieve overall social efficiency is not surprising. One way of expressing Samuelson's condition is that the sum of the benefits to those who benefit from an incremental increase in a public good must equal the marginal cost of the public good plus the sum of the costs to those who bear costs, with both costs and benefits

[20] If a citizen attains his first choice without using all of his influence points, then a more complex description of the conditions at a citizen's optimum is needed. Hylland and Zeckhauser do not consider this complexity, and neither do we.

measured in dollars. For the HZ mechanism, the same condition holds when costs and benefits are measured in points. The two conditions would generally coincide only if the dollar value of a point was the same for all citizens, and it is implausible that the government could have the information necessary to ensure that that condition would hold. Although the HZ mechanism achieves Samuelson's condition only by chance, it still provides citizens with the incentive to state their true preferences for multiple public goods, given exogenously determined shares of their contributions towards financing these public goods.

Ensuring equality in points is much easier than reaching an equitable allocation of wealth. When all goods are public and are produced with existing resources, the HZ mechanism generates a Nash equilibrium that is not influenced by differences in wealth. In such a setting, the HZ equilibrium with equal points is undoubtedly attractive, especially in comparison with the equilibria reached by conventional decision mechanisms. However, as , and the HZ mechanism generates a Nash equilibrium that is not influenced by differences in wealth. In comparison with the equilibria reached by conventional decision mechanisms, the HZ equilibrium with equal points is undoubtedly attractive when all goods are public and are produced with existing resources. As long as there are private goods, the question of the tradability of points arises. If points are not tradable for private goods, then the HZ equilibrium will be Pareto optimal only by chance. If points are tradable, then wealth still affects the provision of public goods.

7. Conclusion

When Vickrey sought a way to implement Lange's idea of efficient counterspeculation, he found that the answer was to pay suppliers according to the marginal social value of what they supplied and to charge buyers the marginal social value of what they bought. An efficient auction, he found, results from charging the highest bidder the marginal social value of the item, represented by the second-highest bid. Coase noted that efficient management of pollution required that both polluters and those harmed by pollution be charged the marginal social costs of their actions. In a variation on this theme, both Vickrey and Zeckhauser noted that efficient deterrence of accidents results from charging each person involved in an accident the marginal social cost of his actions, represented by the full cost of the accident. Groves noted that efficient incentives in teams require that every team member be awarded the full product of the team's efforts. A pattern can be seen: efficiency is achieved by marginal cost pricing.

When Clarke sought a mechanism that would motivate citizens to report truthfully their preferences for public goods, he had the idea of charging people the net social cost of their expressed preferences. Groves generalized the idea, and Green and Laffont showed that Groves' generalization described any mechanism that would motivate truthful reporting of preferences as a dominant strategy. But if one is satisfied with a Bayesian Nash equilibrium rather than a dominant strategy, then, as Arrow as well as D'Aspremont and Gérard-Varet showed, one can motivate people to express their preferences truthfully by requiring them to pay the prior expected value of the net social cost of expressing their preferences. For a binary choice, that expected value is proportional to the square of the expressed preference. Hylland and Zeckhauser showed that people can be motivated to express the relative intensities of their preferences for changes in the quantities of two or more public goods by asking them to allocate points, provided that, in parallel with the Arrow-D'Aspremont-Gérard-Varet proposal, the cost (in points) of a given amount of influence is proportional to the square of the amount of influence.

Finally we come to the Lalley and Weyl proposal for quadratic voting. While their idea is similar to that of Arrow, D'Aspremont, and Gérard-Varet, it is so much simpler because rather than requiring parameter estimates of the prior distribution of outcomes, it simply charges each person the square of the number of votes he wants to cast. The ideas that preceded quadratic voting were often developed by people who were not aware of related work, and the same idea was frequently developed nearly simultaneously by different scholars. The separate strands were brought together once, in the 1977 special issue of *Public Choice*. Here they are brought together again. It will be very interesting to see what further developments emerge.

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