

Trickle-down housing economics

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Abstract

This paper provides a quantitative framework for estimating the effects on house prices and household welfare of building different types of housing within a city or metropolitan area. According to our estimates, low-income households without a college degree benefit more from the construction of low-quality rather than high-quality housing, but low-quality construction makes many other households worse off. These conclusions depend on household mobility across cities, the strength of urban spillovers, the indivisibility of housing, and the differential preferences of households with and without a college degree.

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Since 1980, the inflation-adjusted price of housing has significantly risen in many large cities around the world. In the United States, many households with low incomes or lacking a college degree have migrated away from such cities in response to rising house prices (Gyourko et al., 2013; Diamond, 2016; Ganong and Shoag, 2017). Policymakers have called this situation an “affordability crisis” (e.g., White House, 2016). Several economists recommend that cities ease permitting rules so that the construction of new housing units can bring down house prices. Using fast tracking, inclusionary zoning, or tax credits, many governments relax permitting rules specifically for the construction of smaller units or units in neighborhoods with low-income households. While there exist empirical studies of these policies, little theoretical work has estimated how the type of construction affects house prices and the composition of households within a metropolitan area.

To answer this question, we model a city with different qualities of housing that is home to households of varying education and income. We study the effects of raising the quantity of different types of housing, which we interpret as a targeted relaxation in permitting rules. When we estimate our model using data from Boston in 2016, we find that low-quality construction increases the welfare of low-education, low-income households twice as much as high-quality construction. However, low-quality construction makes many other households worse off because in-migration of low-education, low-income households lowers the city’s wages and amenities.

The model features multiple cities with different amenities, labor markets, and housing stocks. Households take house prices, amenities, and labor prices as given and choose a city and type of housing to maximize utility. Amenities rise with the share of a city’s population with high education, as in Diamond (2016). High- and low-education wages also depend on the composition of city households. High-education households care relatively more about amenities than wages than low-education households do, which is consistent with the evidence in Bayer et al. (2007) and Diamond (2016).

The negative welfare effects of construction depend critically on these assumptions on amenities. When amenities do not depend on the city population—or when high- and low-education households have the same preference for amenities—construction makes few, if any, households worse off. In contrast, the results are robust to whether the neighborhood or the city-wide share of high-education households alters the amenities a household enjoys. Surprisingly, rich households suffer from low-quality construction even more when amenities depend on neighborhood characteristics, even though construction does not directly alter rich neighborhoods.

To study the affordability crisis, we introduce a skill-biased shock that raises high-education productivity and lowers low-education productivity. This shock significantly raises city house prices and drives many poor, low-education households out of the city. Displacing them are middle-income households with a college degree who switch from higher- to lower-quality housing within the city. In this manner, the shock endogenously generates gentrification, as in Guerrieri et al. (2013).

How effective are different policy responses to this shock? The construction we observe in the

data does attenuate the out-migration of poor households without a college degree, but it does not eliminate it. It would take quadruple the amount of construction—a 1.7% expansion of the housing stock—to prevent the shock from driving any households out of the city. In fact, the city could achieve this objective with only a 1.4% expansion of the housing stock by building lower quality housing than current construction. Increasing construction in either manner lowers the welfare of rich households relative to the construction in the data, which may explain why local governments do not allow more construction in Boston. Rent control drives even more poor households out of the city while increasing in-migration of the rich. By lowering the slope of house prices with respect to quality, rent control amplifies switching of households down the quality distribution, who out-compete the poor for low-quality housing.

The paper proceeds as follows. Section 1 relates our work in more detail to the previous literature. Section 2 lays out the economic environment, defines equilibrium, and characterizes equilibrium house prices with and without divisibility. Section 3 theoretically analyzes the equilibrium effects of construction and productivity shocks. Section 4 discusses our strategy for estimating the model, and Section 5 describes the data we use. Section 6 presents the quantitative results, and Section 7 concludes. Supplements to Sections 2–4 appear, respectively, in Appendices A–C.

1 Relation to literature

The key contribution of our paper is incorporating urban spillovers into a quantitative model of heterogeneous housing quality. Spillovers distinguish our paper from an older theoretical literature building on Sweeney (1974b), who theoretically analyzes the effects of constructing different qualities of indivisible housing on house prices (see Arnott, 1987 for a literature review). Braid (1981), for instance, corresponds to the special case of our indivisible model without spillovers and mobility. In these older models, neither incomes nor amenities depend on the composition of households in the city. These spillovers drive our key theoretical and quantitative results.

Our results hold because households are mobile across cities and because housing is indivisible. When households are immobile, construction makes all of the city’s households better off, and building a unit of the highest quality improves welfare more than building a unit of any other quality. The same holds when housing is divisible—households can costlessly divide any housing unit into two new units of lesser quality—for a subset of the spillovers we consider. Papers have studied indivisibility with immobility (Braid, 1981; Määttänen and Terviö, 2014; Landvoigt et al., 2015) and divisibility with mobility (Henderson, 1974; Glaeser and Gottlieb, 2009). The *combination* of mobility and indivisibility breaks the trickle-down mechanism in these frameworks.¹

Another contribution is to explain why certain households oppose construction in their city of

¹Like us, Davis and Dingel (2018) incorporate spillovers into a spatial equilibrium model with heterogeneous and indivisible housing quality. However, their focus is on the sorting of households and firms across cities. They do not analyze increases to housing supply.

residence. According to our estimates, high-income and high-education households oppose low-quality construction because it attracts low-education households, who lower amenities and labor prices. This mechanism differs from that in Hilber and Robert-Nicoud (2013) and Ortalo-Magné and Prat (2014). These papers formalize the “homevoter hypothesis” of Fischel (2001) with models in which homeowners oppose construction in order to increase their home equity wealth. This incentive is absent from our static model. Our results might explain why the restrictiveness of zoning correlates more closely with demographics than with homeownership in multiple empirical studies (Gyourko and Molloy, 2015).

Several empirical papers estimate the effects of subsidies for low-income housing on housing supply and house prices. Schwartz et al. (2006), Baum-Snow and Marion (2009), and Diamond and McQuade (2017) find that these subsidies increase the supply of low-income housing in the neighborhoods they target and increase the value of surrounding homes in low-income and declining neighborhoods. Unlike these studies, our paper estimates the effect of low-quality housing on an entire city or metropolitan area. This distinction matters, as many of these subsidies fail to increase the supply of low-income housing at the metropolitan area level (Eriksen and Rosenthal, 2010; Schuetz et al., 2011). As a result, existing subsidy programs may not identify the effects of constructing low-quality housing on a metropolitan area. Our structural approach does.

Another paper estimating the effect of construction on house prices with a structural approach is Anenberg and Kung (2018). Our papers differ in how we treat the cross-city migration that occurs in response to construction. Anenberg and Kung (2018) assume that households from outside the city occupy all newly built housing, irrespective of any adjustment to house prices. In contrast, migration demand in our model is a downward-sloping function of house prices that we endogenously derive. Anenberg and Kung (2018) estimate that building high-quality housing has no effect on the prices of other types of housing, while we find large effects of high-quality construction on low-quality house prices.

Our theoretical results on the effect of high-quality construction correspond to the empirical findings of Mast (2019). Using micro data on address history, he finds that luxury development frees up housing in poor neighborhoods by inducing a series of moves within a metropolitan area. This channel exists in our model, and its quantitative size is similar to what Mast (2019) finds empirically.

Several papers analyze the distributional consequences of rent control, which we examine as a policy response to house price growth. Diamond et al. (2018) find empirically that rent control increases the population of high income households relative to poor households. We find this effect in our model as well, although the mechanism is different. In the data, rent control leads to housing supply changes that favor the rich, whereas rent control favors the rich in our model without any housing supply changes. Favilukis et al. (2019) present a theoretical model of rent control and other policies that try to address affordability. They focus on the risk-sharing and distortionary effects of these policies with a lifecycle model featuring endogenous housing supply

and location choices within the New York City metropolitan area. While risk and endogenous supply are absent from our model, we have endogenous cross-city mobility, local amenities, and wages. Therefore, our paper and Favilukis et al. (2019) present complementary analyses of a similar problem.

Our framework does not directly model filtering, the process through which high-quality construction eventually houses low-income households after years of depreciation (Rosenthal, 2014). Despite this omission, there is a simple way to map our framework into a filtering model. Filtering models admit a unique steady-state distribution for housing quality that depends on the intensity of construction of each type of housing (Sweeney, 1974a). The stock of housing in our static model corresponds to the steady-state distribution in a filtering model. Increasing the housing stock in our framework maps to the change in construction intensity in a filtering model that would cause the corresponding shift to the steady-state distribution of housing quality.

2 Environment and equilibrium

2.1 Housing supply

The economy consists of T cities indexed by t . In city t , available housing qualities are $q_{j,t} > 0$, where $j \in \mathcal{J}_t = \{0, \dots, J_t\}$ and $q_{j,t}$ strictly increases in j . The measure of housing of quality $q_{j,t}$ is $h_{j,t} > 0$, and this housing trades in competitive markets at a price $p_{j,t}$.

There are two types of agents: households and rentiers. Rentiers are endowed with the entire housing stock and have utility that is a linear function of a composite non-housing consumption good c , whose price we normalize to 1. They take house prices as given and choose how much housing to sell and how much c to consume subject to a budget constraint.

2.2 The distribution of households

Households differ in their education, $e \in \{L, H\}$, labor endowment, $z > 0$, and taste for each city t , ϵ_t . Across households and cities, the ϵ_t are distributed independently as identical Gumbel distributions (McFadden, 1973). The distribution of z among households of education e equals $\tilde{n}_e(z)$, about which we assume the following:

Assumption 1. For each $e \in \{L, H\}$,

- (a) the support of \tilde{n}_e is convex;
- (b) the greatest lower bound of the support of \tilde{n}_e equals zero;
- (c) \tilde{n}_e is continuous; and
- (d) $\int_0^\infty \tilde{n}_e(z) dz > 0$.

Assumptions 1(a) and 1(b) mean that there are no gaps in the distribution of labor endowments

and that there are households with arbitrarily small labor endowments. These conditions ensure that equilibrium house prices are locally unique. Assumption 1(c) rules out mass points, in which a positive measure of households have the same education and labor endowment, as well as jumps in the endowment distributions. This assumption allows us to define comparative statics with respect to an equilibrium.

Each household lives in one city. We denote the measure of households of education e with labor endowment z living in city t by $n_{e,t}(z)$. The population of households of education e in city t equals $N_{e,t} = \int_0^\infty n_{e,t}(z)dz$, and the total population of households in city t is $N_t = N_{L,t} + N_{H,t}$. The total labor endowment of education group e in city t is $Z_{e,t} = \int_0^\infty zn_{e,t}(z)dz$. We restrict attention to allocations of households across cities in which $N_{e,t} > 0$ for each $e \in \{L, H\}$ and $t \in \{1, \dots, T\}$; that is, some households of each education group must live in each city. This restriction allows us to divide by these populations when we specify amenity and productivity spillovers. Such allocations are possible due to Assumption 1(d), which guarantees a nonzero population of households with each education.

2.3 Household preferences and constraints

Households have preferences over four goods—composite non-housing consumption c , housing quality q , city amenities a , and an idiosyncratic taste for each city ϵ —represented by the utility function

$$u_e(c, q, a, \epsilon) = c^{\beta_{c,e}} q^{\beta_{q,e}} a^{\beta_{a,e}} \exp(\beta_{\epsilon,e} \epsilon), \quad (1)$$

where $\beta_{c,e}, \beta_{q,e}, \beta_{a,e}, \beta_{\epsilon,e} > 0$ for each $e \in \{L, H\}$.

Cobb-Douglas preferences over housing and non-housing consumption, such as in (1), appear in many equilibrium models of city choices (e.g., Glaeser and Gottlieb, 2009; Gennaioli et al., 2013; Diamond, 2016), and are consistent with the stability of housing expenditure as a share of income across places and time (Davis and Ortalo-Magné, 2011). The term involving ϵ is present in some recent work (Kline and Moretti, 2014; Hsieh and Moretti, 2018) and limits household mobility across cities in response to changes in utility coming from c , q , and a . As in Diamond (2016), preferences may differ across education groups. For instance, low-education households may care relatively more about non-housing consumption than amenities compared to high-education households. Differences in preferences across the groups are quite important for our results.

Amenities in each city are non-rival and non-excludable to city households. As in Diamond (2016), amenities depend on exogenous characteristics of the city as well as the relative population of households with education H :

$$a_t = \tilde{a}_t \left(\frac{N_{H,t}}{N_{L,t}} \right)^{\gamma_a}, \quad (2)$$

where $\gamma_a \geq 0$ and $\tilde{a}_t > 0$ for each t . When $\gamma_a > 0$, city amenities increase when more high education

households arrive. Households may simply enjoy meeting high-education households. Alternatively, consumption by high-education households may produce non-excludable benefits to other households, as is the case with philanthropy. Diamond (2016) discusses (2) further.

Labor in each city trades in competitive markets, and we denote the price of labor of education e in city t by $w_{e,t}$. A household's income then equals $w_{e,t}z$, which we denote $y_{e,t}(z)$. Each household takes house prices, labor prices, and amenities as given and chooses a city t , non-housing consumption c , and a quantity x_j of each housing quality $q_{j,t}$ subject to the following constraints:

$$c + \sum_{j \in \mathcal{J}_t} p_{j,t} x_j \leq w_{e,t} z \quad (3)$$

$$q = \sum_{j \in \mathcal{J}_t} x_j q_{j,t} \quad (4)$$

$$t \in \{1, \dots, T\} \quad (5)$$

$$0 \leq c \quad (6)$$

$$0 \leq x_j, \quad \forall j \in \mathcal{J}_t. \quad (7)$$

Within these constraints, households can combine fractional amounts of housing of different qualities into a single effective housing unit. These activities might take the form of splitting time between different locations, renting a single room in a larger house, or knocking down walls between neighboring apartment units.

Many of these activities, however, involve costs not present in our model, such as the privacy lost from occupying a single room in a larger house. Another cost, which is particularly relevant in the cities that motivate this paper, are regulations prohibiting divisibility like minimum lot sizes and maximum occupancy constraints (Gyourko et al., 2008; Glaeser and Ward, 2009). To capture these costs, we introduce two additional constraints:

$$(x_0, \dots, x_{J_t}) \in \{0, 1\}^{J_t+1} \quad (8)$$

$$\sum_{j=0}^{J_t} x_j = 1. \quad (9)$$

We call (8) and (9) the *indivisibility constraints*. Within them, each household must choose exactly one unit of one type of housing. Recent papers featuring these constraints include Määttänen and Terviö (2014) and Landvoigt et al. (2015).

2.4 Firms

Firms in each city t combine low-education and high-education labor to produce the non-housing consumption good c according to the production function

$$F_t(Z_L, Z_H) = ((A_{L,t}Z_L)^\rho + (A_{H,t}Z_H)^\rho)^{\frac{1}{\rho}}, \quad (10)$$

where Z_e is the quantity of labor of education e a firm uses, and $0 < \rho \leq 1$. A large literature in labor economics adopts (10) to explain the evolution of wages for workers with and without a college degree (Goldin and Katz, 2008; Card, 2009). Firms in t take $A_{L,t}$ and $A_{H,t}$ as given and choose labor inputs Z_L and Z_H . The resulting profits, $F_t(Z_L, Z_H) - w_{L,t}Z_L - w_{H,t}Z_H$, accrue to the rentiers living in city t .

The only differences in production technology across cities come from variation in $A_{L,t}$ and $A_{H,t}$, which govern the productivity of each type of labor. As with amenities, productivity depends on exogenous characteristics of the city as well as the city's population:

$$A_{e,t} = \tilde{A}_{e,t} N_t^{\gamma_N} \left(\frac{N_{H,t}}{N_t} \right)^{\gamma_H}, \quad (11)$$

where $\gamma_N \geq 0$, $\gamma_H \geq 0$, and $\tilde{A}_{e,t} > 0$ for each t . When $\gamma_N > 0$, productivity increases when the population of the city goes up and the relative share of each education group remains constant. Labor productivity is indeed higher in more populous cities, and an extensive literature in urban economics finds that part of this phenomenon is a causal effect of population on productivity (Combes and Gobillon, 2015).

When $\gamma_H > 0$, productivity increases when the share of high-education households in the city rises. The functional form of this effect matches that in Lucas (1988), who posits a constant elasticity of productivity spillover with respect to the average human capital in the population. While productivity is higher in cities and states with more human capital (Moretti, 2004a,b; Gennaioli et al., 2013), some of this effect may arise from the decisions of relatively productive firms to locate in regions with more human capital. We explore the implications of both positive and zero values.

2.5 Equilibrium definitions

Local equilibrium consists of house prices $p_{j,t}$, labor prices $w_{e,t}$, amenity levels a_t , productivity levels $A_{e,t}$, populations $n_{e,t}(z)$, and housing demands $x_{j,t}$ for each $e \in \{L, H\}$, $j \in \mathcal{J}_t$, and $t \in \{1, \dots, T\}$ satisfying six conditions:

- the measure of households of education e and labor endowment z choosing city t equals $n_{e,t}(z)$, while the sum of x_j across households choosing city t equals $x_{j,t}$;
- households choose c and x_j to maximize utility subject to the constraints (3)–(7) and also (8)–(9) in the case of indivisible housing;

- renters in each city t choose a quantity of housing to sell to maximize utility subject to their budget constraints, and the quantity they choose of housing of quality q_j equals $x_{j,t}$;
- each firm in each city chooses Z_L and Z_H to maximize profits, and the total labor of education e firms in city t choose equals $Z_{e,t}$;
- the population of households of education e , $N_{e,t}$, is positive for each city t ; and
- the amenity and productivity equations, (2) and (11), hold for each city t .

In local equilibrium, markets clear, and households maximize utility conditional on their city choices. An *equilibrium* is a local equilibrium in which household city choices maximize utility.

2.6 Equilibrium characterization

We begin by characterizing equilibrium prices of labor. Firm profit maximization implies that these labor prices coincide with marginal products:

$$w_{e,t} = ((A_{L,t}Z_{L,t})^\rho + (A_{H,t}Z_{H,t})^\rho)^{\frac{1}{\rho}-1} A_{e,t}^\rho Z_{e,t}^{\rho-1} \quad (12)$$

for each $e \in \{L, H\}$ and $t \in \{1, \dots, T\}$. Because the population of each education group in each city, $N_{e,t}$, is positive, the productivities and labor endowments, $A_{e,t}$ and $Z_{e,t}$, for the corresponding households are positive as well. As a result, $w_{e,t} > 0$ for each education group in each city, meaning that all households earn positive income in equilibrium.

We next characterize the equilibrium city choice of each household. To do so, we define indirect utility, $v_{e,t}(z)$ to be the maximized utility of a household in city t whose idiosyncratic taste for that city is zero. Specifically,

$$v_{e,t}(z) = \max_{c, x_j} u_e(c, q, a_t, 0) \quad (13)$$

subject to the relevant constraints between (3) and (9). In the indivisible case, some households may be unable to choose any combinations of c and x_j within these constraints, in which case the right side of (13) does not exist. This situation arises precisely for households too poor to afford even the cheapest housing unit, that is, when $w_{e,t}z < \min(p_{0,t}, \dots, p_{J,t})$, and for these households, we define $v_{e,t}(z) = 0$. Indirect utility also equals zero for households in the indivisible case who must spend all of their income to live in city t , in which case $w_{e,t}z = \min(p_{0,t}, \dots, p_{J,t})$.

To study the average welfare of households in the economy, we introduce another measure of indirect utility, $\bar{v}_{e,t}(z)$, that delivers the average utility of all households in city t with a given e and z . Specifically, we define $\bar{v}_{e,t}(z) = \exp(E(\log u_e(c, q, a_t, \epsilon_t) \mid e, z, t))$ whenever $n_{e,t}(z) > 0$. Putting the log inside the expectation is necessary because the expected level of utility fails to exist due to the thickness of the Gumbel distribution's tail.

Because households choose cities to maximize utility, a household chooses city t when, for each $t' \neq t$, $v_{e,t}(z) \exp(\beta_{\epsilon, e} \epsilon_t) > v_{e,t'}(z) \exp(\beta_{\epsilon, e} \epsilon_{t'})$. Standard results on Gumbel distributions (McFadden,

1978) allow us to solve for the population distributions and average utilities, $n_{e,t}$ and $\bar{v}_{e,t}$, in terms of the indirect utilities, $v_{e,t}$. Lemma 1 provides these solutions and uses Assumptions 1(a) and (c) to prove facts about each $n_{e,t}$ that are useful for characterizing equilibrium.

Lemma 1. *In equilibrium, the following hold for each $z > 0$, $e \in \{L, H\}$, and $t \in \{1, \dots, T\}$:*

- (a) *there exists $t' \in \{1, \dots, T\}$ such that $v_{e,t'}(z) > 0$;*
- (b) *$n_{e,t}$ is a continuous distribution with convex support satisfying*

$$n_{e,t}(z) = \frac{\tilde{n}_e(z)v_{e,t}(z)^{\beta_{\epsilon,e}^{-1}}}{\sum_{t'=1}^T v_{e,t'}(z)^{\beta_{\epsilon,e}^{-1}}}; \quad (14)$$

- (c) *$\inf\{v_{e,t}(z') \mid n_{e,t}(z') > 0\} = 0$; and*
- (d) *if $n_{e,t}(z) > 0$, then $\bar{v}_{e,t}(z) = \exp(\beta_{\epsilon,e}\gamma) \left(\sum_{t'=1}^T v_{e,t'}(z)^{\beta_{\epsilon,e}^{-1}}\right)^{\beta_{\epsilon,e}}$, where γ is Euler's constant.*

Proof. Appendix A.1. □

By Lemma 1(a), every household can achieve positive utility in one of the economy's cities. As shown in the proof, this result depends on Assumption 1(b). Lemma 1(a) guarantees that the denominator in (14) is positive so that the solution for $n_{e,t}(z)$ is well-defined. By Lemma 1(b), there are no mass points, discontinuities, or gaps in the distributions of labor endowments among households of a given education within each city. These properties hold for the distributions within the entire economy, \tilde{n}_e , so Lemma 1(b) proves that the within-city distributions inherit these properties. Lemma 1(c) states that indirect utility comes arbitrarily close to zero within each education group in each city. This result relies on Assumptions 1(a) and 1(b).

The formula that Lemma 1(d) gives for $\bar{v}_{e,t}(z)$ does not depend on t . Therefore, the average utility of households of a given e and z is the same in all cities where they live. In urban models with perfect mobility, utility is equal in all cities for households of a given skill level (Roback, 1982; Glaeser and Gottlieb, 2009). Imperfect mobility generalizes this equivalence by requiring only that the average level of utility is equal across cities (see Hsieh and Moretti, 2018 for further discussion of this point). We label this average $\bar{v}_e(z)$.

To finish characterizing equilibrium, we solve for equilibrium house prices and indirect utilities within each city, $p_{j,t}$ and $v_{e,t}(z)$, as functions of the distributions of households in the city, $n_{e,t}$. We consider the divisible and indivisible cases separately.

2.6.1 Divisible housing

When housing is divisible, the price per unit of quality, $p_{j,t}/q_{j,t}$, must be equal in equilibrium across the different types of housing in the city. To demonstrate this result, we write each house-

hold's first-order condition with respect to x_j (the amount of housing of quality $q_{j,t}$) as

$$\frac{\partial u_e / \partial q}{\partial u_e / \partial c} \leq \frac{p_{j,t}}{q_{j,t}}, \quad (15)$$

with consumption of x_j only in the case of equality. If $p_{j,t}/q_{j,t}$ exceeds $p_{j',t}/q_{j',t}$ for any $j' \neq j$, then no household will choose quality $q_{j,t}$. This situation cannot hold in equilibrium because the market for that type of housing must clear.

Proposition 1 solves for the equilibrium price-to-quality ratio.

Proposition 1. *In equilibrium, $p_{j,t} = \mu_t q_{j,t}$ for each $j \in \mathcal{J}_t$, where*

$$\mu_t = \frac{\sum_{e \in \{L, H\}} (\beta_{c,e} + \beta_{q,e})^{-1} \beta_{q,e} w_{e,t} Z_{e,t}}{\sum_{j' \in \mathcal{J}_t} h_{j',t} q_{j',t}}. \quad (16)$$

Proof. Appendix A.2. □

Each household in city t can purchase a unit of housing quality at an effective price μ_t . Because preferences over c and q are Cobb-Douglas, each household chooses to spend a constant share of income on housing quality, and this share equals $\beta_{q,e}/(\beta_{c,e} + \beta_{q,e})$. As a result, the numerator of (16) gives the total expenditure on housing in city t . Plugging the c and q choices of each household into (1) yields each household's indirect utility:

$$v_{e,t}(z) = \beta_c^{\beta_c} \beta_q^{\beta_q} (\beta_c + \beta_q)^{-(\beta_c + \beta_q)} (w_{e,t} z)^{\beta_{c,e} + \beta_{q,e}} \mu_t^{-\beta_{q,e}} a_t^{\beta_{a,e}}. \quad (17)$$

2.6.2 Indivisible housing

In the indivisible case, each household occupies exactly one housing unit in equilibrium. Because households are optimizing, house prices must strictly increase in quality among occupied units. Furthermore, the set of occupied units must equal the N_t highest quality units in city t . Otherwise, a rentier endowed with a high quality vacant unit would not be optimizing. Finally, the housing quality a household chooses must weakly increase in that household's labor endowment, z , within each education group, e . In other words, households sort on labor endowment, and hence income, within each education group. As shown by Määtänen and Terviö (2014) for more general utility functions, this sorting condition holds whenever the marginal rate of substitution from housing to non-housing consumption increases in non-housing consumption.

Proposition 2 formalizes these statements.

Proposition 2. *In equilibrium,*

$$j_{0,t} = \sup \left\{ j \in \mathcal{J}_t \mid \sum_{j'=j}^{J_t} h_{j',t} \geq N_t \right\} \quad (18)$$

exists, $p_{j,t}$ strictly increases over $j \geq j_{0,t}$ and equals zero for j such that $\sum_{j'=j}^{J_t} h_{j',t} > N_t$, and $x_{j,t}$ equals zero for $j < j_{0,t}$ and $h_{j,t}$ for $j > j_{0,t}$. The quality chosen by a household of education e and labor endowment z weakly increases in z for each e .

Proof. Appendix A.3. □

Proposition 2 pins down the price of the lowest quality occupied unit, $p_{j_{0,t},t}$, when the housing stock exceeds the city's population:

$$p_{j_{0,t},t} = 0 \tag{19}$$

if $\sum_{j \in \mathcal{J}_t} h_{j,t} > N_t$. The prices of the city's higher quality units solve the system that equates household demand for these units to the rentiers' endowments. Under the conditions in the following lemma, this system of equations admits a unique solution.

Lemma 2. *Suppose $n_{L,t}$ and $n_{H,t}$ are continuous distributions whose supports are convex sets with greatest lower bound zero. If $\sum_{j \in \mathcal{J}_t} h_{j,t} > N_t$, then a unique local equilibrium exists.*

Proof. Appendix A.4. □

In equilibrium, $n_{L,t}$ and $n_{H,t}$ satisfy the continuity and convexity properties by Lemma 1(b). Furthermore, when (19) holds, the greatest lower bounds of the distributions' supports are zero by Lemma 1(c). Therefore, in any equilibrium in which $\sum_{j \in \mathcal{J}_t} h_{j,t} > N_t$, the population distributions $n_{L,t}$ and $n_{H,t}$ uniquely determine the local equilibrium in city t .

We characterize this local equilibrium when a positive measure of households from each education group choose each type of occupied housing. Such equilibria are the focus of our comparative statics analysis in Section 3 and our estimation in Section 4. The other case, in which a positive measure of only one education group chooses some types of housing, complicates the comparative statics analysis and does not hold for the data we analyze in Section 6.

For each chosen quality level—that is, for each $j \geq j_{0,t}$ —we define $z_{e,j,t}$ to be the greatest lower bound of labor endowments z among households of education e choosing $q_{j,t}$. When $j > j_{0,t}$, $z_{e,j,t}$ also equals the least upper bound of labor endowments among households of education e choosing the quality one step below, $q_{j-1,t}$, because of sorting and because the support of $n_{e,t}$ is convex. A household with this endowment and education level is indifferent between $q_{j,t}$ and $q_{j-1,t}$:

$$\left(w_{e,t} z_{e,j,t} - p_{j,t}\right)^{\beta_{c,e}} q_{j,t}^{\beta_{q,e}} = \left(w_{e,t} z_{e,j,t} - p_{j-1,t}\right)^{\beta_{c,e}} q_{j-1,t}^{\beta_{q,e}} \tag{20}$$

for each $j \in \{j_{0,t} + 1, \dots, J_t\}$ and $e \in \{L, H\}$. By Proposition 2, the measure of households choosing each such quality level coincides with the total housing stock available:

$$h_{j,t} = \sum_{e \in \{L, H\}} \int_{z_{e,j,t}}^{z_{e,j+1,t}} n_{e,t}(z) dz \tag{21}$$

for each $j \in \{j_{0,t} + 1, \dots, J_t\}$, where $z_{e,j_{t+1},t}$ equals the least upper bound (or ∞ , if no upper bound exists) of the labor endowments of households of education e in city t .

By (20), the endowment cutoffs are linear functions of house prices. Substituting these functions into (21) delivers, together with (19), $J_t - j_{0,t} + 1$ equations in $J_t - j_{0,t} + 1$ unknown house prices. As we show in Appendix A.5, these equations always admit a unique solution for house prices, and prices in this solution strictly increase in quality. When the resulting endowment cutoffs also strictly increase in quality, the unique local equilibrium is one in which households from each education group choose each type of housing. The indirect utility of each household in city t is

$$v_{e,t}(z) = u_e(w_{e,t}z - p_{j,t}, q_{j,t}, a_t, 0), \quad z \in (z_{e,j,t}, z_{e,j+1,t}]. \quad (22)$$

3 Equilibrium effects of construction

In this section, we study the effects of constructing housing in a single city, t^* , on equilibrium house prices and welfare there. Constructing housing of quality q_{j,t^*} in city t^* corresponds to increasing h_{j,t^*} . We interpret such construction as the outcome of a relaxation of permitting rules for housing of quality q_{j,t^*} . House prices exceed the marginal costs of land and structure in many metropolitan areas (Glaeser and Gyourko, 2003), suggesting that easier permitting would lead developers to build more housing.

We later explore the equilibrium effects of exogenous changes to amenities and productivity. To encompass these changes as well, we study the equilibrium effect of marginally increasing each h_{j,t^*} by $\delta_{h,j}$, each $\log \bar{A}_{e,t^*}$ by $\delta_{A,e}$, and $\log a_{t^*}$ by δ_a . We denote the combined equilibrium effect of these changes by ∂ , which we call a comparative static.

The effect of these changes on house prices consists of $\partial p_{j,t^*}$ for $j \in \mathcal{J}_{t^*}$. For households who strictly prefer t^* to all other cities, the effect on the log of their indirect utility equals $\partial \log v_{e,t^*}(z)$. This effect does not depend on the household's idiosyncratic taste for the city, ϵ_{t^*} , which appears as an additive constant in the log of the household's indirect utility. We adopt $\partial \log v_{e,t^*}(z)$ as our measure of the effect of the changes in t^* on welfare.

3.1 Equilibrium assumptions

To avoid edge cases, we proceed under two assumptions about the equilibrium around which we compute comparative statics. First, a positive measure of households of each education choose each occupied quality of housing in city t^* :

Assumption 2. For each $e \in \{L, H\}$ and $j \in \{j_{0,t^*}, \dots, J_{t^*}\}$, $x_{e,j,t^*} > 0$.

Due to Assumption 2, we may differentiate (20) and (21) to calculate comparative statics. The data we analyze in Section 6 satisfy this assumption. Second, some of the lowest quality occupied

housing remains vacant:

Assumption 3. $x_{j_0,t^*,t^*} < h_{j_0,t^*,t^*}$.

If this lowest quality, q_{j_0,t^*,t^*} , represents outdoor locations, then Assumption 3 holds as long as there remain some unoccupied outdoor locations where households could feasibly reside. In the data we present in Section 5, we identify q_{j_0,t^*,t^*} as locations where the homeless live. Assumption 3 implies that $\sum_{j \in \mathcal{J}_t} h_{j,t} > N_t$. As a result, (19) holds, and n_{L,t^*} and n_{H,t^*} uniquely determine local equilibrium by Lemma 2.

3.2 Local approximation

An exact solution for comparative statics in city t^* necessitates computing comparative statics in all other cities as well. This interconnectedness is apparent from (14), which shows that a change to $v_{e,t^*}(z)$, the indirect utility in t^* , alters the population levels in all other cities, $n_{e,t}(z)$. These population changes move indirect utilities, $v_{e,t}(z)$, and these changes feed back to the population levels in t^* , $n_{e,t^*}(z)$.

Computing comparative statics in every city substantially complicates both the theoretical and quantitative analysis. To avoid these complications, we propose an approximation that allows us to solve for comparative statics in t^* exclusively using the equilibrium allocations in t^* :

$$\partial \log \bar{v}_e(z) \approx 0 \tag{23}$$

for each $e \in \{L, H\}$ and z such that $n_{e,t^*}(z) > 0$. When (23) holds, changes in t^* do not affect the average utility in the economy of households of a given education and labor endowment, or they affect this average only a small amount. Models with perfect mobility (e.g., Roback, 1982), sometimes make the analogous approximation that changes in one city do not affect household utility. This approximation may hold because the city is a relatively small share of the economy.

3.3 Derivatives of equilibrium conditions

To solve for the effects on house prices and welfare, we differentiate the model's equilibrium conditions to obtain a system of linear equations that we can directly solve.

Differentiating (14) while applying (23) and Lemma 1(d) yields

$$\partial \log n_{e,t^*}(z) = \beta_{e,e}^{-1} \partial \log v_{e,t^*}(z) \tag{24}$$

for each $e \in \{L, H\}$ and z such that $n_{e,t^*}(z) > 0$. Due to the approximation in (23), changes to the population in city t^* depend only on changes to indirect utility there and not in any other city. The population of households of a given education and labor endowment moves in the same direction

as their indirect utility, rising if it rises and falling if it falls. This relation is stronger when $\beta_{\epsilon,e}$ is smaller because the idiosyncratic city taste, ϵ , has less of an effect on utility.

Differentiating (2) produces

$$\partial \log a_{t^*} = \delta_a + \gamma_a \partial \log N_{H,t^*} - \gamma_a \partial \log N_{L,t^*}. \quad (25)$$

The city's amenities increase with an exogenous shock, δ_a . In the presence of amenity spillovers—that is, when $\gamma_a > 0$ —amenities also increase when the population of high-education households endogenously rises or when the population of low-education households endogenously falls.

Differentiating (11) delivers

$$\partial \log A_{e,t^*} = \delta_{A,e} + (\gamma_N - \gamma_H) \frac{N_{L,t^*}}{N_{t^*}} \partial \log N_{L,t^*} + \left(\gamma_N \frac{N_{H,t^*}}{N_{t^*}} + \gamma_H \frac{N_{L,t^*}}{N_{t^*}} \right) \partial \log N_{H,t^*} \quad (26)$$

for each $e \in \{L, H\}$. Productivity increases with an exogenous shock, $\delta_{A,e}$. If population spillovers are stronger than human capital spillovers, then $\gamma_N > \gamma_H$ and productivity also increases when the population of low-education households endogenously rises. Under the reverse, $\gamma_N < \gamma_H$ and productivity falls when the population of low-education households rises. Productivity rises along with the population of high-education households as long as $\gamma_N > 0$ or $\gamma_H > 0$.

Differentiating (12) gives

$$\partial \log w_{e,t^*} = \partial \log A_{e,t^*} + (1 - \rho) \frac{Y_{\sim e,t^*}}{Y_{t^*}} \partial \log \left(\frac{Z_{\sim e,t^*}}{Z_{e,t^*}} \right) + (1 - \rho) \frac{Y_{\sim e,t^*}}{Y_{t^*}} \partial \log \left(\frac{A_{\sim e,t^*}}{A_{e,t^*}} \right) \quad (27)$$

for each $e \in \{L, H\}$, where $\sim e$ denotes the other education group, $Y_{e,t^*} = w_{e,t^*} Z_{e,t^*}$ is the total income of households in that group, and $Y_{t^*} = Y_{L,t^*} + Y_{H,t^*}$ is the total income of the city's households. When $\rho = 1$, high- and low-education labor are perfect substitutes, so each labor price depends only on that labor's productivity. When $\rho < 1$, each labor price increases with the relative scarcity of that type of labor in the city, given by the endogenous ratio $Z_{\sim e,t^*}/Z_{e,t^*}$. It also decreases with any relative increase in that labor's productivity, $A_{\sim e,t^*}/A_{e,t^*}$, meaning that an increase in $\log A_{e,t^*}$ raises $\log w_{e,t^*}$ more when a similar shock occurs to $\log A_{\sim e,t^*}$.

Equations (25)–(27) express changes to amenities and labor prices in terms of changes to the total populations and labor endowments of each education group in city t^* . The changes in these totals come from aggregating (24), which gives the change in the population of every household in the city by education e and labor endowment z . Integrating (24) over all households in each education group yields

$$\partial \log N_{e,t^*} = N_{e,t^*}^{-1} \int_{z|n_{e,t^*}(z)>0} \partial \log n_{e,t^*}(z) n_{e,t^*}(z) dz, \quad (28)$$

which states that the total log change equals the average of the log changes for households with

each labor endowment. The change in the total labor endowment follows a similar formula, but now the average involves weighting by the income of each household:

$$\partial \log Z_{e,t^*} = Y_{e,t^*}^{-1} \int_{z|n_{e,t^*}(z)>0} \partial \log n_{e,t^*}(z) y_{e,t^*}(z) n_{e,t^*}(z) dz. \quad (29)$$

The remaining equilibrium conditions are those determining house prices and indirect utilities. We differentiate these separately in the divisible and indivisible cases.

3.3.1 Divisible housing

By Proposition 1, the change in equilibrium house prices is given by

$$\partial \log p_{j,t^*} = \partial \log \mu_{t^*} \quad (30)$$

for each $j \in \mathcal{J}_{t^*}$. Across all quality levels present in the city, log house prices move lock-step according to fluctuations in the city's price-to-quality ratio, μ_{t^*} . Proposition 1 allows us to derive the change in this ratio as

$$\partial \log \mu_{t^*} = -\frac{\sum_{j \in \mathcal{J}_{t^*}} \delta_{h,j} q_{j,t^*}}{\sum_{j \in \mathcal{J}_{t^*}} h_{j,t^*} q_{j,t^*}} + \sum_{e \in \{L,H\}} \frac{(\beta_{c,e} + \beta_{q,e})^{-1} \beta_{q,e} Y_{e,t} (\partial \log w_{e,t} + \partial \log Z_{e,t})}{(\beta_{c,L} + \beta_{q,L})^{-1} \beta_{q,L} Y_{L,t} + (\beta_{c,H} + \beta_{q,H})^{-1} \beta_{q,H} Y_{H,t}}. \quad (31)$$

The first term gives the log change to the total housing quality present in city t^* . An log increase to this quantity enters the equation negatively. The effect of building a unit of housing is more negative when that unit is of larger quality, as the new unit then has a larger effect on the total housing quality in the city. The second term gives the log change in the average income in the city. Households spend more on housing when they have more income, so this term is positive.

To calculate the change in indirect utility, we differentiate (17) to obtain

$$\partial \log v_{e,t^*}(z) = (\beta_{c,e} + \beta_{q,e}) \partial \log w_{e,t^*} - \beta_{q,e} \partial \log \mu_{t^*} + \beta_{a,e} \partial \log a_{t^*} \quad (32)$$

for all $z > 0$ and for each $e \in \{L,H\}$. Indirect utility rises with labor prices, falls with the house price-to-quality ratio, and rises with amenities. These changes do not depend on the labor endowment z , so all households of the same education experience the same log change in welfare.

3.3.2 Indivisible housing

In equilibrium, the measure of households choosing each quality level above the lowest occupied one coincides with the measure of housing at that quality level. Differentiating this condition,

which appears in (21), gives

$$\delta_{h,j} = \sum_{e \in \{L,H\}} \left(\underbrace{z_{e,j+1,t^*} n_{e,t^*}(z_{e,j+1,t^*}) \partial \log z_{e,j+1,t^*}}_{\text{trickle-up}} - \underbrace{z_{e,j,t} n_{e,t^*}(z_{e,j,t}) \partial \log z_{e,j,t}}_{\text{trickle-down}} + \underbrace{\int_{z_{e,j,t^*}}^{z_{e,j+1,t^*}} \partial \log n_{e,t^*}(z) n_{e,t^*}(z) dz}_{\text{migration}} \right) \quad (33)$$

when $j_{0,t^*} < j < J_{t^*}$. When $j = J_{t^*}$, (33) holds without the trickle-up term. Three forces combine to absorb the $\delta_{h,j}$ units of new housing. The first, which we call trickle-up, involves marginal households from the next highest quality level switching down to q_{j,t^*} . The second, which we call trickle-down, involves marginal households from the next lowest quality level switching up to q_{j,t^*} . The final term, migration, represents households who move to the city and choose q_{j,t^*} .

We relate the trickle-up and trickle-down terms to house prices by differentiating (20):

$$\partial \log z_{e,j,t^*} = \frac{(y_{e,j,t^*} - p_{j-1,t^*}) \partial p_{j,t^*}}{y_{e,j,t^*} (p_{j,t^*} - p_{j-1,t^*})} - \frac{(y_{e,j,t^*} - p_{j,t^*}) \partial p_{j-1,t^*}}{y_{e,j,t^*} (p_{j,t^*} - p_{j-1,t^*})} - \partial \log w_{e,t^*}, \quad (34)$$

where $y_{e,j,t^*} = y_{e,t^*}(z_{e,j,t^*})$, for each $e \in \{L,H\}$ and $j \in \{j_{0,t^*} + 1, \dots, J_{t^*}\}$. The endowment cutoff—that is, the left side of (34)—rises when the price of the higher quality level rises or the price of the lower quality level falls. Both price changes induce marginal households to switch from the higher quality to the lower one. The endowment cutoff also rises when the price of the education group's labor, w_{e,t^*} , falls, as a decline in income leads marginal households to switch to the lower quality.

Due to Assumption 3, (19) holds. Furthermore, $x_{j,t^*} < h_{j,t^*}$ continues to hold for $j = j_{0,t^*}$ under perturbations to the local equilibrium given the strict inequality. As a result, the identity of the lowest occupied quality does not change, so

$$\partial p_{j,t^*} = 0 \quad (35)$$

for $j = j_{0,t^*}$.

For households not on the margin between two qualities, differentiating (22) yields

$$\partial \log v_{e,t^*}(z) = \frac{\beta_{c,e} y_{e,t^*}(z) \partial \log w_{e,t^*}}{y_{e,t^*}(z) - p_{j,t^*}} - \frac{\beta_{c,e} \partial p_{j,t^*}}{y_{e,t^*}(z) - p_{j,t^*}} + \beta_{a,e} \partial \log a_{t^*}, \quad z \in (z_{e,j,t^*}, z_{e,j+1,t^*}) \quad (36)$$

for each $e \in \{L,H\}$. A household's welfare rises with the price of its labor and with the amenities in the city. Welfare falls with the price of the housing the household currently consumes. Any

changes in the prices of other qualities of housing have no direct effect on the household's welfare. The change for marginal households depends on whether the corresponding endowment cutoff increases or decreases. If $\partial \log z_{e,j,t^*} > 0$, then the marginal households all choose the lower quality level as a result of the changes to primitives, so the relevant house price for them in (36) is p_{j-1,t^*} . If $\partial \log z_{e,j,t^*} < 0$, then the marginal households all choose the higher quality level as a result of the changes to primitives, so the relevant house price in (36) is p_{j,t^*} .

3.4 When trickle-down economics works

We present two special cases in which trickle-down housing economics works. That is, construction improves the welfare of all households in city t^* , and a single new housing unit improves welfare most when the quality of the unit is $q_{J_{t^*},t^*}$, the highest quality available. Both properties fail in the model we estimate in Section 6. For the rest of this section, $\delta_{A,L} = \delta_{A,H} = \delta_a = 0$.

3.4.1 Divisible housing

In the divisible case, the total quality-adjusted housing stock, $\sum_{j \in \mathcal{J}_t} q_{j,t} h_{j,t}$, determines local equilibrium. As a result, the effects of construction depend on the change to this total, $\sum_{j \in \mathcal{J}_t} q_{j,t} \delta_{h,j}$. An increase raises household welfare under two conditions. First, the equilibrium is *locally stable*, meaning that perturbations to the city's population raise the welfare of departing households or decrease the welfare of arriving households. A formal definition appears in Appendix B.1. Second, $\gamma_N = 0$, which limits spillovers to those depending on the relative population of households with different education.

Proposition 3. *In the divisible model, for each $e \in \{L, H\}$ and $z > 0$,*

$$\partial \log v_{e,t^*}(z) \propto \sum_{j \in \mathcal{J}_{t^*}} q_{j,t^*} \delta_{h,j}, \quad (37)$$

where \propto denotes proportionality that is positive if the equilibrium is locally stable and $\gamma_N = 0$.

Proof. Appendix B.2. □

According to Proposition 3, the benefits constructing high quality housing trickle down to lower income households. Furthermore, each household benefits most when the quality of a single new unit is the highest quality in the city. A rich household may move into this new unit, and then poor households may move into partitions of the rich household's vacated unit whose quality exceeds that of their previous housing. This reallocation is one way that the benefits of high quality construction can trickle down to lower income households.

3.4.2 Indivisible housing with nearly no mobility

In the limit without mobility, the outcomes that depend on the city population remain fixed. These fixed outcomes include labor prices and amenities. The effect on welfare, $\partial \log v_{e,t^*}(z)$, depends only on house price changes, as is apparent from (36). Welfare increases more when the price of the housing that a household is choosing decreases more. The following proposition characterizes the effect of construction on house prices and welfare:

Proposition 4. *Suppose $\delta_{h,j} \geq 0$ for $j > j_{0,t^*}$, with at least one strict inequality, in the immobile limit of the indivisible model. For each e and $z \geq z_{e,j_{0,t^*}+1,t^*}$, $\partial \log v_{e,t^*}(z)/\partial \delta_{h,j}$ is positive, increases over $j' \in \{j_{0,t^*} + 1, \dots, j\}$, and stays constant over $j' \in \{j, \dots, J_{t^*}\}$. If $z < z_{e,j_{0,t^*}+1,t^*}$, then $\partial \log v_{e,t^*}(z) = 0$.*

Proof. Appendix B.3. □

An analogous result appears in Section 3C of Braid (1981).

The benefits of constructing high quality housing again trickle down to lower income households, with the exception of non-marginal households choosing the lowest unoccupied quality, who are indifferent. For other households, the decline in one's house price is largest when construction occurs at the city's highest quality, $q_{J_{t^*},t^*}$. As in the divisible case, the strict Pareto optimum for constructing a set amount of housing is to build all of it at this highest quality. Another similarity is that constructing any unit whose price is positive lowers all positive house prices and increases the welfare of all households choosing housing with a positive price.

The mechanism behinds these results is similar to the one in the divisible case. When construction occurs at a high quality, the price of this housing falls so that some households on the margin with the next lowest quality choose the new housing. This choice creates vacancy at the next lowest quality, so the price of this housing falls to induce poorer marginal households to switch. This process continues down to the lowest occupied quality, lowering the price of all occupied housing in the city above the lowest level. Even the prices of housing of quality higher than the construction must fall; otherwise, richer households would choose to switch down to the new housing, which Proposition 2 rules out. However, a given inframarginal household benefits most when construction occurs at or above the quality it currently chooses.

4 Estimation strategy

To solve the system of linear equations determining comparative statics in the indivisible case, we require values of the following parameters: ρ , γ_a , γ_N , γ_H , $\beta_{c,e}/\beta_{e,e}$, and $\beta_{a,e}/\beta_{e,e}$ for $e \in \{L, H\}$. We also require values of the following equilibrium outcomes: $N_{e,t^*}/N_{t^*}$, $Y_{e,t^*}/Y_{t^*}$, y_{e,j,t^*} , and p_{j,t^*} for $e \in \{L, H\}$ and $j \in \{j_{0,t^*}, \dots, J_{t^*}\}$. Two other equilibrium outcomes appear in the equations: z_{e,j,t^*} and n_{e,t^*} for $e \in \{L, H\}$ and $j \in \{j_{0,t^*}, \dots, J_{t^*}\}$. By changing variables from the labor endowment, z ,

to income, y , we replace these outcomes with the probability density functions of income within each education group, f_{L,t^*} and f_{H,t^*} . Details of this change in variables appear in Appendix C.1.

The remainder of Section 4 describes how we estimate the parameters and equilibrium outcome we need using household-level data from city t^* and prior estimates from the literature.

4.1 Observations

We observe a representative sample of households indexed by i in a single metropolitan area with the following data. First is a sample weight, g_i . Second is a dummy variable, e_i , equal to one if the household has education H and zero if the household has education L . Third is the income that the household reports for the prior year, y_i . Fourth is a categorical variable, $o_i \in \{-1, 0, 1\}$, giving the status of the household's ownership of its place of residence. This variable equals minus one for households who reside in a house of the lowest occupied quality, q_{j_0,t^*,t^*} . For households residing in houses of higher quality, o_i equals zero for renters and one for owner-occupants. When $o_i = 0$, we observe the monthly rent that the household pays, r_i , and when $o_i = 1$, we observe the value of the house, v_i .

4.2 House prices and housing demands

Our first step is imputing an annual price of housing for each household, which we call p_i . Due to the assumption above, we know from the model that $p_{j_0,t^*,t^*} = 0$, so we set $p_i = 0$ when $o_i = -1$. We set $p_i = 12r_i$ when $o_i = 0$, meaning we multiply the monthly rent by twelve. For owner-occupants, the annual price of housing equals the cost of capital times v_i , plus maintenance costs and taxes, less expected capital gains. If the latter are proportional to v_i , then there exists some constant, ϕ , such that the annual price for each owner-occupant is $p_i = \phi v_i$. We assume that such proportionality holds, and we take ϕ from data we present in Section 5.

The second step is assigning households, i , to housing quality indices, j . In the absence of measurement error, each distinct value of the annual house price, p_i , corresponds to a distinct quality by Proposition 2. Measurement error may arise for a variety of reasons, including misreporting and unmodeled search frictions leading to price dispersion for similar housing units. To smooth this error, we assign quality indices by binning households along annual house prices, p_i . Specifically, we set $j(i) = 0$ when $o_i = -1$. When $o_i \in \{0, 1\}$, we set $j(i)$ equal to the quantile of p_i among households in this group, out of a total of 50 quantiles. We assume that $\mathcal{J}_t = \{0, \dots, 50\}$, meaning that we observe households in every occupied quality level. We estimate the price for each quality as the average of p_i among households choosing that quality: $\widehat{p}_{j,t^*} = \sum_i g_i \delta_{i,j} p_i / \sum_i g_i \delta_{i,j}$, where $\delta_{i,j}$ is an indicator equal to one if and only if $j(i) = j$. For each $j \in \mathcal{J}_{t^*}$, our estimate of the share of households choosing housing of quality q_{j,t^*} is $\sum_i g_i \delta_{i,j} / \sum_i g_i$.

4.3 The distribution of households

To fit income distributions to the data, we specify f_{L,t^*} and f_{H,t^*} as double Pareto-lognormal distributions, a four-parameter family that Reed (2003) and Reed and Jorgensen (2004) propose to characterize income distributions. Because it allows for Pareto behavior in both the upper and lower tails, the double Pareto-lognormal distribution describes income distributions better than the lognormal distribution.

Proposition 2 sharply restricts the joint distribution of income and housing quality by requiring that $y_i \geq y_{i'}$ whenever $j(i) \geq j(i')$ and $e_i = e_{i'}$. The data violate this restriction if there are any pairs of households of the same education in which the poorer household lives in a higher quality house than the richer household. To fit the model to data that may violate this restriction, we assume that the income we observe, y_i , does not necessarily equal the income from the model, $y_{e_i,t^*}(z_i)$, where z_i is the unobserved labor endowment of household i . Instead, y_i equals $y_{e_i,t^*}(z_i)$ plus noise. This noise may come from temporary fluctuations in income that, due to adjustment costs, do not cause households to change the quality of the housing they choose (Chetty and Szeidl, 2007). Our empirical strategy identifies several parameters of the model under the following assumption about this noise:

Assumption 4. $0 = E\delta_{i,j}e_i(y_i - y_{e_i,t^*}(z_i)) = E\delta_{i,j}(1 - e_i)(y_i - y_{e_i,t^*}(z_i))$ for each $j \in \mathcal{J}_{t^*}$.

By Assumption 4, the noise not only has a mean of zero but also is uncorrelated with a household's education, housing choice, and the interaction of the two.

We exploit Assumption 4 to estimate several of the remaining parameters using the generalized method of moments (Hansen, 1982). These parameters consist of the eight double Pareto-lognormal parameters as well as the rent-to-price ratio, ϕ , the population shares of each education group, $N_{L,t^*}/N_{t^*}$ and $N_{H,t^*}/N_{t^*}$, and the ratio $(\beta_{q,L}/\beta_{c,L})/(\beta_{q,H}/\beta_{c,H})$, which we call ζ . This ratio describes the relative preference of low-education households for housing versus non-housing consumption relative to high-education households. We denote the vector of these twelve parameters by θ , and we denote the set over which we search for this vector by Θ .

Given our estimates of house prices and housing demands, the components of θ uniquely determine the incomes of the households on the margin between the occupied quality levels, y_{e,j,t^*} for $e \in \{L, H\}$ and $j \in \{1, \dots, 50\}$. To illustrate this correspondence, we divide (21) by the city population, N_{t^*} , and change variables from z to y to obtain

$$\frac{\sum_i g_i \delta_{i,j}}{\sum_i g_i} = \sum_{e \in \{L, H\}} \frac{N_{e,t^*}}{N_{t^*}} \int_{y_{e,j,t^*}}^{y_{e,j+1,t^*}} f_{e,t^*}(y) dy \quad (38)$$

for each $j \in \{1, \dots, 50\}$. We also equate the two solutions to (20) for $q_{j,t^*}/q_{j-1,t^*}$:

$$\left(\frac{y_{L,j,t^*} - \widehat{p}_{j,t^*}}{y_{L,j,t^*} - \widehat{p}_{j-1,t^*}} \right)^{\frac{\beta_{c,L}}{\beta_{q,L}}} = \left(\frac{y_{H,j,t^*} - \widehat{p}_{j,t^*}}{y_{H,j,t^*} - \widehat{p}_{j-1,t^*}} \right)^{\frac{\beta_{c,H}}{\beta_{q,H}}} \quad (39)$$

for each $j \in \{1, \dots, 50\}$. For certain θ , these equations give unique solutions for the income cutoffs:

Lemma 3. *If*

$$\frac{\sum_{j' \in \{j, \dots, 50\}} \sum_i g_i \delta_{i,j'}}{\sum_i g_i} \leq \sum_{e \in \{L, H\}} \frac{N_{e,t^*}}{N_{t^*}} \int_{\widehat{p}_{j,t^*}}^{\infty} f_{e,t^*}(y) dy \quad (40)$$

for each $j \in \{1, \dots, 50\}$, then unique values of y_{e,j,t^*} for $e \in \{L, H\}$ and $j \in \{1, \dots, 50\}$ solve (38)–(39). When $\zeta = 1$, these values strictly increase in j for each e .

Proof. Appendix C.2. □

Lemma 3 guarantees unique solutions for the income cutoffs when θ satisfies (40), which states, for each j , that the share of housing of quality at least q_{j,t^*} in the data cannot exceed the share of households in the model with income at least \widehat{p}_{j,t^*} . Assumption 2 requires that the resulting income cutoffs strictly increase in j for each e . To abide by this restriction, we limit Θ to θ that satisfy (40) and for which the resulting income cutoffs strictly increase in j . Because monotonicity holds when $\zeta = 1$, Θ is nonempty. For computational reasons, we further restrict Θ to lie within a neighborhood of our initial guess, θ_0 .

To estimate θ , we compare several conditional expectations of income and education in the model to the data. In the model, the average income of households of education e choosing housing quality q_{j,t^*} , which we denote $\bar{y}_{e,j,t^*}(\theta)$, equals the conditional mean under the distribution f_{e,t^*} over the interval $[y_{e,j,t^*}(\theta), y_{e,j+1,t^*}(\theta))$. Similarly, the average education of households choosing housing quality q_{j,t^*} , which we denote \bar{e}_{j,t^*} , equals the ratio of the measure under f_{e,t^*} of the interval $[y_{e,j,t^*}(\theta), y_{e,j+1,t^*}(\theta))$ to the sum of these measures across both values of e . The following moment conditions equate empirical realizations of these conditional expectations to their model-based counterparts:

$$0 = E \delta_{i,j} (1 - e_i) (y_i - \bar{y}_{L,j,t^*}(\theta)) \quad (41)$$

$$0 = E \delta_{i,j} e_i (y_i - \bar{y}_{H,j,t^*}(\theta)) \quad (42)$$

$$0 = E \delta_{i,j} (e_i - \bar{e}_{j,t^*}(\theta)) \quad (43)$$

for each $j \in \mathcal{J}_{t^*}$. These moment conditions hold due to Assumption 4.

As in Hansen (1982), we estimate θ by minimizing a quadratic form of the realizations of these moments in the data. The covariance of each pair of distinct moments equals zero, so a valid weighting matrix for the estimation is the diagonal matrix consisting of the inverses of the sample variances of each moment under our initial guess, θ_0 . The resulting estimator is then

$$\begin{aligned} \theta(\theta_0) = \arg \min_{\theta \in \Theta} & \sum_{j \in \mathcal{J}_t} \frac{(\sum_i g_i \delta_{i,j} (1 - e_i) (y_i - \bar{y}_{L,j,t^*}(\theta)))^2}{\sum_i g_i \delta_{i,j} (1 - e_i) (y_i - \bar{y}_{L,j,t^*}(\theta_0))^2} \\ & + \frac{(\sum_i g_i \delta_{i,j} e_i (y_i - \bar{y}_{H,j,t^*}(\theta)))^2}{\sum_i g_i \delta_{i,j} e_i (y_i - \bar{y}_{H,j,t^*}(\theta_0))^2} + \frac{(\sum_i g_i \delta_{i,j} (e_i - \bar{e}_{j,t^*}(\theta)))^2}{\sum_i g_i \delta_{i,j} (e_i - \bar{e}_{j,t^*}(\theta_0))^2}. \end{aligned} \quad (44)$$

We iterate the generalized method of moments estimation by using $\theta(\theta_0)$ as the initial guess to arrive at a final estimate of $\widehat{\theta} = \theta(\theta(\theta_0))$. This estimator chooses the value of θ that best fits the joint distribution of income, education, and housing quality we observe in the data.

4.4 Remaining parameters

We take the production function parameters— ρ , γ_N , and γ_H —from prior estimates in the literature. The remaining parameters we need are $\beta_{c,L}/\beta_{\epsilon,L}$, $\beta_{c,H}/\beta_{\epsilon,H}$, $\beta_{a,L}/\beta_{\epsilon,L}$, $\beta_{a,H}/\beta_{\epsilon,H}$, and γ_a .

We rely on three sets of estimates for these remaining parameters. To describe these existing estimates, we let $\beta_{w,e} = \beta_{c,e} + \beta_{q,e}$ denote the sum of the utility weights on non-housing and housing consumption for each education $e \in \{L, H\}$. The first estimate is the average of the ratio $\beta_{q,e}/\beta_{w,e}$ in a subset of the population. This estimate, which we denote α , equals the average share of income spent on housing in the divisible model. In the indivisible model, this ratio satisfies the equation

$$1 - \alpha = (1 - \bar{e}_\alpha) \frac{\beta_{c,L}}{\beta_{w,L}} + \bar{e}_\alpha \frac{\beta_{c,H}}{\beta_{w,H}}, \quad (45)$$

where \bar{e}_α equals the share of households of education H among the subset in which α is the average value of $\beta_{q,e}/\beta_{w,e}$. This education share constitutes the second of the three estimates. Together with the equation defining ζ , (45) determines the ratios $\beta_{c,L}/\beta_{w,L}$ and $\beta_{c,H}/\beta_{w,H}$:

Lemma 4. *Given ζ , unique values of $\beta_{c,L}/\beta_{w,L}$ and $\beta_{c,H}/\beta_{w,H}$ jointly in $(0, 1)$ solve (45).*

Proof. Appendix C.3. □

Using our estimate of ζ from $\widehat{\theta}$, we solve for these unique values of $\beta_{c,L}/\beta_{w,L}$ and $\beta_{c,H}/\beta_{w,H}$.

The final source is a paper that jointly estimates $\beta_{w,L}/\beta_{\epsilon,L}$, $\beta_{w,H}/\beta_{\epsilon,H}$, $\beta_{a,L}/\beta_{\epsilon,L}$, $\beta_{a,H}/\beta_{\epsilon,H}$, and γ_a , the vector of which we denote ψ . These joint estimates identify the remaining parameters when we multiply the first two estimates by the ratios $\beta_{c,L}/\beta_{w,L}$ and $\beta_{c,H}/\beta_{w,H}$. The equilibrium under all of the parameters we have estimated may not be stable. The paper whose estimates we are using provides a sampling distribution for ψ in addition to point estimates. To ensure stability, we sample 10,000 times from the sampling distribution for ψ and use the mean of the estimates for which the marginal effects of δ_a on a_{t^*} and of each $\delta_{A,e}$ on A_{e,t^*} are positive.

5 Data

5.1 American Community Survey

5.1.1 Data description

Household-level data come from the American Community Survey (ACS), which the U.S. Census Bureau has conducted annually since 2005 to provide current economic information about the United States (see U.S. Census Bureau, 2014 for the most recent available documentation). We use the public use microdata sample that is part of the Integrated Public Use Microdata Series (Ruggles et al., 2018). The data are a weighted random sample of the U.S. population.

The Census Bureau categorizes observations depending on the type of residence where the surveyed person lives. The names of the two types of residences are “housing units” and “group quarters.” When a sampled person resides at a housing unit, we observe all persons living at that place of residence. The respondent designates one of these persons—someone owning or renting the unit, if possible—as the “householder.” We observe no other linked persons in the case of a person residing in group quarters.

Group quarters consist of institutional (e.g., prisons and hospitals) and non-institutional facilities. Non-institutional group quarters fall into one of three types: college dormitories, military facilities, and “other.” The ACS excludes seven sub-categories of other non-institutional group quarters. The non-excluded sub-categories are homeless shelters, religious group quarters, adult group homes, adult residential treatment centers, and workers’ living quarters (U.S. Census Bureau, 2012). For sampled persons in other non-institutional group quarters, the ACS sample weights reflect the full population in other non-institutional group quarters. To construct these weights, the Census Bureau uses population estimates from the decennial census, which counts persons in both the excluded and included sub-categories.

We aggregate all persons in a housing unit into a single household observation, i , while a person in group quarters constitutes a single observation. We use the “household weight” variable, which assigns a weight to each group quarters person and housing unit, as g_i . We use total personal income, which is available for all persons at least 15 years old, as y_i for persons in group quarters. For each housing unit, we use the sum of this variable across persons as y_i . We set $e_i = 1$ if a person in group quarters or the householder of a housing unit has a bachelor’s degree. For housing units, we set $o_i = 0$ if the residents rent the housing unit and $o_i = 1$ if the residents own or are purchasing the housing unit. We set $o_i = -1$ for persons in group quarters. For renters paying cash rent, we use monthly contract rent as r_i . We use the survey respondent’s estimate of the house value as v_i when $o_i = 1$. The Census Bureau top-codes rent and home values above separate thresholds in each state with the average of the variable above that threshold in that state. Finally, we label a housing unit as new construction if the household reports the construction year as the one directly before the survey year.

5.1.2 Sample selection

For renter households not paying cash rent, we do not observe the value of any non-cash goods and services they pay to reside in their housing unit. Because our estimation strategy depends on observing this information, we drop such households.

The group quarters observations we keep should correspond to persons living in the lowest quality housing in each city. Ideally, we could limit group quarters persons to the homeless, but we do not observe homelessness in the ACS. To attempt this refinement, we drop persons in institutional group quarters, and then among the remaining group quarters population, we drop minors, students, and the employed (which includes the armed forces). The remaining persons likely reflect adults in other non-institutional group quarters who do not have a job.

We limit our estimation sample to the Boston-Cambridge-Newton, MA-NH metropolitan area in 2016 (the most recent data year when we starting writing this paper). Among the largest 20 metropolitan areas in the United States in 2016, Boston has some of the lowest construction activity and highest home values and rents. Some of the large metropolitan areas more extreme than Boston in these dimensions are New York-Newark-Jersey City, NY-NJ-PA and San Francisco-Oakland-Hayward, CA. We choose Boston over these metropolitan areas because New York and California allow rent control while Massachusetts and New Hampshire prohibit it. Our model assumes that a competitive market determines rents.

5.1.3 Summary statistics

Table 1 lists summary statistics for the data from the Boston metropolitan area in 2016. As shown in Panel A, we drop about 91% of the group quarters persons when selecting our estimation sample. This share is large because most group quarters persons reside in an institution or are minors, students, or employed. In contrast, we drop only about 3% of renter households, corresponding to those not paying cash rent. We keep all owner-occupant households.

Panel B shows weighted means of the variables we use. Income of owner-occupant households is about twice that of renter households, on average. The mean income of group quarters persons is much lower, at \$7,531. Our flag for a college degree is also highest for owner-occupant households and lowest for group quarters persons. Among renters, the average rent is \$1,284, and the Census Bureau censors less than 2% of rents. Among owner-occupants, the average home value is \$491,724, with the Census Bureau censoring less than 1% of home values. Almost all of our weighted sample consists of renter and owner-occupant households, with about twice as many owner-occupant households as renters. The total number of unweighted observations in the estimation sample is 18,269.

5.2 Real Capital Analytics

Real Capital Analytics (RCA) provides quarterly estimates of the capitalization rate (i.e., the annual income return) for multifamily rental properties with at least ten units in the Boston metropolitan area that sold in 2016. RCA also provides quarterly estimates of the rental revenue and net operating income per square foot for multifamily properties held by institutional investors. For each quarter in 2016, we calculate the annual rental return by multiplying the capitalization rate by the institutional rental revenue per square foot and then dividing by the institutional operating income per square foot. The average of the rental yield estimate for each quarter, 0.09, serves as our estimate for ϕ .

5.3 Estimates from other work

Several papers in labor economics estimate the inverse elasticity of substitution between college and non-college labor to be about 0.7 (see the discussion in Card, 2009). This inverse elasticity corresponds to $1 - \rho$, so we set $\rho = 0.3$. Combes and Gobillon (2015) review the empirical literature on productivity spillovers and find that the typical estimate of the elasticity of productivity with respect to population density lies between 0.04 and 0.07. These estimates correspond to γ_N , so we set $\gamma_N = 0.055$, which is the midpoint of this range.

Moretti (2004b) estimates that log output in an industry within a city rises about 0.0055 (“a 0.5–0.6-percentage-point increase”) when the college share in other industries in the same city rises by one percentage point. Interpreting this estimate as 100 times the derivative of log productivity with respect to $N_{H,t}/N_t$, we obtain $0.55 = \gamma_H N_t / N_{H,t}$. The college shares in the two years in the sample in Moretti (2004b) are 0.161 and 0.191. Setting $N_{H,t}/N_H$ equal to the average of these two numbers, 0.176, gives us $\gamma_H = 0.097$.²

Diamond (2016) estimates $\psi = (\beta_{w,L}/\beta_{\epsilon,L}, \beta_{w,H}/\beta_{\epsilon,H}, \beta_{a,L}/\beta_{\epsilon,L}, \beta_{a,H}/\beta_{\epsilon,H}, \gamma_a)$ using the generalized method of moments. Her data include labor incomes, rental payments, and city choices of workers with and without a college degree in the United States between 1980 and 2000, and her model is close to the divisible housing framework in our paper. We use the estimate from her “full model,” corresponding to column 3 of her Table 5 (ignoring differential effects for Blacks and immigrants), which is $\psi = (4.026, 2.116, 0.274, 1.1012, 2.60)$. We obtain the sampling distribution of ψ from her replication files. Under her point estimate, low-education households care more than

²A similar approach to calibrating γ_H uses numbers from Gennaioli et al. (2013), who estimate that an additional year of average schooling in a sub-national region raises log productivity in that region by 0.074. In their data, the average college share is 0.11, the years of schooling for individuals with a college degree is 16, and the average years of schooling for all individuals is 6.52. The average years of schooling among individuals without a college degree is thus $(6.52 - 0.11 * 16) / 0.89 = 5.35$. A one percentage point increase in the college share increases log productivity by $0.01 * (16 - 5.35) * 0.074 = 0.0079$ if we assume that the commensurate one percentage point decrease in the non-college share is orthogonal to their years of schooling. Multiplying this estimate by 100 and by the college share (as we did in the Moretti, 2004b calculation) yields $\gamma_H = 0.087$. Reassuringly, this number is close to the one from the Moretti (2004b) calculation. We use the larger estimate, 0.097, to explore the full effects of human capital spillovers in the model.

high-education households about income when choosing a city. The reverse is true for amenities.

In the divisible model that Diamond (2016) estimates, $\beta_{q,e}/\beta_{w,e}$ coincides with the share of income that a household of education e spends on housing. Davis and Ortalo-Magné (2011) estimate that renters in the United States between 1980 and 2000 (the setting in which Diamond, 2016 estimates her model) spend about 24% of their income on housing, so we set $\alpha = 0.24$. The final estimate is \bar{e}_α , the share of renter households in the United States between 1980 and 2000 in which the householder has a college degree. In the U.S. Census from 1980, 1990, and 2000 (the same data that Davis and Ortalo-Magné (2011) and Diamond (2016) use), $\bar{e}_\alpha = 0.18$.

6 Quantitative results

6.1 Estimation

Table 2 reports our estimates of the model parameters as well as some of the equilibrium outcomes. Households split roughly evenly between low and high education, with about 51% in the former category and 49% in the latter. However, the low education households earn only about one third of the city’s income. On average, then, high education households earn about double what low education households earn.

The components of ψ —the vector of parameters whose sampling distribution we take from Diamond (2016)—substantially differ from the point estimates in Diamond (2016) only for γ_a , the amenity spillover. Our equilibrium is unstable under her point estimates.³ When we take the mean of the resamples that lead to stability, we obtain an estimate for γ_a of 1.103, which is less than half the point estimate of 2.6 from Diamond (2016). The other estimates change much less, indicating that the strength of the amenity spillover in the point estimate prevents stability. These estimates imply that high-education households value amenities versus non-housing consumption 8.20 times more than low-education households do.

Figure 1 displays the average incomes and education shares of households choosing each quality of housing against the price of the corresponding quality. We plot these averages in both the model and the data. Our estimation minimizes the differences between the model and data averages, so the closeness of these outcomes indicates the goodness of fit.

As Panel A shows, the model matches the empirical income averages quite well with the notable exception of over-predicting the incomes of the few low-education households choosing very high quality housing and under-predicting the incomes of the few high-education households choosing very low quality housing. Conditional on housing quality, the incomes of high-education

³Because our models differ, this instability does not imply that the equilibrium in Diamond (2016) is unstable. In particular, by imposing the local approximation in (23), we assume away economy-wide changes in utility levels from migrations into and out of Boston. Diamond (2016) estimates her model for the entire United States, meaning that such migrations change economy-wide utility levels. As a result, re-allocations of households that cause instability in our approximate model may not do so in her more complete framework.

households exceed the incomes of low-education households in both the model and the data. This outcome is consistent with our estimated value of $\zeta = 1.603$, which indicates that low-education households value housing versus non-housing consumption relatively more than high-education households.

Panel B shows that the model likewise closely fits the education shares for each type of housing, with the exception of under-predicting the high-education share choosing very low quality housing and over-predicting the high-education share choosing very high quality housing. Only about 10% of households choosing the lowest quality levels have high education, while more than 80% of the households choosing the highest quality levels have high education.

6.2 Construction effects

6.2.1 Low-quality construction

What is the effect of building low-quality housing on household welfare? To answer this question, we study construction in bin $j = 10$, which represents the 20th percentile of occupied housing units. We consider what would happen if the housing stock of the city expanded by 0.45% solely through new housing of this quality type. We choose 0.45% to match the metro-wide construction we observe in the data. To implement this experiment mathematically, we set $\delta_{h,j} = 0$ for $j \neq 10$ and $\delta_{h,10} = 0.0045$, and then we solve the linear system of equations from Section 2.

Our outcome of interest is $\partial \log v_{e,t^*}(z)$, the welfare change for a household of education e and endowment z who remains in t^* after the construction. According to (24), this welfare change is proportional to $\partial \log n_{e,t^*}(z)$, the change in the population of such households. We average this population response across each education group e and housing bin j . Because households sort by income across housing bins, the effect in low bins delivers the welfare change for the poor. The effect on high bins describes what happens to the rich.

Figure 2 displays the results. Our baseline specification appears in Panel D. According to these results, low-quality construction makes poor non-college households better off. At that same time, it makes rich households worse off. It also hurts middle-income college households. Equivalently, such construction increases the population of poor non-college households while causing out-migration of the rich and college-educated.

The combination of spillovers and preference heterogeneity is key for causing negative welfare effects from construction. To illustrate this point, Panels A–C reproduce Panel D under different parameter values. We turn off spillovers by setting $\gamma_N = \gamma_H = \gamma_a = 0$ and $\rho = 1$. In this case, city amenities and each household’s income no longer depend on the composition of city households. We turn off preference heterogeneity by setting $\beta_{c,L}/\beta_{\epsilon,L}$ and $\beta_{c,H}/\beta_{\epsilon,H}$ (resp. $\beta_{a,L}/\beta_{\epsilon,L}$ and $\beta_{a,H}/\beta_{\epsilon,H}$) equal to their average value in the city before the construction. In this specification, the cross-city migration response to non-housing consumption and amenities is equally sensitive for

college and non-college households.⁴

Panel A displays the results under preference homogeneity without spillovers. Construction improves the welfare of all households by lowering house prices. On a proportional basis, it lowers low-quality house prices more than high quality prices, leading to larger welfare gains for the poor. When we turn on spillovers, construction continues to improve the welfare of college households, as Panel B shows. Some very rich non-college households suffer from labor market competition with poor non-college migrants. Panel C turns off spillovers but re-introduces preference heterogeneity. The picture is largely similar to Panel A—without spillovers, construction makes no one worse off. It is only in Panel D that construction hurts many city households.

Why does the combination of spillovers and preference heterogeneity lead to negative welfare effects? Figure 3 clarifies this point by decomposing the average welfare effect in Panel D for two groups of households: non-college with incomes in the bottom quartile of the city income distribution, and college with incomes in the top quartile. Poor non-college households benefit entirely because house prices fall. In contrast, rich college households suffer solely because city amenities decline. Constructing low-quality housing inherently attracts more non-college households because they constitute a larger share of households living in such housing. In the presence of spillovers, amenities endogenously fall as a result. Because preferences are heterogeneous, college households react more strongly to this decline in amenities, moving out of the city. This migration lowers house prices, stimulating further in-migration of poor non-college households. The result is a strong price welfare effect for these households and a negative amenity welfare effect for rich college households. Consistent with this mechanism, the welfare gain for poor non-college households in Figure 2 is largest in Panel D.

6.2.2 High-quality construction

To study the effects of high-quality construction, we perform a similar experiment where all construction occurs at bin $j = 40$, which is the 80th percentile of occupied housing units. We narrow our attention by further averaging the welfare effect across education groups and city income quartiles. Doing so gives us eight categories (two of which appear in Figure 3). We report results in column (1) of Panel B of 3. Panel A reports analogous results for building in the 20th percentile.

High-quality construction hurts college households much less than low-quality construction. Only the richest quartile suffer losses, which are small enough to round to 0.00%. Conversely, poor non-college households gain much less than before, 0.93% versus 1.66%. Significantly, high-quality construction continues to improve poor non-college welfare, but much less effectively. As a result, rich college households prefer different construction than poor non-college households. By lowering low-quality prices less, high-quality construction is worse for poor households, most of whom are in the non-college group. Less of them migrate to the city, which improves amenities

⁴College and non-college households still differ in $\beta_{q,e}/\beta_{\epsilon,e}$. The estimation forces this difference by selecting a value of $\zeta \neq 1$. When we change $\beta_{c,e}/\beta_{\epsilon,e}$, we keep the ratio $(\beta_{q,L}/\beta_{c,L})/(\beta_{q,H}/\beta_{c,H})$ constant at our estimated value of ζ .

and makes college households better off.

How much low-quality housing does high-quality construction make available? High-quality construction raises the city’s population by 0.33%. Because the housing stock expands by 0.45%, 100 new luxury units bring about 73 new households into the city, freeing up new units for 27 existing residents on net. As a result, many poor households move up to higher quality housing. In aggregate, the measure of existing residents in the bottom quartile and bottom half of the non-homeless housing distribution who move to a nicer house equals 0.19% and 0.29% of the total city population, corresponding to 42 and 64 vacated units. These numbers are close to the 34 and 65 that Mast (2019) calculates.

6.2.3 Extensions

The remaining columns in Table 3 explore welfare effects of construction under different parameters and assumptions. This exercise clarifies the robustness of the results so far and the mechanisms generating them.

In column (2), we set $\gamma_a = 0$ so that amenities no longer depend on the composition of city households. Low-quality construction no longer makes college households worse off. In fact, both types of construction make all four groups of college households better off than in column (1). The endogeneity of amenities is crucial in generating the negative welfare effects on college households. In contrast, low-quality construction continues to improve poor non-college welfare more than high-quality construction. Therefore, this feature of the baseline model does not depend on endogenous amenities.

In the next column, we report results from an extension in which endogenous amenities differ across housing bins. This extension captures the empirical dependence of amenities on the composition of households in one’s neighborhood (Bayer et al., 2007). Specifically, amenities for households choosing $q_{j,t}$ are now

$$a_{j,t} = \tilde{a}_t \left(\frac{N_{H,t}}{N_{L,t}} \right)^{\gamma_a - \gamma_{al}} \left(\frac{N_{H,j,t}}{N_{L,j,t}} \right)^{\gamma_{al}}, \quad (46)$$

where $N_{e,j,t}$ is the measure of households of education e choosing $q_{j,t}$. This specification re-allocates some of the city-wide amenity channel to each housing sub-market, which we think of as a neighborhood. Specifically, an increase in $N_{H,j,t}/N_{L,j,t}$ that is uniform across all j raises amenities the same amount under this new specification as under (2), the baseline amenity specification. We use estimates from Bayer et al. (2007) on the willingness-to-pay for an increase in college households in one’s neighborhood to get a value of $\gamma_{al} = 0.05$.⁵ We then re-estimate θ

⁵According to their Table 8, a 10 percentage point increase in the neighborhood college share increases the willingness-to-pay of college households to live in the neighborhood by \$68.51 per month. Using their summary statistics on mean income, housing costs, and college neighborhood shares, as well as our value for $\beta_{a,H}/\beta_{c,H}$, we calculate $\gamma_{al} = 0.06$. Because they find a negative willingness-to-pay of non-college households for this amenity, we

using this new amenity specification, as the trade-off between adjacent housing bins must now reflect the different college shares in each bin. The new θ is very close to the baseline value. We perform comparative statics with the new θ for internal consistency.

Column (3) displays the results. Unsurprisingly, poor college households are worse off than under the baseline, as the migration of poor non-college households into their bins has a negative effect on local amenities. More notable is that rich college households are also worse off than under the baseline, even though amenities are more localized than before. The in-migration of poor non-college households is stronger because they more easily push out poor college households. This effect is large enough to increase the overall non-college population. Because the rich college amenities still depend on this population (although less than in the baseline), their welfare decreases. The negative effect of construction on college households is therefore robust to this localization of amenities.

We next add to the model local supply of services by non-college households. These jobs have become an increasingly important part of the U.S. labor market (Autor and Dorn, 2013). We follow Autor and Dorn (2013) in modeling the production of services using a linear production function of non-college labor. We allow services to enter the utility function, splitting the log of utility from $\beta_{c,e} \log c$ into $\beta_{cs,e} \log c_s + \beta_{cm,e} \log c_m$, with $\beta_{cs,e} + \beta_{cm,e} = \beta_{c,e}$. Here, c_s denotes consumption of services, and c_m denotes consumption of manufactured goods, which firms in our baseline model produce. Households consume only services that other households in their city supply. Equilibrium pins down the local price of services, the wage paid to labor from L households, and the allocation of this labor across the service and manufacturing sectors in each city.

To calibrate this extended model, we set $\beta_{cs,L} = 0$, meaning that only college households consume low-skilled services. This assumption gives us the greatest chance of attenuating the negative effects of construction on college households. To estimate $\beta_{cs,H}$, we calculate the total wage and salary earnings of non-college workers in service occupations our data. The ratio of this total to the aggregate income less housing costs of college households equals $\beta_{cs,H}$, which we calculate as 8.2%.

The results appear in column (4). As expected, construction harms college households less than in the baseline. The in-migration of poor non-college households lowers the price of services, which makes college households better off. However, the aggregate effect of low-quality construction still makes rich college households worse off. The extent of service consumption is not large enough to attenuate the baseline effects.

Finally, column (5) reports results where we turn off cross-city migration. As Proposition 4 states, construction now makes all households better off, and all households prefer high-quality construction to low-quality construction. We further learn that all households are better off than they are in the baseline. For instance, welfare of poor non-college households improves by more than 6%. Migration significantly attenuates the welfare gains from construction, turning them

conservatively round the number down slightly to 0.05.

negative in some cases.

A final extension, which we do not report in the table, is to include property tax revenue. A local government spends this revenue on a public good that enters the utility function. In this extension, construction at the 80th percentile fails to raise property tax revenue—although there is more housing to tax, house prices fall by so much that total revenue falls. Construction in only the top seven bins raises tax revenue.

6.3 Addressing the affordability crisis

House prices have grown so much in certain cities because productivity has risen while construction has been low (Hsieh and Moretti, 2018). We study a productivity shock that raises house prices in our model, asking how effective different policies are in addressing welfare losses.

6.3.1 Effects of a skill-biased productivity shock

The exogenous productivity shifters for L and H labor are $\delta_{A,L}$ and $\delta_{A,H}$. Diamond (2016) estimates that, for the period between 1980 and 2000, the values of these shifters are -0.314 and 0.075 for the Boston metropolitan area (see Table A.6 of her online appendix). According to these estimates, the Boston production function changed to make high-education labor more productive and low-education labor less. We annualize these numbers by setting $\delta_{A,L} = -0.0157$ and $\delta_{A,H} = 0.0038$. During this period, real house prices in Boston doubled by growing at an annual rate of 3.78%.

Figure 4 plots the house price and population changes that occur in response to this shock. House prices sharply increase relative to their initial value, as Panel A shows. The largest gains occur in the lowest quality bins. House price growth during the 2000–2005 U.S. boom also followed this pattern (Guerrieri et al., 2013; Landvoigt et al., 2015). The remaining panels clarify why this pattern holds in our model.

Panel C shows the net measure of households moving into each bin from outside the city. The high bins accept an influx of college households, while non-college households move out of the low bins. Therefore, the shock generates an affordability crisis: a sharp rise in prices accompanying an out-migration of poor households without a college degree. The combined effect, shown in gray, is in-migration to high bins and out-migration from low bins. Given the out-migration from low bins, why do house prices increase most sharply there?

The answer comes from changes in demand between adjacent bins. Panel D plots the net switching into each bin from households already in the city before the shock. Because the housing market clears, the combined switching effect—appearing in gray—exactly cancels the combined migration effect in Panel C. In particular, college households exert demand on the low bins by switching down into them. To induce this switching, the slope of house prices with respect to

quality cannot increase too much, as switching households are using this slope to decide which bin to choose. Consistent with this idea, the growth in house price levels across bins is relatively constant, as Panel B shows. Only between the lowest bins does this slope increase much. The net demand to switch into these bins is very strong so the slope must rise more there.

The mechanism in Figure 4 resembles the endogenous gentrification that Guerrieri et al. (2013) study. In both cases, richer households drive up low-quality house prices by pushing out lower income households. The importance of switching between adjacent housing qualities is similar to phenomenon that Landvoigt et al. (2015) describe. However, they generate the pattern in Panel A with a housing demand shock that is strongest for poor households. In contrast, we generate the same pattern with a shock that is weakest for the poor.

6.3.2 Policy effectiveness

Table 4 analyzes the effectiveness of various policies in addressing the migratory responses to this shock. Panel A reports the population changes of non-college households in each income quartile, Panel B gives the same information for college households. Panel C summarizes house price and quantity changes from each policy.

Results in column (1) correspond to the case in Figure 4 when there is no policy response. Consistent with Panel C of that figure, the poor non-college population declines while the college population increases at all income levels. House prices rise by 5.78% on average, with a median increase of 4.14%. There is no change in the housing stock (by assumption).

Column (2) presents the case where construction matches what we observe in the data for 2015. Mathematically, we set $\delta_{h,j}$ equal to the measure of new housing in each bin that we observe in our data, and then solve the linear system in Section 3 using the aforementioned values of $\delta_{A,L}$ and $\delta_{A,H}$. This level of construction does curtail the out-migration of poor non-college households, but it does not come close to reducing it to zero. In the lowest income quartile, the population change is -2.78% versus -3.84% in the baseline. At the same time, this construction lowers the welfare of rich college households, curtailing their in-migration from 3.58% to 3.50% . This change is small, which may explain why local governments enacted this policy in reality, as households with more education are more active in the political process (Galston, 2001; Milligan et al., 2004). House price growth falls to 4.26% and 3.06% for the average and median, respectively. These values are close to the empirical house price growth of 3.78% in response to the shock.

How much construction is necessary to stem the out-migration of poor non-college households? Columns (3)–(5) answer this question in different ways. We search for a set of construction amounts, $\delta_{h,j}$, so that the combined effect of the productivity shock and construction makes no household in the city worse off. Such an outcome represents a Pareto improvement over the baseline. In particular, poor non-college households no longer leave the city.

Column (3) finds the minimal number of new housing units necessary to achieve this objective.

Mathematically, we find $\delta_{h,j}$ that minimizes $\sum_{j=1}^{50} \delta_{h,j}$ subject to the constraints that $\partial \log v_{e,t^*}(z) \geq 0$ for all households and that $\delta_{h,j} \geq 0$ for all j . As Panel C reports, this minimal construction expands the housing stock by 1.43%, more than triple the amount in the data. Furthermore, the quality of new housing is almost 17% lower in column (3) than column (2). We compute construction quality as the average price of new housing in the pre-shock equilibrium. The population of all eight household groups increases, which must hold because the shock and construction response makes no one worse off.

Should governments in the Boston metropolitan area change permitting rules to implement the construction in column (3)? Doing so would make poor non-college households better off, but would harm many households relative to the status quo in column (2). In particular, the welfare of households in the upper half of the income distribution would fall. Switching from actual construction to that in column (3) is not a Pareto improvement.

In column (4), we instead minimize the value of new housing using the pre-shock equilibrium prices, $\sum_{j=1}^{50} p_{j,t^*} \delta_{h,j}$. We think of this sum as proportional to the cost of constructing this housing. This minimand addresses the possibility that it is cheaper to supply lower quality housing. The “construction cost” column reports this sum as a fraction of the initial value of the housing stock, $\sum_{j=1}^{50} p_{j,t^*} h_{j,t^*}$. The construction in column (4) involves a cost of only 1.49% of the housing stock, which is lower than the 1.74% cost in column (3). The construction tilts even more toward lower quality housing, with a quality that is 29% lower than the 2015 amount. While cheaper to implement, this policy further lowers the welfare of households in the top income quartile.

Column (5) serves as a placebo check to columns (3) and (4). It requires that the quality distribution match that in column (2), while solving for the total quantity of construction that eliminates out-migration. The resulting optimum expands the housing stock by 1.71%, which is moderately more than in columns (3) and (4). The construction cost, at 2.53% of the housing stock, is much higher than the 1.49% number from column (4). The welfare losses relative to column (2) are much smaller. Therefore, while keeping the quality distribution involves higher construction costs, many households prefer that policy over one that involves building lower-quality units.

Finally, column (6) reports an analysis of rent control. We limit all prices changes to 2%: $\partial \log p_{j,t^*} \leq \log(1.02)$ for $j \in \{1, \dots, 50\}$. For housing bins where demand exceeds supply, we ration households randomly within each bin. To solve the system, we impose 2% growth of prices at certain indices and solve for the rationing at those indices and the price responses at other indices. Using iteration, we find a set of indices where demand exceeds supply at the rationed indices and price growth is less than 2% at the un-rationed indices. The rationed indices are the lowest 22 occupied housing bins.

Consistent with this price cap, price growth is much smaller in column (6) than under the baseline without construction or the 2015 construction policy. Both average and median price growth is between 1.5% and 2%. However, the policy fails to limit the out-migration of the poor. In fact, it accelerates it—the out-migrations in the bottom two quartiles are larger in column

(6) than in column (1). By capping price growth, rent control limits the growth in the slope of prices with respect to quality at the low end of the distribution. As a result, switching down the quality distribution is stronger. Consistent with this mechanism, the population of third quartile non-college households and top quartile college households grows under rent control. These are the households exerting the most downward pressure through switching in Panel D of Figure 4. Rent control makes these households better off even though it does not directly limit the growth of their house prices. Diamond et al. (2018) find empirically that rent control attracted higher income residents to San Francisco, which is consistent with these results.

7 Conclusion

To conclude, we return to the topic with which we began: what can stem the rapid growth of house prices in many cities? Like many economists, our analysis recommends expanding the stock of available housing. The novelty of our approach is to consider the quality of construction. We calculate that expanding the housing stock in Boston by 1.4% annually would eliminate welfare losses if the new units primarily are at the lower end of the housing market. This finding provides one rationale for cities to relax permitting rules specifically for low-quality construction. However, building such units makes high-education and high-income households worse off, which might explain why expensive cities have not already relaxed permitting rules for low-income housing enough to eliminate welfare lost by low-income, low-education households.

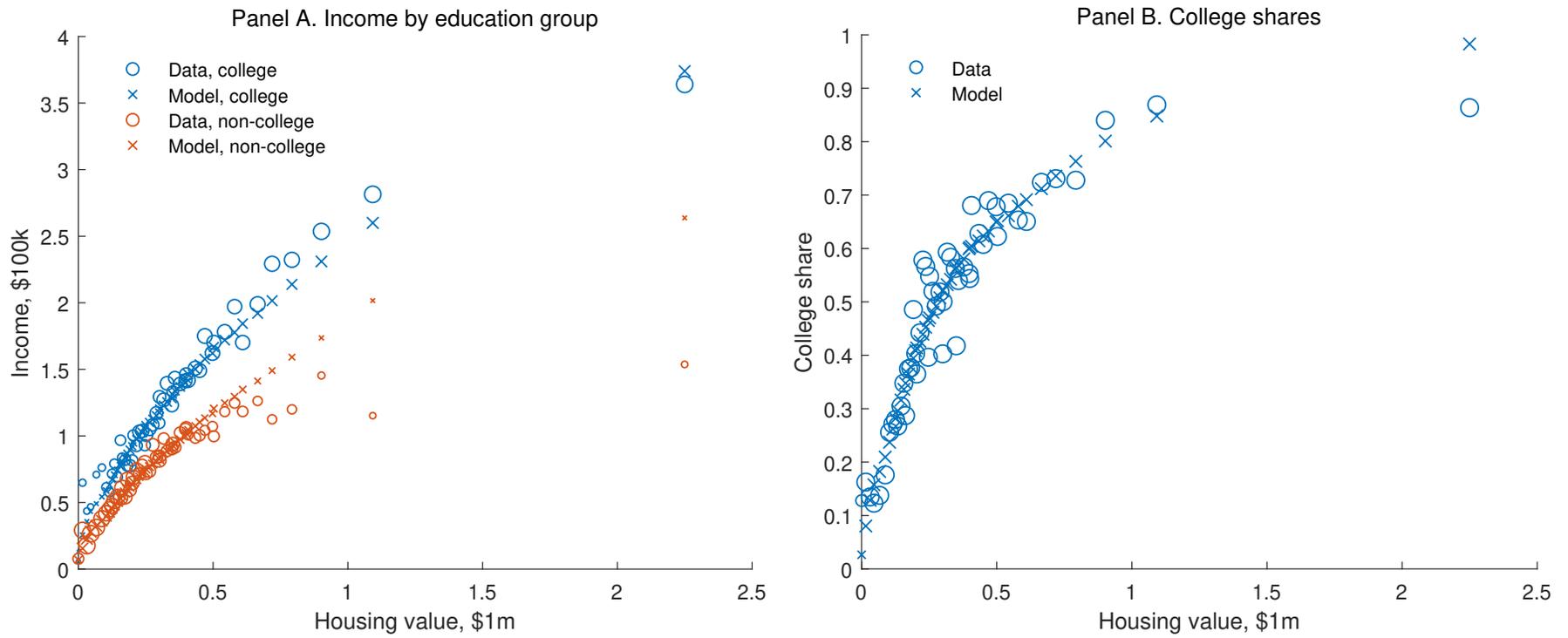
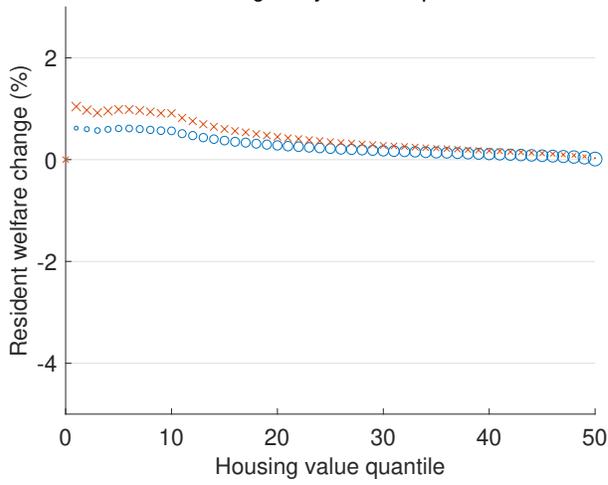


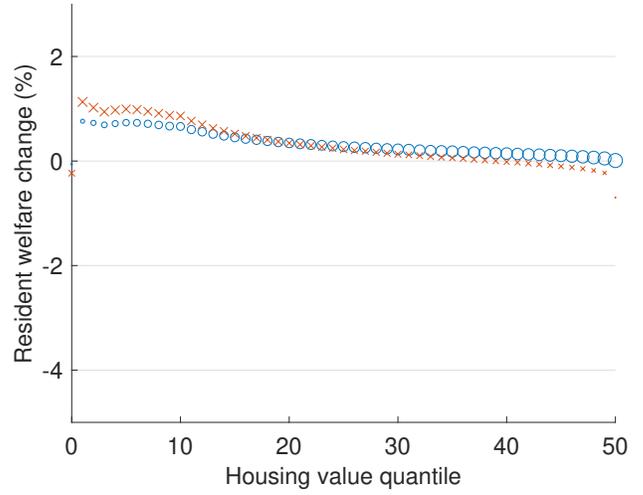
FIGURE 1. GOODNESS OF FIT

Notes: The two-dimensional size of each marker is proportional to the number of housing units in each housing value and education bin.

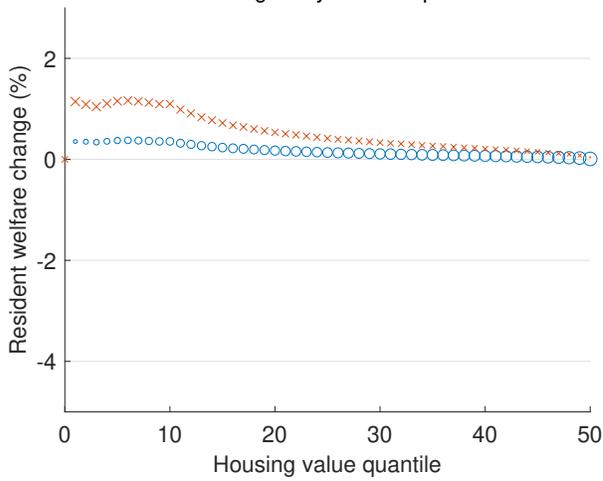
Panel A. Preference homogeneity without spillovers



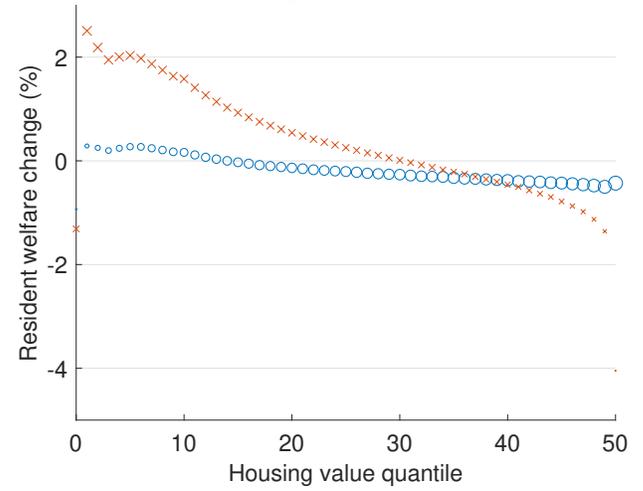
Panel B. Preference homogeneity with spillovers



Panel C. Preference heterogeneity without spillovers



Panel D. Preference heterogeneity with spillovers



○ College × Non-college

FIGURE 2. EFFECT OF 20TH PERCENTILE CONSTRUCTION ON RESIDENT WELFARE

Notes: The size of each marker is proportional to the number of housing units in each housing value and education bin.

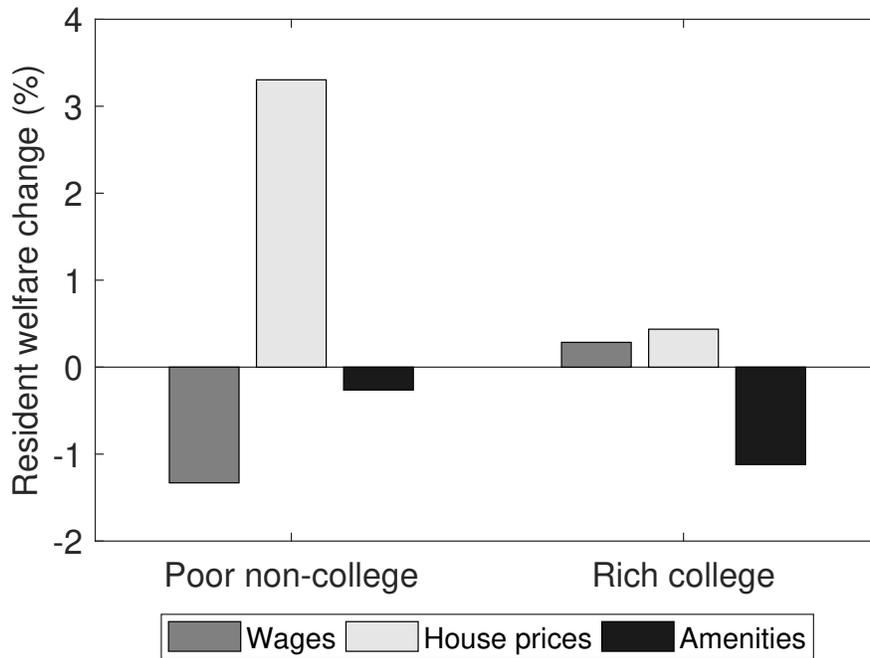


FIGURE 3. COMPONENTS OF WELFARE EFFECT OF 20TH PERCENTILE CONSTRUCTION

Notes: "Poor" and "rich" denote the bottom and top quartiles of the city income distribution, respectively.

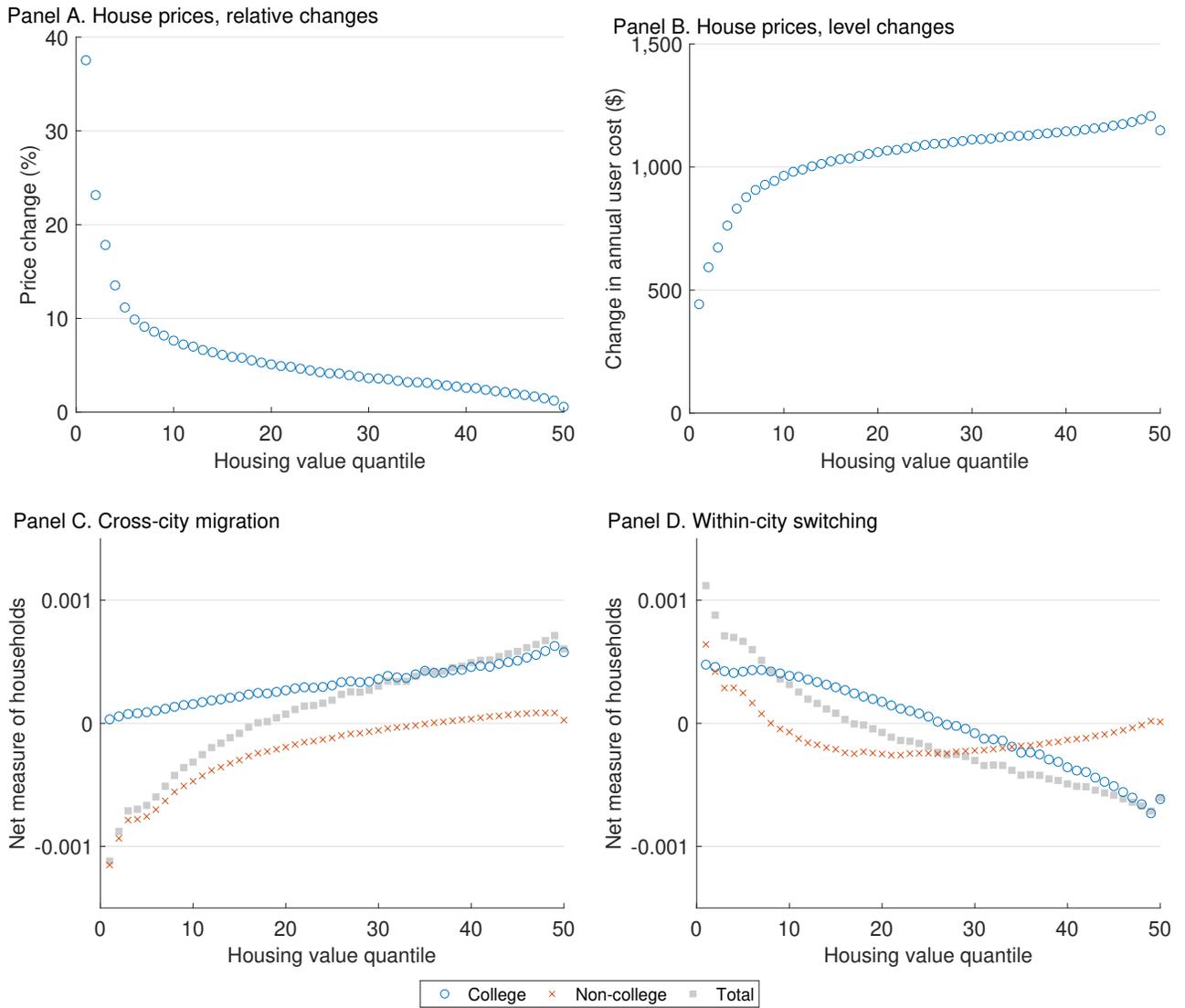


FIGURE 4. EFFECT OF SKILL-BIASED PRODUCTIVITY SHOCK ON HOUSING SUBMARKETS

Notes: Panels C and D plot the change in demand for each house type from outside and inside the city, respectively.

TABLE 1
SUMMARY STATISTICS FOR BOSTON-CAMBRIDGE-NEWTON, MA-NH METROPOLITAN AREA IN 2016

	Group quarters	Renters	Owners
<i>Panel A: All observations</i>			
Present in estimation sample	0.091	0.968	1
Weighted observations	170,602	701,301	1,120,455
Unweighted observations	3,223	6,156	11,893
<i>Panel B: Estimation sample</i>			
Ownership status	-1	0	1
Income	\$7,531	\$64,961	\$138,223
Education	0.128	0.397	0.554
Rent	-	\$1,284	-
Rent censoring	-	0.019	-
Home value	-	-	\$491,724
Home value censoring	-	-	0.009
New construction	-	0.005	0.004
Weighted observations	15,486	678,917	1,120,455
Unweighted observations	430	5,946	11,893

TABLE 2
ESTIMATES OF MODEL PARAMETERS

	Low education	High education
Panel A: Parameters differing by education group		
$N_{e,t^*}/N_{t^*}$	0.509	0.491
$Y_{e,t^*}/Y_{t^*}$	0.334	0.666
$\beta_{w,e}/\beta_{\epsilon,e}$	4.398	2.078
$\beta_{c,e}/\beta_{\epsilon,e}$	3.280	1.713
$\beta_{a,e}/\beta_{\epsilon,e}$	0.245	1.047
Panel B: Parameters not differing by education group		
ζ		1.603
γ_a		1.103
γ_N		0.055
γ_H		0.098

TABLE 3
EFFECT OF CONSTRUCTION ON RESIDENT WELFARE (%)

	Non- college	Col- lege	Non- college	Col- lege	Non- college	Col- lege	Non- college	Col- lege	Non- college	Col- lege
	Baseline (1)		Exogenous amenities (2)		Neighborhood amenities (3)		Local services (4)		Closed city (5)	
<i>Panel A. Construction at 20th percentile</i>										
Income quartile										
1	1.66	0.24	0.95	0.49	1.94	-0.45	1.60	0.27	6.07	2.26
2	0.40	0.06	0.34	0.41	0.30	-0.21	0.39	0.10	3.10	1.50
3	-0.38	-0.22	0.02	0.25	-0.48	-0.41	-0.35	-0.17	1.83	0.90
4	-1.03	-0.41	-0.19	0.16	-1.67	-0.53	-0.97	-0.35	1.11	0.46
<i>Panel B. Construction at 80th percentile</i>										
Income quartile										
1	0.93	0.21	0.54	0.35	1.39	-0.30	0.88	0.24	6.12	2.26
2	0.49	0.15	0.46	0.34	0.43	-0.15	0.48	0.17	3.66	1.56
3	0.27	0.08	0.50	0.34	0.12	-0.17	0.29	0.11	2.69	1.10
4	-0.33	-0.00	0.14	0.31	-1.10	-0.21	-0.29	0.03	1.75	0.70

TABLE 4
EFFECTS OF SKILL-BIASED PRODUCTIVITY SHOCK UNDER DIFFERENT POLICIES (%)

	Construction policies					
	None (1)	2015 (2)	Unit- minimizing optimum (3)	Cost- minimizing optimum (4)	2015 quality optimum (5)	Rent control (6)
<i>Panel A. Non-college population change</i>						
Income quartile						
1	-3.84	-2.78	0.09	0.09	0.26	-4.23
2	-1.43	-0.97	0.02	0.02	0.32	-1.88
3	0.32	0.38	0.03	0.02	0.56	0.55
4	1.89	1.62	0.17	0.05	0.87	0.93
<i>Panel B. College population change</i>						
Income quartile						
1	2.29	2.50	2.93	2.98	3.09	-1.17
2	2.71	2.81	2.89	2.95	3.07	-1.51
3	3.21	3.18	2.88	2.94	3.12	2.90
4	3.58	3.50	2.98	2.94	3.28	4.06
<i>Panel C. Housing market changes</i>						
Average price	5.78	4.26	0.53	0.50	0.13	1.76
Median price	4.14	3.06	0.48	0.44	0.11	1.96
Housing units	0.00	0.45	1.43	1.43	1.71	0.00
Construction cost	0.00	0.66	1.74	1.49	2.53	0.00
Construction quality	-	0.00	-16.95	-29.25	0.00	-

Notes: "Construction cost" gives the total value of new housing as a share of the value of existing housing, using pre-shock prices. "Construction quality" gives the average value of new housing relative to the average value of new housing in 2015, using pre-shock prices.

A Supplements to Section 2

A.1 Proof of Lemma 1

For a contradiction, suppose (a) fails, so that $v_{e,t}(z) = 0$ for all $t \in \{1, \dots, T\}$ for some $e \in \{L, H\}$ and $z > 0$. This situation may hold only in the indivisible and requires that $w_{e,t}z \leq \min(p_{0,t}, \dots, p_{J,t})$ for all $t \in \{1, \dots, T\}$. By Assumption 1(b), there exists households of education e with labor endowment $z' < z$. For such households, $w_{e,t}z' < w_{e,t}z \leq \min(p_{0,t}, \dots, p_{J,t})$ for all $t \in \{1, \dots, T\}$, contradicting the budget constraint for such households and proving (a).

We next prove (14). Consider households of education e with endowment z . No such household chooses t if $v_{e,t}(z)\exp(\beta_{\epsilon,e}\epsilon_t) < v_{e,t'}(z)\exp(\beta_{\epsilon,e}\epsilon_{t'})$ for any $t' \neq t$. If $v_{e,t}(z) = 0$, then this inequality holds by part (a), so $n_{e,t}(z) = 0$. Such households choose city t if $v_{e,t}(z) > 0$ and $\epsilon_t - \epsilon_{t'} > \beta_{\epsilon,e}^{-1} \log v_{e,t}(z') - \beta_{\epsilon,e}^{-1} \log v_{e,t}(z)$ for all $t' \neq t$ such that $v_{e,t'}(z) > 0$. Train (2009), on pages 36 and 74–75, shows that the probability that independent draws from a Gumbel distribution satisfy this inequality is $v_{e,t}(z)^{\beta_{\epsilon,e}^{-1}} / \sum_{t' | v_{e,t'}(z) > 0} v_{e,t'}(z)^{\beta_{\epsilon,e}^{-1}}$. Because the measure of such households in the economy is $\tilde{n}_e(z)$, (14) follows.

To finish proving (b), we note that the denominator of (14) is always positive and that $v_{e,t}(z)^{\beta_{\epsilon,e}^{-1}}$ is either zero or positive. Therefore, $n_{e,t}$ can have a mass point only if \tilde{n}_e does, which Assumption 1(c) rules out, so $n_{e,t}$ is atomless. Consider $z' < z''$ such that $n_{e,t}(z'), n_{e,t}(z'') > 0$. It follows that $\tilde{n}_e(z'), \tilde{n}_e(z''), v_{e,t}(z') > 0$. If $z \in (z', z'')$, then $\tilde{n}_e(z) > 0$ by Assumption 1(a). Furthermore, $v_{e,t}(z) > v_{e,t}(z')$ because $w_{e,t}z > w_{e,t}z'$, meaning that a household with endowment z can choose the same housing bundle and greater non-housing consumption as a household with endowment z' . It follows that $n_{e,t}(z) > 0$, implying that the support of $n_{e,t}$ is convex.

We now prove (c). In equilibrium, $N_{e,t} > 0$ (by definition), so $z_{e,t}^{lb} = \inf\{z \mid n_{e,t}(z) > 0\}$ exists. By Assumption 1(b), $\inf\{z \mid \tilde{n}_e(z) > 0\} = 0$. By (14), $n_{e,t}(z) > 0$ if $v_{e,t}(z) > 0$ and $\tilde{n}_e(z) > 0$. It follows that $z_{e,t}^{lb} = \inf\{z \mid v_{e,t}(z) > 0\}$, which equals $z_{e,t}^{lb} = 0$ in the divisible case and $z_{e,t}^{lb} = w_{e,t}^{-1} \min(p_{0,t}, \dots, p_{J,t})$ in the indivisible case. Consider $\delta_z > 0$. When $z = z_{e,t}^{lb} + \delta_z$, the maximal value of c that satisfies the household constraints is $w_{e,t}\delta_z$, so as $\delta_z \rightarrow 0$, $v_{e,t}(z_{e,t}^{lb} + \delta_z) \rightarrow 0$. It follows that $\inf\{v_{e,t}(z) \mid n_{e,t}(z) > 0\} = \inf\{v_{e,t}(z) \mid v_{e,t}(z) > 0\} = 0$.

To prove (d), we denote $V_{e,t}(z) = \beta_{\epsilon,e}^{-1} \log v_{e,t}(z)$. Using the simplifications on page 74 of Train (2009), we have

$$E(\epsilon_t \mid e, z, t) = \frac{\int_{-\infty}^{\infty} \epsilon_t \exp\left(-\exp(-\epsilon_t) \sum_{t' | v_{e,t'}(z) > 0} \exp(V_{e,t'}(z) - V_{e,t}(z))\right) \exp(-\epsilon_t) d\epsilon_t}{\int_{-\infty}^{\infty} \exp\left(-\exp(-\epsilon_t) \sum_{t' | v_{e,t'}(z) > 0} \exp(V_{e,t'}(z) - V_{e,t}(z))\right) \exp(-\epsilon_t) d\epsilon_t}. \quad (\text{A1})$$

Changing variables to $\tau = \exp(-\epsilon_t)$ and simplifying, as on page 75 of Train (2009), gives

$$E(\epsilon_t \mid e, z, t) = -\frac{\int_0^{\infty} \log(\tau) \exp\left(-\tau \sum_{t' | v_{e,t'}(z) > 0} \exp(V_{e,t'}(z) - V_{e,t}(z))\right) d\tau}{\int_0^{\infty} \exp\left(-\tau \sum_{t' | v_{e,t'}(z) > 0} \exp(V_{e,t'}(z) - V_{e,t}(z))\right) d\tau}. \quad (\text{A2})$$

Evaluating these integrals yields $E(\epsilon_t \mid e, z, t) = \gamma - V_{e,t} + \log \sum_{t' | v_{e,t'}(z) > 0} \exp(V_{e,t'}(z))$. It follows that $\log \bar{v}_{e,t}(z) = \beta_{\epsilon,e} V_{e,t}(z) + \beta_{\epsilon,e} E(\epsilon_t \mid e, z, t) = \beta_{\epsilon,e} \gamma + \beta_{\epsilon,e} \log \sum_{t'} v_{e,t'}(z)^{\beta_{\epsilon,e}^{-1}}$. Exponentiating proves (d).

A.2 Proof of Proposition 1

The text above Proposition 1 proves that $p_{j,t} = \mu_t q_{j,t}$ for each $j \in \mathcal{J}_t$. Using this result, we re-write the budget constraint, (3), as $c + \mu_t q \leq w_{e,t} z$. Given t , maximizing (1) is equivalent to maximizing the Cobb-Douglas utility function $c^{1-\alpha} q^\alpha$ given income $w_{e,t} z$ and prices one and μ_t for c and q , where $\alpha = \beta_{q,e}(\beta_{c,e} + \beta_{q,e})^{-1}$. If $\mu_t \leq 0$, then households cannot optimize, so $\mu_t > 0$ in equilibrium. As shown on pages 55–56 of Mas-Collel et al. (1995), the optimal consumption bundle in this environment consists of expending a share α of income on q , so that $q = \beta_{q,e}(\beta_{c,e} + \beta_{q,e})^{-1} \mu_t^{-1} w_{e,t} z$. Given that $p_{j,t} > 0$ for all $j \in \mathcal{J}_t$, renters maximize utility by selling all of their housing. Equating the total housing quality renters sell to the total quality households demand produces (16).

A.3 Proof of Proposition 2

In equilibrium, the housing market clears, so the measure of households must be no greater than the total measure of housing renters may sell: $\sum_{j \in \mathcal{J}_t} h_{j,t} \geq N_t$. It follows that $j_{0,t}$ exists.

We next prove the statements about housing demand. Because renters optimize, they sell $h_{j,t}$ units of housing of quality $q_{j,t}$ when $p_{j,t} > 0$, they sell any amount of such housing when $p_{j,t} = 0$, and $p_{j,t} < 0$ is impossible. Consider $j < j_{0,t}$ such that $x_{j,t} > 0$. Because $\sum_{j'=j_{0,t}}^{j_t} h_{j',t} \geq N_t$, there exists $j' \geq j_{0,t}$ such that $x_{j',t} < h_{j',t}$. It follows that $p_{j',t} = 0$, which implies that households choosing $q_{j,t}$ are failing to optimize because $q_{j',t} > q_{j,t}$ and $0 = p_{j',t} \leq p_{j,t}$. This contradiction implies that no such j exist, so $x_{j,t} = 0$ when $j < j_{0,t}$. Similarly, consider j such that $j > j_{0,t}$ and $x_{j,t} < h_{j,t}$. Then $p_{j,t} = 0$. Furthermore, because $\sum_{j'=j_{0,t}+1}^{j_t} h_{j',t} < N_t$, there exists $j' \leq j_{0,t}$ such that $x_{j',t} > 0$. Households choosing $q_{j',t}$ cannot be optimizing, as $p_{j,t} \leq p_{j',t}$ and $q_{j,t} > q_{j',t}$. This contradiction implies that there are no such j , proving that $x_{j,t} = h_{j,t}$ when $j > j_{0,t}$.

To prove the statements about house prices, we first consider j such that $\sum_{j'=j}^{j_t} h_{j',t} > N_t$ and $h_{j,t} > 0$. By the definition of $j_{0,t}$, $j \leq j_{0,t}$. If $j < j_{0,t}$, then $0 = x_{j,t} < h_{j,t}$, so $p_{j,t} = 0$. If $j = j_{0,t}$, then $x_{j,t} = N_t - \sum_{j'=j_{0,t}+1}^{j_t} h_{j',t} < h_{j_{0,t},t} = h_{j,t}$, again implying $p_{j,t} = 0$. Next, consider j, j' such that $h_{j,t}, h_{j',t} > 0$ and $j_{0,t} \leq j < j'$. Because $x_{j,t} = h_{j,t} > 0$, there exist households choosing $q_{j,t}$. If $p_{j,t} \geq p_{j',t}$, these households are failing to optimize because $q_{j',t} > q_{j,t}$. It follows that $p_{j,t} < p_{j',t}$.

We prove the final statement by contradiction. Consider two households with education e and respective labor endowments $z < z'$ and quality choices $q_{j,t} > q_{j',t}$. Optimality for each household implies the inequalities $(w_{e,t} z - p_{j,t})^{\beta_{c,e}} q_{j,t}^{\beta_{q,e}} \geq (w_{e,t} z - p_{j',t})^{\beta_{c,e}} q_{j',t}^{\beta_{q,e}}$ and $(w_{e,t} z' - p_{j',t})^{\beta_{c,e}} q_{j',t}^{\beta_{q,e}} \geq (w_{e,t} z' - p_{j,t})^{\beta_{c,e}} q_{j,t}^{\beta_{q,e}}$. If $w_{e,t} z = p_{j,t}$, then the first optimality inequality is violated because $p_{j',t} < p_{j,t}$, which implies that $w_{e,t} z - p_{j',t} > w_{e,t} z - p_{j,t} = 0$. By the budget constraint, $w_{e,t} z \geq p_{j,t}$, so we must have $w_{e,t} z > p_{j,t}$. Because $z < z'$, $w_{e,t} z < w_{e,t} z'$, so $w_{e,t} z' - p_{j,t} > w_{e,t} z - p_{j,t} > 0$. We may therefore rearrange the optimality inequalities to produce

$$\frac{w_{e,t} z - p_{j',t}}{w_{e,t} z - p_{j,t}} \leq \left(\frac{q_{j,t}}{q_{j',t}} \right)^{\frac{\beta_{q,e}}{\beta_{c,e}}} \leq \frac{w_{e,t} z' - p_{j',t}}{w_{e,t} z' - p_{j,t}}. \quad (\text{A3})$$

Cross-multiplying the outer terms yields the contradiction $(p_{j,t} - p_{j',t})(z' - z) \leq 0$.

A.4 Proof of Lemma 2

Denote by $p_t = (p_{0,t}, \dots, p_{J_t,t})$ a vector of potential house prices in city t . Define $\mathcal{P}_t = \{p_t \mid 0 \leq p_{0,t} \leq \dots \leq p_{J_t,t}\}$. We derive housing demand as a function of $p_t \in \mathcal{P}_t$.

For $j, j' \in \mathcal{J}_t$ distinct, a household of education e and endowment z prefers and can afford $q_{j,t}$ over $q_{j',t}$ only if two conditions hold. First, $w_{e,t}z \geq p_{j,t}$. Second, either $w_{e,t}z < p_{j',t}$ or both $w_{e,t}z \geq p_{j',t}$ and

$$(w_{e,t}z - p_{j,t})^{\beta_{c,e}} q_{j,t}^{\beta_{q,e}} \geq (w_{e,t}z - p_{j',t})^{\beta_{c,e}} q_{j',t}^{\beta_{q,e}}, \quad (\text{A4})$$

with strict preference in the case of strict inequality. The point where (A4) becomes an equality is

$$z_{e,j,j',t}(p_t) = \frac{q_{j,t}^{\frac{\beta_{q,e}}{\beta_{c,e}}} p_{j,t} - q_{j',t}^{\frac{\beta_{q,e}}{\beta_{c,e}}} p_{j',t}}{\left(q_{j,t}^{\frac{\beta_{q,e}}{\beta_{c,e}}} - q_{j',t}^{\frac{\beta_{q,e}}{\beta_{c,e}}} \right) w_{e,t}}. \quad (\text{A5})$$

If $j > j'$, then (A4) holds if and only if $z \geq z_{e,j,j',t}(p_t)$. Furthermore, because $p_{j',t} \leq p_{j,t}$, $z_{e,j,j',t}(p_t) \geq w_{e,t}^{-1} p_{j,t} \geq w_{e,t}^{-1} p_{j',t}$. Therefore, if $j > j'$, the household prefers and can afford $q_{j,t}$ over $q_{j',t}$ only if $z_{e,j,j',t}(p_t)$, with strict preference when $z > z_{e,j,j',t}(p_t)$. By symmetry, when $j < j'$, these preference relations hold when $z \leq z_{e,j',j,t}(p_t) = z_{e,j,j',t}(p_t)$, strictly with strict inequality. Let $z_{e,j,t}^{\min}(p_t) = \max\{z_{e,j,j',t}(p_t) \mid j > j'\}$ for $j > 0$ and zero for $j = 0$. Let $z_{e,j,t}^{\max}(p_t) = \min\{z_{e,j,j',t}(p_t) \mid j < j'\}$ for $j < J_t$ and infinity for $j = J_t$. Demand for $q_{j,t}$ equals

$$x_{j,t}(p_t) = \sum_{e \in \{L,H\}} x_{e,j,t}(p_t), \quad (\text{A6})$$

where

$$x_{e,j,t}(p_t) = \int_{z_{e,j,t}^{\min}(p_t)}^{z_{e,j,t}^{\max}(p_t)} n_{e,t}(z) dz. \quad (\text{A7})$$

The following lemma collects useful facts about demand.

Lemma A1. *For each $j \in \mathcal{J}_t$, $x_{j,t}$ is continuous. Let $p_t, p'_t \in \mathcal{P}_t$. If $p'_{j,t} \geq p_{j,t}$ and $p'_{j',t} \leq p_{j',t}$ for $j' \neq j$, then $x_{j,t}(p'_t) \leq x_{j,t}(p_t)$, with strict inequality if $p'_{j,t} < p_{j,t}$ and $x_{j,t}(p_t) > 0$. Finally,*

$$\bar{p}_t = \inf \left\{ \tilde{p} \geq 0 \mid \sum_{e \in \{L,H\}} \int_{w_{e,t}^{-1} \tilde{p}}^{\infty} n_{e,t}(z) dz < h_{J_t,t} \right\} \quad (\text{A8})$$

exists, and if $p_{J_t+1,t} = \bar{p}_t$, then $x_{j,t}(p_t) \leq h_{j,t}$ if $p_{j,t} = p_{j+1,t}$.

Proof. We prove the first two sentences for each $x_{e,j,t}$; they then hold for $x_{j,t}$ immediately. Both $z_{e,j,t}^{\min}$ and $z_{e,j,t}^{\max}$ (the latter only for $j < J_t$), are continuous functions of p_t . Because $n_{e,t}$ is continuous by assumption, and because the composition of continuous functions is continuous, the fundamental theorem of calculus implies that $x_{e,j,t}$ is continuous.

From (A5), $z_{e,j,j',t}(p_t)$ strictly increases in $p_{j,t}$ and decreases in $p_{j',t}$ if $j > j'$ and strictly decreases in $p_{j,t}$ and increases in $p_{j',t}$ if $j < j'$. Therefore, $z_{e,j,t}^{\min}(p'_t) \geq z_{e,j,t}^{\min}(p_t)$ and $z_{e,j,t}^{\max}(p'_t) \leq z_{e,j,t}^{\max}(p_t)$. It

follows that $x_{e,j,t}(p'_t) \leq x_{e,j,t}(p_t)$. Now suppose that $x_{e,j,t}(p_t) > 0$ and that $p'_{j,t} < p_{j,t}$. We claim that $0 \leq z_{e,j,t}^{\min}(p_t) < \sup\{z \mid n_{e,t}(z) > 0\}$. The first inequality follows because for $j > j'$, $z_{e,j,j',t}(p_t) \geq w_{e,t}^{-1}p_{j,t} \geq 0$. If the second inequality fails, then $[z_{e,j,t}^{\min}(p_t), z_{e,j,t}^{\max}(p_t)] \cap \{z \mid n_{e,t}(z) > 0\}$ is either \emptyset or $\{\sup\{z \mid n_{e,t}(z) > 0\}\}$, both of which have measure zero under $n_{e,t}$, contradicting $x_{e,j,t}(p_t) > 0$. Then $\inf\{z \mid n_{e,t}(z) > 0\} = 0 \leq z_{e,j,t}^{\min}(p_t) < \sup\{z \mid n_{e,t}(z) > 0\}$. Therefore, because $z_{e,j,t}^{\min}(p_t) < z_{e,j,t}^{\min}(p'_t)$, the convexity of the support of $n_{e,t}$ implies that $x_{e,j,t}(p'_t) < x_{e,j,t}(p_t)$, as desired.

Let $j < J_t$. If $p_{j,t} = p_{j+1,t}$, then $z_{e,j,j+1,t}(p_t) = w_{e,t}^{-1}p_{j,t}$. A household of education can afford $q_{j,t}$ only if $z \geq w_{e,t}^{-1}p_{j,t}$ and prefers $q_{j+1,t}$ over $q_{j,t}$ only if $z \leq z_{e,j,j+1,t}(p_t) = w_{e,t}^{-1}p_{j,t}$. Therefore, only those with $z = w_{e,t}^{-1}p_{j,t}$ may choose $q_{j,t}$, and the measure of such households equals zero, so $x_{j,t}(p_t) = 0 \leq h_{j,t}$. We turn to the case $j = J_t$. The limit of the integral in (A8) as $\bar{p} \rightarrow \infty$ equals zero, so \bar{p}_t exists. A household chooses $q_{J_t,t}$ only if $z \geq w_{e,t}p_{J_t,t}$, so the summation in (A8) provides an upper bound on $x_{J_t,t}(p_t)$ when $\bar{p} = p_{j,t}$. Therefore, $x_{J_t,t}(p_t) \leq h_{J_t,t}$ when $p_{J_t,t} \leq \bar{p}_t = p_{J_t+1,t}$. \square

We next prove local equilibrium existence by constructing a sequence $p_t^i \in \mathcal{P}_t$ whose limit provides a local equilibrium. For all $i \geq 0$, set $p_{J_t+1,t} = \bar{p}_t$. We set $p_t^0 = 0$. If $i + j \equiv 0 \pmod{J_t + 1}$ and $x_{j,t}(p_t^i) > h_{j,t}$, then $p_{j,t}^i$ is the unique solution to $h_{j,t} = x_{j,t}(p_{j,t}^i; p_{-j,t}^{i-1})$. Uniqueness and existence follow from the intermediate value theorem and Lemma A1 because $x_{j,t}(p_{j,t}^{i-1}; p_{-j,t}^{i-1}) > h_{j,t} \geq x_{j,t}(p_{j+1,t}^{i-1}; p_{-j,t}^{i-1})$ and $h_{j,t} > 0$. Otherwise, we set $p_{j,t}^i = p_{j,t}^{i-1}$. At each step, $p_{j,t}^i \in [p_{j,t}^{i-1}, p_{j+1,t}^{i-1}]$, so $p_t^i \in \mathcal{P}_t$ for all $i \geq 0$. Furthermore, $p_{j,t}^i$ weakly increases in i for each $j \in \{0, \dots, J_t\}$, and $p_{j,t}^i \leq p_{j+1,t}^i = \bar{p}$ for all such j and all $i \geq 0$. By the monotone converge theorem, p_t^i converges. We denote the limit p_t^* .

Consider j such that $p_{j,t}^* > 0$. Let i_j be the first i such that $p_{j,t}^i > 0$. We claim that $x_{j,t}(p_t^i) \geq h_{j,t}$ for $i \geq i_j$. When $i = i_j$, $p_{j,t}^i > p_{j,t}^{i-1} = 0$, so $x_{j,t}(p_t^i) = h_{j,t}$. We proceed by induction. For each $i > i_j$, $p_{j',t}^i \geq p_{j',t}^{i-1}$ for $j' \neq j$. If $p_{j,t}^i = p_{j,t}^{i-1}$, then by Lemma A1, $x_{j,t}(p_t^i) \geq x_{j,t}(p_t^{i-1}) \geq h_{j,t}$. If $p_{j,t}^i > p_{j,t}^{i-1}$, then $x_{j,t}(p_t^i) = h_{j,t}$. Therefore, $x_{j,t}(p_t^i) \geq h_{j,t}$ for all $i \geq i_j$, as claimed. It follows that $x_{j,t}(p_t^i) = h_{j,t}$ for all $i \equiv -j \pmod{J_t + 1}$. Because $x_{j,t}$ is continuous by Lemma A1, $x_{j,t}(p_t^i)$ converges because p_t^i does. This limit must equal $h_{j,t}$ because it appears infinitely often in the sequence. Therefore, $x_{j,t}(p_t^*) = \lim_{i \rightarrow \infty} x_{j,t}(p_t^i) = h_{j,t}$.

Consider j such that $p_{j,t}^* = 0$. For all $i > 0$ such that $i \equiv -j \pmod{J_t + 1}$, $x_{j,t}(p_t^i) \leq h_{j,t}$. Because $x_{j,t}$ is continuous by Lemma A1, $x_{j,t}(p_t^i)$ converges. This limit cannot exceed $h_{j,t}$ because $x_{j,t}(p_t^i) \leq h_{j,t}$ for infinitely many i . Therefore, $x_{j,t}(p_t^*) = \lim_{i \rightarrow \infty} x_{j,t}(p_t^i) \leq h_{j,t}$.

Given that $w_{L,t}$ and $w_{H,t}$ clear the labor market when (12) holds, local equilibrium holds if and only if prices clear the housing market. Rentiers sell all housing at positive prices and any amount of housing at zero prices, so the market clears at p_t^* if and only if $x_{j,t}(p_t^*) = h_{j,t}$ when $p_{j,t}^* > 0$ and $x_{j,t}(p_t^*) \leq h_{j,t}$ when $p_{j,t}^* = 0$. Therefore, p_t^* provides local equilibrium house prices.

We finally prove that this local equilibrium is unique. Consider two local equilibrium price vectors p_t and p'_t . For a contradiction, suppose that $p_t \neq p'_t$. By Proposition 2 and (19), $p_{j,t} = p'_{j,t} = 0$ for $j \leq j_{0,t}$, $p_{j,t}, p'_{j,t} > 0$ for $j > j_{0,t}$, and $p_t, p'_t \in \mathcal{P}_t$. Furthermore, for $j > j_{0,t}$, $x_{j,t}(p_t) = x_{j,t}(p'_t) = h_{j,t}$. Without loss of generality, suppose that $p_{j,t} < p'_{j,t}$ for some $j > j_{0,t}$. Let $\mathcal{J}'_t = \{j \in \{0, \dots, J_t\} \mid p_{j,t} < p'_{j,t}\}$. Given p_t , a household of education e and endowment z prefers and can afford $q_{j,t}$ over all $q_{j',t}$, where $j \in \mathcal{J}'_t$ and $j' \notin \mathcal{J}'_t$, only if $\max\{z_{e,j,j',t}(p_t) \mid j > j' \notin \mathcal{J}'_t\} \leq z \leq \min\{z_{e,j,j',t}(p_t) \mid j < j' \in \mathcal{J}'_t\}$.

Therefore, total demand for qualities $q_{j,t}$, where $j \in \mathcal{J}'_t$, equals

$$\sum_{j \in \mathcal{J}'_t} h_{j,t} = \sum_{j \in \mathcal{J}'_t} x_{j,t}(p_t) = \sum_{e \in \{L,H\}} \int_{\mathcal{Z}_{e,t}(p_t)} n_{e,t}(z) dz, \quad (\text{A9})$$

where

$$\mathcal{Z}_{e,t}(p_t) = \bigcup_{j \in \mathcal{J}'_t} \left[\max \{z_{e,j,j',t}(p_t) \mid j > j' \notin \mathcal{J}'_t\}, \min \{z_{e,j,j',t}(p_t) \mid j < j' \notin \mathcal{J}'_t\} \right]. \quad (\text{A10})$$

We define $\mathcal{Z}_{e,t}(p'_t)$ similarly. Because $p_{j,t} < p'_{j,t}$ for $j \in \mathcal{J}'_t$ and $p_{j,t} \geq p'_{j,t}$ for $j \notin \mathcal{J}'_t$, $z_{e,j,j',t}(p_t) < z_{e,j,j',t}(p'_t)$ if $\mathcal{J}'_t \ni j > j' \notin \mathcal{J}'_t$ and $z_{e,j,j',t}(p_t) > z_{e,j,j',t}(p'_t)$ if $\mathcal{J}'_t \ni j < j' \notin \mathcal{J}'_t$. Therefore, $\mathcal{Z}_{e,t}(p'_t) \subset \mathcal{Z}_{e,t}(p_t)$. Furthermore, $z_{e,j,j',t}(p'_t) > 0$ when $\mathcal{J}'_t \ni j > j'$ because $p'_{j',t} > 0$ and $p'_t \in \mathcal{P}_t$. It follows that $\min \mathcal{Z}_{e,t}(p_t) > \min \mathcal{Z}_{e,t}(p'_t) > 0$. Because the greatest lower bounds of the supports of $n_{L,t}$ and $n_{H,t}$ equal zero,

$$\int_{\mathcal{Z}_{e,t}(p'_t)} n_{e,t}(z) dz \leq \int_{\mathcal{Z}_{e,t}(p_t)} n_{e,t}(z) dz, \quad (\text{A11})$$

with strict inequality if $\mathcal{Z}_{e,t}(z) \neq \emptyset$, which holds for some e due to (A9). Therefore, by (A9), $\sum_{j \in \mathcal{J}'_t} h_{j,t} = \sum_{j \in \mathcal{J}'_t} x_{j,t}(p'_t) < \sum_{j \in \mathcal{J}'_t} x_{j,t}(p_t) = \sum_{j \in \mathcal{J}'_t} h_{j,t}$, a contradiction.

A.5 Solution to (19)–(21)

For each $j \in \{j_{0,t} + 1, \dots, J_t\}$, we rewrite (20) and (21) as

$$N_t - \sum_{j'=j}^{J_t} h_{j',t} = \sum_{e \in \{L,H\}} \int_0^{z_{e,j,t}(p_{j,t}, p_{j-1,t})} n_{e,t}(z) dz, \quad (\text{A12})$$

where $z_{e,j,t}(p_{j,t}, p_{j-1,t}) = z_{e,j,j-1,t}(p_t)$. By the definition of $j_{0,t}$, the minimum value of the left side of (A12) is positive. The maximal value is less than N_t because $h_{J_t,t} > 0$. Therefore, the left side of (A12) lies in $(0, N_t)$ for each $j \in \{j_{0,t} + 1, \dots, J_t\}$. Because $z_{e,j,t}(\cdot, \cdot)$ is an increasing linear function of its first argument, and because the support of each $n_{e,t}$ is convex, the right side of (A12) strictly and continuously increases in $p_{j,t}$ over the range $[0, N_t)$. Therefore, given $p_{j-1,t}$, a unique value of $p_{j,t}$ solves (A12) for each $j \in \{j_{0,t} + 1, \dots, J_t\}$. Because (19) determines $p_{j_{0,t},t}$, induction shows that unique values of $p_{j,t}$ solve (A12).

We now prove that this unique solution for $p_{j,t}$ strictly increases over $j \in \{j_{0,t}, \dots, J_t\}$. Because $z_{e,j_{0,t}+1,t}(p_{j_{0,t},t}, p_{j_{0,t},t}) = 0$ and the left of (A12) exceeds zero, $p_{j_{0,t}+1,t} > 0 = p_{j_{0,t},t}$. Proceeding inductively, we note that

$$z_{e,j,t}(p_{j-1,t}, p_{j-1,t}) = w_{e,t}^{-1} p_{j-1,t} = z_{e,j-1,t}(p_{j-1,t}, p_{j-1,t}) < z_{e,j-1,t}(p_{j-1,t}, p_{j-2,t}) \quad (\text{A13})$$

as $z_{e,j-1,t}(\cdot, \cdot)$ strictly falls in its second argument. The left of (A12) strictly rises in j , so $p_{j,t} > p_{j-1,t}$.

Finally, we characterize the condition under which $x_{e,j,t} > 0$ for each $e \in \{L, H\}$ and $j \in \{j_{0,t}, \dots, J_t\}$. Denote $z_{e,j,t} = z_{e,j,t}(p_{j,t}, p_{j-1,t})$, where the prices are the unique solutions to (19)–(21). We claim that, in local equilibrium, a positive measure of households of each education choose each type of occupied housing if and only if $z_{e,j,t}$ strictly increases over $j \in \{j_{0,t}, \dots, J_t + 1\}$ for each $e \in \{L, H\}$.

Suppose such monotonicity holds. Appendix A.4 shows that for $j \in \mathcal{J}_t$, a household of education e and endowment z prefers and can afford to choose $q_{j,t}$ over $q_{j-1,t}$ if and only if $z \geq z_{e,j,t}$

when $j > 0$; that same household prefers and can afford to choose $q_{j,t}$ over $q_{j+1,t}$ if and only if $z \leq z_{e,j+1,t}$ when $j < J_t$. By transitivity and the assumed monotonicity, a household of education e with endowment $z \in [z_{e,j,t}, z_{e,j+1,t}]$ prefers and can afford to choose $q_{j,t}$ over all $q_{j',t}$ for $j' \in \mathcal{J}_t$ because either $z_{e,j',t} \leq \dots \leq z_{e,j,t} \leq z$ or $z \leq z_{e,j+1,t} \leq \dots \leq z_{e,j',t}$. Therefore, all households optimize their housing choices when prices solve (19)–(21) and a household of education e and endowment z chooses $q_{j,t}$ only if $z \in [z_{e,j,t}, z_{e,j+1,t}]$. Because housing markets clear by (21), these prices and housing demands constitute a local equilibrium, and due to Lemma 2, they give the unique local equilibrium. In this local equilibrium, $x_{e,j,t} > 0$ for each $e \in \{L, H\}$ and $j \in \{j_{0,t}, \dots, J_t\}$ because $\inf\{z \mid n_{e,t}(z) > 0\} = z_{e,j_{0,t}} \leq z_{e,j,t} < z_{e,j+1,t} \leq z_{e,J_t+1,t} = \sup\{z \mid n_{e,t}(z) > 0\}$ and $n_{e,t}$ has convex support.

Conversely, suppose such monotonicity fails to hold. As argued in the text, in any local equilibrium in which $x_{e,j,t} > 0$ for $e \in \{L, H\}$ and $j \in \{j_{0,t}, \dots, J_t\}$, a household of education e chooses $q_{j,t}$ for such j only if $z \in [z_{e,j,t}, z_{e,j+1,t}]$. If monotonicity fails, then there exists some such e and j such that $x_{e,j,t} = 0$, a contradiction.

B Supplements to Section 3

B.1 Local stability in the divisible model

In any equilibrium of the divisible model, $n_{e,t}(z)/N_{e,t} = \tilde{n}_e(z)/\tilde{N}_e$, where \tilde{N}_e equals the mass of households of education e in the economy. This relation follows from substituting (17) into (14). As a result, $Z_{e,t} = \int_0^\infty z n_{e,t}(z) dz = N_{e,t} \int_0^\infty z \tilde{N}_e^{-1} \tilde{n}_e(z) dz = N_{e,t} \bar{z}_e$, where \bar{z}_e equals the average z among households of education e . Therefore, in equilibrium, $N_{L,t}$ and $N_{H,t}$ determine $Z_{L,t}$ and $Z_{H,t}$. Because amenities, productivities, and prices in t are all functions of $N_{L,t}$, $N_{H,t}$, $Z_{L,t}$, and $Z_{H,t}$, the populations $N_{L,t}$ and $N_{H,t}$ pin down the local equilibrium in any equilibrium. By (17), there exist functions $v_{e,t}^N(N_{L,t}, N_{H,t})$ for each $e \in \{L, H\}$ and $t \in \{1, \dots, T\}$ such that $v_{e,t}(z) = z^{\beta_{c,e} + \beta_{q,e}} v_{e,t}^N(N_{L,t}, N_{H,t})$ for all $z > 0$ in any equilibrium.

An equilibrium is *locally stable* in city t if the Jacobian of $v_t^N = (v_{L,t}^N, v_{H,t}^N)$ at $(N_{L,t}, N_{H,t})$ is Volterra-Lyapunov stable (Cross, 1978). Equivalently for a two-by-two Jacobian, the diagonal is negative and its determinant is positive (Cross, 1978).

Because the diagonal is negative, perturbing $N_{e,t}$ moves equilibrium welfare of households of education e in the opposite direction when local stability holds. Furthermore, perturbing both $N_{L,t}$ and $N_{H,t}$ moves equilibrium welfare of at least one of the education groups in the opposite direction. If not, then there exist $dN_{L,t}, dN_{H,t} \neq 0$ and a non-negative diagonal matrix D_t^N such that $(Dv_t^N)(dN_{L,t}, dN_{H,t}) = D_t^N(dN_{L,t}, dN_{H,t})$. As a result, $Dv_t^N - D_t^N$ has an eigenvalue of zero, meaning that dv_t^N is not strongly stable and hence is not Volterra-Lyapunov stable, a contradiction (Cross, 1978). We prove Proposition 3 using these results.

B.2 Proof of Proposition 3

In the divisible case, each $\delta_{h,j}$ enters the derivatives of the equilibrium conditions only in (31), where it appears with a coefficient proportional to q_{j,t^*} . Due to the linearity of the system of equations, $\partial \log v_{e,t^*}(z)$ is proportional to $\sum_{j \in \mathcal{J}_{t^*}} q_{j,t^*} \delta_{h,j}$, as claimed.

Suppose that $\gamma_N = 0$ and that the equilibrium is locally stable in t^* . Define $Q_t = \sum_{j \in \mathcal{J}_t} q_{j,t} h_{j,t}$. To prove positive proportionality in (37), we assume without loss of generality that $\partial Q_{t^*} > 0$. For a

contradiction, suppose that $\partial \log v_{e',t^*}(z) \leq 0$ for some $e' \in \{L, H\}$ and $z > 0$. By (32), $\partial \log v_{e',t^*}(z') = \partial \log v_{e',t^*}(z)$ for all $z' > 0$, which, by (24), implies that $\partial N_{e',t^*} \leq 0$. Consider the following perturbation to the initial equilibrium:

$$d \log N_{e,t^*} = \partial \log N_{e,t^*} - \begin{cases} 0, & \partial \log N_{\sim e',t^*} \in (-\infty, 0] \\ \partial \log N_{\sim e',t^*}, & \partial \log N_{\sim e',t^*} \in (0, \partial \log Q_{t^*}] \\ \partial \log Q_{t^*}, & \partial \log N_{\sim e',t^*} \in (\partial \log Q_{t^*}, \infty) \end{cases} \quad (\text{B1})$$

for each $e \in \{L, H\}$, where $\sim e' = H$ when $e' = L$ and $\sim e' = L$ when $e' = H$. In equilibrium, $Z_{e,t^*} = \bar{z}_e N_{e,t^*}$, so $Z_{L,t^*}/Z_{H,t^*} = N_{L,t^*}/N_{H,t^*}$. As a result, $N_{L,t^*}/N_{H,t^*}$ determines labor prices and amenities in t^* in equilibrium when $\gamma_N = 0$. Because $d \log(N_{L,t^*}/N_{H,t^*}) = \partial \log(N_{L,t^*}/N_{H,t^*})$, $da_{t^*} = \partial a_{t^*}$, $dw_{L,t^*} = \partial w_{L,t^*}$, and $dw_{H,t^*} = \partial w_{H,t^*}$. Combining (31) with (B1) yields

$$d \log \mu_{t^*} = \partial \log \mu_{t^*} + \begin{cases} \partial \log Q_{t^*}, & \partial \log N_{\sim e',t^*} \in (-\infty, 0] \\ \partial \log Q_{t^*} - \partial \log N_{\sim e',t^*}, & \partial \log N_{\sim e',t^*} \in (0, \partial \log Q_{t^*}] \\ 0, & \partial \log N_{\sim e',t^*} \in (\partial \log Q_{t^*}, \infty). \end{cases} \quad (\text{B2})$$

When $\partial \log N_{\sim e',t^*} \leq 0$, $d \log v_{e,t^*}^N < \partial \log v_{e,t^*}^N \leq 0$ for each $e \in \{L, H\}$. These inequality contradicts local stability because $d \log N_{e,t^*} \leq 0$ for each $e \in \{L, H\}$. When $0 < \partial \log N_{\sim e',t^*} \leq \partial \log Q_{t^*}$, $d \log v_{e',t^*}^N < \partial \log v_{e',t^*}^N \leq 0$. This inequality contradicts local stability because $d \log N_{e',t^*} \leq 0$ and $d \log N_{\sim e',t^*} = 0$. When $\partial \log N_{\sim e',t^*} > \partial \log Q_{t^*}$, $d \log N_{e',t^*} < \partial \log N_{e',t^*} \leq 0$ and $d \log N_{\sim e',t^*} > 0$. These inequalities contradict local stability because $d \log v_{e',t^*}^N = \partial \log v_{e',t^*}^N \leq 0$ and $d \log v_{\sim e',t^*}^N = \partial \log v_{\sim e',t^*}^N > 0$.

B.3 Proof of Proposition 4

In the limit, (24) becomes $\partial \log n_{e,t^*}(z) = 0$ for $e \in \{L, H\}$ and $z > 0$. Using this result, we differentiate (A12) for $j \in \{j_{0,t^*} + 1, \dots, J_{t^*}\}$. Because z_{e,j,t^*} linearly increases in p_{j,t^*} and decreases in p_{j-1,t^*} , solving for $\partial p_{j,t^*}$ yields

$$\partial p_{j,t^*} \propto \chi_{j,t^*} \partial p_{j-1,t^*} - \sum_{j'=j}^{J_{t^*}} \delta_{h,j'}, \quad (\text{B3})$$

for each such j , where $\chi_{j,t^*} > 0$. Using this equation, we induct on j' to prove that $\partial p_{j,t^*}/\partial \delta_{h,j'}$ is negative for $j, j' \in \{j_{0,t^*} + 1, \dots, J_{t^*}\}$, strictly decreases over $j' \in \{j_{0,t^*} + 1, \dots, j\}$, and remains constant over $j' \in \{j, \dots, J_{t^*}\}$. The claim is immediate for $j' = j_{0,t^*} + 1$ because of (35). The inductive step follows immediately as well.

Because $\partial \log n_{e,t^*}(z) = 0$ for $e \in \{L, H\}$ and $z > 0$, $\partial N_{e,t^*} = \partial Z_{e,t^*} = 0$ for each $e \in \{L, H\}$, giving $\partial \log a_{t^*} = 0$ from (2), $\partial \log A_{e,t^*} = 0$ from (26), and $\partial \log w_{e,t^*} = 0$ from (27). Therefore, (36) indicates that $\partial \log v_{e,t^*}(z) = -\beta_{c,e} \partial p_{j,t^*}/(y_{e,t^*}(z) - p_{j,t^*})$ when $z \in (z_{e,j,t^*}, z_{e,j+1,t^*})$. This equation holds for $z = z_{e,j,t^*}$ because $\partial \log z_{e,j,t^*} \leq 0$, which comes from differentiating (A12) and applying the assumption that $\delta_{h,j'} \geq 0$ for $j' > j_{0,t^*}$. Proposition 4 follows from the earlier statements proved about $\partial p_{j,t^*}$.

C Supplements to Section 4

C.1 Changing variables from z to y

For each $e \in \{L, H\}$, the cumulative distribution function of income equals $N_{e,t^*}^{-1} \int_0^{w_{e,t^*}^{-1}y} n_{e,t^*}(z) dz$. Differentiating gives the probability density function, $f(y) = N_{e,t^*}^{-1} w_{e,t^*}^{-1} n_{e,t^*}(w_{e,t^*} y)$. For y such that $n_{e,t^*}(w_{e,t^*}^{-1}y) > 0$, define $\sigma_{e,t^*}(y) = (\partial \log n_{e,t^*})(w_{e,t^*}^{-1}y)$. Applying these equations and the change of variables $y = w_{e,t^*} z$, we transform (28) to

$$\partial \log N_{e,t^*} = \int_{y|\sigma_{e,t^*}(y)>0} \sigma_{e,t^*}(y) f_{e,t^*}(y) dy \quad (C1)$$

(29) to

$$\partial \log Z_{e,t^*} = N_{e,t^*} Y_{e,t^*}^{-1} \int_{y|\sigma_{e,t^*}(y)>0} \sigma_{e,t^*}(y) y f_{e,t^*}(y) dy, \quad (C2)$$

and (33) to

$$\frac{\delta_{h,j}}{N_{t^*}} = \sum_{e \in \{L, H\}} \frac{N_{e,t^*}}{N_{t^*}} \left(\underbrace{y_{e,j+1,t^*} f_{e,t^*}(y_{e,j+1,t^*}) \partial \log z_{e,j+1,t^*}}_{\text{trickle-up}} - \underbrace{y_{e,j,t^*} f_{e,t^*}(y_{e,j,t^*}) \partial \log z_{e,j,t^*}}_{\text{trickle-down}} + \underbrace{\int_{y_{e,j,t^*}}^{y_{e,j+1,t^*}} \sigma_{e,t^*}(y) f_{e,t^*}(y) dy}_{\text{migration}} \right). \quad (C3)$$

To calculate $\sigma_{e,t^*}(y)$, we combine (24) and (36) to obtain

$$\sigma_{e,t^*}(y) = \frac{\beta_{c,e}}{\beta_{\epsilon,e}} \frac{y \partial \log w_{e,t^*}}{y - p_{j,t^*}} - \frac{\beta_{c,e}}{\beta_{q,e}} \frac{\partial p_{j,t^*}}{y - p_{j,t^*}} + \frac{\beta_{a,e}}{\beta_{\epsilon,e}} \partial \log a_{t^*}, \quad y \in (y_{e,j,t^*}, y_{e,j+1,t^*}). \quad (C4)$$

We calculate $N_{e,t^*}/Y_{e,t^*} = (N_{e,t^*}/N_{t^*})/(Y_{e,t^*}/Y_{t^*})$. This methodology allows us to measure $\delta_{h,j}/N_{t^*}$ instead of $\delta_{h,j}$, so we consider changes to the housing stock as a share of the initial population.

C.2 Proof of Lemma 3

We rearrange (39) to produce

$$y_{H,j,t^*} = \widehat{p}_{j-1,t^*} + \frac{\widehat{p}_{j,t^*} - \widehat{p}_{j-1,t^*}}{1 - \left(\frac{y_{L,j,t^*} - \widehat{p}_{j,t^*}}{y_{L,j,t^*} - \widehat{p}_{j-1,t^*}} \right)^\zeta} \quad (C5)$$

for each $j \in \{1, \dots, 50\}$. We rewrite the term in parentheses as $1 - (\widehat{p}_{j,t^*} - \widehat{p}_{j-1,t^*})/(y_{L,j,t^*} - \widehat{p}_{j-1,t^*})$, which strictly increases from zero to one in $y_{L,j,t^*} \in [\widehat{p}_{j,t^*}, \infty)$. Therefore, as a function of y_{L,j,t^*} , y_{H,j,t^*} strictly increases from p_{j,t^*} to ∞ for $y_{L,j,t^*} \in [\widehat{p}_{j,t^*}, \infty)$. By taking cumulative sums, we rewrite (38)

as

$$\frac{\sum_{j' \in \{j, \dots, 50\}} \sum_i g_i \delta_{i,j'}}{\sum_i g_i} = \frac{N_{L,t^*}}{N_{t^*}} \int_{y_{L,j,t^*}}^{\infty} f_{L,t^*}(y) dy + \frac{N_{H,t^*}}{N_{t^*}} \int_{y_{H,j,t^*}(y_{L,j,t^*})}^{\infty} f_{H,t^*}(y) dy \quad (\text{C6})$$

for each $j \in \{1, \dots, 50\}$, where we use the fact that $y_{e,51,t^*} = \infty$ because the support of the double Pareto-lognormal distribution is unbounded from above. The right side is defined for $y_{L,j,t^*} \geq \widehat{p}_{j,t^*}$ and strictly increases in y_{L,j,t^*} . Because (40) holds, the intermediate value theorem implies the existence of a unique solution for y_{L,j,t^*} , which then delivers a unique y_{H,j,t^*} . If $\zeta = 1$, then $y_{H,j,t^*} = y_{L,j,t^*}$. The solutions to (C6) then strictly increase in j because the left side does.

C.3 Proof of Lemma 4

Define $\rho_e = \beta_{c,e}/\beta_{w,e}$ for $e \in \{L, H\}$. Because $\beta_{q,e}/\beta_{c,e} = (\beta_{w,e} - \beta_{c,e})/\beta_{c,e} = \beta_{w,e}/\beta_{c,e} - 1$, we may rewrite the equation defining ζ as $\zeta = (\rho_L^{-1} - 1)/(\rho_H^{-1} - 1)$, which reduces to $\rho_L = \rho_H(\zeta + (1 - \zeta)\rho_H)^{-1}$. We substitute this equation into (45), which takes the form $1 - \alpha = (1 - \bar{e}_\alpha)\rho_L + \bar{e}_\alpha\rho_H$. After multiplying through by $\zeta + (1 - \zeta)\rho_H$ and simplifying, we obtain $0 = \bar{e}_\alpha(1 - \zeta)\rho_H^2 + (\zeta + (\alpha - \bar{e}_\alpha)(1 - \zeta))\rho_H - (1 - \alpha)\zeta$. The right side is a quadratic function of ρ_H that changes sign between $\rho_H = 0$, where it equals $-(1 - \alpha)\zeta$, and $\rho_H = 1$, where it equals α . Therefore, there exists a unique solution for $\rho_H \in (0, 1)$. Because $\zeta + (1 - \zeta)\rho_H = \rho_H + \zeta(1 - \rho_H) > \rho_H$, the corresponding solution for ρ_L also lies in $(0, 1)$.

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