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# The Dynamics of Sovereign Debt Crises and Bailouts

*Francisco Roch and Harald Uhlig*

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## Abstract

Motivated by the recent European debt crisis, this paper investigates the scope for a bailout guarantee in a sovereign debt crisis. Defaults may arise from negative income shocks, government impatience or a "sunspot"-coordinated buyers strike. We introduce a bailout agency, and characterize the strategy with the minimal actuarially fair intervention which guarantees the no-buyers-strike fundamental equilibrium, relying on the market for residual financing. The intervention makes it cheaper for governments to borrow, inducing them borrow more, leaving default probabilities possibly rather unchanged. The maximal backstop will be pulled precisely when fundamentals worsen.

JEL classification: F34, F41.

*Keywords:* Default, Bailouts, Self-fulfilling Crises, Endogenous Borrowing Constraints, Long-term Debt, OMT, Eurozone Debt Crisis.

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# 1 Introduction

Since 2010, financial markets have expressed recurrent concerns about risks to debt sustainability in a number of countries. One symptom of these developments is the observed pattern of eurozone members sovereign yields since 2010, as shown in Figure 1. Various bailouts and interventions have been proposed or been executed, with considerable controversy and mixed success<sup>1</sup>. Of particular interest to this paper is the ECB President Mario Draghi's attempt to restore confidence by pledging to do "whatever it takes" to preserve the euro zone. The ECB followed this speech with a program known as outright monetary transactions (OMT) in September 2012, intended to reduce country-specific distress yields per potentially unlimited purchases of the short-term government bonds of that country. Yields subsequently declined, despite such purchases never taking place. While ECB Draghi stated that "OMT has been probably the most successful monetary policy measure undertaken in recent time", it has been attacked at German constitutional court hearings in June 2013 as fiscal policy and outside the legal framework provided by the Maastricht treaty. It received a favorable ruling by the European Court of Justice on June 16th 2015, but the issue has now returned to the German constitutional court, with the latest round of hearings in February 2016. At the heart of the controversy is whether this ECB program represents monetary policy or whether it represents fiscal policy and a bailout, financed by reductions in seignorage revenue for other member countries or an inflation tax.

This paper is motivated by these developments. The analysis presented here played a considerable role in the testimony of the second author at the German constitutional court hearings in May 2013, see Uhlig (2013b, 2015). The paper seeks to understand the dynamics of sovereign default crisis and the potential role of a large, risk-neutral investor or agency in coordinating expectations on a "good equilibrium", when sovereign debt markets might be prone to panics and run. The

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<sup>1</sup>For example, in the summer of 2015, the Greek voters rejected a proposed bailout and its impositions on fiscal policy, only to see it being implemented anyways, with minor changes. It remains to be seen whether this will lead to a sustainable solution in Greece, but doubts persist. Yields on 10 year bonds are 10 percent above those of German bunds at the time of writing these comments.

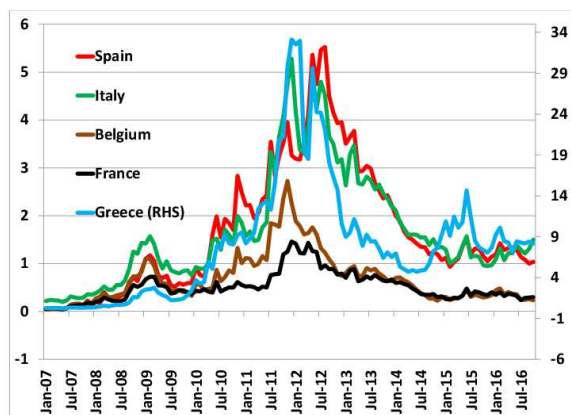


Figure 1: *10yr yield spread to Germany. Source: Bloomberg.*

perspective proposed here can be understood as a benign version of the OMT program. In particular, we characterize the minimal actuarially fair intervention that restores the “good” equilibrium of Cole-Kehoe (2000), relying on the market to provide residual financing. “Fair value” here means that the resources provided by the bail-out fund earn the market return in expectation. We believe this is an important benchmark, shedding light on the OMT program of the ECB. The key issue in this benchmark is that the bail-out agency is able to restore the “good equilibrium” without endangering resources of tax payers in other countries, and it does so just by announcing that it is ready to step in and purchase debt at market prices, which would prevail in the “good equilibrium”. The main insight of the paper is not that the “good equilibrium” can be restored by this agency (to some, this may be fairly obvious), but rather to characterize the implications of the implementation of such a policy.

Our analysis of the dynamics of a sovereign debt crisis builds on and extends three branches of the literature in particular. First, Arellano (2008) has analyzed the dynamics of sovereign default under fluctuations in income, and shown that

defaults are more likely when income is low<sup>2</sup>. Second, Cole and Kehoe (1996,2000) have pointed out that debt crises may be self-fulfilling: the fear of a future default may trigger a current rise in default premia on sovereign debt and thereby raise the probability of a default in the first place. Both theories imply, however, that countries would have a strong incentive to avoid default-triggering scenarios in the first place. We therefore build on the political economy theories of the need for debt constraints in a monetary union of short-sighted fiscal policy makers as in to provide a rationale for a default-prone scenario, see e.g. Beetsma and Uhlig (1999) or Cooper, Kempf and Peled (2010).

We study a dynamic endogenous default model à la Eaton and Gersovitz (1981). This framework is commonly used for quantitative studies of sovereign debt and has been shown to generate a plausible behavior of sovereign debt and spread. Within this framework, we consider a bailout agency, modeled as a particularly large and infinitely lived investor and who is committed to rule out the sunspot-driven defaults of Cole-Kehoe (2000) per debt purchases, even if all other investors do not. We analyze the game between the government, the private sector, and this bailout agency. We show that the intervention only requires knowledge of the amount of revenues needed to prevent a default and whether the country is in the crisis zone or not, in order to avoid potentially bailing out an insolvent government. We also provide practical interpretations of the game, distinguishing between a primary market and a secondary implementation which could have important policy implications in practice. Then, we assume that this bailout agency seeks an actuarially fair return, and characterize the minimal intervention. The bailout agency will not prevent defaults due to fundamental reasons as in Arellano (2008) nor impose additional policy constraints such as conditionality as in e.g. Fink and Scholl (2016).

We find that introducing an actuarially fair bailout agency could effectively serve as a coordination on the “good equilibrium”, by issuing debt purchase guarantees and without incurring losses in expectation. We find that the agency needs to

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<sup>2</sup>That may sound unsurprising, but is actually not trivial and it follows from the assumption of non-contingent bonds. Indeed the recursive contract literature typically implies incentive issues for contract continuation at high rather than low income states, see e.g. Ljungqvist-Sargent (2004).

be willing to potentially purchase (nearly) the entire amount of newly issued debt, casting doubts on proposals that, say, seek to limit the amount the ECB can buy a priori. At that maximum, we find that a small worsening in fundamentals will make the bailout agency jump from the commitment to buy the entire amount of newly issued debt to buying no debt at all and letting the country default: the country is let-go when a future recession becomes more likely than it was. We find that the policy overall leads to higher debt levels and possibly rather small changes in the probability of default, as the probability of default for fundamental reasons is increased. Thus, while the bailout agency intervention may eliminate multiple equilibria, default events may not be reduced as a result of higher debt levels. However, now defaults would only occur due to fundamental reasons. Our numerical analysis shows, that changing the maturity of the debt may have little influence on default probabilities: the main change instead may be the level of debt. Our analysis is “positive”, not “normative”. The impatience of the government and its objectives may well be different from those of the population, which a social planner would take into account. On purpose, we therefore refrain from assessing the efficiency and welfare implications: these would require additional assumptions.

Our study is related to the recent literature on quantitative models of sovereign default that extended the approach developed by Eaton and Gersovitz (1981), starting with Aguiar and Gopinath (2006) and Arellano (2008). Different aspects of sovereign debt dynamics and default have been analyzed in these quantitative studies. Excellent surveys of the literature on sustainable public debt and sovereign default are in the handbook chapters by Aguiar et al (2016) and D’Erasmo et al (2016). However, these studies do not consider defaults driven by a buyers strike and the role of bailouts in eliminating self-fulfilling debt crises.

A few recent papers also analyzed the role of bailouts in models of strategic sovereign default. Boz (2011) introduces a third party that provides subsidized enforceable loans subject to conditionality in order to replicate the procyclical use of market debt but the countercyclical use of IMF loans. Fink and Scholl (2016) also include bailouts and conditionality to reproduce the observed frequency and duration of bailout programs. Juessen and Schabert (2013) include bailout loans

at favorable interest rates but conditional to fiscal adjustments, and show that this could not result in lower default rates. However, these studies do not consider self-fulfilling debt crises. Kirsch and Ruhmkorf (2013) incorporate financial assistance to a multiple equilibrium default model. In contrast to our paper, they model bailouts differently: bailout loans are provided at a fixed price schedule, are senior to market debt, and are subject to conditionality. Furthermore, the scope for the bailout is not to resolve the coordination problem completely as in our paper.

Our paper is closely related to the literature on multiple equilibria in models of sovereign default, most notably Cole and Kehoe (1996, 2000), Conesa and Kehoe (2012, 2014), Corsetti and Dedola (2014), and Broner et al (2014). While we share with these papers that crises can be triggered by a buyers strike, we differ in the focus of our analysis. Cole and Kehoe (1996, 2000) provide a characterization of the crisis zone and optimal policy in a dynamic stochastic general equilibrium model. Conesa and Kehoe (2012) show that under certain conditions government may find optimal not to undertake fiscal adjustments, thus “gambling for redemption”. Conesa and Kehoe (2014) build on the previous paper to also study the role of bailouts. However, they evaluate whether a bail out could induce countries to reduce their debt out of the crisis zone, thus mitigating the gambling for redemption. They find that a bail out with a high penalty interest rate and a large collateral requirement would achieve this goal, but the problem is that the government would rather default than accept that program. Corsetti and Dedola (2014) show that the government’s ability to debase debt with inflation does not eliminate self-fulfilling debt crises, when the government lacks credibility. In many ways, it may be the analysis most closely related to ours, however. Broner et al (2014) propose a model with creditor discrimination and crowding-out effects to show that an increase in domestic purchases of debt may lead to self-fulfilling crises. Uhlig (2003) is one of the early papers to discuss sovereign default aspects in a monetary union. Hatchondo et al (2015) discuss the importance of fiscal rules, while Kriwoluzky et al (2015) discuss currency union exit expectations.

As we do, Aguiar et al (2016) highlights that coordination failures are a significant factor in sovereign bond markets. Their model also features multiplicity

of equilibria but it differs from ours by incorporating time varying probability of rollover crises and stochastic risk premium demanded by foreign investors, which seem important to account for interest rate and debt dynamics in the data. However, they do not discuss the role of a bailout agency in mitigating these coordination failures, which is the main point of our study. Bocola and DAVIS (2015) measure the importance of self-fulfilling crises in driving interest rate spreads during the euro-area sovereign debt crisis.

The rest of the article proceeds as follows. Section 2 introduces the model without bailouts. Section 3 introduces and characterizes the bailout agency. Section 4 presents the numerical results. Section 5 concludes.

## 2 A model of sovereign default dynamics: no bailout agency.

This section closely follows Cole-Kehoe (2000) and Arellano (2008). We assume that there is a single fiscal authority, which finances government consumption  $c_t \geq 0$  with tax receipts  $y_t \geq 0$  and assets  $B_t \in \mathbf{R}$  (with positive values denoting debt), in order to maximize its utility

$$U = \sum_{t=0}^{\infty} \beta^t (u(c_t) - \chi_t \delta_t) \quad (1)$$

where  $\beta$  is the discount factor of the policy maker,  $u(\cdot)$  is a strictly increasing, strictly concave and twice differentiable felicity function,  $\chi_t$  is an exogenous one-time utility cost of default and  $\delta_t \in \{0, 1\}$  is the decision to default in period  $t$ . We assume that tax receipts  $y_t$  are exogenous, while consumption, the level of debt and the default decisions are endogenous and chosen by the government.

In Arellano (2008) as well as Cole and Kehoe (2000), this is the utility of the representative household,  $y_t$  is total output and  $c_t$  is the consumption of the household, i.e. the fiscal authority is assumed to maximize welfare. The structure assumed here is mathematically the same, and consistent with that interpretation. It is also consistent with our preferred interpretation, where the utility function represents the preferences of the policy maker. For example, given the uncertainty of



re-election, a policy maker may discount the future more steeply than would the private sector. Spending may be on groups that are particularly effective in lobbying the government. Finally,  $y_t$  should then be viewed as tax receipts, not national income.

A more subtle difference is the cost of a default, modeled here as a one-time utility cost  $\chi_t$ , while it is modelled as a fractional loss in output in Arellano (2008) with Cole and Kehoe (2000). Note, however, that  $c_t = y_t$  in default, and that at least for log-preferences,  $u(c_t) = \log(c_t)$ , a proportional decline in consumption each period following the default can equivalently be written as a one-time loss in utility. The stochastic utility cost formulation intends to capture the non-pecuniary costs of defaults such as reputation costs and the role of political factors in sovereign defaults episodes. For instance, Sturzenegger and Zettelmeyer (2006) argue that "a solvency crisis could be triggered by a shift in the parameters that govern the country's willingness to make sacrifices in order to repay, because of changes in the domestic political economy (a revolution, a coup, an election, etc.)...". The election of the Syriza government in Greece in January 2015 can be understood as electing a government that was more willing to risk a default than the previous one, and can be captured here by a change in  $\chi_t$ . A similar utility cost formulation has been used in recent studies on personal bankruptcy and mortgage defaults<sup>3</sup>, and in the political economy literature.<sup>4</sup> Technically, it provides a free parameter to fine-tune the quantitative implications of the baseline specification of the model: a feature that we exploit in the numerical analysis. We wish to emphasize, however, that introducing this political-taste feature and its stochastic variability may be quite important on economic grounds for understanding sovereign default.

In each period, the government enters with some debt level  $B_t$  and the tax receipts  $y_t$  as well as some other random variables are realized. Traders on financial markets are assumed to be risk neutral and discount future repayments of debt at

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<sup>3</sup>In this literature the utility cost of declaring bankruptcy or defaulting on a mortgage is meant to capture the social stigma attached to such situations, see Herkenhoff and Ohanian (2012), Chatterjee and Eyigungor (2011), and Luzzetti and Neumuller (2014, 2015).

<sup>4</sup>Beetsma and Ribeiro (2008) assume that the government incurs a utility cost from running a deficit that exceeds a reference level.

some return  $R$ , and price new debt  $B_{t+1}$  according to some market pricing schedule  $q_t(B_{t+1})$ . Given the pricing schedule, the government then first makes a decision whether or not to default on its existing debt. If so, it will experience the one-time exogenously given default utility loss  $\chi_t$ , be excluded from debt markets until re-entry, and simply consume its output,  $c_t = y_t$  in this as well as all future periods, while excluded from debt markets. We assume that re-entry to the debt market happens with probability  $0 \leq \alpha < 1$ , drawn iid each period, and that re-entry starts with a debt level of zero. If the government does not default, it will choose consumption and the new debt level according to the budget constraint

$$c_t + (1 - \theta)B_t = y_t + q_t(B_{t+1})(B_{t+1} - \theta B_t) \quad (2)$$

where  $0 < \theta \leq 1$  is a parameter, denoting the fraction of debt that currently needs to be repaid. The parameter  $\theta$  allows to study the effect of altering the maturity structure: the lower  $\theta$ , the longer the maturity of government debt. The remainder of the debt  $\theta B_t$  will be carried forward, with the government issuing the new debt  $B_{t+1} - \theta B_t$ .

## 2.1 State space representation

We shall restrict attention to the following state-space representations of the equilibrium. At the beginning of a period, the aggregate state  $(B, z)$  describes the endogenous level of debt  $B$  and some exogenous variable  $z \in Z$ . We assume that  $z$  follows a Markov process and that all decisions can be described in terms of the state  $(B, z)$ . The probability measure describing the transition for  $z$  to  $z'$  shall be denoted with  $\mu(dz' | z)$ . More specifically, we shall assume that  $z$  is given by

$$z = (y, \chi, \zeta) \quad (3)$$

We assume that  $y \in [y_L, y_H]$  with  $0 < y_L \leq y_H$  either has a strictly positive and continuous density  $f(y | z_{\text{prev}})$ , given the previous Markov state  $z_{\text{prev}}$ . We assume that  $\chi \in \{\chi_L, \chi_H\}$  takes one of two possible values, with  $0 = \chi_L \leq \chi_H$ . We assume that  $\zeta \in [0, 1]$  is uniformly distributed and denotes a ‘‘crisis’’ sunspot. We assume

that the three entries in  $z$  are independent of each other, given the previous state. For most parts, we shall assume that  $z$  is iid, and that therefore the distributions for  $y$  and  $\chi$  also do not depend on  $z_{\text{prev}}$ .

If the government does not default ( $\delta = 0$ ), the period-per-period budget constraint is

$$c + (1 - \theta)B = y + q(B'; z)(B' - \theta B) \quad (4)$$

where  $B'$  is the new debt level chosen by the government and where  $q(B'; z)$  is the pricing function for the new debt  $B'$ .

If the government defaults ( $\delta = 1$ ), the budget constraint is

$$c = y \quad (5)$$

We assume that the government will be excluded from debt markets until it is given the possibility for re-entry. We assume that re-entry to the debt market happens with probability  $0 \leq \alpha < 1$ , drawn iid each period, and that re-entry starts with a debt level of zero. The default decision of the government is endogenous and (assumed to be) a function of the state  $(B, z)$ ,  $\delta = \delta(B, z)$ .

We can now provide a recursive formulation of the decision problem for the government. The value function in the default state and after the initial default utility loss is given by

$$v_D(z) = u(y) + \beta(1 - \alpha)E[v_D(z') | z] + \alpha E[v_{\text{ND}}(0, z') | z] \quad (6)$$

Given the debt pricing schedule  $q(B; z)$ , the value from not defaulting is

$$v_{\text{ND}}(B, z) = \max_{c, B'} \{u(c) + \beta E[v(B', z') | z] | \\ c + (1 - \theta)B = y + q(B'; z)(B' - \theta B)\}$$

Note that this formulation implicitly allows the buyback of outstanding debt at the market price. The overall value function is given by

$$v(B, z) = \max_{\delta \in \{0,1\}} (1 - \delta)v_{\text{ND}}(B, z) + \delta(v_D(z) - \chi) \quad (7)$$

## 2.2 Debt pricing

Given a level of debt  $B$  and “good standing” (the government was not in default in the previous period), let

$$D(B) = \{z \mid \delta(B, z) = 1\} \quad (8)$$

be the default set, and let

$$A(B) = \{z \mid \delta(B, z) = 0\} \quad (9)$$

be the set of all  $z$ , such that the government will not default and instead, continue to honor its debt obligations: both are (restricted to be) a measurable set, according to our equilibrium definition. The disjoint union of  $D(B)$  and  $A(B)$  is the entire set  $Z$ . Define the market price for debt, in case of no current default, i.e.

$$\bar{q}(B'; z) = \frac{1}{R} \int_{z' \in A(B)} (1 - \theta + \theta q(b(B', z'), z')) \mu(dz' \mid z) \quad (10)$$

where  $b(B', z')$  denotes the debt policy function, and thus the new debt level  $B''$ , given the new state  $(B', z')$ . Due to risk neutral discounting, this is the market price of debt, if there is no default “today”. Define the probability of a continuation **next period** per

$$P(B'; z) = \text{Prob}(z' \in A(B') \mid z) = E [1_{\delta(s')=0} \mid z] \quad (11)$$

If  $\theta = 0$ , i.e., if all debt has the maturity of one period only, then

$$\bar{q}(B'; z) = \frac{1}{R} P(B'; z) \quad (12)$$

We need to check, whether there could be a default “today”. We shall impose the following assumption.

**Assumption A. 1** *Given a state  $s$ , either  $q(B'; z) = \bar{q}(B'; z)$  for all  $B'$  or  $q(B'; z) = 0$  for all  $B'$ .*

This assumption is a selection mechanism that rules out equilibria, where, say, the market expects a current default, if the government tries to finance some future

debt level  $B'$ , but not for others. Cole and Kehoe (2000) finesse this issue with more within-period detail, having the government first sell new debt at some pricing schedule, before taking the default decision. With their timing, the equilibrium price is zero if the government chooses a  $B'$  that does not ensure that the government wants to honour its debt obligations given that the government is able to sell new debt (i.e., if  $B'$  does not satisfy the participation constraint).

Given parameters, a law of motion for  $z$ , an **equilibrium** is defined as measurable mappings  $q(B'; z)$  in  $B'$  and  $z$  as well as  $c(B, z)$ ,  $\delta(B, z)$  and  $B'(B, z)$  in  $(B, z)$ , such that

1. Given the pricing function  $q(B'; z)$ , the government maximizes its utility with the choices  $c(B, z)$ ,  $\delta(B, z)$  and  $B'(B, z)$ , subject to the budget constraint (4) and subject to the exclusion from financial markets for a stochastic number of periods, following a default.
2. The market pricing function  $q(B'; z)$  is consistent with risk-neutral pricing of government debt and discounting at the risk free return  $R$ .

We now turn to analyzing the possibility for a self-fulfilling expectation of a default. Define the value of not defaulting, if the market prices are consistent with current debt repayment,

$$\bar{v}_{\text{ND}}(B, z) = \max_{c, B'} \{u(c) + \beta E[v(B', z') | z] | \\ c + (1 - \theta)B = y + \bar{q}(B'; z)(B' - \theta B)\}$$

where it should be noted that the continuation value function is as before, i.e. given by (7). Define the value of not defaulting, if the market prices are consistent with a current default,

$$\underline{v}_{\text{ND}}(B, z) = \{u(c) + \beta E[v(\theta B, z') | z] | \\ c + (1 - \theta)B = y\}$$

With that, define two bounds for the current debt levels  $B$ . Above the upper bound  $B \geq \bar{B}(z)$ , the government finds it optimal to default today, even if the market

was willing to finance future debt in the absence of a default now, i.e. even if  $q(B'; z) = \bar{q}(B'; z)$ . Above the lower bound  $B \geq \underline{B}(z)$ , the government finds it optimal to default, if the market thinks it will do so and therefore is unwilling to finance further debt,  $q(B'; z) = 0$ . I.e., let

$$\bar{B}(z) = \inf\{B \mid \bar{v}_{\text{ND}}(B, z) \leq v_D(z) - \chi\} \quad (13)$$

as well as

$$\underline{B}(z) = \inf\{B \mid \underline{v}_{\text{ND}}(B, z) \leq v_D(z) - \chi\} \quad (14)$$

Whether or not there will be a default at some debt level  $B$  between these bounds will be governed by the sunspot random variable  $\zeta$ . As in Cole-Kehoe (2000), we assume that the probability of a default in this range is some exogenously given probability  $\pi$ . We could allow this probability to follow a Markov process as some of the recent literature has done (e.g., Bocola and Dovis (2016)) and the results with respect to the equilibrium under assistance would still hold.<sup>5</sup>

**Assumption A. 2** *For some parameter  $\pi \in [0, 1]$ , and all  $B$  with  $\underline{B}(z) \leq B \leq \bar{B}(z)$ , we have  $q(B'; z) = \bar{q}(B'; z)$ , if  $\zeta \geq \pi$  and  $q(B'; z) = 0$ , if  $\zeta < \pi$ .*

The equilibrium will therefore look as follows (up to breaking indifference at the boundary points):<sup>6</sup>

1. If  $B > \bar{B}(z)$ , the government will default now and not be able to sell any debt. The market price for new debt will be zero.
2. If  $\underline{B}(z) \leq B \leq \bar{B}(z)$ , the government will
  - (a) default with probability  $\pi$  (more precisely, for  $\zeta < \pi$ ), and the market price for new debt will be zero,
  - (b) continue with probability  $1 - \pi$  (more precisely, for  $\zeta \geq \pi$ ), and the market price for new debt will be  $\bar{q}(B'; z)$ .

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<sup>5</sup>The stochastic properties of  $\pi$  are important for the no-agency equilibrium which is beyond the scope of this paper.

<sup>6</sup>The resulting equilibrium is similar as in Bocola and Dovis (2016), except that in their model the probability  $\pi$  follows a Markov process. Note that our price schedule  $\bar{q}(B'; z)$  resembles what they refer as the "fundamental price".

3. If  $B < \underline{B}(z)$ , the government will not default, and the market price for debt will be given by  $\bar{q}(B'; z)$ .

Following Cole and Kehoe (2000), we shall use the term “crisis zone” for the maximal range for new debt, for which there might be a “sunspot” default next period, i.e. for

$$B' \in \mathcal{B} = [\min \underline{B}(z), \max \bar{B}(z)]$$

Note that safe debt will be priced at  $q^*$  satisfying

$$q^* = \frac{1}{R}(1 - \theta + \theta q^*)$$

and is therefore given by

$$q^* = \frac{1 - \theta}{R - \theta} \quad (15)$$

Conversely, given some price  $q$ , one can infer the implicit equivalent safe rate

$$R(q) = \theta + \frac{1 - \theta}{q} \quad (16)$$

To denote the dependence of the equilibrium on the sunspot parameter  $\pi$  or the dependence on the debt duration parameter  $\theta$ , we shall use them as superscripts, if needed.

### 3 Bailouts

We now introduce the possibility for a bailout per a large and infinitely lived, risk neutral outside investor. More precisely, we envision an agency with sufficiently deep pockets, possibly backed by, say, governments other than the one under consideration here. In the specific context of the European debt crisis, one may wish to think of this agency as the ECB: given that current inflation levels are low and that a large loss may lead to recapitalization of the ECB by Eurozone member countries, an analysis in real rather than nominal terms appears to be justified. The issue of fiscal support for the balance sheet of a central bank has recently been analyzed by del Negro and Sims (2015).

We assume that this agency aims at ensuring the selection of the “good” equilibrium. We seek to characterize the minimal intervention necessary to achieve this outcome. Throughout our benchmark case, we assume that the buyers’ strikes last for one period, but discuss the extension to more periods in a later subsection.

We consider the following game between the government, the private sector, and a bailout agency. We provide an interpretation of the game in the next subsection. Recall, that at the beginning of the period the aggregate state is  $(B, z)$ , where  $z = (y, \chi, \zeta)$ . Assume that both the government and the bailout agency know  $(B, y, \chi)$ , but do not know  $\zeta$  in steps 1 and 2 of the following game:

1. The government picks a real number  $B'$  as its new debt level.
2. The bailout agency picks a pair  $(B'_a, q_a)$ , where  $B'_a$  is a real number and where  $q_a$  is a non-negative real number. This will denote the willingness of the bailout agency to purchase debt up to  $B'_a$  at price  $q_a$ .
3. The private sector investors learn  $\zeta$ . Then they pick a non-negative number  $q_p$ , the price per unit of debt. We assume they all pick the same number, given  $\zeta$ , i.e. using  $\zeta$  potentially as a coordination mechanism.
4. The government decides whether to default or not.

The payoffs of this game are then as follows. If the government defaults, then we are in the “default” situation that was described in the previous section. However, if the government does not default, there are two cases:

1. **Case A.** If the private sector is willing to buy debt at positive prices, i.e.  $q_p > 0$ , then
  - the government reaches the new debt level  $B'$ , receiving a revenue  $q_p(B' - \theta B)$ , or paying this amount, if negative (i.e. if the government buys back debt).
  - the bailout agency receives and pays nothing.



- the private sector pays  $q_p(B' - \theta B)$ , or receives this amount, if negative.

2. **Case B.** If there is a buyers' strike, i.e.  $q_p = 0$ , then

- the government reaches the new debt level  $B'_a$ , receiving a revenue  $q_a(B'_a - \theta B)$ , or paying this amount, if negative (i.e. if the government buys back debt).
- the bailout agency pays  $q_a(B'_a - \theta B)$ , or receives this amount, if negative.
- the private sector receives and pays nothing.

We shall restrict attention to strategies that insist of buying at the good equilibrium price schedule, and we seek to characterize “good”-equilibrium-restoration intervention strategies. For that, the bailout agency shall pick  $q_a = 0$  outside the crisis zone, but pick  $(q_a, B'_a)$  smartly, in case  $B$  is in the crisis zone for  $z$ ,  $B \in [\underline{B}(z), \bar{B}(z)]$ . More precisely, define the “case B” no default value under assistance as

$$\begin{aligned} \underline{v}_{\text{ND};a}(B, z) = \max_{c, B'} \{ & u(c) + \beta E [v^{(\pi=0)}(B', z') \mid z] \mid \\ & c + (1 - \theta)B = y + q_a(B' - \theta B) \\ & B' \leq B'_a \} \end{aligned} \quad (17)$$

Note the second constraint, encapsulating the limit of the assistance. One can therefore solve for  $B'_a = B'_a(B, z)$  for any state  $(B, z)$  with  $B \in [\underline{B}(z), \bar{B}(z)]$  such that<sup>7</sup>

$$\underline{v}_{\text{ND};a}(B, z) = v_D(z) - \chi \quad (18)$$

With (18) we either assume that the government chooses no-default and “case B” rather than default in case of indifference, or that  $B'_a$  is chosen slightly higher than the value calculated above, in order to break this indifference.

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<sup>7</sup>We assume that if the government is indifferent between two levels of debt issuances, it will always choose the smallest amount. Also, the agency does not need to intervene at all in states where the country is buying back debt, as there will never be a run

One can then solve the game backwards as follows. Assume that the value function for the government from the next period onwards is as in the no-bailout-agency-always-good-sunspot equilibrium, i.e. the  $\pi = 0$  equilibrium described in the previous section. Put differently, along the equilibrium path agents understand that the bailout agency is ready to assist in subsequent buyers' strikes.<sup>8</sup> If the current level of debt is outside the crisis zone for the current  $z$ , then the default/no-default decision will be as in the  $\pi = 0$  equilibrium before. Suppose then, that  $B$  is in the crisis zone for  $z$ . Consider the decision of the government in step 4. Suppose first, that we arrive there with  $q_p > 0$  in step 3. The government will then choose between "default" or proceeding to case A above. If "default" is preferred by the government, then investors should not pick  $q_p > 0$  in step 3, as the debt at that point is valueless. Therefore,  $q_p > 0$  can only arise together with a no-default decision, and pricing as in the  $\pi = 0$  equilibrium. Suppose instead that we arrive at the default decision node in step 4 after a buyers' strike  $q_p = 0$  in step 3. If the government does not choose default, it will enter "case B" and receive the revenue  $q_a(B'_a - \theta B)$ . Note that we have assumed that  $(q_a, B'_a)$  is such that the government (weakly) prefers this outcome to defaulting. It thus chooses to not default in step 4. Thus, regardless of the choice for  $q_p$  in step 3, the government will not default in step 4. Given that investors know this, the debt of the government will be worth more than zero on the market,  $q_p > 0$ . With  $q_p > 0$ , competition then ensures the  $\pi = 0$  outcome. With that, "case B" is off-equilibrium and a buyer strike is always avoided.

In sum:

**Proposition 1** *The bailout policy implements the equilibrium allocation that arises when  $\pi = 0$ .*

This intervention strategy does not require knowledge of the  $\pi = 0$ -equilibrium pricing function. However, the intervention agency needs to know, whether  $B$  is in a crisis zone  $B \in [\underline{B}(z), \bar{B}(z)]$  for  $z$  or not, in order to avoid potentially bailing out an

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<sup>8</sup>If there is uncertainty about the agency assistance in the future, debt prices may not coincide with those in the  $\pi = 0$  equilibrium. This is an interesting avenue for future research beyond the scope of the paper.

insolvent government. Furthermore, the bailout agency needs to know a price-debt  $(q_a, B'_a)$  combination, which avoids the default by the government in step 4 with case B. It could obviously accomplish this by picking hugely favorable terms and buying large amounts of debt. It is fairly apparent, that such generous guarantees might in practice be undesirable for political reasons. For that reason, we have characterized the minimal guarantee level  $B'_a = B'_a(B, z)$ , given the assisted price schedule  $q_a = q_a(B, z)$  in equation (18).

### 3.1 Primary market implementations

One may wonder how the game sketched above could be implemented in practice. For that, note that the choices of the agency no longer play a role in the outcomes, once the game reaches “case A”.

Thus, one practical way of implementing the game would be to have the bailout agency always buy  $(B'_a - \theta B)$  of new debt at price  $q_a$  directly from the government, provided that the country is in the crisis zone, and  $q_a = 0$  otherwise. The private sector then purchases debt from the government or from the bailout agency at a price  $q_p$ . The bailout agency sells its stock of newly purchased debt to the private sector at price  $q_p$ , receiving a reimbursement of  $(q_a - q_p)(B'_a - \theta B)$  from the government, if it paid too much or reimbursing the government for  $(q_a - q_p)(B'_a - \theta B)$ , if it paid too little.

Alternatively, one can think of this game as an auction, with both the bailout agency and the private sector putting in bids, with the somewhat unusual provision that the private sector price prevails, even if  $q_p < q_a$ , provided that  $q_p > 0$ . The provision would be unnecessary, if  $q_a < q_{\pi=0}(B')$ . The choice of  $q_a$  matters for Case B and  $q_p = 0$ . A smaller  $q_a$  then would seem to require a larger  $B'_a$ , i.e. a larger penalty rate requires a more generous credit, akin to the Bagehot principle, in order to avoid a default by the government.

For both of these implementations, the bailout-agency commitment for the off-equilibrium case B amounts to  $q_a(B'_a - \theta B)$ . In particular, if the difference between the new and required debt level  $B'_a$  and the legacy debt  $\theta B$  is small, then the bailout

agency does not require particularly deep pockets to credibly commit to its policy.

### 3.2 The Maastricht treaty, secondary markets and OMT

The Maastricht treaty stipulates, that the ECB can buy government debt only on the secondary market, and thus not directly from the government. In line with that, consider the following variant of the game above. Suppose that the bailout agency announces a policy  $(B'_a, q_a)$ . Let there be a primary market auction where private sector agents buy the newly issued debt directly from the government at some price  $\tilde{q}_p$ . Let there be a secondary market, in which old and new debt is traded, and in which newly issued debt cannot be distinguished from older debt.<sup>9</sup> Let  $q_p$  denote the secondary market price. If  $q_p > 0$ , then standard no-arbitrage arguments result in the primary market price  $\tilde{q}_p = q_p$ , and we are in “case A” as described above. Suppose instead, that the secondary market price is zero,  $q_p = 0$ . Assume that the bailout agency stands ready to buy debt at the announced price  $q_a > 0$  (i.e., above the prevailing secondary market price). Per assumption regarding the secondary market, the bailout agency cannot distinguish sellers of old debt from sellers, who have just obtained newly issued debt in the primary market auction. Suppose all sellers sell their securities with equal probability to the bailout agency. If the total outstanding debt is  $B'$  and given the policy  $(B'_a, q_a)$  of the bailout agency, the risk-neutral participants in the primary market value their bonds at

$$\tilde{q}_p = q_a(B'_a/B') \tag{19}$$

The government picks  $B'$  optimally, taking into account equation (19): a step absent in the game description above. Assume that the bailout agency understands that modified maximization problem and fixes  $(B'_a, q_a)$  in such a way, that the value-maximization choice of the government makes the government choose not to default after the primary auction. It suffices for the bailout agency to make the government just indifferent between defaulting or proceeding with these somewhat meager proceeds from the primary auction, see equation (18). With that, we are

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<sup>9</sup>This may be a legal, rather than a practical restriction.

now back to game as described above. “Case B” is once again off-equilibrium, and the debt purchase above the secondary market price is not necessary. We view this as a stylized description of the OMT (“outright monetary transactions”) program of the ECB, implemented in the fall of 2012.

Two caveats are in order, however. Note that the bailout agency now needs to credibly commit to the purchases of  $B'_a$  rather than just the purchase  $B'_a - \theta B$  at the price  $q_a$ . With longer-maturity debt,  $B'_a$  will be considerably larger than  $B'_a - \theta B$ . Consequently, such a bailout agency must be able to command considerably larger resources in order to credibly execute this policy, should case B come to pass. This distinction between a primary market implementation and secondary market implementation can be of considerable importance in practice.

The second caveat is that the non-intervention by the ECB and the decline in yields is not per se evidence of a successful implementation of the  $\pi = 0$  equilibrium, and could instead signal market expectations of a bailout of a future, insolvent government, i.e. for debt levels exceeding  $\bar{B}(z)$ . Suppose, in the extreme, the bailout agency announces that it will always be ready to buy all government debt at the riskless rate, regardless of whether the government is able to repay or not. In that case, government debt in the hands of private investors is entirely safe, and yields will fall to their risk-free equivalent. For a number of periods, the government may keep repaying its debt obligation, and its debt will continue to trade on private markets. Eventually and perhaps with some probability, however, the government may be insolvent, the secondary market price is zero, all debt gets sold to the bailout agency, and the bailout agency is then stuck with worthless debt<sup>10</sup>. At that point, a version of “case B” materializes, and is no longer “off equilibrium”.

These considerations played a considerable role in the testimony of the second author at the German constitutional court hearings in May 2013, see Uhlig (2013b, 2015).

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<sup>10</sup>This may not be quite true. The bailout agency could keep rolling over this debt. With some luck, some future realization of  $y$  might enable the government to repay its debt at that point, seemingly justifying the intervention ex post. Pursuing this line of reasoning is beyond the scope of this paper.

### 3.3 Avoiding defaults and avoiding bailouts

In terms of practical implementations and with a memory of the negotiations of the IMF with debtor countries or the troika in Europe with Greece, it generally seems rather feasible to find modest  $(q_a, B'_a)$  combinations, which avoid a government default. What seems hard in practice, however, is to determine when this aid is offered, if  $B$  is in a crisis zone  $B \in [\underline{B}(z), \bar{B}(z)]$ , and when it is offered outside of it. It is this key difficulty of telling apart "liquidity" from "solvency" crises, which may make implementing it hard in practice to implement the strategy described above. It is very important for the bailout agency to know if the danger of default is due to sunspots or fundamentals. Bocola and DAVIS (2016) and Juvenal and Wiseman (2015) provide interesting avenues to address this issue in practice. Bocola and DAVIS (2016) use a similar theoretical framework to ours but they allow for endogenous maturity structure. In order to assess whether movements in interest rate spreads are self-fulfilling, they note that the maturity structure chosen by the government differs if a default is likely due to sunspots or fundamentals. Then, they use this restriction to estimate the probability of a self-fulfilling crisis. Juvenal and Wiseman (2015) use the sovereign spread to evaluate Portugal's fiscal position. In particular, they studied the reduction in spreads observed after the winter of 2012 and analyze to what extent the lower yields were driven by global factors and country-specific fundamentals.

### 3.4 Buyers' strikes lasting beyond a single period

In the benchmark case, we assumed that the buyers' strike only last for one period. This may be appear to be a strong assumption, at first blush. What, if the buyer strike continues longer than a single period? For that, we could interpret the length of a period as the maximal time that such a buyer strike may last, provided there is a finite upper bound: this upper bound is then the essential assumption we are making here. With that a buyer strike then does not last more than one period by definition: changes to the interpretation of the length of a period "only" change the quantitative implications. In principle, one could conceive of a situation without such an upper

bound. In that case, the bailout agency would be the ultimate long-term lender, and markets might no longer provide a guide to the appropriate terms.

While it could be interesting to calculate the total depth of the pockets needed to withstand a buyers strike potentially lasting forever, it may be most appropriate to think of the analysis here as the choices for a central bank (as the bailout agency) that takes “one step at a time”. Suppose that this central bank has arbitrary amounts at its disposal *in principle* and is committed to ensuring the  $\pi = 0$  equilibrium even against a prolonged buyers strike. Our analysis then reveals, how much resources that agency has to currently commit to guarantee that outcome. The analysis will then also apply to all future periods, giving the path of future necessary commitments.<sup>11</sup>

### 3.5 Actuarially fair bailouts

Case B is “off-equilibrium”. Nonetheless, one may wish to insist on the bailout agency earning the market rate of return in expectation on its bond holdings, if it ever were to end up holding government bonds.

We view the actuarially fair “restoration-of-the-good-equilibrium” as an important benchmark. It furthermore may be of political relevance for practical applications of our framework, and where one might consider that the case-B branch will be visited “by mistake”.

In order to implement the actuarially-fair implementation even in branch B, when market prices do not provide a guide, we have to assume the bailout agency knows the  $\pi = 0$  pricing schedule. If it is willing to buy the entire debt, then the solution is easy in principle. It should price debt accordingly, and can just let the country choose the debt level it wants, given this pricing schedule. Moreover, since the bailout agency is always there, also in the future, to guarantee the “good” equilibrium, the pricing is actuarially fair. However, as discussed above, one may wish to

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<sup>11</sup>It may be interesting to consider the case where the agency has limited resources (e.g., one period worth of total debt) and the buyers’ strike could last longer than one period. In this case, the proposed bailout may fail to restore the good equilibrium. This extension is beyond the scope of the paper and left for future research.

learn the minimum intervention necessary. The bailout agency then picks a level  $B'_a$  and the corresponding actuarially fair assistance price  $q_a = q^{(\pi=0)}(B'_a; z)$ . Assuming that the pricing schedule is downward sloping in  $B'$ , equation 17 now becomes<sup>12</sup>

$$\begin{aligned} \underline{v}_{\text{ND};a}(B, z) = \max_{c, B'} \{ & u(c) + \beta E [v^{(\pi=0)}(B', z') \mid z] \mid \\ & c + (1 - \theta)B = y + q^{(\pi=0)}(B'; z)(B' - \theta B) \\ & B' \leq B'_a(B, z) \} \end{aligned} \quad (20)$$

Criterion (18) now becomes an equation in one dimension, i.e. in the determination of the unknown  $B'_a$ . We use this approach in our numerical calculations. The following propositions establish some properties of this actuarially-fair intervention solution.

**Proposition 2** *Suppose  $B'_a(B, z)$  satisfies (18). Then:*

1. *There will not be a default, unless debt exceeds  $\bar{B}(z)$ .*
2. *If  $q_a \geq q_{\pi=0}(B'_a)$ , then  $B'_a = B'_a(B, z) \leq B^*(B, z)$ .*
3. *In the state  $(\bar{B}(z), z)$ ,  $B'_a(B, z) = B^*(B, z)$*
4. *For two states  $(B_1, z), (B_2, z)$  with the same exogenous state  $z$ , if  $B_1 > B_2$ , then  $B'_a(B_1, z) \geq B'_a(B_2, z)$ .*

**Proof:** *Appendix. •*

In the iid case and with a constant embarrassment utility costs  $\chi > 0$  of defaulting, a bit more can be said. In that case, some constant value  $\beta \tilde{v}_D$

$$\beta E[v_D(z')] \equiv \beta \tilde{v}_D$$

is the continuation value from defaulting. Likewise, when receiving the full guarantee  $B'_a(B, z)$ , the continuation value of not defaulting is  $\beta \tilde{v}_{\text{ND}}(B'_a(B, z))$ , given by

$$\beta E[v(B'_a(s), z')] = \beta \tilde{v}_{\text{ND}}(B'_a(B, z))$$

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<sup>12</sup>The pricing schedule is downward sloping in  $B'$  in our numerical simulations.



Criterion (18) becomes

$$\begin{aligned} u(y) - u(y + q^{(\pi=0)}(B'_a(B, z); z)(B'_a(B, z) - \theta B) - (1 - \theta)B) \\ = \beta \tilde{v}_{\text{ND}}(B'_a(B, z)) - \beta \tilde{v}_D + \chi - \epsilon \end{aligned} \quad (21)$$

comparing the current utility gain from defaulting to the utility continuation loss from defaulting, including the embarrassment cost  $\chi$ .

**Proposition 3** *In the iid and constant- $\chi$  case, we have*

1. *If  $B > 0$  and the default set is nonempty<sup>13</sup>, then*

$$q^{(\pi=0)}(B'_a(B, z); z)(B'_a(B, z) - \theta B) < (1 - \theta)B$$

2. *For two states  $(B_1, z_1), (B_2, z_2)$ , if  $y_1 > y_2$ , then  $B'_a(B_1, z_1) \leq B'_a(B_2, z_2)$ .*

3. *For two states  $(B_1, z_1), (B_2, z_2)$ , if  $\chi_1 > \chi_2$ , then  $B'_a(B_1, z_1) \leq B'_a(B_2, z_2)$ .*

**Proof:** Appendix. •

## 4 A numerical example

This section presents the results of a numerical exercise, where the model is solved using value function iteration. First we discuss the functional forms and parametrization, and then we give the results.

The government's within period utility function has the CRRA form

$$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}$$

We assume that the income process is a log-normal autoregressive process with unconditional mean  $\mu$

$$\log(y_{t+1}) = (1 - \rho)\mu + \rho \log(y_t) + \varepsilon_{t+1}$$

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<sup>13</sup>This comes from Proposition 2 in Arellano (2008): "Default arises only when the borrower does not have access to a contract that lets him roll over the current debt due. If the borrower could roll over the current debt, then he would simply consume more today and default tomorrow on a higher debt."

Government's risk aversion	$\sigma$	1/2
Interest rate	$r$	3.0%
Income autocorrelation coefficient	$\rho$	0.945
Standard deviation of innovations	$\sigma_\varepsilon$	3.4%
Mean log income	$\mu$	$(-1/2)\sigma_\varepsilon^2$
Exclusion	$\alpha$	0.2
Maturity structure	$\theta$	0.8
Discount factor	$\beta$	0.4
Cost	$\chi_L$	0
Cost	$\chi_H$	0.5
SFC sunspot probability	$\pi$	0.05
Income grid	$y_1, \dots, y_{20}$	[0.73, ..., 1.37]
debt grid	$B_1, \dots, B_{1000}$	

Table 1: Parameter values for the calibration. One period is one year.

with  $E(\varepsilon) = 0$ ,  $E(\varepsilon^2) = \sigma_\varepsilon^2$ .

A period in the model refers to a year. Table 1 summarizes the key parameters used in this exercise. Most of the parameters are standard in the literature except for the discount factor, the sunspot probability, and the utility cost of default. As mentioned in the introduction, we build on political economy theories of short-sighted fiscal policy makers as in to provide a rationale for a default-prone scenario. Thus, we assume that the government discounts the future sufficiently highly to perch itself at a precarious point with an amount of debt in the crisis zone. Our parameter value for  $\pi$  is within the estimated ranges of the probability of a sudden stop in studies such as Jeanne and Ranciere (2001) and Calvo et al (2004). Additionally, as transition matrix between the two  $\chi$ -states, we choose

$$\begin{bmatrix} 0 & 1 \\ 0.04 & 0.96 \end{bmatrix}$$

Both the value for  $\chi_H$  as well as the transition probability from  $\chi_H$  to  $\chi_L$  was chosen after some experimentation to hit two target properties. First, we aimed at a debt-to-tax ratio somewhere between two and three, which is a plausible range of values for European economies. Second, we aimed at default rates between 5

	Target	$\theta = 0.8$
Debt/Tax ratio	2 .. 3	2.4
Default rate	5% .. 8%	6.6%

Table 2: *Targets and numerical results for the debt/tax ratio and the default rate*

	Buyers present	Buyers' strike
$\chi_L$	38%	2%
$\chi_H$	12%	48%

Table 3: *The structure of defaults.*

and 8 percent. While it tends to be hard to hit these numerical targets with, say, the assumption that the only penalty to default is higher consumption variability, it is comparatively easy to do it here, with these two additional free parameters, see table 2.

Table 3 shows the “anatomy” of defaults. One can see that 12 percent of the defaults happen due to fundamental problems, even with a “responsible”  $\chi_H$  government and despite buyers willing to buy the bonds in principle. However, nearly half of all defaults occur due to a buyers’ strike: it is these occurrences which the bailout agency shall help to avoid.

Figure 2 shows the resulting crisis zones. The intervals in the figure denote the pairs of income and debt levels for which the government would only default in the case of a buyers’ strike. For any debt level to the left of the interval, the government always repays independently of whether there is a buyers strike or not. Similarly, for debt levels to the right of the interval, the government will always find it optimal to default. Figure 3 shows the debt purchase assistance policy by the bailout agency. Over a fairly narrow range, the guaranteed purchases quickly rise until they reach 100%. At that point, the risk and incentive of a default due to fundamental reasons tomorrow is so large, that the failure to sell a small fraction of the new debt will be enough to trigger a default. If the current debt is even higher,

the fundamental debt price collapses all the way to zero, and so does the bailout guarantee. The country will not be willing to repay or will be unable to repay in the future, and purchasing debt at any positive price will result in expected losses. Thus, the bailout guarantee is only positive for pairs of income and debt levels in the crises zones, shown in Figure 2. Figure 4 shows the dependence of this policy on income. With currently higher income, it may well be worth guaranteeing debt purchases, that would lead to default at lower income levels. In other words, the bailout agency should rather support the country during a boom than a recession. This result may be counterintuitive from a policy perspective. What happens here, is rather intuitive, however: at some given debt level, worsening the fundamentals moves the country out of the crisis zone, where a purchase guarantee can restore the fundamental equilibrium, to the default-for-sure region, where any purchase guarantee would now result in a subsidy and would be avoided by a risk-neutral investor. Put differently, if the agency would commit to possibly purchasing nearly the entire quantity of new debt at some level of fundamentals, a small worsening in fundamentals will make the bailout agency jump to buying no debt at all and letting the country default. The country is let-go when a future recession becomes more likely than it was, making a fundamental default more likely than before. Finally, Figure 5 shows that when the agency intervenes, the guaranteed purchases range from over 50% up to 100%, and the size of the commitment could be as high as 40% of mean annual income.<sup>14</sup>

Table 4 shows the impact of varying the maturity of debt. As the maturity of debt is increased, the threat from a buyers strike in any given period declines, as an ever smaller fraction of the debt needs to be rolled over. As a result, the incentive to maintain higher debt levels rises, and not much changes with the default rates, as the overall result, while the length of the crisis zones shrink.

The corresponding shift in the debt purchase assistance policy is shown in the left panel Figure 6. Additionally, the right panel of Figure 6 shows the average size of the bailout at the stationary distribution for different levels of  $\theta$ . Note that

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<sup>14</sup>In Figure 5  $\Delta(B, z)$  denotes the size of the commitment.

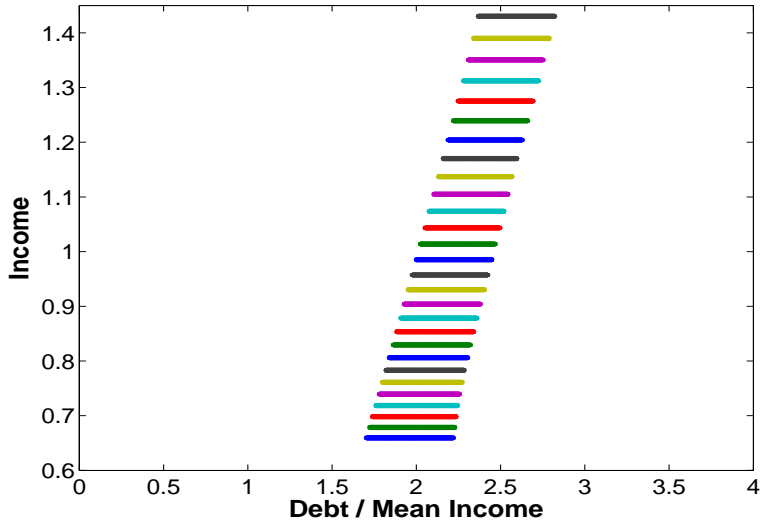


Figure 2: *Crisis zones*

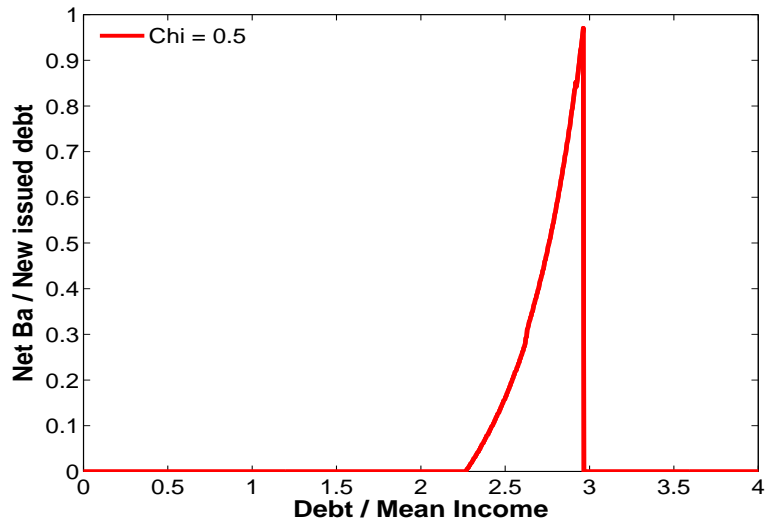


Figure 3: *Debt purchase assistance policy by the bailout agency.*

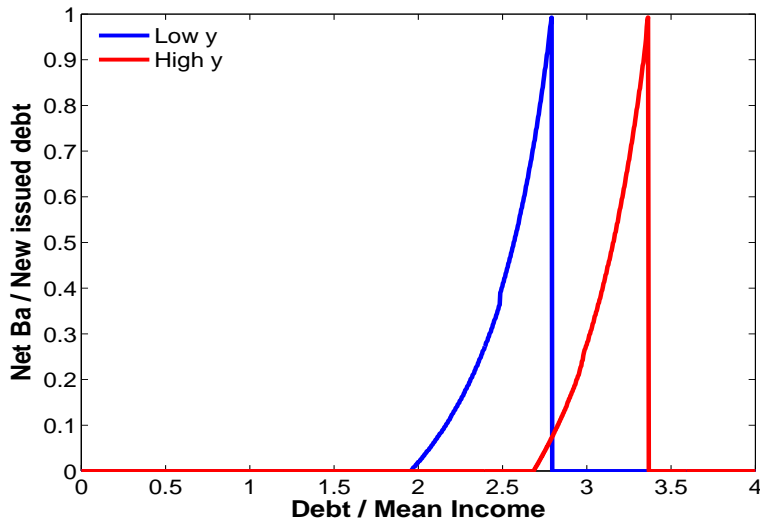


Figure 4: Income and debt purchase assistance

the level of commitment necessary to remove a buyers' strike falls significantly as maturity increases.

Table 5 shows that the change in the sunspot probability  $\pi$  for a buyers strike has only a modest impact on the overall default probability, while the debt level increases. With the fear of a default due to buyer's strike gone, debt becomes more attractive. Indeed, as table 6 shows, the default probability mass now shifts from the "buyer strike" scenario to the default due to fundamental reasons.

There is a conundrum for the bailout agency here. As that agency is successful in reducing the sunspot default probability from, say, 20 percent to zero percent, the overall default rates only decline modestly from 5% to 4%. In some ways, the problem gets postponed: the government gets a bit more time to accumulate more debt. As far as default rates are then concerned after this transition, not much will have changed.

Figure 7 shows the pricing function for debt at our benchmark value for  $\theta$ .

Indeed, debt prices rise and thus the sovereign spread decline, as the bailout agency assures the  $\pi = 0$  equilibrium through its purchase guarantees. The self-congratulatory remarks by Draghi and the ECB in 2015 might have been about point-

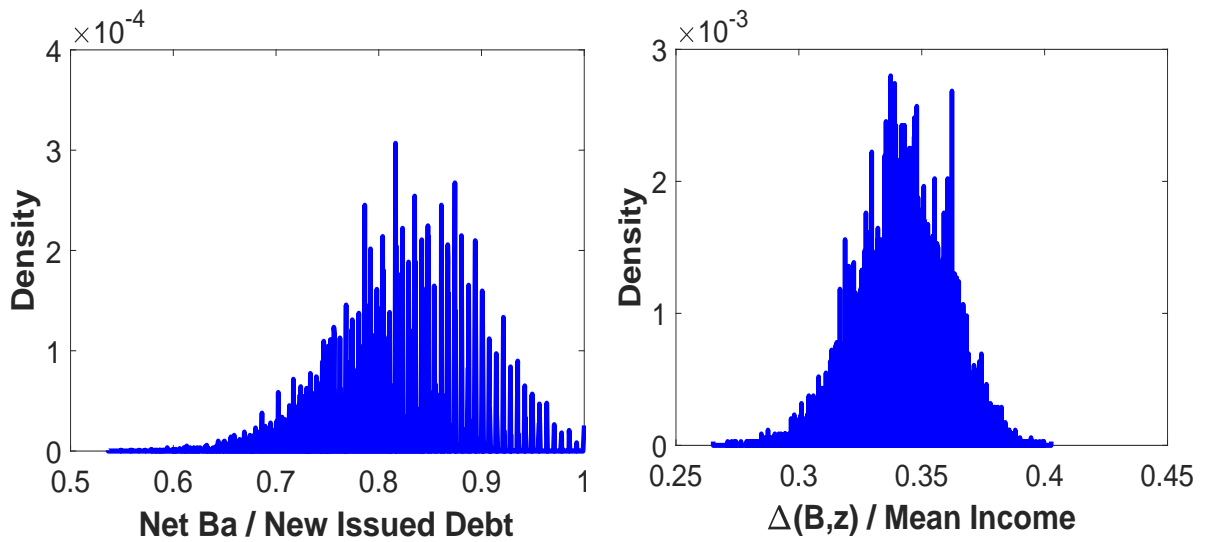


Figure 5: *Distribution of the debt purchase assistance and size of funds committed.*

Targets:

	Target	$\theta = 0.9$	$\theta = \mathbf{0.8}$	$\theta = 0.5$	$\theta = 0$
Debt/Tax ratio	2 .. 3	3.3	<b>2.4</b>	1.8	1.6
Default rate	5% .. 8%	6.6%	<b>6.6%</b>	6.2%	6.2%

Defaults: $\theta = 0.9$ :		
	Buyers present	Buyers' strike
$\chi_L$	38%	2%
$\chi_H$	16%	44%

Defaults: $\theta = 0$ :		
	Buyers present	Buyers' strike
$\chi_L$	42%	2%
$\chi_H$	2%	54%

Table 4: *Variations in maturity and their impact on defaults.  $\theta = 0$  is one-period debt, whereas  $\theta = 0.9$  is essentially 10-period debt.*

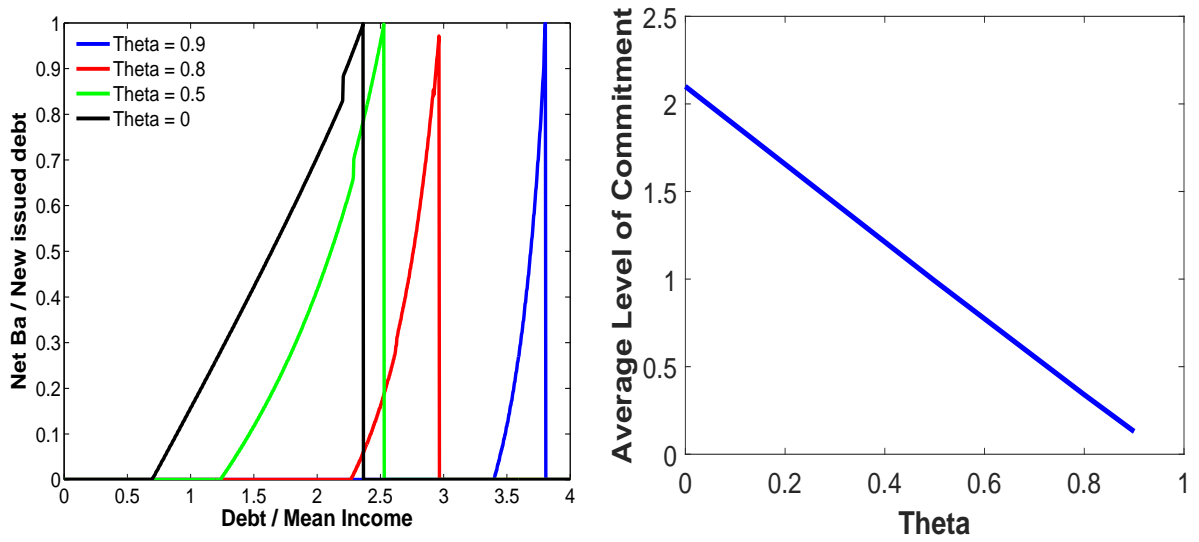


Figure 6: *Maturity and debt purchase assistance*

	Target	$\pi = 0.2$	$\pi = 0.1$	$\pi = \mathbf{0.05}$	$\pi = 0$
Debt/Tax ratio	2 .. 3	1.8	2.1	<b>2.4</b>	2.9
Default rate	5% .. 8%	5%	8%	<b>6.6%</b>	4%

Table 5: *Sunspot probabilities and debt levels*

Defaults for  $\pi = 0.1$ : total prob = 8%:

	Buyers present	Buyers' strike
$\chi_L$	27%	3%
$\chi_H$	8%	62%

Defaults for  $\pi = 0.05$  (Benchmark): total prob = 6.6%:

	Buyers present	Buyers' strike
$\chi_L$	38%	2%
$\chi_H$	12%	48%

Defaults for  $\pi = 0$ : total prob = 4%:

	Buyers present	Buyers' strike
$\chi_L$	81%	0%
$\chi_H$	19%	0%

Table 6: *Sunspot probabilities and default details*



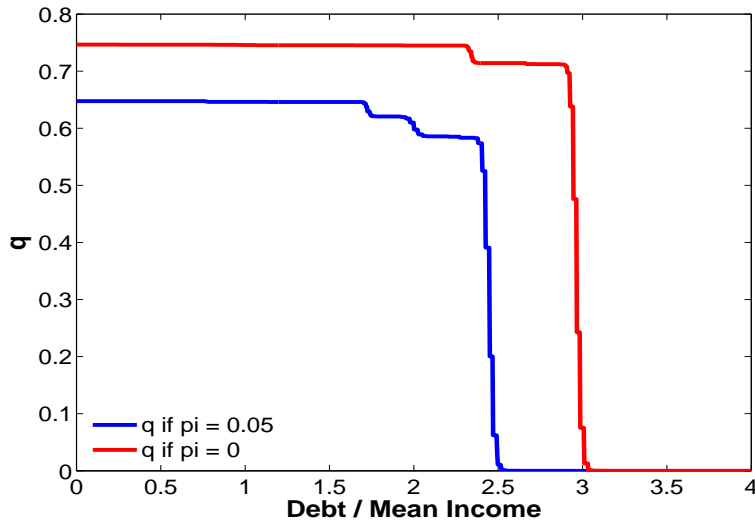


Figure 7: *Debt pricing function,  $\pi = 0.05$  vs  $\pi = 0$ .*

ing to such lower yields after the "whatever it takes" pledge. The government, now facing more favourable yields, is tempted to increase its indebtedness. The resulting debt buildup is rather fast, as figure 8 shows. Figures 9 and 10 show how the stationary debt distribution is shifted to the right, inducing the higher occurrences of defaults due to fundamental reasons. A graphical representation of the decision rules underlying the increased debt accumulation under debt purchase assistance is shown in figure 11: the decision rule shifts upwards, indicating a larger willingness of the government to incur debt.

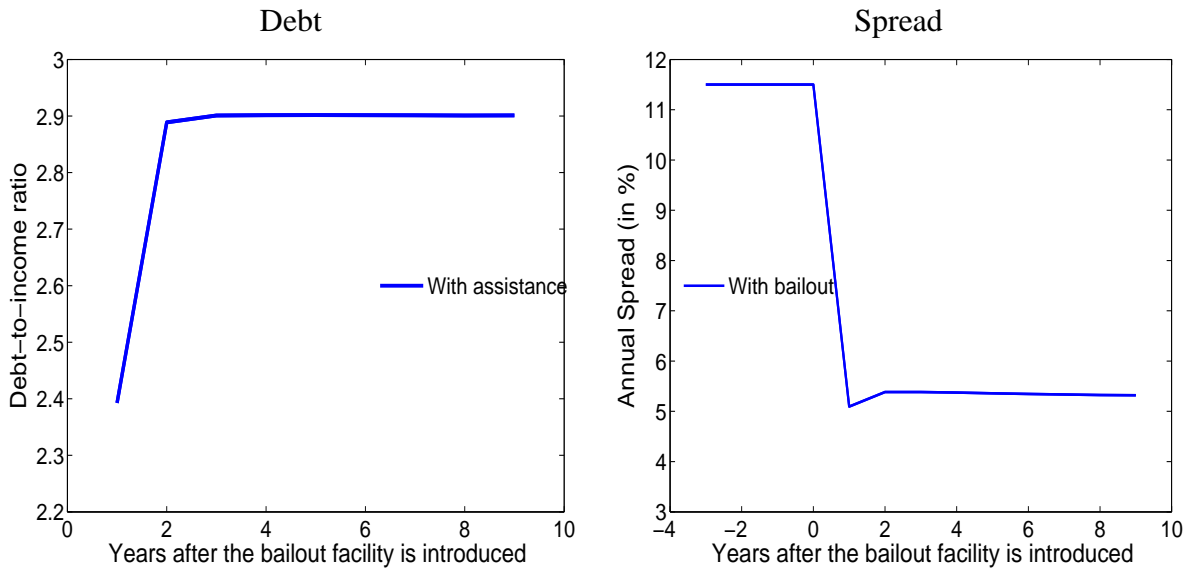


Figure 8: *Debt and Spread dynamics after the assistance agency is introduced. Starting point:  $\pi = 0.05$ , mean income, mean debt/gdp ratio.*

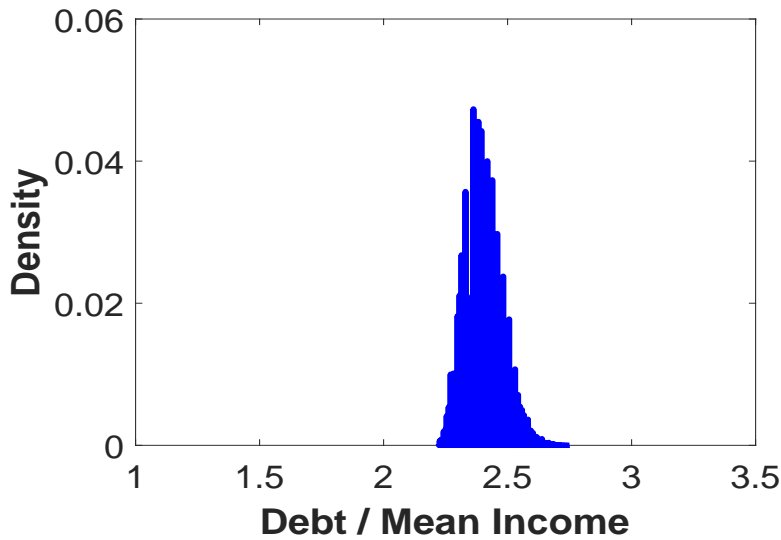


Figure 9: *Debt Distribution with sunspots:  $\pi = 0.05$*

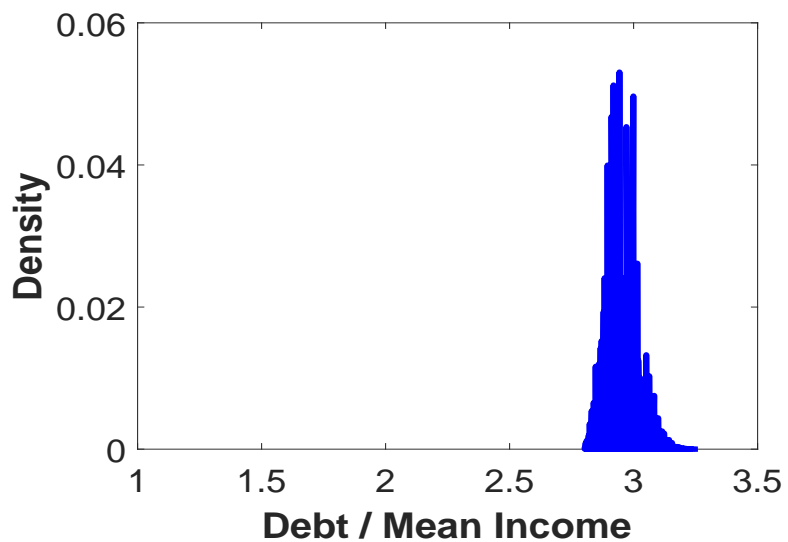


Figure 10: *Debt Distribution without sunspots or with debt purchase assistance:*  
 $\pi = 0$

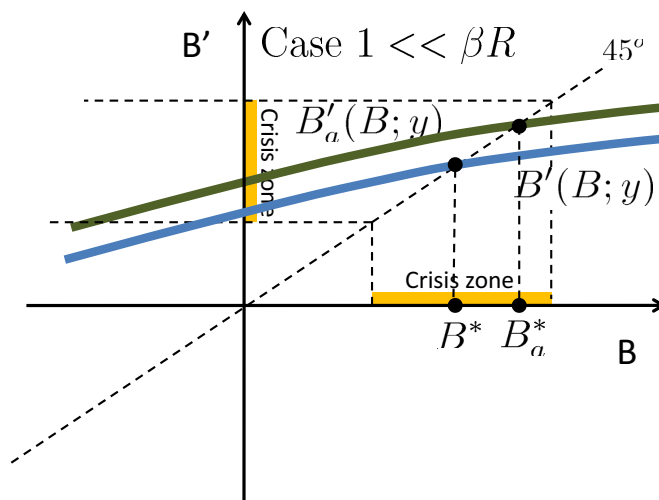


Figure 11: *Stationary debt dynamics, permanent assistance*

## 5 Conclusions

Motivated by the recent Eurozone debt crisis and the OMT program of the ECB to promise purchasing government bonds in unlimited quantity, if their yields are distressed, we have analyzed the dynamics of sovereign debt defaults and the scope for coordination on a “good” equilibrium by a large risk-neutral investor or agency. The analysis has implications beyond current events of the European debt crisis. The issue of belief coordination and the scope for policy intervention by large agencies such as the IMF or a coalition of partner countries is of generic interest. Our analysis has extended insights from three literatures, particularly Arellano (2008), Cole-Kehoe (2000) and Beetsma-Uhlig (1999). More precisely, we have analyzed the dynamics of sovereign debt, when politicians discount the future considerably more than private markets and when there are possibilities for both a “sunspot-”driven default as well as a default driven by worsening of economic conditions or weakening of the resolve to continue with repaying the country debt. We have shown how this can lead to a scenario, where the country perches itself in a precarious position, with the possibility of defaults imminent. We characterized the minimal actuarially fair intervention that restores the “good” equilibrium of Cole-Kehoe, relying on the market to provide residual financing.

Three messages and conclusions emerge. First, an actuarially fair bailout agency may be able to restore the “fundamentals-only” equilibrium, by issuing debt purchase guarantees and without incurring losses in expectation. Second, these guarantees need to go far enough, but not too far. Defaults due to fundamental reasons still lurk around the corner, and excessive debt purchase guarantees would then invariably lead to losses for the bailout agency. Third, the overall default rates may not change much, as the higher guarantees and the lower yields mean that the current government can relax a bit in its efforts to repay its debt level and incur more deficits instead. The resulting higher debt levels in the future will then make future defaults inevitable on occasions, but this time due to fundamental reasons rather than buyers’ strike.

The restoration of the “fundamentals-only” equilibrium may be one interpreta-

tion of why yields have declined in the Eurozone, following the OMT announcement. This coordination on the “good equilibrium” does not imply transfers to the distressed country, as many critiques of the OMT program continue to fear. The devil, however, is in the details, and it will be up to careful implementation of the OMT program and tying purchases to market prices to avoid such transfers.

Our analysis is “positive”, not “normative”. The impatience of the government and its objectives may well be different from those of the population, which a social planner would take into account. On purpose, we therefore refrain from assessing the efficiency and welfare implications: these would require additional assumptions.

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## A Proofs

### Proposition 1

**Proof:**

1. Suppose that  $\underline{B}(z) \neq \bar{B}(z)$ . Then, (13) and (14) imply that  $\underline{B}(z) < \bar{B}(z)$ . It follows that for every  $B \in (\underline{B}(z), \bar{B}(z))$ ,  $\bar{v}_{\text{ND}}(B, z) > v_D(z) - \chi$  and  $\underline{v}_{\text{ND};a}(B, z) < v_D(z) - \chi$ . However, if  $B'_a(B, z)$  satisfies (18), then  $\underline{v}_{\text{ND};a}(B, z) > v_D(z) - \chi$  for all  $0 \leq B \leq \bar{B}(z)$ , which is a contradiction.
2. Suppose that  $B'_a(B, z) > B^*(B, z)$ . For  $B > \bar{B}(z)$ ,  $q^{(\pi=0)}(B'; z) = 0$  for any  $B' > 0$  per definition of  $\bar{B}(z)$ . For  $B \leq \bar{B}(z)$ ,  $q^{(\pi=0)}(B^*(B, z); z)$  and  $B^*(B, z)$  are such that  $\bar{v}_{\text{ND}}(B, z) \geq v_D(z) - \chi + \epsilon(B, z)$ . However, given that utility is increasing and strictly concave, if  $B'_a(B, z) > B^*(B, z)$  then  $\bar{v}_{\text{ND}}(B, z) < \underline{v}_{\text{ND};a}(B, z)$ , which is a contradiction given (18).
3. By definition of  $\bar{B}(z)$ , the government chooses  $B^*(\bar{B}(z), z)$  such that  $\bar{v}_{\text{ND}}(\bar{B}(z), z) = v_D(z) - \chi$ . The previous point shows that  $B'_a(B, z)$  cannot be greater than  $B^*(B, z)$ . If  $B'_a(B, z) < B^*(B, z)$ , then  $\bar{v}_{\text{ND}}(\bar{B}(z), z) < v_D(z) - \chi$ , given that utility is increasing and strictly concave, which is a contradiction.
4. Suppose, to get a contradiction, that  $B'_a(B_1, z) < B'_a(B_2, z)$ . Denote the consumption level associated to  $(B_1, B'_a(B_1, z))$ ,  $(B_2, B'_a(B_2, z))$ , and  $(B_2, B'_a(B_1, z))$  by  $c_1$ ,  $c_2$ , and  $\tilde{c}_2$  respectively. Criterion (18) becomes

$$u(c_2) + \beta E [v(B'_a(B_2, z), z') \mid z] = v_D(z) - \chi + \epsilon$$

Then, by definition of  $B'_a$ , we have

$$u(c_2) + \beta E [v(B'_a(B_2, z), z') \mid z] > u(\tilde{c}_2) + \beta E [v(B'_a(B_1, z), z') \mid z]$$

Given that  $\tilde{c}_2 > c_1$ , we have

$$u(\tilde{c}_2) + \beta E [v(B'_a(B_1, z), z') \mid z] > u(c_1) + \beta E [v(B'_a(B_1, z), z') \mid z]$$

But, by definition of  $B'_a$ , we have

$$u(c_1) + \beta E [v(B'_a(B_1, z), z') \mid z] = v_D(z) - \chi + \epsilon$$

which is a contradiction.

•

### Proposition 2

**Proof:**

1. From proposition 2 in Arellano (2008) it follows that there is no contract available  $\{q^{(\pi=0)}(B'; z), B'\}$  such that  $q^{(\pi=0)}(B'; z)(B' - \theta B) - (1 - \theta)B > 0$ . The definition of our minimal guarantee implies that  $B'_a(B, z) \leq B'$ . Thus, the contract  $\{q^{(\pi=0)}(B'_a(B, z); z), B'_a(B, z)\}$  is available to the economy and it must be the case that  $q^{(\pi=0)}(B'_a(B, z); z)(B'_a(B, z) - \theta B) < (1 - \theta)B$ .
2. Suppose, to get a contradiction, that  $B'_a(B_1, z_1) > B'_a(B_2, z_2)$ . Denote the consumption level associated to  $(y_1, B'_a(B_1, z_1))$ ,  $(y_2, B'_a(B_1, z_1))$ ,  $(y_2, B'_a(B_1, z_1))$ , and  $(y_1, B'_a(B_2, z_2))$  by  $c_1$ ,  $c_2$ ,  $\tilde{c}_2$ , and  $\tilde{c}_1$  respectively. By definition of  $B'_a$ ,  $B'_a(B_1, z_1) > B'_a(B_2, z_2)$  implies

$$u(\tilde{c}_1) + \beta \tilde{v}_{ND}(B'_a(B_2, z_2)) < u(c_1) + \beta \tilde{v}_{ND}(B'_a(B_1, z_1)) = u(y_1) + \beta \tilde{v}_D - \chi + \epsilon$$

$$u(\tilde{c}_2) + \beta \tilde{v}_{ND}(B'_a(B_1, z_1)) > u(c_2) + \beta \tilde{v}_{ND}(B'_a(B_2, z_2)) = u(y_2) + \beta \tilde{v}_D - \chi + \epsilon$$

Also, by concavity of the utility function and part 2 of this proposition, we have

$$u(y_2) - u(\tilde{c}_2) > u(y_1) - u(\tilde{c}_1) = \beta(\tilde{v}_{ND}(B'_a(B_1, z_1)) - \tilde{v}_D) + \chi - \epsilon$$

This implies that  $u(y_2) + \beta \tilde{v}_D - \chi + \epsilon > u(\tilde{c}_2) + \beta \tilde{v}_{ND}(B'_a(B_1, z_1))$ , which is a contradiction.

3. This follows from criterion (18).

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