Optimal Forbearance of Bank Resolution

Linda Schilling
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Linda M. Schilling*

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Abstract

We analyze optimal strategic delay of bank resolution (‘forbearance’) and deposit insurance in a setting where, after bad news on the bank, depositors fear for the uninsured part of their deposit and withdraw while the regulator observes withdrawals and needs to decide when to intervene. Under low insurance coverage the optimal intervention policy is to walk away. Optimal deposit insurance coverage is always interior. Fast intervention cannot minimize public losses and be optimal at the same time. The paper sheds light on the differences between the U.S. and the European Monetary Union in terms of their bank resolution policies.

Key words: Bank resolution, suspension of convertibility, mandatory stay, forbearance, bank run, deposit insurance, deposit freeze, recovery rates, global games

JEL Classification: G28,G21,G33, D8, E6

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1 Motivation

Banking is a highly regulated industry. Regulators not only set deposit insurance levels, they also decide when to resolve banks (Martin et al., 2017). Once an institution is perceived as failing, the regulator through its resolution authority (RA) can intervene and organize a sale of the bank’s assets. The delay of intervention (‘forbearance’) is at RA’s discretion.1 This paper studies the interaction between the level of deposit insurance and the degree of intervention delay. By examining this two-dimensional policy choice the paper breaks new ground in the analysis of the regulator’s double role and thus provides a novel perspective on this topic.

The question how to resolve banks is important since resolution procedures impose large losses on tax payers and public funds, see Figure 1 and White and Yorulmazer (2014). Cases of bank resolution are common, not only during times of crises. Alone the FDIC’s ’Failed Bank List’ shows 553 entries of failed banks under U.S. FDIC supervision for the years 2001-2017. Prominent recent cases of bank resolution in Europe include the bail-out of Monte dei Paschi di Siena in Italy, the sale of Banco Popular in Spain, both in 2017, and the partial sale of Laiki Bank in 2013 during the Cypriot banking crises. Important differences exist between the European Monetary Union and United States in

![Figure 1: Number and costs of U.S bank failures](image)

(a) Number of failed U.S banks under FDIC receivership, source: FDIC failed bank list  
(b) Costs of bank failure to FDIC’s deposit insurance fund (DIF), source: Bankrate.com

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1 The resolution procedure by the FDIC is initiated once a financial institution’s chartering authority sends a Prompt Corrective Action letter to the failing institution advising that it is critically undercapitalized or insolvent, see the FDIC’s Resolutions Handbook (FDIC RH). The FDIC either organizes a Purchase & Assumptions transaction or a deposit payoff to resolve banks. Both methods are comprised by the model outlined here. By FDIC RH ‘Section 38 of the Federal Deposit Insurance Act (FDI Act) generally requires that an insured depository institution be placed in receivership within 90 days after the institution has been determined to be critically undercapitalized.’
terms of their bank resolution policies. In the U.S., the Federal Deposit Insurance Corporation (FDIC) acts as RA and is appointed as receiver if an FDIC insured depository institution or a non deposit making but systemically relevant institution becomes critically undercapitalized. The FDIC operates under the least cost resolution requirement to minimize net losses to the deposit insurance, regardless of factors such as maintaining market discipline, or prevention of contagion (Bennett, 2001). In contrast to the U.S., Article 31 of the European 'Bank Recovery and Resolution Directive' (BRRD) mentions competing objectives for bank resolution\(^2\) such that the European resolution policy is potentially softer compared to the U.S. policy. Furthermore, the BRRD entails the option to divert bank resolution away from the centralized authority 'Single Supervisory Mechanism' (SSM) to national resolution authorities. This can be seen as both a delay of and diversion to resolution under distinct conditions on request\(^3\). To the best of our knowledge, there is no explanation for these differences in the literature so far. This paper sheds light on these differences. We explain under what circumstances it is optimal to exercise the European option to divert resolution and discuss conditions under which the U.S approach to minimize public losses is desirable from a social perspective.

In our setting, a bank finances a risky asset with deposits where deposits are only partially insured at a level set by the regulator\(^4\). As in Goldstein and Pauzner (2005), depositors observe about the fundamental of the bank and may decide to withdraw early. These withdrawals potentially impose losses on the deposit insurance fund. The RA observes withdrawals at the bank level. Should withdrawals exceed a critical level set beforehand by RA, RA intervenes. In that case, RA suspends convertibility of deposits such that depositors can no longer withdraw (mandatory stay). She seizes remaining bank assets which she then liquidates to evenly distribute proceeds to all depositors who were not served so far. If proceeds are below the insured amount of the deposit, the insurance fund is obliged to pay the difference.

RA’s role as insurer interferes with her role as resolution authority. If RA intervenes later, she seizes a smaller proportion of the asset which diminishes the pro rata share to depositors under resolution. If the pro rata share is below the insured fraction of the

\(^2\) 'to ensure continuity of critical functions' and 'to avoid a significant adverse effect on the financial system' in addition to the objective to protect depositors and public funds

\(^3\)If the European Council or Commission objects a resolution scheme proposed by the Single Resolution Board, resolution of the bank in question will be implemented by national resolution authorities 'in accordance with national law' transposing the Bank Recovery and Resolution Directive. The European option to divert bank resolution to national authorities has as further implication that recovery rates to creditors after resolution differ not only across bank asset classes Gupton et al. (2000) but also across national resolution authorities by bankruptcy code (Davydenko and Franks, 2008), see also (Bris et al., 2006).

\(^4\) Despite the existence of (partial) deposit insurance in many countries, the possibility of bank runs persists since only about 59% of U.S. domestic deposits are insured as of 2016, see appendices (FDIC, 2016).
deposit the insurance fund becomes liable, thus losses to the insurance fund increase as RA intervenes later. On the other hand, as RA raises insurance coverage the exposure of the insurance fund increases which may affect RA’s forbearance policy to limit losses. Not only is RA’s role as insurer intertwined with her role as resolution authority but also the bank’s depositors are affected by and react to changes in deposit insurance coverage and timing of intervention in different ways.

The question we ask in this paper is, what is the welfare maximizing measure of withdrawals RA should tolerate before intervening (‘forbearance policy’) and how much insurance coverage should she provide.

To the best of our knowledge, this is the first paper that considers a strategic resolution authority which fully internalizes the impact of her twofold policy on the endogenous probability that the bank is resolved.\(^5\) This allows to answer questions such as, (i) given a cut in deposit insurance, how would RA need to adapt her intervention delay to keep run probability constant (ii) given the authority wants to pursue a more lenient intervention policy, how does the insurance level need to change to maintain welfare at a particular level (iii) how does maximization of welfare interfere with minimization of losses to the insurance fund?

As main contribution, this paper points out hidden trade-offs and dependencies in resolving banks. Late intervention imposes losses on the deposit insurance fund while early intervention increases the likelihood that the bank is resolved since depositors run for smaller solvency shocks. This trade-off crucially depends on the amount of deposit insurance coverage provided. Under too low insurance coverage, inefficient runs may occur no matter when RA intervenes. The optimal forbearance policy by RA is to walk away, i.e. never intervene. This means, even when inefficient runs occur and thus intervention is ex post optimal a stricter policy to intervene will alter depositors’ behavior in a way that inefficient runs become more likely ex ante. RA fully anticipates this change in behavior and optimaially commits to never intervene. On the other hand, under too high insurance inefficient investment exists no matter the timing of intervention.\(^6\) This is to the best of our knowledge the first theory paper which shows that as deposit insurance coverage increases, equilibrium outcomes shift from exhibiting inefficient runs to inefficient investment because depositors pay less attention to information on solvency shocks (gradual decline of market discipline). As a consequence, a higher probability of

\(^5\) In Diamond and Dybvig (1983) for instance there is multiplicity of equilibria. Thus, marginal changes of run probability cannot be analyzed since the likelihood of runs cannot be determined from within the model unless the regulator sets a policy such that running is a dominated action. Thus, there is no feedback from depositors to the regulator unless the occurrence of a run can be excluded. In the paper here instead, marginal changes in resolution probability from within the model feed back into RA’s objective function due to altered depositor behavior. This allows in particular to analyze interaction of the two policy parameters which has not been done before.

\(^6\) In our model, a bank run with subsequent bank resolution is the only mechanism to trigger liquidation of assets.
runs can be desirable from a social perspective to enforce liquidation. By this, the paper provides a theory foundation for the finding in Iyer et al. (2016) that propensity to run increases as insurance goes down, see also Calomiris and Jaremski (2016); Goldberg and Hudgins (2002); Baer et al. (1986); Goldberg and Hudgins (1996). One main implication of these results is, if RA does not fine tune the amount of insurance coverage, there can be inefficiencies, thus the forbearance policy alone is a weak policy parameter.

If RA jointly sets forbearance and insurance coverage, then for every forbearance level there exists a unique interior level of insurance coverage which implements the first best outcome. That is, RA can always achieve that the bank is resolved if and only if it is efficient to liquidate the asset. To achieve optimality, RA balances prevention of both inefficient runs and inefficient investment. She does so by altering information aggregation among depositors through her policy.\(^7\) In particular, all runs which occur under the optimal policy are efficient meaning that ex post RA has no incentive to deviate from her policy.\(^8\) Runs in our setting can be efficient because in contrast to previous work (Diamond and Dybvig, 1983; Ennis and Keister, 2009) our model features aggregate uncertainty. That is, prevention of runs today is inefficient if chances are high that assets do not pay off tomorrow.\(^9\) Depositors may further run inefficiently seldom, a case that does not arise in Diamond and Dybvig (1983); Ennis and Keister (2009) or Goldstein and Pauzner (2005).

We show, the optimal insurance coverage strictly decreases as RA intervenes later. This may explain why US insurance levels are higher compared to European levels given that in the U.S. the FDIC potentially intervenes faster than the European counterpart Single Resolution Board (SRB).\(^10\) Multiplicity of optimal pairs of forbearance and insurance coverage implies that RA can either do optimal policy using insurance coverage only or minimize public losses conditional on using optimal policies. In the latter case we show, an optimal and loss-minimizing policy requires forbearance. A policy to intervene as fast as possible, is either not optimal or not loss minimizing. Further, we point out that minimizing public losses is equivalent to maximizing depositors’ direct utility from the deposit contract.

Finally, the paper discusses extensions such as the case where RA liquidates at different efficiency as opposed to the market, intervention by a lender of last resort, and cascading forbearance along the European resolution directive.

Since we can set insurance coverage to zero, the paper not only applies to banks but also to non deposit making institutions which are supervised by resolution authorities.

\(^7\)In our setting, depositors are risk-neutral. Deposit insurance thus serves no risk sharing purpose but impacts welfare by modifying information aggregation. Also, the only mechanism which enforces liquidation here is a run with subsequent bank resolution.

\(^8\)As a consequence, the time-inconsistency problem discussed in Ennis and Keister (2009) vanishes.

\(^9\)In Goldstein and Pauzner (2005) for instance, there are efficient runs but no intervention.

\(^10\)In the U.S. insurance coverage is $250,000 per account holder, in Europe it is €100,000.
due to systemical relevance since Dodd-Frank and the inception of the European BRRD.

The paper is structured as follows: Section two describes the model, section three solves the interim stage of the three period game while section four solves the ex ante stage. Section five considers extensions and applications, section seven concludes.

1.1 Literature

Our paper is connected to the literature stand on bank runs, liquidity risk and self-fulfilling beliefs. The three papers closest to ours are Diamond and Dybvig (1983), Goldstein and Pauzner (2005) and Ennis and Keister (2009). Similar to Diamond and Dybvig (1983), we study bank runs and how welfare is affected when introducing a resolution authority which can impose a mandatory stay or deposit insurance. As opposed to Diamond and Dybvig (1983), we use a global games approach to obtain a unique equilibrium such that we can analyze how propensity to run changes and feeds back into RA’s objective function as she varies the intervention threshold and insurance coverage. Concerning the model, we are closest to Goldstein and Pauzner (2005) who use a global game to analyze the optimality of risk sharing via demand deposit contracts in the context of bank runs. As opposed to Goldstein and Pauzner (2005) we add a strategic resolution authority who can intervene and (partial) deposit insurance. Similar to Ennis and Keister (2009), we analyze how anticipation of intervention can generate and affect depositors’ incentives to participate in the run. In a Diamond and Dybvig type model, Ennis and Keister (2009) focus on ex post efficient intervention during runs. As opposed to Ennis and Keister (2009) and Diamond and Dybvig (1983), our model features aggregate uncertainty, runs can be efficient and depositors may run inefficiently seldom which impacts optimal intervention policies.

Also closely related are Morris and Shin (2016), Rochet and Vives (2004) and Eisenbach (2016) who consider credit risk, respectively interventions by a lender of last resort or efficiency of asset liquidation, all of them in a global games context. Calomiris and Kahn (1991) study the role of demandable debt as a disciplining device. Runs can enforce liquidation when an informed bank may act against the interests of uninformed depositors. Our paper instead describes how RA can exploit the disciplining feature of demandable debt to alter depositors behavior in a way that they run if and only if liquidation is efficient.\(^{11}\) Cooper and Ross (2002) analyze how deposit insurance affects risk shifting of banks while we focus on how insurance alters depositors’ information aggregation and thus likelihood to run on banks.

Further related are Allen et al. (2017), Ahnert and Kakhbod (2017) and Matta and Perotti (2017) who analyze government guarantees, amplification mechanism of financial crises respectively secured repo funding under roll over risk using global games. To

\(^{11}\)The bank is non strategic in our setting.
obtain an equilibrium selection this paper uses global games technique (Carlsson and Van Damme, 1993; Morris and Shin, 2001).

Important in our framework is that withdrawals occur and are observed gradually but depositors make their roll over decision simultaneously without yet knowing their position in the queue. As a consequence, the chance to recover the entire deposit is always strictly positive and the incentive to run exists, see also He and Manela (2016); Green and Lin (2003) and Peck and Shell (2003).


2 Model

We extend the model set out by Goldstein and Pauzner (2005). There are three time-periods, \( t = 0, 1, 2 \) and no discounting. There are four kinds of agents, a bank, depositors, outside investors and a resolution authority (RA). The bank invests in a risky and illiquid asset and finances her entire investment with short-term debt.\(^{12}\) There are constant returns to scale, thus we normalize initial bank investment to one unit. Depositors are risk-neutral, symmetric, each endowed with one unit to invest and given by a continuum \([0, 1]\). There is free entry, thus the bank is in perfect competition to other banks and makes zero profit.

**Investment and Financing** For each unit invested at time zero, the asset pays off \( H \) at time two with probability \( \theta \) and zero otherwise, where \( \theta \sim U[0, 1] \) is the unobservable, random state of the economy. We assume \( H > 2 \) such that the asset has positive net present value. \( H > 2 \). At time one, the asset can be either sold at value \( l < 1 \) or be pledged to outside investors to raise cash. To raise funds, in \( t = 0 \) the bank offers a demand deposit contract which for each initially invested unit, promises to pay a coupon of one unit if the contract is liquidated at time one (“withdraw”), by this the contract mimics storage. If the deposit is ’rolled over” until time two, the contract promises coupon \( H \).\(^{13}\)

**Outside Investors** To refinance withdrawals of depositors at \( t = 1 \), the bank can

\(^{12}\) Results are fully robust to financing only a fraction \( \delta < 1 \) with short-term debt.

\(^{13}\) We fix the demand deposit contract at \((1, H)\) here for sake of simplicity. In subsection 5.5, we will consider a general contract \((R_1, R_2)\) and explain why our results remain valid.
raise cash by approaching a representative outside investor C with deep pockets. The bank can borrow up to \( l \in (0, 1) \) fast and short-term for one period ahead at interest rate \( i = H \). C is non strategic. We give a micro foundation for the assumptions on the interest rate and on the maximum amount C is willing to borrow in subsection 5.4 and discuss general interest rates in subsection 5.5. In section 5.3, we discuss changes if C is the lender of last resort.

**Signals and actions (interim)** Before depositors decide whether or not to withdraw they observe noisy, private information signals about the state \( \theta \) of the world, given by

\[
\theta_i = \theta + \varepsilon_i
\]

where the idiosyncratic noise is independent of state \( \theta \) and iid distributed according to \( \varepsilon_i \sim U[-\varepsilon, +\varepsilon] \). For \( \varepsilon \) small, signals become precise. The signal contains information on how likely the asset pays off high return \( H \) at time \( t_2 \). Since signals are correlated through the state, each signal also conveys information on signals and beliefs of other agents. Depositors’ strategies map their private signal \( \theta_i \) to an action.

**Deposit Insurance** Each deposit is insured up to fraction \( \gamma \in (0, 1) \). If the bank becomes illiquid or insolvent, the insurance repays depositors \( \gamma \) for each unit invested at time zero. The bank is prone to runs if \( l < 1 \), that is overall debt claims exceed the amount of cash the bank can raise by pledging the asset. We maintain the assumption \( l < 1 \) throughout the paper. Since insurance coverage is partial, bad news on asset return probability \( \theta \) can trigger a run, since by withdrawing a depositor has the chance to recover the entire deposit instead of only a fraction. The deposit insurance fund is financed via lump-sum taxation of depositors at the time the depositor demands repayment from the bank.\(^{14}\)

**Resolution Authority (RA)** Our model adds new to the literature a strategic resolution authority (RA). RA acts in two ways, she provides deposit insurance and has the legal authority to protect the deposit insurance fund by intervention: She may take over control, impose a mandatory stay for depositors, by this stopping runs on the bank (suspension on convertibility). Given intervention, the bank stops both the service of withdrawing depositors and the pledging of assets in the market. RA seizes and liquidates remaining assets at exogenous recovery rate \( r \) and evenly allocates realized proceeds among all remaining bank depositors who were not paid so far. If proceeds per depositor are below the insured fraction of the deposit, the insurance fund becomes liable. Intervention prevents depositors to run at the expense of other depositors and the deposit insurance fund since by withdrawing depositors also obtain the uninsured part of the deposit which could have otherwise been redistributed to remaining depositors. The

\(^{14}\)In fact, in Germany for instance deposit insurance is financed by charging not depositors but banks a fraction of their total deposits (tax on deposits). Since the bank here makes zero profits, the bank forwards this tax lump-sum to depositors.
RA may liquidate more or less efficient than the market. We discuss the case $r = l$ as the benchmark and consider $r \neq l$ in subsection 5.1. RA cannot observe the state but

**Figure 2: Forbearance-weighted liquidation procedure of assets:** Forbearance determines the proportion of the asset liquidated during the run versus under bank resolution.

![Diagram showing liquidation process](image)

perfectly observes withdrawals at the bank level.\footnote{In equilibrium RA could infer the state from observing $n$. However due to the sequential nature of withdrawals we assume that she cannot set her policy depending on the state realization.} By observing depositors’ behavior she makes inferences about the state. If at $t = 1$ RA observes withdrawals in excess of a particular threshold, which she optimally sets, RA infers that the state is ‘low’ and intervenes. More concrete, if the bank is forced to pledge a fraction larger than ‘$a$’ of assets to serve withdrawing depositors, RA intervenes. We call $a \in (0, 1)$ the RA’s forbearance policy. Forbearance $a$ and insurance coverage $\gamma$ are common knowledge among all agents and are set by RA at time zero before depositors decide whether to roll over. RA fully commits to policy $(a, \gamma)$. The forbearance policy can be understood as a reduced form of ‘timing’ of intervention in the sense that admitting few withdrawals corresponds to ‘early’ intervention while allowing many withdrawals corresponds to ‘late’ intervention. Since RA does not observe the state $\theta$ at $t = 0$, RA’s policy $(a, \gamma)$ does not convey information. Denote by $n \in [0, 1]$ the endogenous equilibrium proportion and measure of depositors who decide to withdraw at the interim period. Since withdrawing depositors claim one unit each, $n$ is also the realized measure of claimed funds at $t = 1$. For given forbearance policy, the event ‘bank resolution’ is triggered if the measure of claimed funds exceeds the critical level of cash withdrawals RA tolerates.

$$n \geq al \iff \{\text{Bank resolution}\}$$ (2)
For $a = 1$, RA does not intervene. For $a < 1$, RA intervenes and secures fraction $1 - a$ of the asset after observing how the bank pledged fraction $a$ to serve withdrawing depositors, see Figure 2. Liquidation of this remaining fraction leads to proceeds $r(1 - a)$ which are evenly allocated to measure $1 - la$ of depositors who were not served so far\textsuperscript{16}. Denote by

$$s(a) := \frac{r(1 - a)}{1 - la} \in (0, 1)$$

the pro rata share RA recovers when resolving the bank. The pro rata share decreases as RA shows more forbearance since the asset is illiquid. For $s(a) < \gamma$, the insurance fund is liable. The pro rata share and the insured part of the deposit always undercut the short-term coupon of one unit which gives incentive to run if resolution is anticipated. If RA’s recovery rate exceeds asset’s liquidation value, RA can set forbearance such that the pro rata share depositors obtain under resolution exceeds the claim they have towards the deposit insurance.\textsuperscript{17} In that case, the insurance fund runs no loss given resolution. We call this case 'early intervention'. Define the maximum forbearance RA can grant such that insurance runs no loss as

$$\overline{a}(r, \gamma) := \max \left( 0, \frac{r - \gamma}{r - l\gamma} \right) \in [0, 1)$$

Forbearance level $\overline{a}(r, \gamma)$ increases in recovery rate $r$ and decreases in insurance coverage $\gamma$. Under 'late intervention' $a \in (\overline{a}, 1]$, the pro rata share undercut the insured amount of the deposit and the insurance has to pay the difference $\gamma - s(a) \in [0, \gamma]$ to each depositor. Under resolution, each depositor obtains

$$s_\gamma(a) := \max(s(a), \gamma) = \begin{cases} s(a), & a \in (\overline{a}, \overline{a}) \\ \gamma, & a \in (\overline{a}, 1] \end{cases}$$

We assume that RA obeys a forbearance minimum $a > 0$ which can be interpreted in the sense that RA observes withdrawals with a delay and cannot intervene immediately. This assumption is grounded in legal constraints since the bank has to be insolvent given bank resolution occurs, see later discussion on 'minimum forbearance'.\textsuperscript{18}

RA’s objective is to set the welfare maximizing policy $(a^*, \gamma^*)(r)$ under the constraint $a^* \in (\overline{a}, 1] \subset (0, 1]$. In subsection 4.4, we discuss refinements of RA’s objective function

\textsuperscript{16}This formulation is equivalent to saying that proceeds are allocated pro rata to depositors holding remaining $1 - al$ debt claims.

\textsuperscript{17} $r \geq l$ implies that the denominator $r - l\gamma$ is positive by $\gamma \leq 1$. In the case $r < l$, it can be that $l\gamma > r$ such that $\frac{(1-a)r}{1-la} < \gamma$ for all $a \in (0, 1]$ since $\frac{2-r}{r-r} > 1$. Thus, the deposit insurance fund is always liable given resolution for any $a$. Therefore, in the case of $l\gamma > r$ we set $a = 0$.

\textsuperscript{18} In the U.S. the FDIC is allowed to intervene only if the asset to debt ratio has fallen below a critical threshold. To give an example, in September 2017, bondholders of failed Banco Popular filed an appeal against Spain’s banking bailout fund which followed European authorities (Single Resolution Board) and wiped out equity and junior bond holders before selling the bank to Banco Santander, see Bloomberg (2017) and Reuters (2017).
to minimize public losses. We take RA’s recovery rate as exogenously given.\footnote{This can be justified when seeing recovery rates as being asset and country specific depending on national bankruptcy laws.}

**Payoffs** Given bank resolution occurs, depositors who roll over receive share $s_\gamma(a)$. Depositor’s payoff from ‘withdrawing’ given bank resolution equals

$$\frac{la}{n} \cdot 1 + (1 - \frac{la}{n}) s_\gamma(a)$$ (6)

Analogous to Goldstein and Pauzner (2005), this payoff mirrors the bank’s sequential service constraint. To withdraw, depositors queue and are sequentially served the coupon of one unit until the bank hits the RA’s tolerance threshold.\footnote{In fact, RA observes overall realized claimed funds $n$ and randomly selects measure $al$ out of $n$ to serve, thus $n - al$ are not served.} Depositors’ positions in the queue are random. The probability to be served before resolution takes place is $\frac{la}{n}$ while with probability $1 - \frac{la}{n}$ a queuing depositor is not served and becomes involved in the resolution process where she is treated as if she rolled over her deposit. We define the haircut

$$H(a) = 1 - s_\gamma(a) \in (0, 1]$$ (7)

as the difference between the face value of a deposit and share obtainable under resolution (deviation loss). The haircut is bounded by the uninsured part of the deposit

$$H(a) \leq 1 - \gamma$$ (8)

As coverage increases, the haircut goes to zero, meaning that payoffs from rolling over and withdrawing given resolution become more alike.\footnote{More intuitively, we can now rewrite the payoff from withdrawing as $s_\gamma(a) + \frac{la}{n} \cdot H(a)$ where a withdrawing depositor receives $s(a)$ for sure and with probability $la/n$ she receives the haircut on top.} Given no resolution occurs, the bank can finance all withdrawals at $t = 1$ by borrowing cash $x = n$ from outside investor C. Her time two return, if the asset pays off equals $H - ix = H(1 - n)$. Since the bank is all debt financed, this return is equally pro rated to depositors who roll over, they receive

$$\frac{H - ix}{1 - n} = H$$ (9)

as pinned down in the contract. If the asset does not pay off, depositors who roll over receive the insured fraction of their deposit.\footnote{The simplification in (9) is the reason why we assume $i = H$ in the benchmark model. By this, we obtain a debt-like payoff to depositors who roll over instead of having an equity like payoff $\frac{H - ix}{1 - n}$ as for instance in Goldstein and Pauzner (2005) and Diamond and Dybvig (1983). As a consequence, the model applies to the case where the bank is partially financed with debt for arbitrarily high debt ratios and general debt contracts, see subsection 5.5 where we also generalize the repo rate. The reason for the simplification here is to reduce parameters by two.}

RA raises overall measure $\gamma$ via taxation of depositors. Each depositor is charged $\gamma$. The tax applies at the time depositors demand repayment from the bank, withdrawing
depositors are taxed at $t_1$, depositors who roll over are taxed at $t_2$. The payoff table before taxation is given as

<table>
<thead>
<tr>
<th>Event/Action</th>
<th>Withdraw</th>
<th>Roll-over</th>
</tr>
</thead>
<tbody>
<tr>
<td>No resolution, $n \in [0, la]$</td>
<td>1</td>
<td>$H, p = \theta$</td>
</tr>
<tr>
<td>Bank resolution, $n \in (la, 1]$</td>
<td>$\frac{la}{n} \cdot 1 + (1 - \frac{la}{n})s_\gamma(a)$</td>
<td>$s_\gamma(a)$</td>
</tr>
</tbody>
</table>

Payoffs after taxation equal payoffs given above less $\gamma$, thus the tax will not affect depositors’ behavior.

**Information structure** We follow the information structure in Goldstein and Pauzner (2005) to obtain a unique equilibrium. We assume there are states $\theta$ and $\bar{\theta}$ which mark the bounds to dominance regions: For states in the range $[0, \theta]$ withdrawing is dominant while for high states $[\theta, 1]$ rolling over is dominant. Boundary state $\theta$ is defined by the equation $H\theta + \gamma(1 - \theta) = 1$. That is,

$$\theta = \frac{1 - \gamma}{H - \gamma} \tag{10}$$

By $H > 1 > \gamma$, for states below $\theta$ the expected value of rolling over and either receiving the high coupon or the insured fraction $\gamma$ undercut the payoff from withdrawing. For the upper dominance region, we assume that for states $\theta > \bar{\theta}$ the asset pays off $H$ for sure and already at time one.\(^{23}\) We assume further that in this case the RA is not authorized to intervene \(a = 1\) since the bank is solvent for sure.\(^{24}\) As a consequence, the coordination problem vanishes since bank resolution is never triggered and the bank can always repay all withdrawing depositors. We further assume that the support of the noise $\varepsilon$ is sufficiently small such that depositors can infer from their signals whether the state is located in either of the dominance regions.

**Timing** At $t = 0$ the random state realizes unobservably, depositors invest in the contract and RA sets her forbearance policy and deposit insurance coverage. At $t = 1$, all depositors observe RA’s forbearance policy, insurance coverage and signals about the state. Then they decide whether to withdraw, aggregate withdrawals $n$ realize. RA observes whether withdrawals exceed threshold $al$. If yes, the bank is put into receivership and gets resolved. Otherwise the game proceeds to period two and the asset pays off or not.

The analysis proceeds as follows. Via backward induction, we first analyze the interim stage where depositors take as given RA’s forbearance policy, recovery rate $r$ and insurance coverage $\gamma$. We analyze how depositors’ behavior alters as RA shifts her policy. At

\(^{23}\) This assumption is equivalent to a shift in interim liquidation value from $l$ to $H$.
\(^{24}\) The FDIC is only appointed as receiver if a bank’s capital to asset ratio falls below two percent (12 U.S. Code 1831o), i.e. the bank is close to insolvency.
the ex ante stage, we consider socially optimal policies \((a^*, \gamma^*)\) where RA takes as given the coordination behavior of depositors that will follow in the subgame. All proofs can be found in the appendix. The equilibrium concept is perfect Bayes Nash.

3 Equilibrium coordination game - interim stage

At the interim stage, depositors take RA’s forbearance policy \(a\), deposit insurance coverage \(\gamma\) and recovery rate \(r\) as given when deciding whether to roll over their deposit. All following results are at the limit as noise vanishes.

**Proposition 3.1**

*The game played by depositors has a unique equilibrium which is in trigger strategies. All depositors withdraw if they observe a signal below threshold signal \(\theta^*(a, \gamma, r)\) and roll over otherwise.*

This existence and uniqueness result was first derived in Goldstein and Pauzner (2005). The trigger signal at which a depositor’s belief is such that she is indifferent between rolling over and withdrawing at the limit is explicitly given by

**Lemma 3.1.**

\[
\theta^* = \frac{(1 - \gamma) - H(a) \ln(la)}{H - \gamma}
\]

where \(H(a) = 1 - s_\gamma(a) \geq 0\) is the haircut (deviation loss) given the bank is resolved.

Bank resolution occurs, if the measure of funds withdrawn by depositors with signals below threshold \(\theta^*\), exceeds the critical value \(al\). Denote by \(\theta_b\) the critical state such that bank resolution occurs if the true state realizes below \(\theta_b\). Then \(\theta_b\) is implicitly given by

\[
n(\theta_b, \theta^*) = la
\]

where the function \(n(\theta, \theta^*)\) is the endogenous equilibrium measure of withdrawn funds at state \(\theta\) and trigger \(\theta^*\), see (48). Since the random asset return is uniformly distributed
and bank resolution occurs if the state realizes below the critical state, the probability
that bank resolution occurs is just equal to $\theta_b$. This motivates the following definition,

**Definition 3.1.** We say bank stability increases if the ex ante probability of bank resolution $\theta_b$ goes down.

The fact that the RA intervenes if aggregate withdrawals exceed the critical measure
of withdrawals RA tolerates, has two consequences. First, depositors care for what other
depositors believe and do since optimality of the action to 'withdraw' depends on whether
resolution takes place or not. Second, as RA changes her forbearance policy she changes
strategic uncertainty among depositors: As RA becomes more tolerant and sets a higher
forbearance policy, the run needs to be larger to trigger bank resolution. Therefore, the
action to withdraw is optimal only for larger runs. All depositors internalize this fact
which lowers depositors’ propensity to run. The coordination problem among depositors
relaxes, ex ante the event bank resolution becomes less likely. In particular, RA’s change
in forbearance policy feeds back into depositors’ behavior and thus bank stability.

**Proposition 3.2 (Bank stability - Benchmark ($r = l$))**

Bank stability monotonically improves in forbearance.

The result holds independently of whether RA sets forbearance such that she imposes
losses on the insurance fund ('late intervention') or not ('early intervention'). If RA’s for-
bearance policy exceeds bound $\bar{a}$, conditional on resolution the proceeds from liquidating
remaining assets undercut the insured amount of deposits, and the deposit insurance fund
becomes liable. Even if RA sets forbearance at the highest possible value $a = 1 > \bar{a}$,
the coordination problem among depositors will prevail since by assumption the asset
is not sufficiently liquid to cover the face value of debt at the interim stage $l < 1$. By
setting her forbearance policy, RA balances the haircut and strategic uncertainty among
depositors. A forbearance policy of $a = 1$ corresponds to the standard case that the bank
is on her own when facing a run, there is no intervention, see Goldstein and Pauzner
(2005) and the baseline model of Diamond and Dybvig (1983). Depositor’s behavior is
further sensitive to deposit insurance coverage.

**Lemma 3.2 (Decline of market discipline).** Bank stability monotonically increases in
deposit insurance coverage. As deposit insurance coverage becomes full, depositors always
roll over and bank runs cannot occur.

This result provides a theory foundation for observations in Iyer et al. (2016) who
show that less insured depositors are more prone to run than higher insured depositors.
Deposit insurance coverage bounds the haircut (deviation loss) and thus the downside

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25 The marginal depositors’ posterior belief that the bank is resolved decreases as RA shows more
forbearance.
risk to the action of rolling over, see equation (8). As coverage increases, the maximum loss a depositor faces when being involved in the resolution proceedings, the uninsured part of the deposit, declines while the upside, earning $H$ remains constant. The incentive to withdraw thus goes down. As the depositor becomes fully insured, she rolls over her deposit for every signal no matter how large the inferred solvency shock on the bank and market discipline exercised by withdrawing collapses. As a consequence of this result, under full insurance coverage, the investment in the risky asset is always continued. We discuss implications of this result in a later subsection.

Alterations in deposit insurance coverage have an effect on the maximum forbearance policy $\overline{a}$ at which the insurance runs no losses. Intuitively, as the insurance company has higher obligations, for RA to fully protect the insurance fund she has to intervene sooner thus the maximum lenience RA can show towards the bank to prevent losses to the insurer goes down.

4 Welfare - Ex ante stage

We now proceed to the ex ante stage at which the RA sets her forbearance level and the level of insurance coverage taking as given depositors’ behavior in the following period. We start with the benchmark case $r = l$ and discuss varying levels of RA efficiency $r \neq l$ in later extensions.

4.1 Efficient Liquidation

Since the asset is risky and RA liquidates at market price $l$, liquidation of the asset is efficient when the continuation value drops below the liquidation value, i.e. if the asset return likelihood realizes below the efficiency cut-off

$$\theta_e = \frac{l}{H}$$

In our model, the only mechanism which enforces liquidation of investment is a bank run with subsequent bank resolution. Bank resolution takes place for state realizations below critical state $\theta_b$. Define welfare at RA’s policy $(a, \gamma)$ as the total value from investment realized at forbearance policy $a$ and insurance coverage $\gamma$. For states below the critical state the bank is resolved which results in realized liquidation value $l$, while for states above the critical state investment is continued.

\footnote{In particular, the bank does not liquidate assets voluntarily at the interim stage since she cannot observe the state, only the sequence of withdrawals to which she responds by pledging assets to repay. Even if she could observe the state she would not liquidate voluntarily (when depositors roll over), since for every unit she liquidates she realizes proceeds $l$ which are not sufficient to cover claims $\delta k > \delta > l$. This is result is due to a lack of reinvestment opportunities and the bank’s liquidity mismatch, see Schilling (2017a).}
\[ W(a, \gamma) = l \theta_b(a, \gamma) + \int_{\theta_b(a, \gamma)}^{1} \theta H \, d\theta \]  

(14)

Since the bank is all debt financed, welfare equals the total value of debt inferred from the demand deposit contract \( V_D \) plus the value of the deposit insurance fund \( \Gamma \).

**Lemma 4.1.**

\[ W = V_D + \Gamma \]  

(15)

The deposit insurance fund is financed and therefore owned by depositors. A policy which maximizes welfare is therefore a policy that maximizes depositors joint value of the insurance fund and the given contract. Further, a policy that implements the first best outcome is such that the given demand deposit contract is the optimal contract given the insurance fund and RA’s policy. When imposing zero deposit insurance, i.e. in case of money market funds and investment banks, the welfare maximizing policy set by RA is equal to the policy which maximizes the value of debt at the given contract. If the bank was only partially financed with deposits and otherwise with equity, total value of investment would equal the value of the bank, i.e. value of equity and debt plus value of the insurance fund, see later extension.

In the first best case, liquidation takes place only for states below \( \theta_e \),

\[ W^{FB} = l \theta_e + \int_{\theta_e}^{1} \theta H \, d\theta \]  

(16)

Define the *deadweight loss* at policy \( (a, \gamma) \) as the difference

\[ D(a, \gamma) = W^{FB} - W(a, \gamma) = \int_{\theta_e}^{\theta_b(a, \gamma)} (\theta H - l) \, d\theta \]  

(17)

The deadweight loss is minimal if RA can set her policy \( (a, \gamma) \) in a way such that depositors run on the bank and cause bank resolution if and only if liquidation of investment is efficient. In particular, bank runs are not generically welfare deteriorating in our set up since enforcement of liquidation can be efficient.

We will show that RA’s optimal policy always attains first best, thus the contract \((1, H)\) is an optimal contract at RA’s optimal policy. RA’s policy can therefore be understood as a mechanism to alter depositors behavior in a way that makes the contract \((1, H)\) optimal. In a later extension we discuss robustness of our results to a general deposit contract.

The change in forbearance and insurance coverage *indirectly* impact welfare and the deadweight loss via the change in depositors behavior and thus critical bankruptcy state \( \theta_b \). Here, the relative position of the critical bankruptcy state compared to the efficient
liquidation cut-off becomes crucial. If the chosen policy \((a, \gamma)\) is such that the resulting bankruptcy state \(\theta_b\) undercuts the efficient liquidation cut-off, 'overinvestment' occurs in the range \([\theta_b, \theta_e]\). This is, when depositors are not sufficiently responsive to bad news and roll over their deposit although liquidation of assets is efficient, see Figure 3. As a consequence, stability improvements (decrease in critical bankruptcy state) can harm welfare if inefficient continuation of investment becomes more pronounced. If on the other hand the bankruptcy state exceeds the efficient liquidation cut-off, \([\theta_e, \theta_b]\), depositors are overly sensitive to bad news and run inefficiently often. A raise in bank stability would therefore lower the chance of inefficient runs and increase welfare.

![Figure 3: Realized welfare in blue. For \(\theta_b < \theta_e\) investment is continued too often due to too high deposit insurance. For \(\theta_b > \theta_e\) there are inefficient runs due to too low insurance.](image)

The following result tells us that the first best outcome is not attainable if insurance coverage is too high or too low, no matter when RA intervenes.

**Lemma 4.2.** If deposit insurance is low, inefficient asset liquidation enforced by runs occurs for every forbearance policy and for every recovery rate \(r \in (0, 1)\). If deposit insurance is high, inefficient continuation of investment occurs for every forbearance policy and for every recovery rate \(r \in (0, 1)\).

The negative result of Lemma 4.2 makes clear, the optimal forbearance policy crucially depends on the amount of insurance coverage and foremost, RA’s forbearance policy alone is not as strong a policy parameter. Insurance coverage fundamentally impacts the relative position of the bankruptcy state to the efficiency cut-off, see Figure 3. The result is intuitive when considering that by Lemma 3.2 depositors become unresponsive to bad news on the bank fundamental as insurance coverage becomes high, see Figure 4. Further, by Lemma 3.2 we know that bank runs become more likely as deposit insurance becomes low. This is, since the amount of deposit insurance coverage provides a bound for the haircut. As insurance coverage goes to zero, depositors are exposed to a potentially full loss of their deposit and are therefore overly 'sensitive' to bad news, they withdraw too often which gives rise to inefficient runs, see Figure 4. As coverage goes to one (full coverage), depositors face no losses when taking 'wrong' actions, and thus pay only little attention to bad news on the bank fundamental. Depositors roll over even for news
Figure 4: As insurance coverage increases, the critical state decreases and liquidation of investment enforced by runs becomes less likely. For high coverage, the bankruptcy state undercuts the efficiency cutoff $\theta_e$ and there is inefficient continuation of investment for state realizations in $(\theta_b, \theta_e)$. For low coverage, the critical state exceeds the efficiency state and there are inefficient runs for states in $(\theta_e, \theta_b)$.

on large solvency shocks which gives rise to inefficient investment such that a higher propensity to run is socially desirable. To the best of our knowledge this result is new to the literature. In both Diamond and Dybvig (1983) and Goldstein and Pauzner (2005) there is no inefficient investment since in Diamond and Dybvig the asset is safe while in Goldstein and Pauzner (2005) there is no deposit insurance such that the bound to the lower dominance region agrees with the efficiency cut-off and as a consequence runs enforce liquidation if liquidation is efficient. See also Iyer et al. (2016) and Goldberg and Hudgins (2002); Baer et al. (1986); Goldberg and Hudgins (1996) on evidence that insured as opposed to uninsured depositors are less likely to withdraw if a bank is hit by a solvency shock.

Lemma 4.2 is in contrast to Diamond and Dybvig (1983) where suspension of convertibility can prevent inefficient runs and implement first best despite no deposit insurance provision. The difference stems from two differences in the model. First, the asset here is risky thus RA liquidates while in Diamond and Dybvig (1983) the asset is safe thus liquidation was inefficient and the resolution authority continues investment. Our assumption that RA liquidates immediately can be justified when assuming that RA does not have the expert knowledge (Diamond and Rajan, 2001) to manage investment in the asset.\footnote{Even though RA learns the state by observing $n$, it can be that RA cannot reap high return $H$ by continuing investment.}

Second, in our model depositors who try to withdraw but are not served in the queue go into resolution proceedings where they are treated as equal to, i.e. receive same payoffs, as depositors who did not try to withdraw. By this, depositors who were not served in the queue receive no 'punishment' for causing resolution and are always weakly better
off compared to depositors who roll over. In Diamond and Dybvig (1983) in contrast,
depositors not served in the queue receive zero while depositors who roll over receive a
pro rata share under resolution which deters depositors from withdrawing ex ante if RA
plays her game right.

Putting Lemma 4.2 and Proposition 5.1 together, RA’s forbearance affects the level
of bank stability but deposit insurance coverage determines additionally whether the
increase in stability is desirable from a social perspective or not.

4.2 Optimal forbearance policy

For given deposit insurance coverage define the optimal forbearance policy \( a^*(\gamma) \) as

\[
a^*(\gamma) \in \arg \min D(a, \gamma) \quad \text{subject to feasibility } a^*(\gamma) \in (a, 1]
\]

The change of deadweight loss in forbearance depends on changed coordination be-
havior and on whether at the provided insurance coverage level inefficient investment or
liquidation occurs:

\[
\frac{\partial}{\partial a} D(a, \gamma) = \frac{\partial \theta_b}{\partial a} \cdot (\theta_b H - l)
\]

As forbearance increases, bank stability improves since depositors’ propensity to run
decreases, but more stability lowers the deadweight loss if and only if inefficient runs exist,
i.e. if deposit insurance coverage is low. In that case, more forbearance makes inefficient
runs ex ante less likely. If insurance coverage is high, there is inefficient investment
since depositors roll over despite bad news on the bank fundamental. Since stability
increases in forbearance, higher forbearance in combination with high deposit insurance
is detrimental to welfare since inefficient investment becomes more likely. Consequently,

**Theorem 1** (Optimal Forbearance - Benchmark Case: \( r = 1 \))

a) If deposit insurance is low, the deadweight loss monotonically decreases in forbearance
and is minimized by never intervening \( a^* = 1 \).

b) If deposit insurance coverage is high, the deadweight loss monotonically increases in
forbearance and is minimized by intervening as soon as feasible \( a^* = a \).

The Theorem considers the entire range of forbearance policies and therefore the
imposition of losses on the insurance fund in case of (a). To understand result (a), under
low deposit insurance coverage, depositors are very sensitive to bad news on the bank
fundamental since they face a loss of a large fraction of their deposit when choosing the
Figure 5: Change of deadweight loss in forbearance for low and high insurance coverage. In the right graph we see, as coverage increases, the deadweight loss changes monotonicity in forbearance. Further we see that the deadweight loss is not monotone in insurance coverage: In the left graph the curve for deadweight loss shifts down as coverage goes up but in the right graph the curve starts shifting upwards as coverage becomes very high. Parameters: $l = 0.3$, $H = 4$

If insurance coverage is 'high', the result reverts. Under high coverage, depositors are insensitive to bad news on bank solvency and do not withdraw even for severe solvency shocks on the bank. Thus, investment is continued inefficiently often and less stability in form of a higher propensity to withdraw is desirable from a social perspective. This is achieved by setting a policy to intervene as soon as feasible by this minimizing the
likelihood of inefficient investment ex ante. In Figure 6, we see as insurance coverage approaches full coverage, the slope of the deadweight loss switches from negative to positive and fast intervention is desirable from a social perspective, by this minimizing public losses\textsuperscript{28}. The U.S. forbearance policy to intervene fast is thus optimal, under high insurance coverage and if RA liquidates at market conditions. To put this in perspective, note that as of 2016, insurance coverage of US domestic deposits is 59\% (FDIC, 2016).

By setting the tolerance threshold to zero however, the least desirable outcome is realized under high insurance. This is since under no tolerance the game structure changes since depositors’ incentive to withdraw vanishes. Under zero forbearance, the event bank resolution is caused for arbitrarily small but nonzero measure of withdrawals. In addition, conditional on resolution a withdrawing investor obtains the pro rata share for sure since the probability of getting served one unit is zero. As a consequence, under resolution depositors who try to withdraw and those who roll over are treated equally and the first come first serve incentive to withdraw before resolution takes place vanishes. When rolling over, a depositor can obtain the convex combination \( \theta H + (1 - \theta) \gamma \) if all other depositors roll over with her. The value of the convex combination always exceeds the value of the pro rata share for high enough insurance coverage such that rolling over is a dominant action for each depositor. For high insurance coverage the critical state increases and approaches the efficiency cut-off from below as forbearance goes down to \( a \) but then jumps down to zero in \( a = 0 \). Thus, there is inefficient continuation of investment in the full range \([0, \theta_e]\). While minimum forbearance \( a = a^* \) is the optimal forbearance policy in case of high insurance, where \( a \) can be arbitrarily close to zero, no forbearance implements the worst outcome under high insurance. Lower bound \( a > 0 \) but arbitrarily close to zero maintains market discipline exercised by depositors, i.e. depositors respond to information on solvency shocks conveyed by their signals.

The imposition of a minimum forbearance level has also legal reasons. First, in the U.S., the FDIC has to obey a forbearance minimum, the asset to debt ratios has to be below a critical threshold otherwise interventions is not legally justified. One can interpret \( a \) as the delay with which RA monitors withdrawals.

\subsection{4.3 Optimal policy}

Given that RA may not attain the first best outcome by choosing forbearance if insurance coverage is too high or low we now allow RA to not only set forbearance but also the

\textsuperscript{28} We have \( \theta < \pi \), thus the pro rata share recovered under \( a^* = a \) exceeds the insured fraction of the deposit.
amount of deposit insurance coverage. Define RA’s optimal policy \((a^*, \gamma^*)\) as

\[
(a^*, \gamma^*) \in \arg \min D(a, \gamma) \quad \text{subject to } a^* \in (a, 1]
\]  

(20)

**Theorem 2 (Optimal insurance coverage - Multiplicity of optimal policy)**

*For every forbearance policy \(a \in (a, 1]\) there exists a unique interior level of insurance coverage \(\gamma^*(a) \in (0, 1)\) such that the pair \((a, \gamma^*(a))\) achieves the first best outcome*

\[
\theta_b(a, \gamma^*(a)) = \theta_e
\]

(21)

and the deadweight loss strictly decreases in insurance for \(\gamma < \gamma^*(a)\) and increases in insurance for \(\gamma > \gamma^*(a)\). The optimal coverage level \(\gamma^*(a)\) decreases in forbearance. In particular, there are infinitely many optimal policy pairs \((a, \gamma^*(a))\).

![Change of deadweight loss in insurance coverage](image)

\[\text{Figure 6: Change of deadweight loss in insurance coverage for different degrees of forbearance. At each forbearance level } a \text{ the deadweight loss is minimized and brought to zero at some unique, interior coverage level } \gamma^*, \text{ marked by vertical lines. The minimizer } \gamma^* \text{ decreases (moves to the left) as forbearance goes up, that is the vertical line moves to the left for higher } a.\]

Optimal insurance coverage is interior since for too high coverage, depositors ignore their signals and inefficient investment exists, but for too low coverage depositors pay too much attention to their signals and withdraw too often thus there are inefficient runs, see Figure 6. Here, most notably is the difference to Lemma 4.2: While for too high or too low insurance coverage there exists no forbearance policy that could prevent inefficient investment respectively runs, for every forbearance policy there exists a unique, optimal amount of insurance coverage RA needs to provide in order to implement that depositors coordinate on enforcing bank resolution if and only if liquidation of investment is efficient. As a consequence, deposit insurance coverage is the stronger policy parameter as opposed
to the forbearance policy. Since RA can always achieve the first best outcome by setting forbearance and insurance coverage, this means that the initially imposed debt contract \((1, H)\) is optimal to depositors at RA’s optimal policy in the sense that the combined value of debt inferred from the contract and the insurance fund owned by depositors is at maximum, see (15). To give intuition for why the optimal amount of insurance coverage has to decrease in forbearance, assume forbearance and insurance coverage are such that the first best outcome obtains. As RA shows more forbearance, depositors’ propensity to withdraw goes down which gives rise to inefficient investment. To maintain depositors’ propensity to withdraw, the deviation loss has to go up, RA has to lower insurance coverage, see Figure 6.

The multiplicity of optimal pairs of forbearance and insurance coverage implies that RA can achieve the first best outcome by arbitrarily fixing the forbearance threshold and only optimizing via insurance coverage alone, by this reducing the number of policy parameters. Multiplicity has another consequence when reconsidering identity (15). Since there are infinitely many policies which achieve first best, RA can maximize the direct utility from the contract conditional on having found an optimal policy which is equivalent to minimizing losses of the insurance fund.

### 4.4 Minimizing public losses

Given that there is an infinity of optimal policy pairs, we can refine RA’s objective to choose the pair \((a, \gamma^*(a))\) among all optimal policy pairs to minimize losses to the deposit insurance fund. Payments from the insurance fund to depositors at optimal pair \((a, \gamma^*(a))\) equal

\[
(1 - la) \max(0, (\gamma^*(a) - s(a))) \int_0^{\theta_e} d\theta + \gamma^*(a) \int_{\theta_e}^1 (1 - \theta) d\theta
\]

(22)

This is since resolution occurs for optimal pairs only for state realizations below the efficient cut-off, the measure of depositors involved in resolution is \(1 - al\) and the fund compensates each depositor up to \(\gamma\) less the pro rata share RA recovers from seizing assets. On the other hand, if no resolution takes place, there is still the chance that the risky asset does not pay off in which case depositors are also compensated, see the second term in (22).

**Lemma 4.3.** Public losses are strictly increasing in insurance coverage and increasing in forbearance.

Intuitively, as insurance coverage increases, expenditures to the fund go up in case it becomes liable. Further, the fund becomes liable more often since the fund is obliged to pay if the pro rata share falls below the covered part of the deposit. As forbearance
increases, conditional on resolution the seized fraction of the asset is smaller which lowers the pro rata share and again the fund becomes liable more often.

We obtain as a corollary

**Proposition 4.1**

*Conditional on the policy \((a, \gamma)\) being optimal, minimization of public losses requires 'late' intervention \(a \in [\overline{a}, 1]\).*

Optimal policies which imply early intervention are not minimizing public losses. Consequently, 'early' intervention \(a \in (a, \overline{a})\) is either not optimal or imposes unnecessary losses on the insurance fund. Remember, \(\overline{a}\) is the forbearance level at which the pro rata share equals insurance coverage and the insurance fund just becomes liable. Note, since the optimal insurance coverage level \(\gamma^*(a)\) strictly decreases in forbearance, it is not possible to maintain optimality and simultaneously lower both forbearance and insurance coverage. To see the result, as the resolution authority chooses to intervene 'early', \(a \in (a, \overline{a})\), the insurance fund is not liable in case of resolution since the pro rata share exceeds the insured amount of the deposit, the first term in \((22)\) is zero. As long as the pro rata share is above the covered amount, losses to the insurance fund can be strictly diminished by lowering insurance coverage. For the policy to remain optimal, as insurance coverage goes down forbearance needs to increase. Insurance coverage can be decreased to the point where the required value of forbearance to maintain optimality hits the bound \(\overline{a}\). Note, \(\overline{a}\) is a decreasing function of insurance coverage, see 4, thus as coverage declines, the range of values for forbearance at which the insurance fund is not liable \((a, \overline{a})\) widens.

In the U.S it is the FDIC's declared objective to intervene fast in order to minimize public losses. Lemma 4.2 tells us that this policy is not socially optimal under low insurance coverage. Corollary 4.1 in addition tells us, that fast intervention cannot be optimal and loss minimizing at the same time. If fast intervention is optimal at the provided amount of insurance coverage, there exists another optimal policy pair which requires more forbearance and offers lower insurance coverage at which public losses are strictly lower.

## 5 Extensions

### 5.1 Efficiency of Liquidation

In this subsection we analyze changes to our results when RA liquidates assets at different terms from the market \(r \neq l\). Reasons for this can be that RA has more time to resolve assets and therefore finds a buyer with higher willingness to pay \(r > l\) or RA is less skilled in searching for a buyer as opposed to the bank \(r < l\). In general, for the case \(r \neq l\), the
forbearance policy now determines the overall realized proceeds from liquidation

\[ T(a) = al + (1 - a)r \]  

available to depositors at the interim period should resolution occur. We first discuss the case \( r > l \).

5.1.1 Case: \( r \geq l \)

The efficient liquidation cut-off becomes

\[ \theta_e = \frac{r}{H} \]  

As RA’s forbearance with the bank increases, more assets are pledged during the run\(^{29}\) at low liquidation value as opposed to being liquidated by RA during resolution at higher recovery rate. Denote by

\[ \theta_r(a) = \frac{T(a)}{H} \]  

the state at which realized liquidation value \( T(a) = al + r(1 - a) \) equals continuation value. As RA liquidates at higher efficiency than the market, depositors’ coordination behavior changes.

**Proposition 5.1** (Comparative statics - critical state)

Assume RA intervenes before the insurance runs a loss \( a \in (\overline{a}, \overline{a}] \).

1a) For recovery rate equal or close to liquidation value in the market, bank stability monotonically improves in forbearance \((r = l \text{ benchmark case})\).

1b) As RA’s recovery rate increases, stability can become non-monotone in forbearance with a unique interior, stability maximizing policy. As recovery rate increases further, stability becomes monotone decreasing in forbearance.

2) As RA sets forbearance such that the insurance runs a loss \( a \in (\overline{a}, 1] \), stability monotonically improves in forbearance.

The results are depicted in Figure 7 and are due to a shift in trade-off as RA liquidates more efficiently. In general, depositors face a trade-off as RA’s forbearance to resolve a bank increases. The more withdrawals RA tolerates, the larger becomes the haircut (deviation loss) depositors have to take if the bank is resolved. This is, since under more tolerance the bank is exposed to the run for a longer time\(^{30}\). The proportion of the asset which RA seizes and liquidates during bank resolution is therefore smaller which decreases the pro rata share. Shortly, as forbearance increases, more depositors

\(^{29}\) We assume, if the bank is resolved the outside investor liquidates the pledged fraction of the asset \( a \) at market value \( l \) since the bank is an investment expert and stops managing the asset.

\(^{30}\) In terms of depositors she has to serve until intervention.
obtain the full coupon when withdrawing at the expense of either depositors who roll over or the deposit insurance fund. As the haircut increases, the coordination problem among depositors is aggravated and bank stability goes down. On the other hand, as RA’s forbearance increases and tolerates more withdrawals, the run has to be larger for bank resolution to take place, thus rolling over is the optimal action ‘more often’ which relaxes the coordination problem.\textsuperscript{31} Allover, more lenience with the bank can have a non-monotone impact on bank stability.

![Figure 7](image)

(a) $r = 0.5 = l$

(b) $r = 0.75$

(c) $r = 0.9$

(d) $r = 0.99$

Figure 7: Monotonicity of trigger varies in forbearance as recovery rate changes for different levels of deposit insurance $\gamma$. The kinks arise as the pro rata share $s(a)$ falls below the insured fraction of the deposit $\gamma$ as forbearance becomes high. Consequently the haircut is at $1 - \gamma$ and stops changing in $a$. Since the trigger monotonically decreases in insurance coverage $\gamma$, monotonicity is preserved as $\gamma$ changes. Held fixed through all graphs: $H = 4$, $l = 0.5$.

The last effect is independent of RA’s liquidation efficiency while the first effect becomes stronger as RA’s recovery rate goes up, the haircut increases faster in forbearance.

\textsuperscript{31}The marginal depositors’ posterior belief that the bank is resolved decreases as RA shows more forbearance.
As a consequence, the relative strength of these effects alters in recovery rate. Intuitively, as RA liquidates more efficiently than the market late intervention becomes more costly to depositors since overall realized proceeds $T(a)$ are highest if RA would intervene right away. In the benchmark case the RA liquidates at market terms. Consequently, to depositors there is no efficiency loss as RA intervenes sooner or later since overall funds raised from liquidating the asset, before and after intervention, remain the same and since depositors are risk-neutral. Thus, forbearance determines how funds are reallocated from depositors who roll over to withdrawing depositors. As RA liquidates at market terms, the change in haircut is slow as RA shows more forbearance and the decrease in depositors’ propensity to withdraw dominates, see Proposition 5.1. As RA liquidates more efficiently this result reverts and the increase in haircut becomes dominant.

Further, as recovery rate increases, RA’s maximum forbearance level $\overline{\alpha}$ to prevent losses to the insurer increases since proceeds she recovers rise. As RA sets a level such that the insurance runs a loss, the haircut becomes constant in forbearance and just equals the uninsured fraction of the deposit. Thus, the trade-off between an increase in haircut and decrease in belief that resolution occurs vanishes.

As RA liquidates more efficiently than the market, there is a direct deadweight loss due to forbearance if the bank is resolved since proceeds $r - T(a) > 0$ are lost. Redefine the deadweight loss at forbearance policy $a$ as the difference between first best and welfare at forbearance policy $a$

$$D_{(r, \gamma)}(a) = W^{FB} - W = (r - T(a)) \theta_{\overline{\alpha}}(a) + \int_{\theta_{\alpha}}^{\theta_{\overline{\alpha}}} (\theta H - r) d\theta \tag{26}$$

As before, the relative position of the critical state to the efficiency cut-off matters for welfare and optimal policy. The critical state is additionally influenced by RA’s recovery rate.

**Lemma 5.1.** Bank stability monotonically increases in RA’s recovery rate.

Lemma 4.2 holds for general $r \in (0, 1)$ and thus also for $r > l$. One difference is that the efficiency cut-off is no longer constant but increases with recovery rate, see Figure 8. As a consequence, for general $r > l$ we maintain the negative result from the benchmark case that for too high or low provision of insurance coverage there exists no forbearance level which implements the first best outcome.

The optimal forbearance policy for given insurance coverage is

**Theorem 3** (Case: $r > l$)

Assume low deposit insurance. For $r$ close to $l$, $r > l$, the deadweight loss increases in forbearance globally for all $a \in (\underline{a}, 1]$ and no intervention is optimal $a^* = 1$. For $r$ high compared to $l$ the deadweight loss increases in forbearance, as long as the insurer runs no losses $a \in (\underline{a}, \overline{a})$. Immediate intervention $a^* = \overline{a}$ minimizes the deadweight loss locally.
Figure 8: As recovery rate increases, the efficient liquidation cut-off increases while the critical state goes down. This is since under higher recovery rate, asset liquidation is efficient for a greater range of states while bank stability improves in recovery rate since haircuts decrease. As insurance coverage increases, inefficient continuation already occurs for lower recovery rates as opposed to under low insurance coverage since depositors become less sensitive to bad news.

Figure 9: Change of deadweight loss in forbearance for varying recovery rate functions across different deposit insurance coverage. As recovery rate increases away from the asset’s liquidation value the deadweight loss changes monotonicity in forbearance. The deadweight loss goes from decreasing to increasing in forbearance independently of insurance coverage. For low insurance coverage the deadweight loss decreases pointwise at
The results are demonstrated in Figure 9. For intuition, as recovery rates are close to the asset’s liquidation value, the direct loss is small since RA liquidates at terms similar to the market and we are therefore in or close to the benchmark case. As recovery rates go up, two new effects come into play, first, RA liquidates more efficiently than the bank, thus the direct loss goes up. Second, bank stability no longer improves but deteriorates in forbearance as long as the insurance runs no losses (early intervention), see Proposition 5.1. Further, under low deposit insurance inefficient runs exist, thus less stability is detrimental to welfare. Allover, under low deposit insurance and high recovery rates the deadweight loss increases in forbearance as long as the insurance runs no losses, $a \in (a, \bar{a})$. The effect becomes ambiguous as RA imposes losses on the insurance, $a > \bar{a}$. Under late intervention $a > \bar{a}$, depositors receive the insured amount under resolution independently of forbearance and stability changes monotonically and increases as forbearance exceeds $\bar{a}$. Thus, inefficient runs become less likely but the direct loss continues to increase. As we transition to higher insurance coverage and maintain high recovery rates the case becomes even less clear. The direct loss continues to increase in forbearance which lowers welfare. But under high insurance there is inefficient investment. Since more forbearance lowers stability for high recovery rates as long as the insurance runs no loss $a < \bar{a}$, more forbearance makes inefficient investment less likely and increases welfare. This effect opposes the effect of the direct loss. In Figure 9, for high insurance the slope hardly reacts as recovery rates increase.

For general $r \neq l$, we obtain a version of Theorem 5.2

**Proposition 5.2**

Let $r, l \in (0, 1)$ general. For every given forbearance level, the deadweight loss is minimized when insurance coverage is such that the critical state matches

$$\theta_b = \frac{T(a)}{H}$$

(27)

For every forbearance level there exists a unique interior insurance coverage level $\gamma^*(a) \in (0, 1)$ which satisfies (27), and the deadweight loss strictly decreases in insurance for $\gamma < \gamma^*(a)$ and increases in insurance for $\gamma > \gamma^*(a)$. The optimal level $\gamma^*(a)$ decreases in forbearance if RA liquidates at $r < l$ or $r > l$ and sufficiently close to $l$ but increases in forbearance if $r >> l$.

There are two distinctions to the benchmark case. First, for general $r \geq l$ the deadweight loss is not minimized as the critical state matches the efficiency cut-off $\theta_b = r/H$ but as it takes the lower value $T(a)/H$. This is, because under $r > l$ the deadweight loss has a second component, namely the direct loss from showing forbearance, see the first term in (26). Second, for high recovery rate, stability deteriorates in forbearance, thus the efficient insurance coverage level may become increasing in forbearance. For
the general case $r > l$, no level of deposit insurance coverage can implement the first best outcome, i.e. for every optimal pair $(a, \gamma^*(a))$ we have $D(a, \gamma^*(a)) > 0$. This is since the RA has to obey a minimum forbearance level $a \geq a$ which implies $r > T(a)$ and $\theta_r(a) = \theta_b(a, \gamma^*(a)) < \theta_e$. Therefore, in the case $r > l$, all optimal policies imply inefficient investment, the interval $(\theta_b(a, \gamma^*(a)), \theta_e)$ is non-empty. Intuitively, if RA would intervene sooner a higher proportion of the asset could be seized and liquidated at RA’s higher liquidation rate.

**Lemma 5.2.** For given forbearance level, the deadweight loss decreases in recovery rate if insurance coverage is low but increases in recovery rate if insurance coverage is high.

The result is visible in Figure 9 and holds since stability monotonically improves in recovery rate but more stability increases welfare only if there are inefficient runs, that is under low insurance.

5.1.2 Case: $r < l$

In the case $r < l$, the efficient liquidation cut-off is

$$\theta_e = \frac{l}{H}$$

(28)

Redefine the *deadweight loss at forbearance policy $a$* as

$$\hat{D}(r, \gamma)(a) = \hat{W}^{FB} - W = (l - T(a)) \theta_b(a) + \int_{\theta_e}^{\theta_b(a)} (\theta H - l) \, d\theta$$

(29)

We have

**Lemma 5.3.** Bank stability monotonically improves in forbearance, if $r \leq l$.

The comparative statics are thus as in the benchmark case. The trade-off between the increase in haircut but decrease in belief that the bank will be resolved still exists but the effect of the haircut is always dominated since the recovery rate is small. By Lemma 4.2, the negative result prevails for $r < l$. For insurance coverage high there is inefficient continuation of investment for every forbearance policy while for low insurance coverage there are inefficient runs independently of forbearance. As in the case with $r \geq l$, the deadweight loss entails an additional term compared to the benchmark case. With $r < l$, there is a direct efficiency gain as RA shows more forbearance since the bank liquidates more efficiently in the market than RA does. We obtain

**Proposition 5.3**

*Under an inefficient RA, $r < l$, if deposit insurance is low, maximum forbearance is optimal $a^* = 1$.***
Thus, committing ex ante to not intervene during runs is optimal for all recovery rates $r \leq l$ if insurance is low. The result is intuitive since under maximum forbearance the direct gain is highest. Further under low insurance there exist inefficient runs. By Proposition 5.3, the likelihood of inefficient runs is minimized by intervening late. Both effects go in the same direction. Under higher insurance the effect of more forbearance on the deadweight loss is ambiguous. Inefficient investment becomes more likely as stability increases in forbearance. This effect lowers welfare. On the other hand, there is a direct efficiency gain from showing more forbearance.

Considering the provision of optimal deposit insurance coverage, Theorem 5.2 holds for $r < l$, thus the deadweight loss is minimized when the critical state below which depositors enforce liquidation of assets is at the state $\theta_r(a) = \frac{T(a)}{H}$. When choosing $a = 1$, RA can match the efficient liquidation state $\theta_r(a)$ to the state $\theta_r(a)$ which puts the deadweight loss at zero. For all other forbearance policies $a \in (a, 1)$, first best is not attainable. In the case $r < l$, there is thus a unique policy $(a, \gamma^*(a)) = (1, \gamma^*(1))$ which implements the first best outcome, while first best is not attainable in the case $r > l$ and is attainable for infinitely many pairs $(a, \gamma^*(a))$ in the benchmark case $r = l$.

**Theorem 4**

In the case where RA liquidates less efficient than the market, there is a unique optimal policy which implements the first best outcome. Under this optimal policy RA never intervenes $a^* = 1$ and in return provides low insurance coverage $\gamma^*(1)$.

Under the policy to never intervene, optimal insurance coverage $\gamma^*(1)$ is low since optimal insurance coverage strictly decreases in forbearance by Theorem 5.2. Since the pro rata share recovered under no intervention equals zero the unique optimal policy imposes maximum losses on the insurance fund should resolution occur but in return a low level of insurance coverage is provided.

### 5.2 Cascading Forbearance in the European Monetary Union

So far we have analyzed the model from the perspective that there exists one resolution authority which can decide whether and how much forbearance to exercise. In Europe however, the case is that national authorities can decide to divert and by this cascade resolution from one central authority (European Stability Mechanism) to a national resolution authority. We assume that the national and central authority provide equal insurance coverage. Forbearance and recovery rate of the central and the national resolution authority may however in general differ. Denote by $(r_1, a_1)$ the exogenously given recovery rate and forbearance level of the centralized resolution authority and let $(r_n, a_n)$ the recovery rate and forbearance level of a national resolution authority. Given that national politics opt to divert resolution to the national authority, the bank is resolved at
recovery rate $r_n$ and forbearance level $a_2 = a_1 + a_n > a_1$. Exercising the option to delay and divert is optimal if and only if $a_n$ can be chosen such that

$$D(r_1, a_1) > D(r_n, a_2)$$

(30)

**Application 1 (Theorem 3)** Assume $r_n = r_1$. Assume insurance coverage is low. If recovery rate $r_1$ is close to market value $l$ of the asset, it is strictly optimal to exercise the option to delay and divert resolution to national authorities. If recovery rates are high though, is is strictly optimal to not delay and let the centralized resolution authority resolve the bank at lower forbearance level, see Figure 9.

**Application 2 (Lemma 5.2)** Assume the national RA works highly effective such that $a_2$ is close to $a_1$ and resolution takes place at higher recovery rate $r_n > r_1$ if diversion of resolution takes place. Then exercising the option to delay and divert resolution is strictly optimal if insurance coverage is low but not optimal if insurance coverage is high.

**Application 3 (Theorem 1)** Assume both the national and the centralized resolution authority liquidate as efficient as the market, then exercising the option to delay is not optimal if insurance coverage is low, but is strictly optimal under close to full insurance coverage.

### 5.3 Emergency Liquidity Assistance (ELA) and the lender of last resort

A different situation compared to the previous subsection emerges if the bank taps ELA instead of borrowing from outside investors directly. In Europe, ELA is paid by the national central bank to banks which are illiquid but solvent if the bank in question has trouble to raise cash in the market. In return, the bank has to provide assets as collateral which may be of ‘inferior’ quality, i.e. of quality not accepted by the market. If the bank accesses emergency liquidity assistance, the institution with whom the bank pledges the asset is the same as the resolution authority, since ELA is paid under supervision of the European Central Bank. In that case, the process of pledging the asset already increases proceeds available to depositors under resolution, should bank resolution occur. Assume that the outside investor $C$ refuses to lend cash to the bank when more than a critical proportion $h \in (0, a_l)$ of deposits are withdrawn. Consequently, for $n \in (h, a_l)$ the bank accesses ELA and in return pledges assets with the LoLR instead of with $C$. The LoLR in general provides no deposit insurance and does not close banks as the RA does. A LoLR therefore acts as a constrained version of the RA with $(\gamma, a) = (0, 1)$, i.e. no insurance coverage and allowing the bank to pledge assets up to $\ell$ without closing the bank. We
know, under no deposit insurance, there are inefficient runs independently of the level of forbearance. Thus, LoLR alone cannot achieve the first best outcome. Combining the LoLR with the RA, the case changes. As withdrawals exceed RA’s forbearance threshold, the bank is resolved and RA seizes remaining assets $1-a$ but also liquidates assets already pledged $(a-h/l)$ such that the pro rata share to depositors becomes

$$s_{ELA}(a) = \frac{r(1 - \frac{h}{l})}{1 - al}$$ (31)

Thus, the pro rata share to depositors under resolution now increases in forbearance, the haircut decreases in forbearance for every $r \in (0,1)$ while it was increasing under the previous model set-up without ELA. Thus, bank stability becomes monotonically increasing in forbearance for all $a \in (a,1]$ independently of whether RA liquidates more or less efficient than the market since the trade-off between haircuts and belief that bank resolution occurs vanishes.

**Lemma 5.4.** Under ELA, bank stability monotonically improves in forbearance for every $r,l \in (0,1)$.

We know however from previous results that RA can achieve the first best outcome already without LoLR. The results therefore show that an RA which sets both a forbearance policy and deposit insurance coverage makes a LoLR redundant.

### 5.4 Strategic Outside Investors

We have assumed that investor C is willing to borrow to the bank as much cash as the bank could access when selling the asset, maximum amount $l \in (0,1)$ at time $t_1$, and at interest rate $i = H$. We now address why C is willing to do this and why the bank prefers borrowing from C at rate $i = H$ instead of selling the asset. Assume outside investors are competitive and borrowing to the bank has to be individually rational, i.e. C has to break even in expectation. Assume, riskiness of the asset is common knowledge, that is C knows the distribution but does not observe the realization of the state $\theta$. Assume the bank can choose between selling assets and borrowing from C to prevent selling assets as she faces withdrawals at the interim period.

Schilling (2017b) shows, C breaks even and participates at every interest rate if she provides just as much liquidity as the bank can raise through selling the asset. If C provides more liquidity, there exists no interest rate at which she breaks even. She voluntarily provides up to $l$, the sale value of the asset since C sells the asset if the bank is wound down and thus stops managing the asset. Thus, if the bank faces a run, the bank will always access up to $l$ instead of accepting to default. The bank in return prefers borrowing from C as opposed to selling for every interest rate $i < H/l$ since then she partially avoids the liquidation cost. The reason we choose repo rate $i = H$ in this paper
is to obtain a parameter reduction. The bank is all debt financed, depositors who roll over would thus obtain an equity like pro rata share of total asset return which depends on the total measure of depositors who roll over, see for instance Diamond and Dybvig (1983) or Goldstein and Pauzner (2005). To prevent equity like payoffs and obtain results which generalize to banks which are partially financed with debt, the payoff to rolling over has to be constant in aggregate withdrawals which is achieved for $i = H$.\footnote{Under repo financing, the pro rata share to depositors who roll over when the bank borrows $x = n$ is $\frac{n - x}{1 - x} = H$} Alternatively to the outside investor, the lender of last resort can provide liquidity at the interim stage, see extension above. We below discuss what will change if $C$ sets rate $i \neq H$.

5.5 General Setting

In the model we have fixed demand deposit contract coupons at $(1, H)$ and the refinancing interest rate at $i = H$ to obtain a parameter reduction. For general interest rate $i \in (1, H/l)$, contract $(R_1, R_2)$, $l \leq R_1 < R_2 < H$ and debt ratio $\delta \in (0, 1)$ the bank is prone to runs if and only if

$$\delta R_1 > l$$  \hspace{1cm} (32)

We maintain this assumption from here on. Let again $n$ denote the proportion of withdrawing depositors. Then the measure of withdrawn funds equals $\delta n R_1$. Let $a \in (a, 1]$ RA’s forbearance policy. Resolution takes place if and only if

$$\delta R_1 n > la$$  \hspace{1cm} (33)

The payoff table becomes

<table>
<thead>
<tr>
<th>Event/ Action</th>
<th>Withdraw</th>
<th>Roll-over</th>
</tr>
</thead>
<tbody>
<tr>
<td>No resolution</td>
<td>$R_1$</td>
<td>$\begin{cases} R_2, p = \theta \ \gamma, p = 1 - \theta \end{cases}$</td>
</tr>
<tr>
<td>Bank resolution</td>
<td>$\frac{la}{\delta R_1 n} \cdot R_1 + (1 - \frac{la}{\delta R_1 n}) s_\gamma(a)$</td>
<td>$s_\gamma(a)$</td>
</tr>
</tbody>
</table>

with

$$s_\gamma(a) = \max \left( \gamma, \frac{r(1 - a)}{\delta - la/R_1} \right) = \max \left( \gamma, \frac{\frac{r}{2}(1 - a)}{1 - a \cdot \left( \frac{1}{\delta R_1} \right)} \right)$$  \hspace{1cm} (34)

since the measure of depositors served before resolution is $la/R_1$ thus the measure of depositors involved in resolution is $\delta - la/R_1$. For the payoff table to be consistent, conditional on no resolution taking place and the asset paying off high, the bank has to
be able to repay depositors who roll over the coupon $R_2$. In other words equity value has to be positive,

$$H - i\delta R_1 n > (1 - n)\delta R_2$$

for all $n > \frac{la}{\delta R_1}$ (35)

Assume that refinancing via outside investors is more expensive than financing via deposits, $i > R_2/R_1$. Condition $iR_1 > R_2$ is an incentive condition on the bank and says that the bank cannot make money by encouraging withdrawals by depositors in $t_1$. Then condition (35) holds if the asset return over total short-term exposure exceeds the repo rate.

$$\frac{H}{\delta R_1} > i$$

In that case, the game is consistent and can be analyzed as before. Define $\tilde{l} = \frac{l}{\delta R_1}$, recovery rate $\tilde{r} = \frac{r}{\delta R_1}$, $\tilde{\gamma} = \frac{\gamma}{\delta R_1}$, $\tilde{R}_2 = \frac{R_2}{\delta R_1}$. Since incentives are robust under rescaling of payoffs, the game above is equivalent to the game

<table>
<thead>
<tr>
<th>Event/Action</th>
<th>Withdraw</th>
<th>Roll-over</th>
</tr>
</thead>
<tbody>
<tr>
<td>No resolution $n \in [0, a \cdot \tilde{l}]$</td>
<td>1</td>
<td>$\bar{R}_2$, $p = \theta$</td>
</tr>
<tr>
<td>Bank resolution $n \in (a \tilde{l}, 1]$</td>
<td>$\frac{la}{n} \cdot 1 + (1 - \frac{la}{n})s_{\gamma,R_1}(a)$</td>
<td>$s_{\gamma,R_1}(a)$</td>
</tr>
</tbody>
</table>

where

$$s_{\gamma,R_1}(a) = \max \left( \tilde{\gamma}, \frac{\tilde{r}(1 - a)}{1 - a} \right)$$

Thus, all results from previous sections go through under the renamed parameters.

6 Conclusion

This paper has analyzed optimal strategic delay of bank resolution in combination with provision of deposit insurance. A resolution authority (RA) monitors withdrawals of depositors at the bank level. If withdrawals are 'abnormally high' due to a solvency shock, she has the authority to suspend conversion of deposits by putting the bank into receivership and seizing bank assets to protect a deposit insurance fund. We show, if RA can only set the forbearance level, inefficient runs or inefficient investment may exist, independently of when she intervenes, if insurance coverage is too low respectively too high. If RA liquidates at market terms and can set both the intervention threshold and insurance coverage she can always implement the first best outcome. This means, she can steer depositors’ incentives to withdraw in a way that bank resolution occurs if and only if asset liquidation is efficient.
Various avenues for future research exist. Crucial to our results is that RA can credibly commit ex ante to her policy. If commitment is not possible, RA may not want to enforce her policy, depositors anticipate this and change behavior accordingly. Ennis and Keister (2009) for instance study this time-inconsistency, focusing on ex post efficient intervention in a Diamond and Dybvig type model. In the setting here, RA’s major objective is to maximize welfare. The set-up can however be used to study outcomes under different objective functions such as minimizing public losses. The paper assumes a uniform distribution for the state to ease the analysis. Goldstein and Pauzner (2005) use a generalization, and we believe that our findings are robust when applying this generalization here as well. In the setting here, insurance coverage is partial as percentage of the deposit. In "real life", a fixed amount of the deposit is insured. If depositors hold deposits in excess of the fixed insured amount our setting fully applies. Similarly, we discuss the case of full insurance under which depositors become unresponsive. In a setting where some depositors are fully insured while others are only partially so, fully insured deposits are like equity to partially insured depositors. We have analyzed this case in an extension as well.

7 Appendix

7.1 Proof: Existence and Uniqueness

Proof. [proof Proposition 3.1] We first show equivalence of this game to a version of the game in Goldstein Pauzner: Conditional on a run occurring, the payoff difference from rolling over versus withdrawing equals

\[
\Delta = s_\gamma(a) - \left[ \frac{la_n}{n} \cdot 1 + (1 - \frac{la_n}{n})s_\gamma(a) \right]
\]

(38)

\[
= -\frac{la_n}{n} (1 - s_\gamma(a))
\]

(39)

set \( f(r, a, \gamma) = al \ (1 - s_\gamma(a)) > 0 \), then

\[
\Delta = -\frac{f(r, a, \gamma)}{n}
\]

(40)

and the game described is equivalent to a game which has close similarity to the model analyzed in Goldstein and Pauzner (2005)

<table>
<thead>
<tr>
<th>Event / Action</th>
<th>Withdraw</th>
<th>Roll-over</th>
</tr>
</thead>
<tbody>
<tr>
<td>no Run ( n \in [0, la] )</td>
<td>1</td>
<td>( { H, p = \theta } )</td>
</tr>
<tr>
<td>Run ( n \in (la, 1] )</td>
<td>( \frac{f}{n} )</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
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</tr>
<tr>
<td>Run ( n \in (la, 1] )</td>
<td>( \frac{f}{n} )</td>
<td>0</td>
</tr>
</tbody>
</table>
A: Existence and uniqueness of a trigger equilibrium  Closely following Goldstein and Pauzner (2005): For fixed contract \((1, H)\), recovery rate \(r\), forbearance policy \(a\), and insurance coverage \(\gamma\), a Bayesian equilibrium is a strategy profile such that each investor chooses the best action given her private signal and her beliefs about other players strategies. In equilibrium, an investor decides to withdraw when her expected payoff from rolling over versus withdrawing given her signal is negative, decides to roll over when it is positive and is indifferent if the expected payoff is zero. Since investors are identical ex ante, investors strategies can only differ at signals that make an investor indifferent between rolling over and withdrawing.

In a trigger equilibrium around trigger signal \(\theta^*\), all investors withdraw when they observe signals below \(\theta^*\) and roll over if they observe signals above \(\theta^*\). In case of directly observing \(\theta^*\) investors are indifferent and we specify here that they will roll over. A threshold equilibrium around trigger \(\theta^*\) exists if and only if given that all other investors use a trigger strategy around signal \(\theta^*\) an investor finds it optimal to also use a trigger strategy around trigger \(\theta^*\).

If all investors follow the same strategy, the proportion of investors who withdraw at each state is deterministic. Define \(n(\theta, \theta^*)\) as the proportion of investors who observe signals below signal \(\theta^*\) and thus withdraw if the state is \(\theta\), \(n(\theta, \theta^*) = \mathbb{P}(\theta_i < \theta^*|\theta)\). We can explicitly calculate \(n(\theta, \theta^*)\) using the distribution function of noise as given in (48). Note that if a continuum of investors but one single investor follow the same strategy this result continues to hold. Denote by \(D(\theta_i, n(\cdot, \theta^*))\) the expected payoff difference from rolling over versus withdrawing when the investor observes signal \(\theta_i\), and other investors follow a trigger strategy around \(\theta^*\). Since a run is triggered if the measure of withdrawing depositors \(n\) exceeds \(a\), we have

\[
D(\theta_i, n(\cdot, \theta^*)) = \frac{1}{2\varepsilon} \int_{\theta_i-\varepsilon}^{\theta_i+\varepsilon} (H\theta + \gamma(1 - \theta) - 1) \mathbf{1}_{\{n(\theta, \theta^*) \leq a\}} - \frac{f}{n(\theta, \theta^*)} \mathbf{1}_{\{n(\theta, \theta^*) > a\}} d\theta \quad (41)
\]

For existence of a trigger equilibrium we need to show

\[
D(\theta_i, n(\cdot, \theta^*)) < 0 \quad \text{for all } \theta_i < \theta^* \quad (42)
\]

\*

and existence and uniqueness of a signal \(\theta^*\) for which an investor is indifferent between rolling over and withdrawing (payoff indifference equality)
0 = D(\theta^*, n(\cdot, \theta^*)) = \frac{1}{2\varepsilon} \int_{\theta^* - \varepsilon}^{\theta^* + \varepsilon} (H \theta + \gamma(1 - \theta) - 1) \mathbf{1}_{\{n(\theta, \theta^*) \leq \alpha\}} - \frac{f}{n(\theta, \theta^*)} \mathbf{1}_{\{n(\theta, \theta^*) > \alpha\}} d\theta
\quad (44)

The function \( D(\theta^*, n(\cdot, \theta^*)) \) is continuous in \( \theta^* \). By existence of dominance regions, \( D(\theta^*, n(\cdot, \theta^*)) < 0 \) for \( \theta^* < \bar{\theta} - \varepsilon \) and \( D(\theta^*, n(\cdot, \theta^*)) > 0 \) for \( \theta^* > \bar{\theta} + \varepsilon \). By the Intermediate value Theorem there exists at least one \( \theta^* \in [\bar{\theta} - \varepsilon, \bar{\theta} + \varepsilon] \) for which (44) holds. To see uniqueness, since all other agents use a threshold strategy around \( \theta^* \), substitute for \( n(\theta, \theta^*) = \frac{1}{2} + \frac{\theta - \theta^*}{2\varepsilon} \) and derive

\[
D(\theta^*, n(\cdot, \theta^*)) = \int_{\theta}^{\alpha} (H\theta(n, \theta^*) + \gamma(1 - \theta(n, \theta^*)) - 1) \, dn - \int_{\alpha}^{1} \frac{f}{n} \, dn
\quad (45)
\]

where \( \theta(n, \theta^*) = \theta^* + \varepsilon(1 - 2n) \), \( \theta^* \in [\bar{\theta} - \varepsilon, \bar{\theta} + \varepsilon] \) is the inverse of the function \( n(\theta, \theta^*) \). For uniqueness, \( D(\theta^*, n(\cdot, \theta^*)) \) is strictly increasing in signal \( \theta^* \) for \( \theta^* < \bar{\theta} + \varepsilon \) which gives single-crossing.

Next, show (42): Following Goldstein and Pauzner (2005), let \( \theta_i < \theta^* \). Decompose the intervals \([\theta_i - \varepsilon, \theta_i + \varepsilon]\) and \([\theta^* - \varepsilon, \theta^* + \varepsilon]\) over which the integrals \( D(\theta_i, n(\cdot, \theta^*)) \) and \( D(\theta^*, n(\cdot, \theta^*)) \) are calculated into a potentially empty common part \( c = [\theta_i - \varepsilon, \theta_i + \varepsilon] \cap [\theta^* - \varepsilon, \theta^* + \varepsilon] \) and the disjoint parts \( d^i = [\theta_i - \varepsilon, \theta_i + \varepsilon] \setminus c \) and \( d^* = [\theta^* - \varepsilon, \theta^* + \varepsilon] \setminus c \). Then,

\[
D(\theta_i, n(\cdot, \theta^*)) = \frac{1}{2\varepsilon} \int_{\theta \in c} v(\theta, n(\theta, \theta^*)) \, d\theta + \frac{1}{2\varepsilon} \int_{\theta \in d^i} v(\theta, n(\theta, \theta^*)) \, d\theta
\quad (46)
\]

\[
D(\theta^*, n(\cdot, \theta^*)) = \frac{1}{2\varepsilon} \int_{\theta \in c} v(\theta, n(\theta, \theta^*)) \, d\theta + \frac{1}{2\varepsilon} \int_{\theta \in d^*} v(\theta, n(\theta, \theta^*)) \, d\theta
\quad (47)
\]

Considering (47), the integral \( \int_{\theta \in c} v(\theta, n(\theta, \theta^*)) \, d\theta \) has to be negative since by (44) \( D(\theta^*, n(\cdot, \theta^*)) = 0 \) and since the fundamentals in range \( d^* \) are higher than in \( c \). This is, since we assumed \( \theta_i < \theta^* \) and because in interval \([\theta^* - \varepsilon, \theta^* + \varepsilon]\) the payoff difference \( v(\theta, n) \) is positive for high values of \( \theta \), negative for low values of \( \theta \) and satisfies single-crossing. In addition, the function \( n(\theta, \theta^*) \) equals one over the interval \( d^i \), since \( d^i \) is below \( \theta^* - \varepsilon \) and thus all other investors withdraw. Therefore, the integral \( \int_{\theta \in d^i} v(\theta, n(\theta, \theta^*)) \, d\theta \) is negative too which with (46) implies that \( D(\theta_i, n(\cdot, \theta^*)) \) is negative. The proof for \( \theta_i > \theta^* \) proceeds analogous.

**B No existence of non-monotone equilibria**

See Goldstein and Pauzner, proof of Theorem 1, first page of part C.
7.2 Proof: Comparative statics Trigger

Proof. [Lemma 3.1] 
First, by uniqueness of a trigger equilibrium the proportion of withdrawing depositors $n$ is a deterministic function of the state and is given by

$$n(\theta, \theta^*) = \mathbb{P}(\theta_i < \theta^* | \theta) = \mathbb{P}(\varepsilon_i < \theta^* - \theta | \theta) = \left\{ \begin{array}{ll} \frac{1}{2} + \frac{\theta^* - \theta}{2\varepsilon}, & \theta_i \in [\theta^* - \varepsilon, \theta^* + \varepsilon] \\ 1, & \theta_i < \theta^* - \varepsilon \\ 0, & \theta_i > \theta^* + \varepsilon \end{array} \right. \quad (48)$$

Given signal $\theta_i$ a depositors posterior on $\theta$ is uniform on $[\theta_i - \varepsilon, \theta_i + \varepsilon]$. The payoff difference between rolling over and withdrawing given bank resolution by (39) equals

$$\Delta = -\frac{la}{n} H(a) \quad (49)$$

The expected payoff difference at a signal $\theta_i$ equals

$$0 = \frac{1}{2\varepsilon} \int_{\theta_i - \varepsilon}^{\theta_i + \varepsilon} (H \theta + (1 - \theta)\gamma - 1) 1_{\{n \in [0,la]\}} - \frac{la}{n} H(a) 1_{\{n \in [la,1]\}} d\theta$$

substituting using the function $n(\theta, \theta^*)$, this is equivalent to

$$0 = \int_0^{la} ((H - \gamma)\theta(n, \theta^*) - (1 - \gamma)) dn - H(a) \int_{la}^{1} \frac{la}{n} dn$$

$$= la ((H - \gamma)\theta^* - (1 - \gamma)) + (H - \gamma)\varepsilon la (1 - la) + la H(a) \ln(la)$$

where

$$\theta(n, \theta^*) = \theta^* + \varepsilon(1 - 2n), \theta^* \in [\theta - \varepsilon, \theta + \varepsilon] \quad (50)$$

is the inverse of the function $n(\theta, \theta^*)$. Canceling terms, and solving for the trigger yields

$$\theta^* = \frac{(1 - \gamma) - H(a) \ln(la)}{H - \gamma} - \varepsilon(1 - la) \quad (51)$$

Since the noise term enters linearly, we can take partial derivatives directly from the limit of the trigger.

Proof. [Proposition 5.1] By (50), we have $\theta_b = \theta^* + \varepsilon(1 - 2\frac{la}{\theta})$. Thus, at the limit $\varepsilon \to 0$, we have $\theta_b = \theta^*$ and also the partial derivatives coincide. Set

$$n^* = la \quad (52)$$
\[
\frac{\partial}{\partial a} \theta^* = -\frac{1}{k-\gamma} \left( H'(a) \ln(n^*) + H(a) \frac{1}{n^*} \frac{\partial n^*}{\partial a} \right) \tag{53}
\]

where
\[
\frac{\partial n^*}{\partial a} = l \tag{54}
\]

and thus
\[
H(a) \frac{1}{n^*} \frac{\partial n^*}{\partial a} = \frac{1}{a} H(a) > 0 \tag{55}
\]

while
\[
H'(a) = -\frac{r(1-la) + lr(1-a)}{(1-la)^2} = \frac{r(1-l)}{(1-la)^2} > 0 \tag{56}
\]

But \(\ln(n^*) < 0\), thus in general, the change of the trigger in forbearance can be non-monotone. For \(r = l\), using the logarithm inequality \(\ln(1 + x) > x/(x + 1)\),

\[
\frac{\partial}{\partial a} \theta^* = -\frac{1}{H-\gamma} \left( \frac{1-l}{1-la} \left( \frac{l}{(1-la)} \ln(n^*) + \frac{1}{a} \right) \right) \tag{57}
\]

\[
< -\frac{1}{H-\gamma} \left( \frac{1-l}{1-la} \left( \frac{l}{(1-la)} \frac{la-1}{la} + \frac{1}{a} \right) \right) \tag{58}
\]

\[= 0 \tag{59}\]

Thus, for \(r\) close to \(l\), \(\frac{\partial}{\partial a} \theta^* \leq 0\). The cross derivative of the trigger with respect to forbearance and recovery rate is positive:

\[
\frac{\partial}{\partial r} \frac{\partial}{\partial a} \theta^* = -\frac{1}{H-\gamma} \left[ \left( \frac{\partial}{\partial r} \frac{\partial}{\partial a} H(a) \right) \cdot \ln(n^*) + \frac{1}{a} \left( \frac{\partial}{\partial r} H(a) \right) \right] \tag{60}
\]

\[= -\frac{1}{H-\gamma} \left[ \frac{(1-l)}{(1-la)^2} \cdot \ln(n^*) + \frac{1}{a} \left( -\frac{1-a}{1-la} \right) \right] > 0 \tag{61}\]

since \(\ln(n^*) < 0\), while the cross derivative of the trigger with respect to forbearance and deposit insurance has the same sign as the partial derivative \(\frac{\partial}{\partial a} \theta^*\):

\[
\frac{\partial}{\partial r} \frac{\partial}{\partial a} \theta^* = -\frac{1}{(H-\gamma)^2} \left( H'(a) \ln(n^*) + H(a) \frac{1}{n^*} \frac{\partial n^*}{\partial a} \right) \tag{62}
\]

since signs depend on the bracket. Thus, if \(\frac{\partial}{\partial a} \theta^* > 0\), as \(\gamma\) increases, the trigger increases faster in \(a\) while if \(\frac{\partial}{\partial a} \theta^* < 0\), the trigger falls faster in \(a\) as deposit insurance goes up.

\[
\frac{\partial}{\partial r} \theta^* = -\frac{\ln(la)}{H-\gamma} \cdot \left( \frac{\partial}{\partial r} H(a) \right) < 0 \tag{63}
\]

since

\[
\frac{\partial}{\partial r} H(a) = -\frac{1-a}{1-la} < 0 \tag{64}\]
and $-\ln(la) > 0$ by $1 > l > la$ and $K > 1 > \gamma$

Last,

$$\frac{\partial \theta^*}{\partial \gamma} = -\frac{(H - \gamma) + (1 - \gamma) - H(a) \ln(n^*)}{(H - \gamma)^2} = \frac{1 - H - H(a) \ln(n^*)}{(H - \gamma)^2}$$

(63)

Plugging in the equilibrium condition (70) for $-H(a) \ln(n^*)$, at the limit we obtain

$$\frac{\partial \theta^*}{\partial \gamma} = \frac{1 - H + (H - \gamma) \theta^* - (1 - \gamma)}{(H - \gamma)^2} = \frac{(\theta^* - 1)}{(H - \gamma)} < 0$$

(64)

\[\square\]

### 7.3 Proof: Efficiency

**Proof.** [Lemma 4.1] Total debt value from the contract net off taxation equals

$$V_D = \int_{n}^{\theta^*} n(\theta) \cdot \left(\frac{la}{n(\theta)} + (1 - \frac{la}{n(\theta)})s_\gamma(a)\right) + (1 - n(\theta))s_\gamma(a) d\theta$$

(65)

$$+ \int_{\theta^*}^{1} n(\theta) \cdot 1 + (1 - n(\theta)) \cdot (\theta H + (1 - \theta)\gamma) d\theta - \gamma$$

(66)

while value of the deposit insurance fund equals

$$\Gamma = \gamma - \left[(1 - la) \max(0, \gamma - s(a)) \int_{0}^{\theta^*} d\theta + \int_{\theta^*}^{1} (1 - \theta)\gamma d\theta\right]$$

(67)

where the latter term is the expected payments to depositors by the insurance fund ("public losses of insurance fund").

**Proof.** [Lemma 4.2] We have for every $r, l \in (0, 1)$,

$$\lim_{\gamma \to 0} \theta^* = \frac{1 - H(a) \ln(la)}{H} > \frac{1}{H} > \frac{\max(r, l)}{H} = \theta_e$$

(68)

since $-\ln(la) > 0$, and $l, r < 1$, independently of the ordering of $r$ over $l$. On the other hand, independently of whether $a < \bar{a}$ or $a \in (\bar{a}, 1)$. For every recovery rate $r \in (0, 1)$,

$$H(a) = 1 - s(a) \leq 1 - \gamma$$

(69)

Thus, by $-\ln(la) > 0$ and $k > 1$,

$$\theta^* = \frac{(1 - \gamma) + H(a)(-\ln(la))}{H - \gamma} \leq \frac{(1 - \gamma)(1 - \ln(la))}{H - \gamma} \to 0 \quad \text{as } \gamma \to 1$$

(70)

Thus, we have found an upper majorant for the trigger which converges to zero. By the sandwich lemma therefore $\lim_{\gamma \to 1} \theta^* = 0 < \max(r, l)/H = \theta_e$.  

\[\square\]
7.4 Proof: Change Deadweight loss

Proof. [Theorem 5.2 and 2] Let \( r \geq l \). For \( r = l \), \( T(a) = l = r \). If \( r > l \), \( \theta_e = \frac{r}{H} \) and thus

\[
\frac{\partial}{\partial \gamma} D = (r - T(a)) \frac{\partial \theta_b}{\partial \gamma} + \frac{\partial \theta_b}{\partial \gamma} (\theta_b H - r) 
\]

(71)

\[
\frac{\partial}{\partial \gamma} = (\theta_b H - T(a)) \frac{\partial \theta_b}{\partial \gamma} 
\]

(72)

The critical state strictly decreases in coverage \( \frac{\partial \theta_b}{\partial \gamma} < 0 \). Further, by Lemma 4.2, \( \theta_b \) goes to zero for coverage to one and exceeds \( \theta_e > \frac{T(a)}{H} \) for coverage to zero for any \( a \in (a_0, 1] \). Therefore, we have strict single-crossing: For \( \gamma \) small, \( (\theta_b H - T(a)) > 0 \) and thus \( \frac{\partial}{\partial \gamma} D > 0 \). For coverage high, \( (\theta_b H - T(a)) < 0 \) and thus \( \frac{\partial}{\partial \gamma} D < 0 \). The deadweight loss is minimized if insurance is such that the critical state equals

\[
\theta_b = \frac{T(a)}{H} 
\]

(73)

and since the critical state is continuous and \( \frac{T(a)}{H} \) is constant in insurance, there is exactly one insurance level for given forbearance \( a \) which satisfies this condition, i.e. for every \( a \) there exists a unique insurance level \( \gamma^*(a) \) such that

\[
\theta_b(a, \gamma^*(a)) = \frac{T(a)}{H} 
\]

(74)

which minimizes the deadweight loss. Further, for \( \gamma < \gamma^*(a), \theta_b(a, \gamma) > \frac{T(a)}{H} \) and thus the deadweight loss is decreasing on \( (0, \gamma^*(a)) \), \( \frac{\partial D}{\partial \gamma} < 0 \), while for \( \gamma > \gamma^* \) the deadweight loss is increasing. To determine the change of \( \gamma^*(a) \), consider the total derivative. As \( a \) increases, \( \gamma^* \) has to change in a way that the total change in \( \theta_b \) is zero

\[
\frac{d}{da} \theta_b(a, \gamma^*(a)) = \frac{\partial \theta_b}{\partial \gamma} + \frac{\partial \theta_b}{\partial \gamma} \frac{\partial \gamma^*}{\partial a} = 0 
\]

(75)

Thus,

\[
\frac{\partial \gamma^*}{\partial a} = -\frac{\frac{\partial \theta_b}{\partial \gamma}}{\frac{\partial \theta_b}{\partial a}} 
\]

(76)

We know \( \frac{\partial \theta_b}{\partial \gamma} < 0 \), thus \( \frac{\partial \gamma^*}{\partial a} > 0 \) if and only if \( \frac{\partial \theta_b}{\partial a} > 0 \) that is for \( r \) large. For \( r \) close to \( l \) (benchmark case), \( \frac{\partial \theta_b}{\partial a} < 0 \), thus \( \frac{\partial \gamma^*}{\partial a} < 0 \). Now assume \( r < l \), then \( \theta_e = \frac{l}{H} \). We have

\[
\frac{\partial}{\partial \gamma} D = (l - T(a)) \frac{\partial \theta_b}{\partial \gamma} + \frac{\partial \theta_b}{\partial \gamma} (\theta_b H - l) 
\]

(77)

\[
= (\theta_b H - T(a)) \frac{\partial \theta_b}{\partial \gamma} 
\]

(78)

thus the slope is the same as in the case \( r > l \). Independently of the relation of \( r \) to
We have \( \frac{\partial}{\partial r} D(a, \gamma(a)) = 0 \). For \( r < l \), \( \frac{\partial \theta}{\partial a} < 0 \), thus \( \frac{\partial \gamma^*}{\partial a} < 0 \).

Proof. Lemma 5.2

\[
\frac{\partial}{\partial r} D = \frac{\partial}{\partial r} (r - T(a)) \theta_b + (r - T(a)) \frac{\partial \theta_b}{\partial r} + \frac{\partial \theta_b}{\partial r} (\theta_b H - r) - \frac{\partial \theta_e}{\partial r} (\theta_e H - r) - (\theta_b - \theta_e)
\]

(79)

since \( \theta_e H = r \). We have \( \frac{\partial \theta}{\partial r} < 0 \) and for insurance coverage sufficiently small inefficient runs exist, \( \theta_b H - T(a) > \theta_b H - r > 0 \) and \( \theta_b > \theta_e \). Thus, under low insurance coverage, \( \frac{\partial}{\partial r} D < 0 \). Under high insurance however, the result can revert, since underliquidation occurs. If coverage is sufficiently high, such that \( \theta_b < T(a) / H < r / H \) it follows \( \theta_b < \theta_e \) and thus \( \theta_b (1 - a) < \theta_e \), thus both terms become positive, \( \frac{\partial}{\partial r} D > 0 \).

Proof. [Theorem 3] To see how the change of deadweight loss in forbearance depends on changed coordination behavior and shifts in liquidation proceeds, it is instructive to analyze the derivative of the deadweight loss directly

\[
\frac{\partial}{\partial a} D(a) = \left( \frac{\partial}{\partial a} (r - T(a)) \right) \theta_b(a) + \left( r - T(a) \right) \frac{\partial \theta_b}{\partial a} + \frac{\partial \theta_b}{\partial a} (\theta_b H - r)
\]

(81)

A: Direct efficiency loss

B: Change in direct efficiency due to change in stability

C: Change in liquidation due to change in stability

We have

\[
\frac{\partial}{\partial r} \frac{\partial}{\partial a} D = \theta_b + (r - l) \frac{\partial \theta_b}{\partial r} + \frac{\partial \theta_b}{\partial a} (a + \frac{\partial \theta_b}{\partial r} H - 1) + \frac{\partial}{\partial r} \frac{\partial \theta_b}{\partial a} (\theta_b H - T(a))
\]

(82)

by previous results, the first term is positive, the second term is negative by \( \frac{\partial \theta}{\partial a} < 0 \) but small for recovery rate sufficiently close to liquidation value, the derivative \( \frac{\partial \theta_b}{\partial a} \) is negative for \( r \) small and positive for \( r \) large, the bracket \( (a + \frac{\partial \theta_b}{\partial r} H - 1) \) is always negative by \( \frac{\partial \theta_b}{\partial r} < 0 \) and \( a < 1 \), \( \frac{\partial}{\partial r} \frac{\partial \theta_b}{\partial a} \) is always positive and \( \theta_b H - T(a) \) is positive for deposit insurance sufficiently small. Thus, for \( r \) sufficiently small, and deposit insurance low, only the second term is negative but small and \( \frac{\partial}{\partial r} \frac{\partial}{\partial a} D > 0 \).

To show that \( \frac{\partial}{\partial a} D \) satisfies single-crossing in \( r \) for \( \gamma \) low, fix \( a \) and consider \( r \to l \) in (81). Then, \( \frac{\partial \gamma^*}{\partial a} \leq 0 \) by Proposition 5.1. Further, \( T'(a) = (l - r) \to 0 \), thus term A in (81) is small since \( \theta_b \in [0, 1] \) is bounded. Also \( (r - T(a)) \to 0 \) as \( r \to l \) and \( \frac{\partial \theta_b}{\partial a} \) is bounded since the derivative is a continuous function on a compact interval \( r \in [l, 1] \). Therefore also term B in (81) is small. The sign of term C depends, for low deposit insurance we
know that inefficient runs take place, θ_b > θ_e, thus θ_bH > r and term C becomes negative, thus \( \lim_{r \to 1} \frac{\partial}{\partial a} D < 0 \) for \( \gamma \) low. On the other hand, as \( r \) increases, the comparative statics of the trigger flip. For given \( a \) consider the unique value \( r^*(a) \) such that \( \frac{\partial}{\partial a} (r^*) = 0 \) and \( \frac{\partial}{\partial r} (r) \geq 0 \) for all \( r > r^*(a) \). Then, in (81) for all \( r \geq r^* \), \( -T'(a) = -(l - r) > 0 \), thus term A in (81) is positive. By \( r - T(a) > 0 \), term B is positive if too. Term C is positive under inefficient runs (low deposit insurance), thus under low deposit insurance, all terms are positive, \( \lim_{r \to 1} \frac{\partial}{\partial a} D > 0 \). Thus, by continuity of \( \frac{\partial}{\partial a} D \) there exists at least one value \( \hat{r} \) at which \( \frac{\partial}{\partial a} D(\hat{r}) = 0 \). If in addition, \( \frac{\partial}{\partial r} \frac{\partial}{\partial a} D \geq 0 \) even for \( r > r^* \), then \( \hat{r} \) is unique. Thus, for \( \gamma \) fixed and sufficiently low, by the Intermediate value Theorem for every forbearance level there exists a unique \( \hat{r} \) such that \( \frac{\partial}{\partial a} D(a, \hat{r}) = 0 \) and \( \frac{\partial}{\partial a} D(a, r) < 0 \) for \( r < \hat{r} \), \( \frac{\partial}{\partial a} D(a, r) > 0 \) for \( r > \hat{r} \).

The issue is here, that for \( r > r^* \) the third term in (82) switches sign and becomes negative which either slows down the increase of deadweight loss in forbearance or reverts it. Under reversion, single-crossing can get lost, i.e. there can exist a larger \( r^{**} > r^* \) such that \( D \) decreases again in \( a \) for \( r > r^{**} \).

Under high insurance coverage, the bracket in term four becomes negative \( \theta_bH - T(a) < 0 \). Thus for low recovery rates, terms one and three are positive, term two and four are negative which slows down the change of \( \frac{\partial}{\partial a} D(a, r) \) in \( r \). As \( r \) increases, term three turns negative too such that the slope of deadweight loss in forbearance may even decrease as \( r \) goes up.

\section{7.5 Proof: Inefficient RA}

\textit{Proof.} [Lemma 5.3] From (53), using the logarithm equation again

\[ \frac{\partial}{\partial a} \theta^* = -\frac{1}{H - \gamma} \left( \frac{r(1 - l)}{(1 - la)^2} \ln(la) + \frac{1}{a} (1 - \frac{r(1 - a)}{1 - la}) \right) \]  
\[ < -\frac{1}{H - \gamma} \left( \frac{r(1 - l)}{(1 - la)^2} \frac{la - 1}{la} + \frac{1}{a} (1 - \frac{r(1 - a)}{1 - la}) \right) \]  
\[ = -\frac{1}{H - \gamma} \frac{1}{a} \left( 1 - \frac{r}{l} \right) < 0 \]  

since \( r \leq l \). □

\section{7.6 Proof: Minimizing public losses}

\textit{Proof.} 4.3

\[ (1 - la) \max(0, \gamma - s(a)) = \max(0, (1 - la)\gamma - l(1 - a)) = \max(0, \gamma - l + al(1 - \gamma)) \]

which is increasing in \( a \) by \( \gamma < 1 \). □
References


Phoebe White and Tanju Yorulmazer. Bank resolution concepts, tradeoffs, and changes in practices. 2014.