A Macroeconomic Framework for Quantifying Systemic Risk

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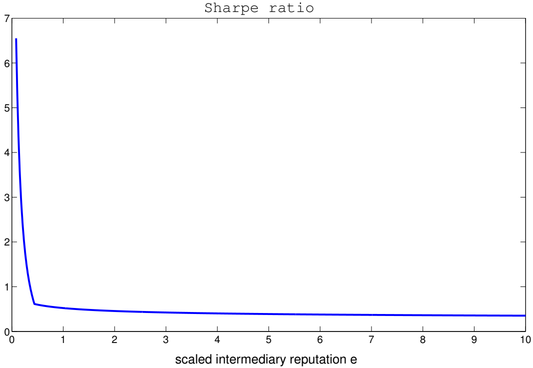
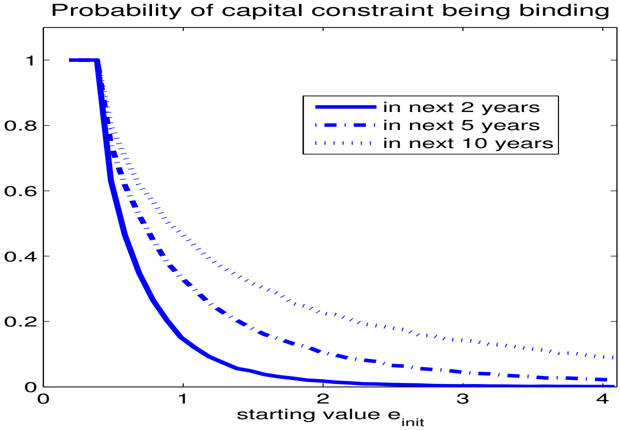
Financial Crisis in the Model



Note: Capital constraint binds for $e < 0.44$

Systemic Risk



Outline

1. Nonlinear macro model of a financial crisis
   - Occasionally binding constraint; global solution method.
   - He-Krishnamurthy, Brunnermeier-Sannikov, Adrian-Boyarchenko
2. Calibration and Data
   - Nonlinearity in model and data
   - Match conditional moments of the data, conditioning on negative (i.e., recession) states
3. Quantify systemic risk
   - What is the ex-ante (e.g., initial conditions of 2007Q2) likelihood of crisis states? (... low)
   - What makes the probability higher?
   - Economics of stress tests (as opposed to accounting of stress tests).

Agents and Technology

- Two classes of agents: households and bankers
  - Households:

$$\mathbb{E}\left[\int_0^\infty e^{-\rho t}\left(c_t^y\right)^{1-\phi}\left(c_t^h\right)^{\phi} dt\right],$$

- Two types of capital: productive capital K_t and housing capital H.
  - Fixed supply of housing $H \equiv 1$
  - Price of capital q_t and price of housing P_t determined in equilibrium

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- Production $Y = AK_t$, with A being constant
- Fundamental shocks: stochastic capital quality shock dZ_t. TFP shocks

$$\frac{dK_t}{K_t} = i_t dt - \delta dt + \sigma dZ_t$$

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$$\frac{dK_t}{K_t} = i_t dt - \delta dt + \sigma dZ_t$$
- Investment/Capital i_t, quadratic adjustment cost
$$\Phi(i_t, K_t) = i_t K_t + \frac{\kappa}{2}\left(i_t - \delta\right)^2 K_t$$
$$\max_{i_t}\ q_t i_t K_t - \Phi(i_t, K_t) \ \Rightarrow\ i_t = \delta + \frac{q_t - 1}{\kappa}$$

Aggregate Balance Sheet

Loans to Capital Producers i_t

Intermediary Sector

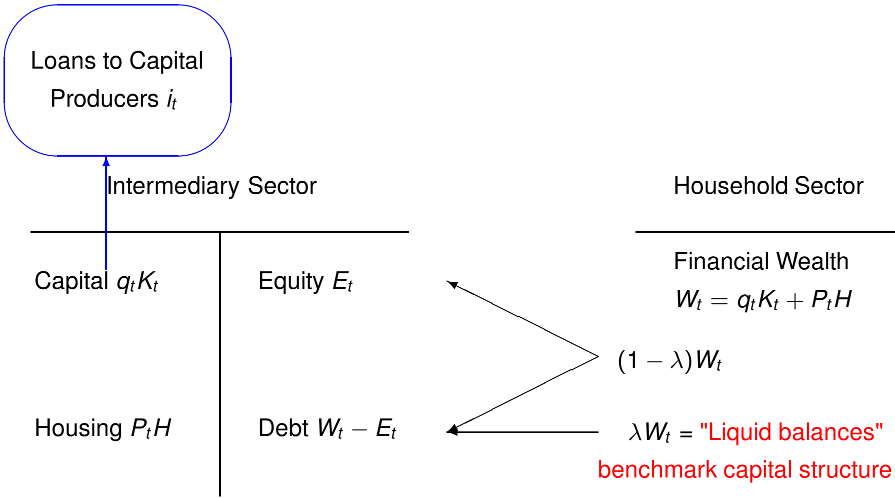
Capital $q_t K_t$	Equity E_t
Housing $P_t H$	Debt $W_t - E_t$

Household Sector

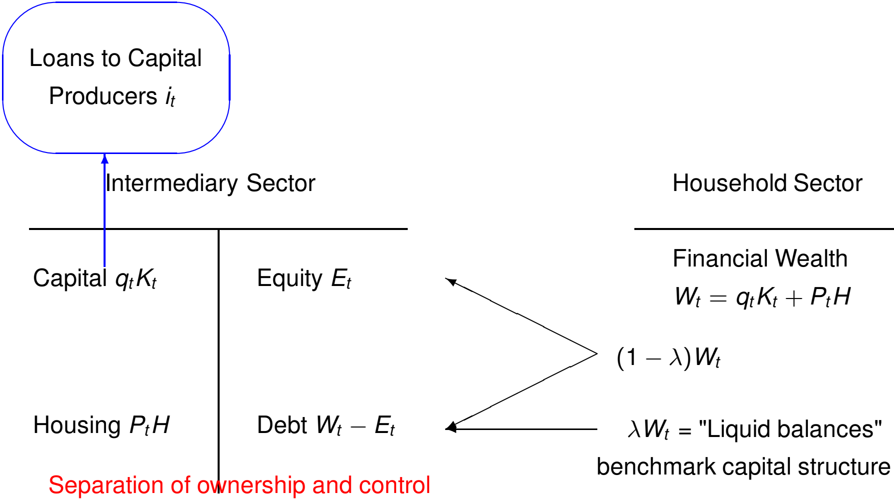
Financial Wealth

$W_t = q_t K_t + P_t H$

Aggregate Balance Sheet



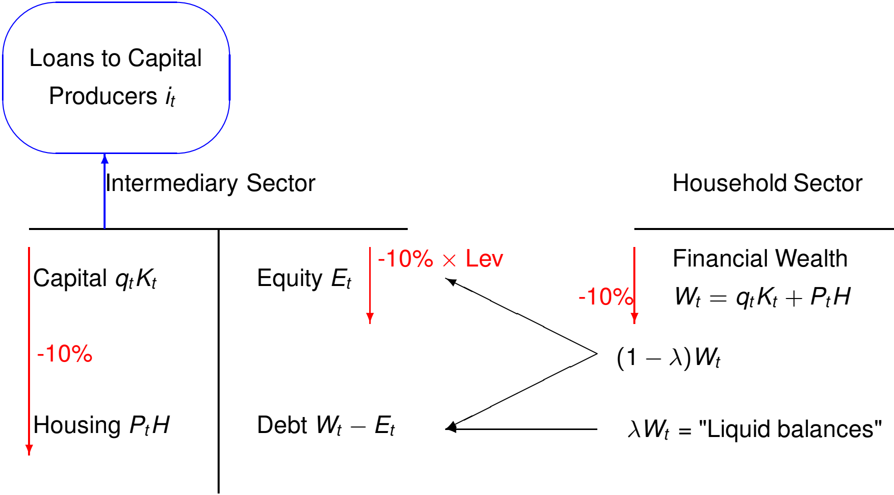
Equity Matters



Separation of ownership and control

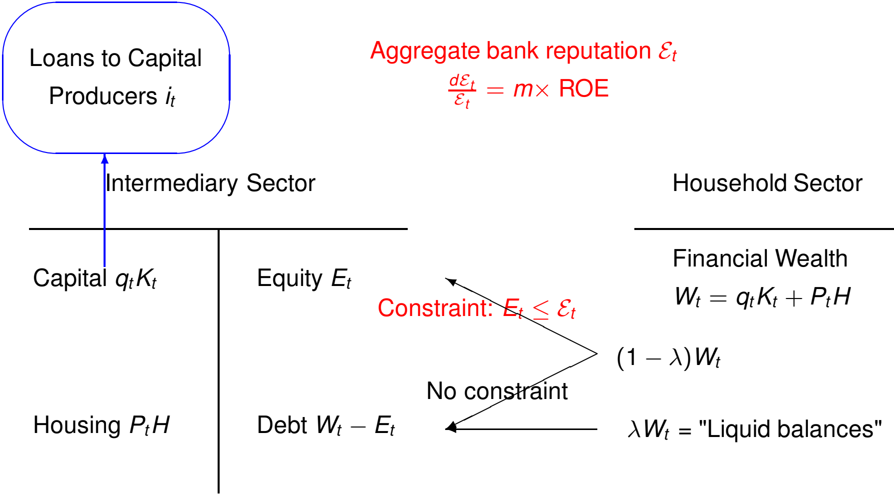
Banker maximizes $E[ROE] - \frac{m}{2} Var[ROE]$

Equity Dynamics



Banker maximizes $E[ROE] - \frac{m}{2} Var[ROE]$

Equity Constraint



Banker maximizes $E[ROE] - \frac{m}{2} Var[ROE]$

Intermediary Reputation

- Single bank has “reputation" (skill, etc.) ϵ_t linked to intermediary performance (constant m)

$$\frac{d\epsilon_t}{\epsilon_t} = md\tilde{R}_t.$$

  - Poor returns reduce reputation: Berk-Green, 04; flow-performance relationship, Warther 95; Chevalier-Ellison, 97
  - Or, ϵ_t as banker's “net worth" fluctuating with performance
    - Kiyotaki-Moore 97, He-Krishnamurthy 12, Brunnermeier-Sannikov 12
- Household invests a maximum of ϵ_t dollars of equity capital with this banker

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- Household invests a maximum of ϵ_t dollars of equity capital with this banker
- $\mathcal{E}_t$: aggregate reputation. Aggregate dynamics of $\mathcal{E}_t$

$$\frac{d\mathcal{E}_t}{\mathcal{E}_t} = m d\tilde{R}_t - \eta dt + d\psi_t$$

- Exogenous death rate η. Endogenous entry $d\psi_t > 0$ of new bankers in extreme bad states

Equity Capital Constraint

- Representative household with W_t, split between bond household λW_t and equity household $(1-\lambda)W_t$ (Lucas 1990)
- Benchmark capital structure: λW_t of Debt, $(1-\lambda)W_t$ of Equity
  - if there is no capital constraint ($\mathcal{E}_t$ is infinite)...
- Intermediary equity capital:

$$E_t = \min\left[\mathcal{E}_t, (1-\lambda)W_t\right]$$

- Suppose a -10% shock to real estate and price of capital:
- $W_t \downarrow 10\%$ (Household wealth = aggregate wealth)
- Reputation: $\frac{d\mathcal{E}_t}{\mathcal{E}_t} = m d\tilde{R}_t + \ldots$ Two forces make $\mathcal{E}_t \downarrow$ more than 10%:
  1. Return on equity = $d\tilde{R}_t < -10\%$: equity is levered claim on assets
  2. $m > 1$ in our calibration

Solving for Equilibrium

- Markov equilibrium with state variables $(\mathcal{E}_t, K_t)$
- Scale invariance. Unidimensional state variable $e_t = \mathcal{E}_t / K_t$, with endogenous evolution

$$de_t = \mu_e dt + \sigma_e dZ_t$$

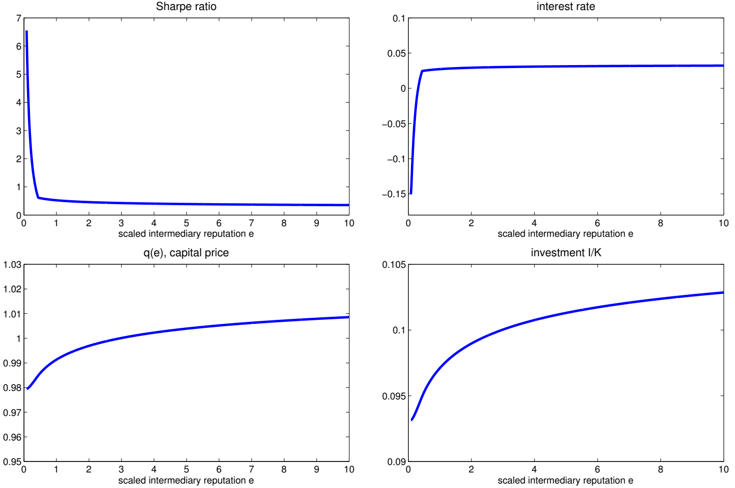
- Asset prices: land price/rent ratio ($p(e)$); capital price ($q(e)$); interest rate (r_t).
- Quantities: Investment is q-theory, but based on intermediary pricing
- We solve for $q(e)$ and $p(e)$ as solutions to a system of ODEs
- Boundary Conditions
  - When $e = \infty$, $\mathcal{E}_t > (1 - \lambda) W_t$ always, frictionless economy. Solving $p(\infty)$, $q(\infty)$ analytically
  - As $e \to 0$, intermediaries' portfolio volatility, i.e. Sharpe ratio, rises
  - New bankers enter if $e = \underline{e}$ (Sharpe ratio hits γ, exogenous constant)
  - $\underline{e}$ is a reflecting boundary, with

$$q'(\underline{e}) = 0,\ p'(\underline{e}) = \frac{p(\underline{e})\beta}{1 + \underline{e}\beta}, \text{ and } Sharpe\_Ratio(\underline{e}) = \gamma$$

Calibration: Baseline Parameters

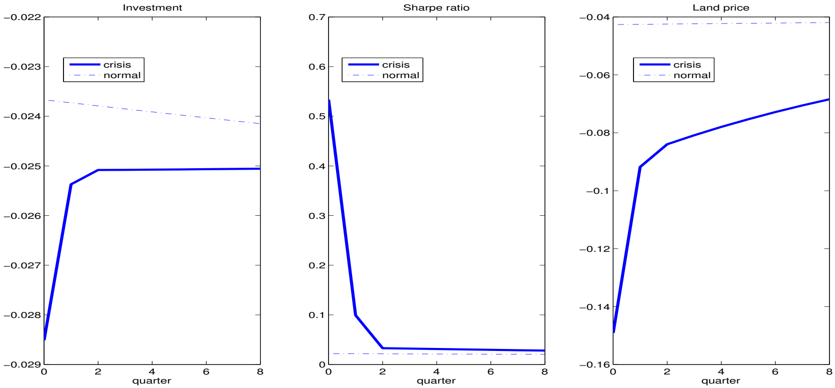
	Parameter	Choice	Targets (Unconditional)
Panel A: Intermediation			
m	Performance sensitivity	2	Average Sharpe ratio
λ	Debt ratio	0.67	Average intermediary leverage
η	Banker exit rate	17%	Good model dynamics
γ	Entry trigger	6.5	Highest Sharpe ratio
β	Entry cost	2.34	Average land price volatility
Panel B: Technology			
σ	Capital quality shock	4%	Average consumption volatility
δ	Depreciation rate	10%	Literature
κ	Adjustment cost	3	Literature
A	Productivity	0.148	Average investment-to-capital ratio
Panel C: Others			
ρ	Time discount rate	3%	Literature
ϕ	Housing share	0.5	Housing-to-wealth ratio

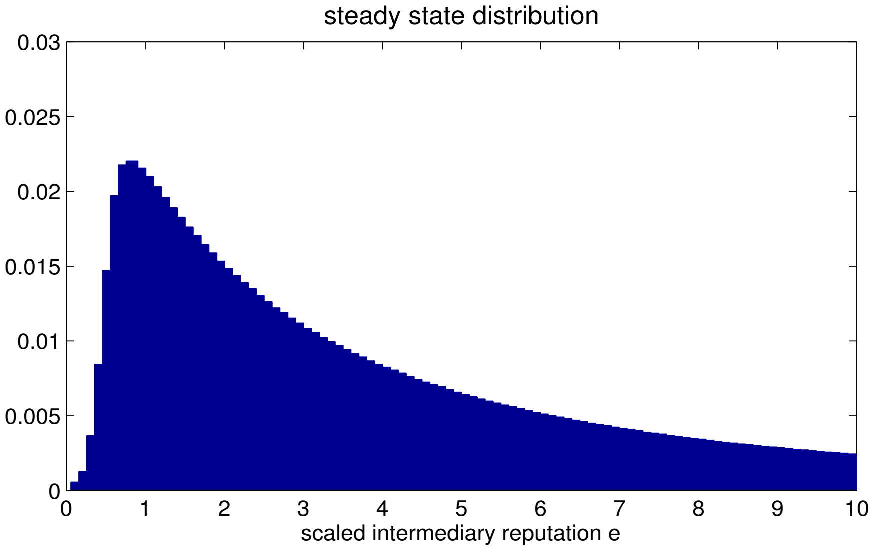
Results(1): State variable is $e_t = \frac{\mathcal{E}_t}{K_t}$



- Capital constraint binds for $e < 0.44$
- Without the possibility of the capital constraint, all of these lines would be flat. Model dynamics would be i.i.d., with vol=4%

State-dependent Impulse Response: -2% Shock ($= \sigma dZ_t$) ▸ VARdata




steady state distribution
0.03
0.025
0.02
0.015
0.01
0.005
0
0
1
2
3
4
5
6
7
8
9
10
scaled intermediary reputation e

Nonlinearities in Model and Data

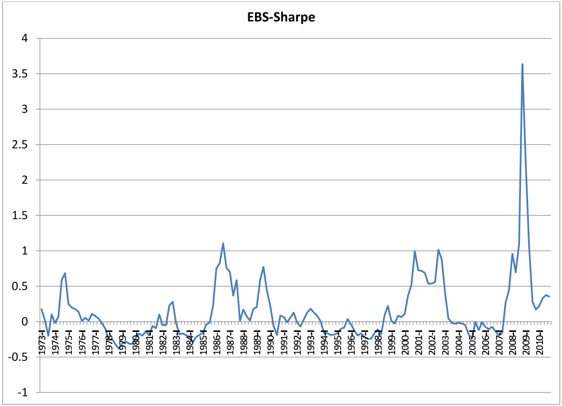
Model:

- Distress states = worst 33% of realizations of e ($e < 2.14$)
- Compute conditional variances, covariances of intermediary equity growth with other key variables

Data:

- Distress states = worst 33% of realizations of credit spread
  - We use Gilchrist-Zakrajsek (2011) Excess Bond Premium, which we convert to a Sharpe ratio
  - Excess Bond Premium: risk premium of corporate bonds, presumably reflects distress of financial sector
  - Similar results if using NBER recessions
- Compute conditional variances, covariances of intermediary equity growth with other key variables

EBS time series



Distress Classification

Distress Periods	NBER Recessions
1974Q3 - 1975Q4	11/73 - 3/75
1982Q3 - 1982Q4	7/81 - 11/82
1985Q4 - 1987Q3	
1988Q4 - 1990Q1	
	7/90 - 3/91
1992Q4 - 1993Q2	
2001Q2 - 2003Q1	3/01 - 11/01
2007Q3 - 2009Q3	12/07 - 6/09

Covariances in Data

	EB	NBER Recession	NBER+,-2Qs	EB, Drop Crisis
Panel A: Distress Periods				
vol(Eq)	31.48	32.40	31.78	22.19
vol(I)	8.05	8.79	7.44	4.56
vol(C)	1.71	1.54	1.59	0.95
vol(PL)	21.24	23.34	21.07	7.91
vol(EB)	60.14	93.59	74.57	28.69
cov(Eq, I)	1.31	1.08	0.84	0.37
cov(Eq, C)	0.25	0.16	0.13	0.04
cov(Eq, PL)	4.06	5.61	4.39	-0.63
cov(Eq, EB)	-6.81	-10.89	-7.57	-2.12
Panel B: Non-distress Periods				
vol(Eq)	17.54	19.42	17.11	17.26
vol(I)	6.61	5.97	4.91	6.60
vol(C)	1.28	0.98	0.91	1.28
vol(PL)	9.79	10.00	8.46	9.34
vol(EB)	12.72	30.93	30.42	12.78
cov(Eq, I)	0.07	0.09	-0.06	0.03
cov(Eq, C)	0.03	0.01	0.01	0.03
cov(Eq, PL)	0.12	0.07	-0.31	-0.01
cov(Eq, EB)	-0.14	-0.81	-0.78	-0.19

Matching State-Dependent Covariances

	Distress		Non Distress	
	Data	Baseline	Data	Baseline
$vol\,(Eq)$	31.48%	31.2	17.54	6.4
$vol\,(I)$	8.05%	5.4	6.61	4.8
$vol\,(C)$	1.71%	1.8	1.28	2.4
$vol\,(LP)$	21.24%	22.1	9.79	9.8
$vol\,(EB)$	60.14%	71.1	12.72	8.7
$cov\,(Eq, I)$	1.31%	0.90	0.07	0.3
$cov\,(Eq, C)$	0.25%	0.0	0.03	0.1
$cov\,(Eq, LP)$	4.06%	5.6	0.12	0.6
$cov\,(Eq, EB)$	-6.81%	-13.0	-0.14	-0.2

- Note: without the capital constraint, all volatilities would be 4%, and have no state dependence.

Probability of Systemic Event

- Based on EBS classification, we cross the 33% boundary ($e = 2.14$) between 2007Q2 and 2007Q3
- What is the likelihood of the constraint binding (“systemic crisis") assuming $e = 2.14$ currently:
  - 1.12% in next 2 years
  - 9.12% in next 5 years
  - 20.73% in next 10 years

Small...

VIX


VIX
90
80
70
60
50
40
30
20
10
0
Jan-06
Mar-06
May-06
Jul-06
Sep-06
Nov-06
Jan-07
Mar-07
May-07
Jul-07
Sep-07
Nov-07
Jan-08
Mar-08
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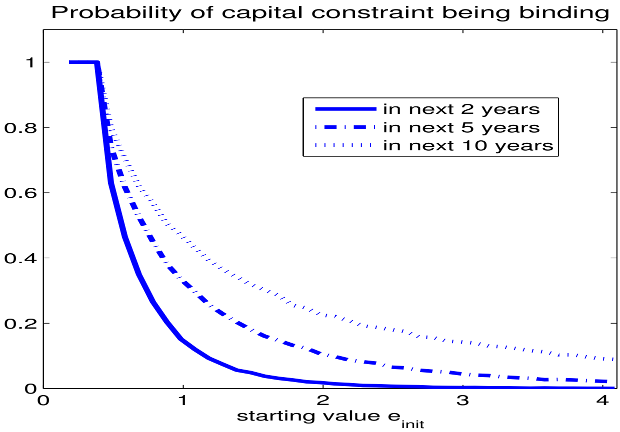
Nov-09

Stress testing

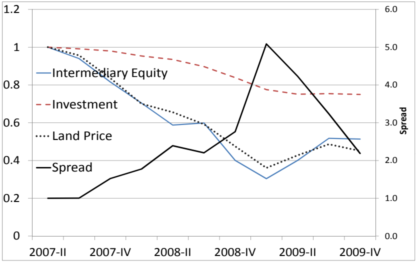


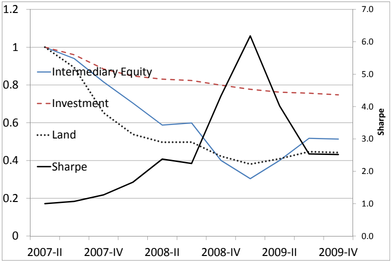
Map “stress test" into a shock to *e*.

Stress testing: “Hidden" Leverage

- Financial sector aggregate leverage fixed at 3 in model
- Pushed to crisis boundary after a -13% shock. 1.12% Prob of crisis in next 2 years.
- Suppose “hidden" leverage: leverage was 4.5 but agents take as given price functions and returns at leverage=3
- Prob of hitting crisis rises to 60%!

Matching Recent Crisis: *Data(L) and Model(R)*





- Based on realized equity return we uncover fundamental shocks to K

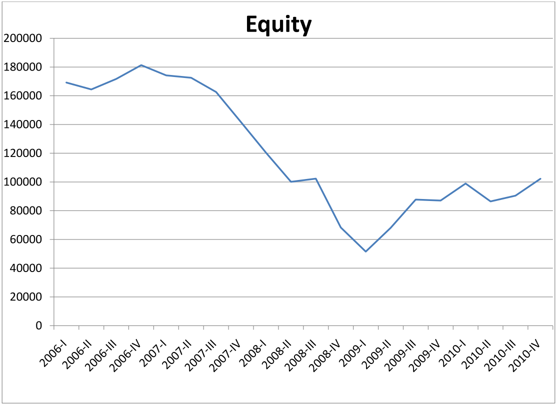
07QIII	07QIV	08QI	08QII	08QIII	08QIV	09QI	09QII	09QIII	09QIV
-3.1%	-5.5	-3.0	-1.4	-0.8	-2.2	-2.3	-2.2	-1.0	-1.0

- Total -19%. Capital constraint binds after 08QI—systemic crisis
  - In the model (data), land price falls by 56.2% (55%)
  - In the model (data), investment falls by 25.4% (25%)

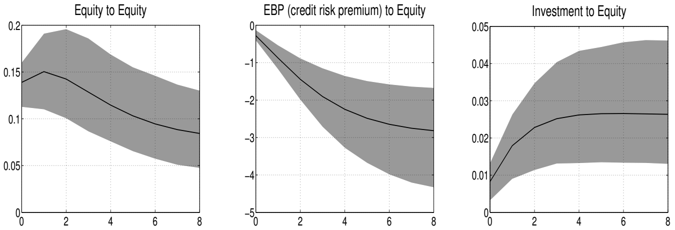
Conclusion

- We develop a fully stochastic model of a systemic crisis, with an equity capital constraint on the intermediary sector
- The model quantitatively matches the differential comovements in distress and non-distress periods
- Is able to replicate 2007/2008 period with only intermediary capital shocks
- Offers a way of mapping macro-stress tests into probability of systemic states.

Equity series



Panel A: Distress Periods



Panel B: Non Distress Periods