A Macroeconomic Framework for Quantifying Systemic Risk

Zhiguo He, University of Chicago and NBER
Arvind Krishnamurthy, Northwestern University and NBER

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Financial Crisis in the Model

Note: Capital constraint binds for \( e < 0.44 \)
Systemic Risk

Probability of capital constraint being binding

- in next 2 years
- in next 5 years
- in next 10 years

starting value $e_{\text{init}}$

He and Krishnamurthy (Chicago, Northwestern)
Outline

1. Nonlinear macro model of a financial crisis
   - Occasionally binding constraint; global solution method.
   - He-Krishnamurthy, Brunnermeier-Sannikov, Adrian-Boyarchenko

2. Calibration and Data
   - Nonlinearity in model and data
   - Match conditional moments of the data, conditioning on negative (i.e., recession) states

3. Quantify systemic risk
   - What is the ex-ante (e.g., initial conditions of 2007Q2) likelihood of crisis states? (... low)
   - What makes the probability higher?
   - Economics of stress tests (as opposed to accounting of stress tests).
Agents and Technology

- Two classes of agents: households and bankers
  - Households:
    \[\mathbb{E} \left[ \int_0^\infty e^{-\rho t} (c_t^y)^{1-\phi} (c_t^h)^{\phi} \, dt \right],\]

- Two types of capital: productive capital \(K_t\) and housing capital \(H\).
  - Fixed supply of housing \(H \equiv 1\)
  - Price of capital \(q_t\) and price of housing \(P_t\) determined in equilibrium
Agents and Technology

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- Production $Y = AK_t$, with $A$ being constant

- Fundamental shocks: stochastic capital quality shock $dZ_t$. TFP shocks
  $$\frac{dK_t}{K_t} = i_t \, dt - \delta \, dt + \sigma dZ_t$$
Agents and Technology

- Two classes of agents: households and bankers
  - Households:
    \[ \mathbb{E} \left[ \int_0^\infty e^{-\rho t} \left( c_t^y \right)^{1-\phi} \left( c_t^h \right)\phi \, dt \right], \]

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- Fundamental shocks: stochastic capital quality shock \( dZ_t \). TFP shocks
  \[ \frac{dK_t}{K_t} = i_t dt - \delta dt + \sigma dZ_t \]

- Investment/Capital \( i_t \), quadratic adjustment cost
  \[ \Phi(i_t, K_t) = i_t K_t + \frac{\kappa}{2} (i_t - \delta)^2 K_t \]
  \[ \max_{i_t} q_t i_t K_t - \Phi(i_t, K_t) \Rightarrow i_t = \delta + \frac{q_t - 1}{\kappa} \]
Aggregate Balance Sheet

Loans to Capital Producers $i_t$

Intermediary Sector

Capital $q_t K_t$

Equity $E_t$

Housing $P_t H$

Debt $W_t - E_t$

Household Sector

Financial Wealth

$W_t = q_t K_t + P_t H$
Aggregate Balance Sheet

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Financial Wealth

\[ W_t = q_t K_t + P_t H \]

\[ (1 - \lambda) W_t \]

\[ \lambda W_t = \text{"Liquid balances" benchmark capital structure} \]
Equity Matters

Loans to Capital Producers $i_t$

Intermediary Sector

Capital $q_t K_t$

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Separation of ownership and control

Banker maximizes $E[ROE] - \frac{m}{2} \text{Var}[ROE]$

Household Sector

Financial Wealth

$W_t = q_t K_t + P_t H$

$(1 - \lambda) W_t$

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Financial Wealth \( W_t = q_t K_t + P_t H \)

Banker maximizes \( E[ROE] - \frac{m}{2} \text{Var}[ROE] \)

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Equity Constraint

Loans to Capital Producers $i_t$

Intermediary Sector

Capital $q_t K_t$

Equity $E_t$

Constraint: $E_t \leq \varepsilon_t$

Debt $W_t - E_t$

Banker maximizes $E[ROE] - \frac{m}{2} Var[ROE]$

Aggregate bank reputation $\varepsilon_t$

$\frac{d\varepsilon_t}{\varepsilon_t} = m \times ROE$

Household Sector

Financial Wealth

$W_t = q_t K_t + P_t H$

(1 - $\lambda$) $W_t$

$\lambda W_t = "Liquid balances"

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Systemic Risk
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Intermediary Reputation

- Single bank has “reputation” (skill, etc.) $\epsilon_t$ linked to intermediary performance (constant $m$)

\[
\frac{d\epsilon_t}{\epsilon_t} = md\tilde{R}_t.
\]

- Poor returns reduce reputation: Berk-Green, 04; flow-performance relationship, Warther 95; Chevalier-Ellison, 97

- Or, $\epsilon_t$ as banker’s “net worth” fluctuating with performance
  - Kiyotaki-Moore 97, He-Krishnamurthy 12, Brunnermeier-Sannikov 12

- Household invests a maximum of $\epsilon_t$ dollars of equity capital with this banker
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- Household invests a maximum of $\epsilon_t$ dollars of equity capital with this banker
- $E_t$: aggregate reputation. Aggregate dynamics of $E_t$

$$\frac{dE_t}{E_t} = md\tilde{R}_t - \eta dt + d\psi_t$$

- Exogenous death rate $\eta$. Endogenous entry $d\psi_t > 0$ of new bankers in extreme bad states
Equity Capital Constraint

- Representative household with $W_t$, split between bond household $\lambda W_t$ and equity household $(1 - \lambda) W_t$ (Lucas 1990)
- Benchmark capital structure: $\lambda W_t$ of Debt, $(1 - \lambda) W_t$ of Equity
  - if there is no capital constraint ($E_t$ is infinite)...
- Intermediary equity capital:
  \[ E_t = \min [E_t, (1 - \lambda) W_t] \]

Suppose a $-10\%$ shock to real estate and price of capital:
- $W_t \downarrow 10\%$ (Household wealth = aggregate wealth)
- Reputation: \[ \frac{dE_t}{E_t} = md\tilde{R}_t + \ldots \] Two forces make $E_t \downarrow$ more than $10\%$:
  1. Return on equity $= d\tilde{R}_t < -10\%$: equity is levered claim on assets
  2. $m > 1$ in our calibration
Solving for Equilibrium

- Markov equilibrium with state variables \((E_t, K_t)\)
- Scale invariance. Unidimensional state variable \(e_t = E_t/K_t\), with endogenous evolution

\[
de_t = \mu_e dt + \sigma_e dZ_t
\]

- Asset prices: land price/rent ratio \((p(e))\); capital price \((q(e))\); interest rate \((r_t)\).
- Quantities: Investment is \(q\)-theory, but based on intermediary pricing
- We solve for \(q(e)\) and \(p(e)\) as solutions to a system of ODEs

Boundary Conditions

- When \(e = \infty\), \(E_t > (1 - \lambda) W_t\) always, frictionless economy. Solving \(p(\infty), q(\infty)\) analytically
- As \(e \to 0\), intermediaries’ portfolio volatility, i.e. Sharpe ratio, rises
- New bankers enter if \(e = \underline{e}\) (Sharpe ratio hits \(\gamma\), exogenous constant)
- \(\underline{e}\) is a reflecting boundary, with

\[
q'(\underline{e}) = 0, \ p'(\underline{e}) = \frac{p(\underline{e}) \beta}{1 + \underline{e} \beta}, \text{ and } Sharpe\_Ratio(\underline{e}) = \gamma
\]
## Calibration: Baseline Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Choice</th>
<th>Targets (Unconditional)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Intermediation</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$m$ Performance sensitivity</td>
<td>2</td>
<td>Average Sharpe ratio</td>
</tr>
<tr>
<td>$\lambda$ Debt ratio</td>
<td>0.67</td>
<td>Average intermediary leverage</td>
</tr>
<tr>
<td>$\eta$ Banker exit rate</td>
<td>17%</td>
<td>Good model dynamics</td>
</tr>
<tr>
<td>$\gamma$ Entry trigger</td>
<td>6.5</td>
<td>Highest Sharpe ratio</td>
</tr>
<tr>
<td>$\beta$ Entry cost</td>
<td>2.34</td>
<td>Average land price volatility</td>
</tr>
<tr>
<td><strong>Panel B: Technology</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma$ Capital quality shock</td>
<td>4%</td>
<td>Average consumption volatility</td>
</tr>
<tr>
<td>$\delta$ Depreciation rate</td>
<td>10%</td>
<td>Literature</td>
</tr>
<tr>
<td>$\kappa$ Adjustment cost</td>
<td>3</td>
<td>Literature</td>
</tr>
<tr>
<td>$A$ Productivity</td>
<td>0.148</td>
<td>Average investment-to-capital ratio</td>
</tr>
<tr>
<td><strong>Panel C: Others</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho$ Time discount rate</td>
<td>3%</td>
<td>Literature</td>
</tr>
<tr>
<td>$\phi$ Housing share</td>
<td>0.5</td>
<td>Housing-to-wealth ratio</td>
</tr>
</tbody>
</table>
Results(1): State variable is $e_t = \frac{E_t}{\varphi}$

- Capital constraint binds for $e < 0.44$
- Without the possibility of the capital constraint, all of these lines would be flat. Model dynamics would be i.i.d., with vol=4%
State-dependent Impulse Response: -2% Shock \((= \sigma dZ_t)\)

<table>
<thead>
<tr>
<th>quarter</th>
<th>Investment</th>
<th>Sharpe ratio</th>
<th>Land price</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-0.029</td>
<td>0.000</td>
<td>-0.040</td>
</tr>
<tr>
<td>2</td>
<td>-0.026</td>
<td>-0.050</td>
<td>-0.070</td>
</tr>
<tr>
<td>4</td>
<td>-0.024</td>
<td>-0.060</td>
<td>-0.080</td>
</tr>
<tr>
<td>6</td>
<td>-0.023</td>
<td>-0.070</td>
<td>-0.090</td>
</tr>
<tr>
<td>8</td>
<td>-0.022</td>
<td>-0.080</td>
<td>-0.100</td>
</tr>
</tbody>
</table>

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Steady State Distribution

Scaled intermediary reputation $e$

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Nonlinearities in Model and Data

Model:

- Distress states = worst 33% of realizations of $e (e < 2.14)$
- Compute conditional variances, covariances of intermediary equity growth with other key variables

Data:

- Distress states = worst 33% of realizations of credit spread
  - We use Gilchrist-Zakrajsek (2011) Excess Bond Premium, which we convert to a Sharpe ratio
  - Excess Bond Premium: risk premium of corporate bonds, presumably reflects distress of financial sector
  - Similar results if using NBER recessions
- Compute conditional variances, covariances of intermediary equity growth with other key variables
EBS time series

EBS-Sharpe

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## Distress Classification

<table>
<thead>
<tr>
<th>Distress Periods</th>
<th>NBER Recessions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1974Q3 - 1975Q4</td>
<td>11/73 - 3/75</td>
</tr>
<tr>
<td>1982Q3 - 1982Q4</td>
<td>7/81 - 11/82</td>
</tr>
<tr>
<td>1985Q4 - 1987Q3</td>
<td></td>
</tr>
<tr>
<td>1988Q4 - 1990Q1</td>
<td>7/90 - 3/91</td>
</tr>
<tr>
<td>1992Q4 - 1993Q2</td>
<td></td>
</tr>
<tr>
<td>2001Q2 - 2003Q1</td>
<td>3/01 - 11/01</td>
</tr>
<tr>
<td>2007Q3 - 2009Q3</td>
<td>12/07 - 6/09</td>
</tr>
</tbody>
</table>
## Covariances in Data

<table>
<thead>
<tr>
<th></th>
<th>EB</th>
<th>NBER Recession</th>
<th>NBER+,-2Qs</th>
<th>EB, Drop Crisis</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Distress Periods</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>vol(Eq)</td>
<td>31.48</td>
<td>32.40</td>
<td>31.78</td>
<td>22.19</td>
</tr>
<tr>
<td>vol(I)</td>
<td>8.05</td>
<td>8.79</td>
<td>7.44</td>
<td>4.56</td>
</tr>
<tr>
<td>vol(C)</td>
<td>1.71</td>
<td>1.54</td>
<td>1.59</td>
<td>0.95</td>
</tr>
<tr>
<td>vol(PL)</td>
<td>21.24</td>
<td>23.34</td>
<td>21.07</td>
<td>7.91</td>
</tr>
<tr>
<td>vol(EB)</td>
<td>60.14</td>
<td>93.59</td>
<td>74.57</td>
<td>28.69</td>
</tr>
<tr>
<td>cov(Eq, I)</td>
<td>1.31</td>
<td>1.08</td>
<td>0.84</td>
<td>0.37</td>
</tr>
<tr>
<td>cov(Eq, C)</td>
<td>0.25</td>
<td>0.16</td>
<td>0.13</td>
<td>0.04</td>
</tr>
<tr>
<td>cov(Eq, PL)</td>
<td>4.06</td>
<td>5.61</td>
<td>4.39</td>
<td>-0.63</td>
</tr>
<tr>
<td>cov(Eq, EB)</td>
<td>-6.81</td>
<td>-10.89</td>
<td>-7.57</td>
<td>-2.12</td>
</tr>
<tr>
<td><strong>Panel B: Non-distress Periods</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>vol(Eq)</td>
<td>17.54</td>
<td>19.42</td>
<td>17.11</td>
<td>17.26</td>
</tr>
<tr>
<td>vol(I)</td>
<td>6.61</td>
<td>5.97</td>
<td>4.91</td>
<td>6.60</td>
</tr>
<tr>
<td>vol(C)</td>
<td>1.28</td>
<td>0.98</td>
<td>0.91</td>
<td>1.28</td>
</tr>
<tr>
<td>vol(PL)</td>
<td>9.79</td>
<td>10.00</td>
<td>8.46</td>
<td>9.34</td>
</tr>
<tr>
<td>vol(EB)</td>
<td>12.72</td>
<td>30.93</td>
<td>30.42</td>
<td>12.78</td>
</tr>
<tr>
<td>cov(Eq, I)</td>
<td>0.07</td>
<td>0.09</td>
<td>-0.06</td>
<td>0.03</td>
</tr>
<tr>
<td>cov(Eq, C)</td>
<td>0.03</td>
<td>0.01</td>
<td>0.01</td>
<td>0.03</td>
</tr>
<tr>
<td>cov(Eq, PL)</td>
<td>0.12</td>
<td>0.07</td>
<td>-0.31</td>
<td>-0.01</td>
</tr>
<tr>
<td>cov(Eq, EB)</td>
<td>-0.14</td>
<td>-0.81</td>
<td>-0.78</td>
<td>-0.19</td>
</tr>
</tbody>
</table>
## Matching State-Dependent Covariances

<table>
<thead>
<tr>
<th></th>
<th>Distress</th>
<th></th>
<th>Non Distress</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Baseline</td>
<td>Data</td>
<td>Baseline</td>
</tr>
<tr>
<td>vol (Eq)</td>
<td>31.48%</td>
<td>31.2</td>
<td>17.54%</td>
<td>6.4</td>
</tr>
<tr>
<td>vol (I)</td>
<td>8.05%</td>
<td>5.4</td>
<td>6.61</td>
<td>4.8</td>
</tr>
<tr>
<td>vol (C)</td>
<td>1.71%</td>
<td>1.8</td>
<td>1.28</td>
<td>2.4</td>
</tr>
<tr>
<td>vol (LP)</td>
<td>21.24%</td>
<td>22.1</td>
<td>9.79</td>
<td>9.8</td>
</tr>
<tr>
<td>vol (EB)</td>
<td>60.14%</td>
<td>71.1</td>
<td>12.72</td>
<td>8.7</td>
</tr>
<tr>
<td>cov (Eq, I)</td>
<td>1.31%</td>
<td>0.90</td>
<td>0.07</td>
<td>0.3</td>
</tr>
<tr>
<td>cov (Eq, C)</td>
<td>0.25%</td>
<td>0.0</td>
<td>0.03</td>
<td>0.1</td>
</tr>
<tr>
<td>cov (Eq, LP)</td>
<td>4.06%</td>
<td>5.6</td>
<td>0.12</td>
<td>0.6</td>
</tr>
<tr>
<td>cov (Eq, EB)</td>
<td>-6.81%</td>
<td>-13.0</td>
<td>-0.14</td>
<td>-0.2</td>
</tr>
</tbody>
</table>

Note: without the capital constraint, all volatilities would be 4%, and have no state dependence.
Based on EBS classification, we cross the 33% boundary \((e = 2.14)\) between 2007Q2 and 2007Q3

What is the likelihood of the constraint binding ("systemic crisis") assuming \(e = 2.14\) currently:

- 1.12% in next 2 years
- 9.12% in next 5 years
- 20.73% in next 10 years

Small...
Stress testing

Map “stress test” into a shock to $e$. 
Financial sector aggregate leverage fixed at 3 in model

Pushed to crisis boundary after a -13% shock. 1.12% Prob of crisis in next 2 years.

Suppose “hidden” leverage: leverage was 4.5 but agents take as given price functions and returns at leverage=3

Prob of hitting crisis rises to 60%! 
Based on realized equity return we uncover fundamental shocks to $K$

<table>
<thead>
<tr>
<th></th>
<th>07QIII</th>
<th>07QIV</th>
<th>08QI</th>
<th>08QII</th>
<th>08QIII</th>
<th>08QIV</th>
<th>09QI</th>
<th>09QII</th>
<th>09QIII</th>
<th>09QIV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Return</td>
<td>-3.1%</td>
<td>-5.5</td>
<td>-3.0</td>
<td>-1.4</td>
<td>-0.8</td>
<td>-2.2</td>
<td>-2.3</td>
<td>-2.2</td>
<td>-1.0</td>
<td>-1.0</td>
</tr>
</tbody>
</table>

Total -19%. Capital constraint binds after 08QI—systemic crisis

- In the model (data), land price falls by 56.2% (55%)
- In the model (data), investment falls by 25.4% (25%)
We develop a fully stochastic model of a systemic crisis, with an equity capital constraint on the intermediary sector.
The model quantitatively matches the differential comovements in distress and non-distress periods.
Is able to replicate 2007/2008 period with only intermediary capital shocks.
Offers a way of mapping macro-stress tests into probability of systemic states.
Panel A: Distress Periods

Panel B: Non Distress Periods